CSE 150 hw5

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CSE 150 hw 5 May 22^{nd}, 2018 Jiajin Liang

#1

(a) $$P(B=b|A=a) = \frac{\sum_{t=1}^{T} I(b,b_t)\, I(a,a_t)}{\sum_{t=1}^{T} I(a,a_t)}$$

$$P(C=c|A=a,B=b) = \frac{\sum_{t=1}^{T} I(c,c_t)\, I(a,a_t)\, I(b,b_t)}{\sum_{t=1}^{T} I(a,a_t)\, I(b,b_t)}$$

$$P(D=d|A=a,C=c) = \frac{\sum_{t=1}^{T} I(a,a_t)\, I(c,c_t)\, I(d,d_t)}{\sum_{t=1}^{T} I(a,a_t)\, I(c,c_t)}$$

(b) $$P(a,c|b,d) = \frac{P(c,d,b|a)\, P(a)}{P(b,d)} \quad \text{(Baye's)}$$

$$= \frac{P(b|a)\, P(c|a,b)\, P(d|a,b,c)\, P(a)}{\sum_{a'} P(a')\, P(b|a')\, P(d|a',b)} \quad \text{(product) (normaliz.)}$$

$$= \frac{P(b|a)\, P(c|a,b)\, P(d|a,c)\, P(a)}{\sum_{a'} P(a')\, P(b|a') \left[\sum_{c'} P(c'|a',b)\, P(d|a',c')\right]}$$

(D and B are C.I. when A, C is given by d-sep. #1&2) (normalization)

(c) $$P(a|b,d) = \frac{P(a,c|b,d)}{P(c|a,b,d)} \quad \text{(product Rule)}$$

$$= \frac{P(a,c|b,d)}{\dfrac{P(c,d|a,b)}{P(d|a,b)}} = \frac{P(a,c|b,d)}{\dfrac{P(c|a,b)\, P(d|a,b,c)}{\sum_{c'} P(c'|a,b)\, P(d|a,b,c')}}$$

(product Rule, normalization)

$$= \frac{P(a,c|b,d) \cdot \sum_{c'} P(c'|a,b)\, P(d|a,c')}{\ldots}$$

(D and B are C.I. when A, C

$$= \frac{P(a,c|b,d) \cdot \sum_{c'} P(c'|a,b)\, P(d|a,c')}{P(c|a,b)\, P(d|a,c)}$$

(D and B are C.I. when A, C are given by d-sep. #1 & 2)

$$P(c|b,d) = \sum_a P(a,c|b,d)$$ (marginalization)

(d) $$\mathcal{L} = \sum_t \log P(B=bt, D=dt)$$

(margin.)

$$= \sum_t \log \sum_{a,c} P(A=a, B=bt, C=c, D=dt)$$

$$= \sum_t \log \sum_{a,c} P(a)\, P(bt|a)\, P(c|a,bt)\, P(dt|a,c)$$

(Product Rule)
(C.I.)

(e) $$P(A=a) \leftarrow \frac{1}{T} \sum_t P(A=a|B=bt, D=dt)$$

$$P(B=b|A=a) \leftarrow \frac{\sum_t P(A=a, B=b|B=bt, D=dt)}{\sum_t P(A=a|B=bt, D=dt}$$

$$P(B=b|A=a) \leftarrow \frac{\sum_t I(b,bt)\, P(a|bt,dt)}{\sum_t P(a|bt,dt)}$$

$$P(C=c|A=a, B=b) \leftarrow \frac{\sum_t P(A=a, B=b, C=c|B=bt, D=dt)}{\sum_t P(A=a|B=bt, D=dt)\, P(b|a,bt,dt)}$$

$$P(C=c|A=a, B=b) \leftarrow \frac{\sum_t I(b,bt)\, P(a,c|bt,dt)}{\sum_t P(a|bt,dt) \cdot I(b,bt)}$$

$$P(D=d \mid A=a, C=c) \leftarrow \frac{\sum_t P(A=a, C=c, D=d \mid B=b_t, D=d_t)}{\sum_t P(A=a \mid B=b_t, D=d_t)\, P(c \mid a, b_t, d_t)}$$

$$P(D=d \mid A=a, C=c) \leftarrow \frac{\sum_t I(d, d_t)\, P(a, c \mid b_t, d_t)}{\sum_t P(a, c \mid b_t, d_t)}$$

$$P(D=d \mid A=a, C=c) \leftarrow \sum_t I(d, d_t)$$

#2

iteration 0 log likehood is -0.958085408216 M is 175
iteration 1 log likehood is -0.495916394078 M is 56
iteration 2 log likehood is -0.408220817058 M is 43
iteration 4 log likehood is -0.3646149825 M is 42
iteration 8 log likehood is -0.347500616209 M is 44
iteration 16 log likehood is -0.334617048959 M is 40
iteration 32 log likehood is -0.322581403167 M is 37
iteration 64 log likehood is -0.314826698363 M is 37
iteration 128 log likehood is -0.311155847215 M is 36
iteration 256 log likehood is -0.310161353474 M is 36
iteration 512 log likehood is -0.309990302985 M is 36

Final estimate of p_i for i from 1 to 23

[7.825910611671573e-05, 0.0047896936438733455,
2.4572053572601886e-11, 0.2653480592892651,
1.4641986190239666e-05, 0.009414189848976004,
0.24031335014859453, 0.11340199970656507,
0.0001416199333200315, 0.5234838385255078,
0.4073197394282568, 8.818891591636871e-08, 0.6158119943991922,
5.837772618077978e-06, 0.044823707802064225,
0.5899862302524816, 0.9999999999999938, 0.9999999828376271,
4.011172119017799e-09, 0.46299769189725615,
0.35319531211378413, 0.5248673231383694, 0.19475780199532694]

iteration	number of mistakes M	log conditional likelihood $\mathcal{L}$
0	175	-0.9581
1	56	-0.4959
2	43	-0.4082
4	42	-0.3646
8	44	-0.3475
16	40	-0.3346
32	37	-0.3226
64	36	-0.3148
128	36	-0.3112
256	36	0.3102
512	36	-0.3100

#3 (a) movies with mean popularity rate sorted from low to high

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[(0.13114754098360656, 'The_Last_Airbender\t\n'), (0.15151515151515152, 'Fifty_Shades_of_Grey\t\n'), (0.25, 'Phantom_Thread\t
\n'), (0.4, 'I_Feel_Pretty\t\t\n'), (0.43478260869565216, 'Magic_Mike\t\n'), (0.4444444444444444, 'Bridemaids\t\n'), (0.4905660
3773584906, 'Man_of_Steel\t\n'), (0.5116279069767442, 'World_War_Z\n'), (0.6153846153846154, 'The_Perks_of_Being_a_Wallflower\t
\n'), (0.631578947368421, 'Prometheus\t\n'), (0.6363636363636364, 'Manchester_by_the_Sea\t\n'), (0.6666666666666666, 'American_
Hustle\t\n'), (0.6666666666666666, 'Midnight_in_Paris\t\n'), (0.6792452830188679, 'Pitch_Perfect\t\n'), (0.6836734693877551, 'I
ron_Man_2\t\n'), (0.6976744186046512, 'Fast_Five\t\n'), (0.6979166666666666, 'The_Hunger_Games\t\n'), (0.7272727272727273, 'Dar
kest_Hour'), (0.7307692307692307, 'Her\t\n'), (0.7333333333333333, 'Room\t\n'), (0.7391304347826086, 'The_Shape_of_Water\t\n'),
(0.7407407407407407, 'Mad_Max:_Fury_Road\t\n'), (0.7424242424242424, 'The_Great_Gatsby\t\n'), (0.7468354430379747, 'Frozen\t
\n'), (0.76, 'Avengers:_Age_of_Ultron\t\n'), (0.7666666666666667, 'Captain_America:_The_First_Avenger\t\n'), (0.766666666666666
7, 'Ready_Player_One\t\n'), (0.7704918032786885, 'Now_You_See_Me\t\n'), (0.7741935483870968, 'Gone_Girl\t\n'), (0.8, '12_Years_
a_Slave\t\n'), (0.8, 'Harry_Potter_and_the_Deathly_Hallows:_Part_1\t\n'), (0.8024691358024691, 'Thor\t\n'), (0.805555555555555
6, 'Dunkirk\t\n'), (0.8125, 'The_Revenant\t\n'), (0.825, 'Star_Wars:_The_Force_Awakens\t\n'), (0.8333333333333334, 'Harry_Potte
r_and_the_Deathly_Hallows:_Part_2\t\n'), (0.8333333333333334, 'The_Girls_with_the_Dragon_Tattoo\t\n'), (0.8333333333333334, 'Th
ree_Billboards_Outside_Ebbing\t\n'), (0.8349514563106796, 'The_Avengers\t\n'), (0.8360655737704918, 'Wolf_of_Wall_Street\t\n'),
(0.8387096774193549, 'Toy_Story_3\t\n'), (0.8421052631578947, 'Jurassic_World\t\n'), (0.8421052631578947, 'The_Help\t\n'), (0.8
59375, 'X-Men:_First_Class\t\n'), (0.8648648648648649, 'La_La_Land\t\n'), (0.8771929824561403, 'Avengers:_Infinity_War\t\n'),
(0.8823529411764706, 'The_Social_Network\t\n'), (0.8863636363636364, 'Les_Miserables\t\n'), (0.8888888888888888, 'Hidden_Figure
s\t\n'), (0.8939393939393939, '21_Jump_Street\t\n'), (0.8939393939393939, 'The_Martian\t\n'), (0.896551724137931, 'The_Hateful_
Eight\t\n'), (0.9107142857142857, 'Django_Unchained\t\n'), (0.9111111111111111, 'The_Dark_Knight_Rises\t\n'), (0.93333333333333
33, 'Black_Swan\t\n'), (0.9404761904761905, 'Interstellar\t\n'), (0.9411764705882353, 'Drive\t\n'), (0.9459459459459459, 'Ex_Ma
china\t\n'), (0.95, 'The_Theory_of_Everything\t\n'), (0.96875, 'Shutter_Island\t\n'), (0.978494623655914, 'Inception\t\n')]
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(b) $$P(\{R_j = r_j^{(t)}\}_{j\in\Omega_t}) = \sum_{i=1}^{K} P(Z=i, \{R_j=r_j^{(t)}\}_{j\in\Omega_t})$$ (margin.)

$$= \sum_{i=1}^{K} P(Z=i)\, P(\{R_j=r_j^{(t)}\}_{j\in\Omega_t} \mid Z=i)$$ (product Rule)

$$= \sum_{i=1}^{K} P(Z=i) \prod_{j\in\Omega_t} P(R_j = r_j^{(t)} \mid Z=i)$$ (C.I.)

(c) $P(Z=i \mid \{R_j = r_j^{(t)}\}_{j\in\Omega_t})$

(c) $P(Z=i \mid \{R_j = r_j^{(t)}\}_{j\in\Omega_t})$

$$= \frac{P(\{R_j = r_j^{(t)}\}_{j\in\Omega_t} \mid Z=i)\, P(Z=i)}{P(\{R_j = r_j^{(t)}\}_{j\in\Omega_t})} \quad \left(\begin{matrix}\text{Baye's}\\ \text{Rule}\end{matrix}\right)$$

$$= \frac{P(Z=i) \prod_{j\in\Omega_t} P(R_j = r_j^{(t)} \mid Z=i)}{P(\{R_j = r_j^{(t)}\}_{j\in\Omega_t})} \quad \left(\begin{matrix}\text{Product}\\ \text{Rule,}\\ \text{C.I.}\end{matrix}\right)$$

$$= \frac{P(Z=i) \prod_{j\in\Omega_t} P(R_j = r_j^{(t)} \mid Z=i)}{\sum_{i'=1}^{K} P(Z=i') \prod_{j\in\Omega_t} P(R_j = r_j^{(t)} \mid Z=i')} \quad \left(\begin{matrix}\text{normal-}\\ \text{ization}\end{matrix}\right)$$

(d) $$P(Z=i) \leftarrow \frac{1}{T}\sum_{t=1}^{T} P(Z=i \mid \{R_j = r_j^{(t)}\}_{j\in\Omega_t}) = \rho_{it}$$

$$P(R_j=1 \mid Z=i) \leftarrow \frac{\sum_{t=1}^{T} P(R_j=1, Z=i \mid \{R_j = r_j^{(t)}\}_{j\in\Omega_t})}{\sum_{t=1}^{T} P(Z=i \mid \{R_j, r_j^{(t)}\}_{j\in\Omega_t})}$$

(product)

$$= \frac{\left[\sum_{\{t \mid j\in\Omega_t\}} I(r_j^{(t)}, 1)\, P(Z=i \mid \{R_j = r_j^{(t)}\}_{j\in\Omega_t}) + \sum_{\{t \mid j\notin\Omega_t\}} P(Z=i, R_j=1 \mid \{R_j = r_j^{(t)}\}_{j\in\Omega_t})\right.}{\sum_{t=1}^{T} P(Z=i \mid \{R_j, r_j^{(t)}\}_{j\in\Omega_t})}$$

$$= \frac{\qquad}{\sum_{t=1}^{T} P(Z=i \mid \{R_j, r_j^{(t)}\}_{j\in\Omega_t})}$$

$$= \frac{\sum_{\{t \mid j\in\Omega_t\}} I(r_j^{(t)}, 1)\, \rho_{it} + \sum_{\{t \mid j\notin\Omega_t\}} P(Z=i \mid \{R_j=r_j^{(t)}\}_{j\in\Omega_t})\, P(R_j=1 \mid Z=i, \{R_j=r_j^{(t)}\}_{j\in\Omega_t})}{\sum_{t=1}^{T} \rho_{it}}$$

(Product)

$$= \frac{\sum_{\{t \mid j\in\Omega_t\}} I(r_j^{(t)}, 1)\, \rho_{it} + \sum_{\{t \mid j\in\Omega_t\}} \rho_{it}\, P(R_j=1 \mid Z=i)}{\sum_{t=1}^{T} \rho_{it}}$$

(C.I.)