1) $f(x) = 3x_1^2 + 2x_2^2 + 4x_1x_2 - 5x_1 + 6$

$$\begin{bmatrix} \frac{\partial f}{x_1} \\ \frac{\partial f}{x_2} \end{bmatrix} = \begin{bmatrix} 6x_1 + 4x_2 - 5 \\ 4x_1 + 4x_2 \end{bmatrix}$$

$4x_1 + 4x_2 = 0$
$\Rightarrow 4x_1 = -4x_2$
$\Rightarrow x_1 = -x_2$

$6x_1 + 4(-x_1) - 5 = 0$
$2x_1 - 5 = 0$
$x_1 = 5/2$
$x_2 = -5/2$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{5}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$x_1 \neq x_2$ no unique solutions

check if max or min

$H = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f_{xy}(x_0, x_0)^2$

$H = 6 \cdot 4 - 4 = 24 - 4 = 20$

$\frac{\partial f}{x_1}(6x_1 + 4x_2 - 5) = 6$.

$\frac{\partial f}{\partial x_2}(4x_1 + 4x_2) = 4$

Since $H > 0$ and f_{xx} is > 0 then it is a minimum.

$f_{xy}(3x_1^2 + 2x_2 + 4x_1x_2 - 5x_1 + 6)$
$= f_{x_2}(6x_1 + 4x_2 - 5) = 4$

2) $f(x) = \sin(x) + 0.3x$
$\partial f / \partial x = \cos(x) + 0.3 = 0$
$\cos^{-1}(-0.3) = 1.87549$

GD $w \leftarrow w - 2(\cos(x) + 0.3)$

$4.408 - 1.875 = 2.533$

next gradient point