Enhanced Aerial Reorientation Performance Using a 3-DoF Morphable Inertial Tail Inspired by Kangaroo Rats

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I. MODELING

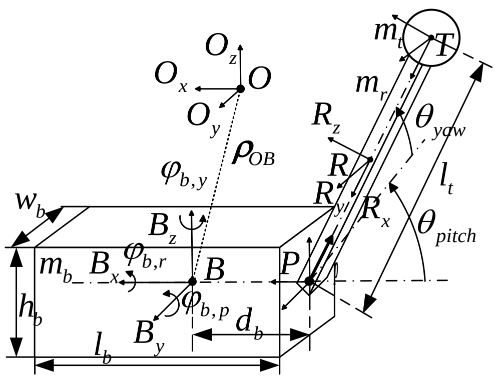

Fig. 1. Analytical models of the kangaroo rat and our tailed robot with a 3-DoF tail.

We start with an angular momentum equation expressed in the inertial frame $\{O\}$:

$$H^O = I_b^O \boldsymbol{\omega}_b^O + I_R^O \boldsymbol{\omega}_r^O + m_b \boldsymbol{\rho}_{OB}^O \times \dot{\boldsymbol{\rho}}_{OB}^O + m_r \boldsymbol{\rho}_{OR}^O \times \dot{\boldsymbol{\rho}}_{OR}^O + m_t \boldsymbol{\rho}_{OT}^O \times \dot{\boldsymbol{\rho}}_{OT}^O, \tag{1}$$

where I_b^O and I_r^O are the inertias of the robot body and tail link in the inertial frame $\{O\}$. m_t, m_r, m_b denote the mass of the body, the tail link, and the tail end mass. $\rho_{OT}^O, \rho_{OR}^O, \rho_{OB}^O$ are the vectors from the origin of the frame $\{O\}$ to the origins of the tail end frame $\{T\}$, the tail link frame $\{R\}$, and the body frame $\{B\}$. It can be also expressed in the body frame as:

$$H^B = I_b^B \boldsymbol{\omega}_b^B + I_R^B \boldsymbol{\omega}_r^B + m_b (R_B^O)^T \boldsymbol{\rho}_{OB}^O \times \dot{\boldsymbol{\rho}}_{OB}^O + m_r (R_B^O)^T \boldsymbol{\rho}_{OR}^O \times \dot{\boldsymbol{\rho}}_{OR}^O + m_t (R_B^O)^T \boldsymbol{\rho}_{OT}^O \times \dot{\boldsymbol{\rho}}_{OT}^O, \tag{2}$$

where R_B^O is the rotation matrix from frame $\{B\}$ to frame $\{O\}$. Based on $\boldsymbol{\rho}_{OB}^O = R_B^O \boldsymbol{\rho}_{OB}^B$, we have $\dot{\boldsymbol{\rho}}_{OB}^O = \dot{R}_B^O \boldsymbol{\rho}_{OB}^B + R_B^O \dot{\boldsymbol{\rho}}_{OB}^B$. Then, we have:

$$\begin{aligned}
&(R_B^O)^T \boldsymbol{\rho}_{OB}^O \times \dot{\boldsymbol{\rho}}_{OB}^O \\
&= (R_B^O)^T \left(R_B^O \boldsymbol{\rho}_{OB}^B\right) \times \left(\dot{R}_B^O \boldsymbol{\rho}_{OB}^B + R_B^O \dot{\boldsymbol{\rho}}_{OB}^B\right) \\
&= (R_B^O)^T \left(R_B^O \boldsymbol{\rho}_{OB}^B\right) \times \left(\dot{R}_B^O \boldsymbol{\rho}_{OB}^B\right) + (R_B^O)^T \left(R_B^O \boldsymbol{\rho}_{OB}^B\right) \times \left(R_B^O \dot{\boldsymbol{\rho}}_{OB}^B\right) \\
&= \left((R_B^O)^T R_B^O \boldsymbol{\rho}_{OB}^B\right) \times \left((R_B^O)^T \dot{R}_B^O \boldsymbol{\rho}_{OB}^B\right) + \left((R_B^O)^T R_B^O \boldsymbol{\rho}_{OB}^B\right) \times \\
&\left((R_B^O)^T R_B^O \dot{\boldsymbol{\rho}}_{OB}^B\right) \\
&= \boldsymbol{\rho}_{OB}^B \times \left((R_B^O)^T \left(\left[\boldsymbol{\omega}_b^O \times\right] R_B^O\right) \boldsymbol{\rho}_{OB}^B\right) + \boldsymbol{\rho}_{OB}^B \times \dot{\boldsymbol{\rho}}_{OB}^B \\
&= \boldsymbol{\rho}_{OB}^B \times \left((R_B^O)^T \left(\boldsymbol{\omega}_b^O \times \boldsymbol{\rho}_{OB}^O\right)\right) + \boldsymbol{\rho}_{OB}^B \times \dot{\boldsymbol{\rho}}_{OB}^B \\
&= \boldsymbol{\rho}_{OB}^B \times \left(\boldsymbol{\omega}_b^B \times \boldsymbol{\rho}_{OB}^B\right) + \boldsymbol{\rho}_{OB}^B \times \dot{\boldsymbol{\rho}}_{OB}^B \\
&= \left(\boldsymbol{\rho}_{OB}^B\right)^2 \boldsymbol{\omega}_b^B - (\boldsymbol{\rho}_{OB}^B \cdot \boldsymbol{\omega}_b^B) \boldsymbol{\rho}_{OB}^B + \boldsymbol{\rho}_{OB}^B \times \dot{\boldsymbol{\rho}}_{OB}^B \\
&= -\left[\boldsymbol{\rho}_{OB}^B \times\right]\left[\boldsymbol{\rho}_{OB}^B \times\right] \boldsymbol{\omega}_b^B + \boldsymbol{\rho}_{OB}^B \times \dot{\boldsymbol{\rho}}_{OB}^B,
\end{aligned} \tag{3}$$

where $R(x \times y) = (Rx) \times (Ry)$ and $\Omega_b^O = \left[\boldsymbol{\omega}_b^O \times\right] = \dot{R}_B^O (R_B^O)^T$ are used. Here $\left(\boldsymbol{\rho}_{OB}^B\right)^2 = \left(\boldsymbol{\rho}_{OB}^B\right) \cdot \left(\boldsymbol{\rho}_{OB}^B\right)$. Then Eq. 2 becomes:

$$\begin{aligned}
H^B =& I_b^B \boldsymbol{\omega}_b^B + I_R^B \boldsymbol{\omega}_r^B + m_b \boldsymbol{\rho}_{OB}^B \times \dot{\boldsymbol{\rho}}_{OB}^B + m_r \boldsymbol{\rho}_{OR}^B \times \dot{\boldsymbol{\rho}}_{OR}^B + \\
& m_t \boldsymbol{\rho}_{OT}^B \times \dot{\boldsymbol{\rho}}_{OT}^B + m_b \left(\boldsymbol{\rho}_{OB}^B\right)^2 \boldsymbol{\omega}_b^B + m_r \left(\boldsymbol{\rho}_{OR}^B\right)^2 \boldsymbol{\omega}_b^B + m_t \\
& \left(\boldsymbol{\rho}_{OT}^B\right)^2 \boldsymbol{\omega}_b^B - m_b (\boldsymbol{\rho}_{OB}^B \cdot \boldsymbol{\omega}_b^B) \boldsymbol{\rho}_{OB}^B - m_r (\boldsymbol{\rho}_{OR}^B \cdot \boldsymbol{\omega}_b^B) \boldsymbol{\rho}_{OR}^B - \\
& m_t (\boldsymbol{\rho}_{OT}^B \cdot \boldsymbol{\omega}_b^B) \boldsymbol{\rho}_{OT}^B.
\end{aligned} \tag{4}$$

At the robot center of mass (CoM), we have:

$$m_b \boldsymbol{\rho}_{OB}^O + m_r \boldsymbol{\rho}_{OR}^O + m_t \boldsymbol{\rho}_{OT}^O = \boldsymbol{0}, \tag{5}$$

here

$$\begin{aligned}
&\boldsymbol{\rho}_{OR}^O = \boldsymbol{\rho}_{OT}^O - \boldsymbol{\rho}_{RT}^O = \boldsymbol{\rho}_{OT}^O - R_B^O \boldsymbol{\rho}_{RT}^B = \\
&\boldsymbol{\rho}_{OT}^O - \frac{1}{2} l_t R_B^O \begin{bmatrix} -c_{pitch} c_{yaw} & -s_{yaw} & s_{pitch} c_{yaw} \end{bmatrix}^T,
\end{aligned} \tag{6}$$

where $c_{pitch} = \cos\theta_{pitch}$, $s_{pitch} = \sin\theta_{pitch}$, $c_{yaw} = \cos\theta_{yaw}$, $s_{yaw} = \sin\theta_{yaw}$. θ_{pitch} is the tail swing angle in the body pitch direction and θ_{yaw} is the tail swing angle in the body yaw direction. $\boldsymbol{p}^B = \begin{bmatrix} -c_{pitch} c_{yaw} & -s_{yaw} & s_{pitch} c_{yaw} \end{bmatrix}^T$ is unit tail reorientation vector in frame $\{B\}$. l_t denotes the tail length. Substitute Eq. 6 into Eq. 5, we get:

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$$
\begin{aligned}
\boldsymbol{\rho}_{OT}^{B} &= -\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t \\
&\begin{bmatrix} -c_{pitch}c_{yaw} & -s_{yaw} & s_{pitch}c_{yaw} \end{bmatrix}^T, \\
\boldsymbol{\rho}_{OR}^{B} &= -\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} - \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t \\
&\begin{bmatrix} -c_{pitch}c_{yaw} & -s_{yaw} & s_{pitch}c_{yaw} \end{bmatrix}^T.
\end{aligned}
\tag{7}
$$

We also have:

$$
\dot{\boldsymbol{\rho}}_{OT}^{B} = -\frac{m_b}{(m_r+m_t)}\dot{\boldsymbol{\rho}}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)}\dot{l}_t
$$

$$
\begin{bmatrix} -c_{pitch}c_{yaw} & -s_{yaw} & s_{pitch}c_{yaw} \end{bmatrix}^T + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t [s_{pitch}c_{yaw}\dot{\theta}_{pitch}
$$

$$
+c_{pitch}s_{yaw}\dot{\theta}_{yaw} \quad -c_{yaw}\dot{\theta}_{yaw} \quad c_{pitch}c_{yaw}\dot{\theta}_{pitch} - s_{pitch}s_{yaw}\dot{\theta}_{yaw}]^T,
$$

$$
\dot{\boldsymbol{\rho}}_{OR}^{B} = -\frac{m_b}{(m_r+m_t)}\dot{\boldsymbol{\rho}}_{OB}^{B} - \frac{1}{2}\frac{m_t}{(m_r+m_t)}\dot{l}_t
$$

$$
\begin{bmatrix} -c_{pitch}c_{yaw} & -s_{yaw} & s_{pitch}c_{yaw} \end{bmatrix}^T - \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t [s_{pitch}c_{yaw}\dot{\theta}_{pitch}
$$

$$
+c_{pitch}s_{yaw}\dot{\theta}_{yaw} \quad -c_{yaw}\dot{\theta}_{yaw} \quad c_{pitch}c_{yaw}\dot{\theta}_{pitch} - s_{pitch}s_{yaw}\dot{\theta}_{yaw}]^T.
$$

Eq. 4 can be updated as:

$$
\begin{aligned}
H^B =& I_b^B\boldsymbol{\omega}_b^B + I_r^B\boldsymbol{\omega}_r^B + m_b\boldsymbol{\rho}_{OB}^{B}\times\dot{\boldsymbol{\rho}}_{OB}^{B} + m_r\boldsymbol{\rho}_{OR}^{B}\times\dot{\boldsymbol{\rho}}_{OR}^{B} + \\
& m_t\boldsymbol{\rho}_{OT}^{B}\times\dot{\boldsymbol{\rho}}_{OT}^{B} + m_b\left(\boldsymbol{\rho}_{OB}^{B}\right)^2\boldsymbol{\omega}_b^B + m_r\left(\boldsymbol{\rho}_{OR}^{B}\right)^2\boldsymbol{\omega}_b^B + m_t \\
& \left(\boldsymbol{\rho}_{OT}^{B}\right)^2\boldsymbol{\omega}_b^B - m_b(\boldsymbol{\rho}_{OB}^{B}\cdot\boldsymbol{\omega}_b^B)\boldsymbol{\rho}_{OB}^{B} - m_r(\boldsymbol{\rho}_{OR}^{B}\cdot\boldsymbol{\omega}_b^B)\boldsymbol{\rho}_{OR}^{B} - \\
& m_t(\boldsymbol{\rho}_{OT}^{B}\cdot\boldsymbol{\omega}_b^B)\boldsymbol{\rho}_{OT}^{B} \\
=& I_b^B\boldsymbol{\omega}_b^B + m_b\boldsymbol{\rho}_{OB}^{B}\times\dot{\boldsymbol{\rho}}_{OB}^{B} + m_r\begin{pmatrix} -\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} - \\ \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t\boldsymbol{p}^B \end{pmatrix}\times \\
& \begin{pmatrix} -\frac{m_b}{(m_r+m_t)}\dot{\boldsymbol{\rho}}_{OB}^{B} - \frac{1}{2}\frac{m_t}{(m_r+m_t)}\dot{l}_t\boldsymbol{p}^B \\ -\frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t\dot{\boldsymbol{p}}^B \end{pmatrix} \\
& +m_t\left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t\boldsymbol{p}^B\right)\times \\
& \begin{pmatrix} -\frac{m_b}{(m_r+m_t)}\dot{\boldsymbol{\rho}}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)}\dot{l}_t\boldsymbol{p}^B \\ +\frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t\dot{\boldsymbol{p}}^B \end{pmatrix} + m_b\left(\boldsymbol{\rho}_{OB}^{B}\right)^2\boldsymbol{\omega}_b^B \\
& +m_r\begin{pmatrix} \left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} - \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t\boldsymbol{p}^B\right)\cdot \\ \left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} - \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t\boldsymbol{p}^B\right) \end{pmatrix}\boldsymbol{\omega}_b^B + \\
& m_t\begin{pmatrix} \left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t\boldsymbol{p}^B\right)\cdot \\ \left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t\boldsymbol{p}^B\right) \end{pmatrix}\boldsymbol{\omega}_b^B - \\
& m_b(\boldsymbol{\rho}_{OB}^{B}\cdot\boldsymbol{\omega}_b^B)\boldsymbol{\rho}_{OB}^{B} - m_r\left(\begin{pmatrix} -\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} - \\ \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t\boldsymbol{p}^B \end{pmatrix}\cdot\boldsymbol{\omega}_b^B\right) \\
& \left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} - \frac{1}{2}\frac{m_t}{(m_r+m_t)} l_t\boldsymbol{p}^B\right) - \\
& m_t\left(\left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t\boldsymbol{p}^B\right)\cdot\boldsymbol{\omega}_b^B\right) \\
& \left(-\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t\boldsymbol{p}^B\right) \\
=& I_b^B\boldsymbol{\omega}_b^B + I_r^B\boldsymbol{\omega}_r^B + \frac{m_b(m_r+m_t+m_b)}{(m_r+m_t)}(\boldsymbol{\rho}_{OB}^{B}\times\dot{\boldsymbol{\rho}}_{OB}^{B} + \\
& \left(\boldsymbol{\rho}_{OB}^{B}\right)^2\boldsymbol{\omega}_b^B - (\boldsymbol{\rho}_{OB}^{B}\cdot\boldsymbol{\omega}_b^B)\boldsymbol{\rho}_{OB}^{B}) + \frac{1}{4}\left(\frac{m_t m_r}{(m_r+m_t)}\right) l_t^2(\boldsymbol{p}^B \\
& \times\dot{\boldsymbol{p}}^B + \boldsymbol{\omega}_b^B - (\boldsymbol{p}^B\cdot\boldsymbol{\omega}_b^B)\boldsymbol{p}^B) \\
=& I_b^B\boldsymbol{\omega}_b^B + I_r^B\boldsymbol{\omega}_r^B + \frac{m_b(m_r+m_t+m_b)}{(m_r+m_t)}(-\left[\boldsymbol{\rho}_{OB}^{B}\times\right] \\
& \left[\boldsymbol{\rho}_{OB}^{B}\times\right]\boldsymbol{\omega}_b^B + \boldsymbol{\rho}_{OB}^{B}\times\dot{\boldsymbol{\rho}}_{OB}^{B}) + \frac{1}{4}\left(\frac{m_t m_r}{(m_r+m_t)}\right) l_t^2(- \\
& \left[\boldsymbol{p}^B\times\right]\left[\boldsymbol{p}^B\times\right]\boldsymbol{\omega}_b^B + \boldsymbol{p}^B\times\dot{\boldsymbol{p}}^B).
\end{aligned}
\tag{8}
$$

A closed path starting from the origin of frame $\{O\}$ passing through body CoM B, tail base P, and tail end mass CoM P can be expressed as:

$$
\boldsymbol{\rho}_{OB}^{O} + \boldsymbol{\rho}_{BP}^{O} + \boldsymbol{\rho}_{PT}^{O} - \boldsymbol{\rho}_{OT}^{O} = \boldsymbol{0},
$$

where

$$
\begin{aligned}
\boldsymbol{\rho}_{BP}^{O} &= R_B^O\boldsymbol{\rho}_{BP}^{B} = R_B^O\boldsymbol{\rho}_{BP}^{B} = R_B^O\begin{bmatrix} -d_b & 0 & 0 \end{bmatrix}^T, \\
\boldsymbol{\rho}_{PT}^{O} &= R_B^O\boldsymbol{\rho}_{PT}^{B} = R_B^O\left(l_t\boldsymbol{p}^B\right).
\end{aligned}
\tag{9}
$$

d_b denotes the distance between the body CoM and the tail pivot. Combining Eq. 7 and Eq. 9 gives:

$$\boldsymbol{\rho}_{OT}^{B} = -\frac{m_b}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B} + \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t \boldsymbol{p}^B$$
$$= R_O^B\left(\boldsymbol{\rho}_{OB}^{O} + \boldsymbol{\rho}_{BP}^{O} + \boldsymbol{\rho}_{PT}^{O}\right)$$
$$= \boldsymbol{\rho}_{OB}^{B} + \begin{bmatrix} -d_b & 0 & 0 \end{bmatrix}^T + l_t \boldsymbol{p}^B.$$

Then:

$$\boldsymbol{\rho}_{OB}^{B} = \frac{m_r+m_t}{m_b+m_r+m_t}\begin{pmatrix} \frac{1}{2}\frac{m_r}{(m_r+m_t)} l_t \boldsymbol{p}^B + \begin{bmatrix} d_b & 0 & 0 \end{bmatrix}^T \\ -l_t \boldsymbol{p}^B \end{pmatrix}$$
$$= \frac{m_r+m_t}{m_b+m_r+m_t}\left(\frac{-\frac{1}{2}m_r - m_t}{(m_r+m_t)} l_t \boldsymbol{p}^B + \begin{bmatrix} d_b & 0 & 0 \end{bmatrix}^T\right)$$
$$\dot{\boldsymbol{\rho}}_{OB}^{B} = -\frac{\frac{1}{2}m_r + m_t}{m_b+m_r+m_t}(\dot{l}_t \boldsymbol{p}^B + l_t \dot{\boldsymbol{p}}^B)$$

Besides, we have:

$$I_r^B \boldsymbol{\omega}_r^B = R_R^B I_r^R \left(R_R^B\right)^T \boldsymbol{\omega}_r^B,$$
$$R_R^B = \begin{bmatrix} c_{pitch}c_{yaw} & -c_{pitch}s_{yaw} & s_{pitch} \\ s_{yaw} & c_{yaw} & 0 \\ -s_{pitch}c_{yaw} & s_{pitch}s_{yaw} & c_{pitch} \end{bmatrix}$$
$$\boldsymbol{\omega}_r^B = \begin{bmatrix} s_{pitch}\dot{\theta}_{yaw} & \dot{\theta}_{pitch} & c_{pitch}\dot{\theta}_{yaw} \end{bmatrix}^T$$
$$I_r^R = \frac{m_r}{12}\begin{bmatrix} w_t^2+h_t^2 & 0 & 0 \\ 0 & l_t^2+h_t^2 & 0 \\ 0 & 0 & l_t^2+w_t^2 \end{bmatrix}$$

Eq. 8 can be expressed as:

$$H^B = A\boldsymbol{\omega}_B^B + F\boldsymbol{u}, \tag{10}$$

$$A = I_b^B - \frac{m_b(m_r+m_t+m_b)}{(m_r+m_t)} S(\boldsymbol{\rho}_{OB}^{B})S(\boldsymbol{\rho}_{OB}^{B})$$
$$-\frac{m_t m_r l_t^{\,2}}{4(m_r+m_t)} S(\boldsymbol{p}^B)S(\boldsymbol{p}^B),$$

$$F\boldsymbol{u} = \frac{m_b(m_r+m_t+m_b)}{(m_r+m_t)}\boldsymbol{\rho}_{OB}^{B}\times\dot{\boldsymbol{\rho}}_{OB}^{B} + \frac{m_t m_r l_t^{\,2}}{4(m_r+m_t)}\boldsymbol{p}^B\times\dot{\boldsymbol{p}}^B$$
$$+R_R^B I_r^R \left(R_R^B\right)^T \boldsymbol{\omega}_r^B,$$

And

$$F = \begin{bmatrix} -\frac{(c_{yaw}c_{pitch}s_{yaw}(l_t^2 m_r^2 - m_r^2 w_t^2 + 4l_t^2 m_b m_r + 12 l_t^2 m_b m_t + 4l_t^2 m_r m_t - m_b m_r w_t^2 - m_r m_t w_t^2))}{12(m_b+m_r+m_t)}, & \frac{(s_{pitch}(m_r^2 w_t^2 + l_t^2 m_r^2 + 4l_t^2 m_b m_r + 12 l_t^2 m_b m_t + 4l_t^2 m_r m_t + m_b m_r w_t^2 + m_r m_t w_t^2))}{12(m_b+m_r+m_t)}, & 0 \\ \frac{m_r s_{yaw}{}^2(h_t^2+w_t^2) + m_r c_{yaw}{}^2(h_t^2+l_t^2)}{12} + \frac{l_t m_b c_{yaw}(m_r+2m_t)\begin{pmatrix} 2d_b m_r c_{pitch} + 2d_b m_t c_{pitch} + l_t m_r c_{yaw} + \\ 2l_t m_t c_{yaw} \end{pmatrix}}{4(m_b+m_r+m_t)(m_r+m_t)} + \frac{l_t^2 m_r m_t c_{yaw}{}^2}{4(m_r+m_t)}, & -\frac{(d_b l_t m_b s_{yaw} s_{pitch}(m_r+2m_t))}{2(m_b+m_r+m_t)}, & \frac{(d_b m_b c_{yaw} s_{pitch}(m_r+2m_t))}{2(m_b+m_r+m_t)} \\ \frac{(c_{yaw}s_{yaw}s_{pitch}(l_t^2 m_r^2 - m_r^2 w_t^2 + 4l_t^2 m_b m_r + 12 l_t^2 m_b m_t + 4l_t^2 m_r m_t - m_b m_r w_t^2 - m_r m_t w_t^2))}{12(m_b+m_r+m_t)}, & \frac{(l_t^2 m_r^2 c_{pitch} + m_r^2 w_t^2 c_{pitch} + 4l_t^2 m_b m_r c_{pitch} + 12 l_t^2 m_b m_t c_{pitch} + 4l_t^2 m_r m_t c_{pitch} + m_b m_r w_t^2 c_{pitch} + m_r m_t w_t^2 c_{pitch} + 6d_b l_t m_b m_r c_{yaw} + 12 d_b l_t m_b m_t c_{yaw})}{12(m_b+m_r+m_t)}, & \frac{(d_b m_b s_{yaw}(m_r+2m_t))}{2(m_b+m_r+m_t)} \end{bmatrix}$$

$$\boldsymbol{u} = \begin{bmatrix} \dot{\theta}_{pitch} & \dot{\theta}_{yaw} & \dot{l}_t \end{bmatrix}.$$

II. TAIL MOTION ANALYSIS

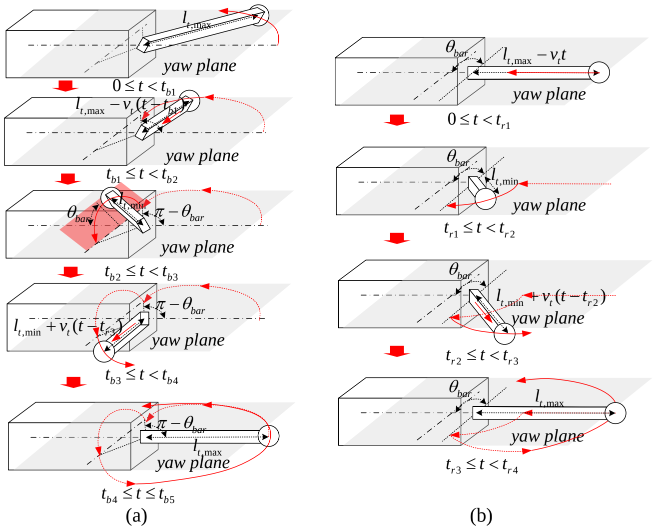

Fig. 2. Tail motion trajectories in the analysis. (a) During TBBMP. (b) During TRMP.

In the analysis, we assume $H^B = 0$ and is subjected to no external forces during aerial reorientation. The center of the whole robot keeps still:

$$m_b \boldsymbol{\rho}_{OB}^O + m_r \boldsymbol{\rho}_{OR}^O + m_t \boldsymbol{\rho}_{OT}^O = \mathbf{0},$$

Then:

$$\begin{aligned} &m_b \boldsymbol{\rho}_{OB}^B + m_r(\boldsymbol{\rho}_{OB}^B + \boldsymbol{\rho}_{BR}^B) + m_t(\boldsymbol{\rho}_{OB}^B + \boldsymbol{\rho}_{BT}^B) = \mathbf{0}, \\ &\boldsymbol{\rho}_{OB}^B = -\frac{m_r \boldsymbol{\rho}_{BR}^B + m_t \boldsymbol{\rho}_{BT}^B}{m_b + m_r + m_t} \end{aligned} \tag{11}$$

A. *Tail Bypass Barrier Motion Pattern (TBBMP)*

As shown in Fig. 2(a), the tail motion trajectory in frame $\{B\}$ can be expressed as:

$$\boldsymbol{\rho}_{PT}^B = \begin{cases} \begin{bmatrix} -l_{t,\max}\cos(\omega_t t), & -l_{t,\max}\sin(\omega_t t), & 0, \end{bmatrix}^T, 0 \le t < t_{b1} \\ \begin{bmatrix} -(l_{t,\min} + v_t(t_{b2} - t))\cos(\omega_t t), & -(l_{t,\min} + v_t(t_{b2} - t))\sin(\omega_t t), & 0, \end{bmatrix}^T, t_{b1} \le t < t_{b2} \\ \begin{bmatrix} l_{t,\min}\cos\theta_{bar}, & -l_{t,\min}\sin\theta_{bar}\cos(\omega_t(t - t_{b2})), & l_{t,\min}\sin\theta_{bar}\sin(\omega_t(t - t_{b2})), \end{bmatrix}^T, \\ \quad t_{b2} \le t < t_{b3} \\ \begin{bmatrix} -(l_{t,\min} + v_t(t - t_{b3}))\cos(\omega_t(t - t_{b3}) + \pi + \theta_{bar}), & -(l_{t,\min} + v_t(t - t_{b3}))\sin(\omega_t(t - t_{b3}) + \pi + \theta_{bar}), & 0, \end{bmatrix}^T, t_{b3} \le t < t_{b4} \\ \begin{bmatrix} -l_{t,\max}\cos(\omega_t(t - t_{b3}) + \pi + \theta_{bar}), & -l_{t,\max}\sin(\omega_t(t - t_{b3}) + \pi + \theta_{bar}), & 0, \end{bmatrix}^T, t_{b4} \le t \le t_{b5} \end{cases}$$

where

$$t_{b1} = \frac{\pi - \theta_{bar}}{\omega_t} - \frac{l_{t,\max} - l_{t,\min}}{v_t}, \quad t_{b2} = \frac{\pi - \theta_{bar}}{\omega_t}, \quad t_{b3} = \frac{2\pi - \theta_{bar}}{\omega_t},$$

$$t_{b4} = \frac{2\pi - \theta_{bar}}{\omega_t} + \frac{l_{t,\max} - l_{t,\min}}{v_t}, \text{ and } t_{b5} = \frac{4\pi - 3\theta_{bar}}{\omega_t}.$$

At stage 1, during $t \in [0, t_{b1})$,

$$\begin{aligned} \boldsymbol{\rho}_{BR}^B &= \frac{1}{2} l_{t,\max} \boldsymbol{p}^B + [-d_b \quad 0 \quad 0]^T \\ &= \left[-d_b - \frac{1}{2} l_{t,\max}\cos(\omega_t t) \quad -\frac{1}{2} l_{t,\max}\sin(\omega_t t) \quad 0\right]^T, \\ \boldsymbol{\rho}_{BT}^B &= [-d_b - l_{t,\max}\cos(\omega_t t) \quad -l_{t,\max}\sin(\omega_t t) \quad 0]^T. \end{aligned}$$

$$\begin{aligned} \boldsymbol{\rho}_{OB}^B &= -\frac{m_r \boldsymbol{\rho}_{BR}^B + m_t \boldsymbol{\rho}_{BT}^B}{m_b + m_r + m_t} \\ &= \frac{1}{m_b + m_r + m_t} \begin{bmatrix} m_r\left(d_b + \frac{l_{t,\max}\cos(\omega_t t)}{2}\right) + m_t(d_b + l_{t,\max}\cos(\omega_t t)) \\ l_{t,\max}\sin(\omega_t t)\left(\frac{m_r}{2} + m_t\right) \\ 0 \end{bmatrix} \end{aligned}$$

Based on Eq. 10, we can get the expression of $\boldsymbol{\omega}_b^B$. Here to get simple expression in the analysis, we assume $m_r = 0$. We have:

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{l_{t,\max} m_b m_t \omega_t (l_{t,\max} + d_b cos(\omega_t t))}{(m_b m_t d_b^2 + 2 m_b m_t cos(\omega_t t) d_b l_{t,\max} + m_b m_t l_{t,\max}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)} \end{bmatrix}$$

The changed angle in yaw direction at $t = t_{b1}$ is:

$$\Delta\varphi_{b,y,1} = \omega_t \int_0^{t_{b1}} -\frac{l_{t,\max} m_b m_t (l_{t,\max} + d_b cos(\omega_t t))}{(m_b m_t d_b^2 + 2 m_b m_t cos(\omega_t t) d_b l_{t,\max} + m_b m_t l_{t,\max}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)} dt$$

At stage2, during $t \in [t_{b1}, t_{b2})$,

$$\boldsymbol{\rho}_{OB}^{B} = -\frac{m_r\boldsymbol{\rho}_{BR}^{B} + m_t\boldsymbol{\rho}_{BT}^{B}}{m_b + m_r + m_t}$$

$$= \frac{1}{m_b + m_r + m_t}\begin{bmatrix} m_r(d_b + \frac{l_t\cos(\omega_t t)}{2}) + m_t(d_b + l_t \\ \cos(\omega_t t)) \\ l_t\sin(\omega_t t)(\frac{m_r}{2} + m_t) \\ 0 \end{bmatrix}$$

We can get:

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ -\frac{(m_b m_t(\omega_t l_t^2 + d_b\omega_t cos(\omega_t t)l_t - d_b v_t sin(\omega_t t)))}{(m_b m_t d_b^2 + 2m_b m_t cos(\omega_t t)d_b l_t + m_b m_t l_t^2 + I_{b,y}^B(m_b + m_t))} \end{bmatrix},$$

The changed angle in yaw direction at $t = t_{b2}$ is:

$$\varphi_{b,y,2}(t) = \int_{t_{b1}}^{t_{b2}} -\frac{(m_b m_t(\omega_t l_t^2 + d_b\omega_t cos(\omega_t t)l_t - d_b v_t sin(\omega_t t)))}{(m_b m_t d_b^2 + 2m_b m_t cos(\omega_t t)d_b l_t + m_b m_t l_t^2 + I_{b,y}^B(m_b + m_t))}dt$$
$$+\omega_t\int_0^{t_{b1}} -\frac{l_{t,\max} m_b m_t(l_{t,\max} + d_b cos(\omega_t t))}{(m_b m_t d_b^2 + 2m_b m_t cos(\omega_t t)d_b l_{t,\max} + m_b m_t l_{t,\max}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)}dt$$
$$l_t = l_{t,\min} + v_t(t_{b2} - t).$$

At stage 3, during $t \in [t_{b2}, t_{b3})$

$$\boldsymbol{\rho}_{OB}^{B} = -\frac{m_r\boldsymbol{\rho}_{BR}^{B} + m_t\boldsymbol{\rho}_{BT}^{B}}{m_b + m_r + m_t}$$

$$= -\frac{1}{m_b + m_r + m_t}\begin{bmatrix} (\frac{m_r}{2} + m_t)l_t c_{bar} - (m_r + m_t)d_b \\ -(\frac{m_r}{2} + m_t)l_t s_{bar}\cos(\omega_t(t - t_{r2})) \\ (\frac{m_r}{2} + m_t)l_t s_{bar}\sin(\omega_t(t - t_{r2})) \end{bmatrix}$$

Here to get simple expression for the analysis, we assume $m_r = 0$. Especially, in specific case $d_b = l_t\cos\theta_{bar}$ for creating pure body roll rotation. We can get:

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} \frac{l_t^2 m_b m_t\omega_t sin^2\theta_{bar}}{m_b m_t l_t^2 sin^2\theta_{bar} + I_{b,r}^B(m_b + m_r)} \\ 0 \\ 0 \end{bmatrix}$$

When $t = t_{r3}$, the changed angle in the body roll direction is:

$$\Delta\varphi_{b,r} = \frac{\pi l_t^2 m_b m_t sin^2\theta_{bar}}{m_b m_t l_t^2 sin^2\theta_{bar} + I_{b,r}^B(m_b + m_r)}$$

Similarly, at stage 4, during $t \in [t_{b3}, t_{b4})$,

$$\boldsymbol{\rho}_{OB}^{B} = -\frac{m_r\boldsymbol{\rho}_{BR}^{B} + m_t\boldsymbol{\rho}_{BT}^{B}}{m_b + m_r + m_t}$$

$$= \frac{1}{m_b + m_r + m_t}\begin{bmatrix} (m_r + m_t)d_b - (\frac{m_r}{2} + m_t)l_t \\ \cos(\omega_t(t - t_{b3}) + \theta_{bar}) \\ -(\frac{m_r}{2} + m_t)l_t\sin(\omega_t(t - t_{b3}) + \theta_{bar}) \\ 0 \end{bmatrix},$$

$$l_t = l_{t,\min} + v_t(t - t_{b3}).$$

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ -\frac{(m_b m_t(\omega_t l_t^2 - d_b\omega_t cos(\theta_{bar} + \omega_t t - \omega_t t_{b3})l_t - d_b v_t sin(\theta_{bar} + \omega_t t - \omega_t t_{b3})))}{(m_b m_t d_b^2 - 2m_b m_t cos(\theta_{bar} + \omega_t t - \omega_t t_{b3})d_b l_t + m_b m_t l_t^2 + I_{b,y}^B(m_b + m_t))} \end{bmatrix},$$

Similarly, at stage 5, during $t \in [t_{b4}, t_{b5}]$,

$$\boldsymbol{\rho}_{OB}^{B} = -\frac{m_r \boldsymbol{\rho}_{BR}^{B} + m_t \boldsymbol{\rho}_{BT}^{B}}{m_b + m_r + m_t}$$

$$= \frac{1}{m_b + m_r + m_t}\begin{bmatrix} (m_r + m_t)d_b - (\frac{m_r}{2} + m_t)l_{t,\max} \\ \cos(\omega_t(t - t_{b3}) + \theta_{bar}) \\ -(\frac{m_r}{2} + m_t)l_{t,\max}\sin(\omega_t(t - t_{b3}) + \theta_{bar}) \\ 0 \end{bmatrix}$$

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ -\frac{(l_{t,\max} m_b m_t \omega_t (l_{t,\max} - d_b cos(\theta_{bar} + \omega_t t - \omega_t t_{b3})))}{(m_b m_t d_b^2 - 2m_b m_t cos(\theta_{bar} + \omega_t t - \omega_t t_{b3})d_b l_{t,\max} + m_b m_t l_{t,\max}{}^2 + I_{b,y}^B (m_b + m_t))} \end{bmatrix},$$

The final changed angle after TBBMP is presented in the paper.

B. Tail Bypass Barrier Motion Pattern (TBBMP)

As shown in Fig. 2(b), the tail motion trajectory in frame $\{B\}$ can be expressed as:

$$\boldsymbol{\rho}_{PT,r}^{B} = \begin{cases} [-l_t, \quad 0, \quad 0,]^T, 0 \le t < t_{r1}, l_t = l_{t,\max} - v_t t, \\ [-l_{t,\max}\cos(-\omega_t(t - t_{r1})), \quad -l_{t,\max}\sin(-\omega_t(t - t_{r1})), \quad 0,]^T, \\ t_{r1} \le t < t_{r2}, \\ [-l_t\cos(\omega_t(t - t_{r2})), \quad -l_t\sin(\omega_t(t - t_{r2})), \quad 0,]^T, \\ t_{r2} \le t < t_{r3}, l_t = l_{t,\min} + v_t(t - t_{r1}), \\ [-l_{t,\max}\cos(\omega_t(t - t_{r2})), \quad -l_{t,\max}\sin(\omega_t(t - t_{r2})), \quad 0,]^T, \\ t_{r3} \le t \le t_{r4}, \end{cases}$$

At stage 1, during $t \in [0, t_{r1})$, there is no yaw angle change.

At stage 2, during $t \in [t_{r1}, t_{r2})$. Similar to the method during Section A, the changed angle in yaw direction is:

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ \frac{l_{t,\min} m_b m_t \omega_t (l_{t,\min} + d_b cos(\omega_t t))}{(m_b m_t d_b^2 + 2m_b m_t cos(\omega_t t) d_b l_{t,\min} + m_b m_t l_{t,\min}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)} \end{bmatrix}$$

The changed angle in yaw direction at $t = t_{r2}$ is:

$$\Delta\varphi_{b,y} = \omega_t \int_{t_{r1}}^{t_{r2}} \frac{l_{t,\min} m_b m_t \omega_t (l_{t,\min} + d_b cos(\omega_t t))}{(m_b m_t d_b^2 + 2m_b m_t cos(\omega_t t) d_b l_{t,\min} + m_b m_t l_{t,\min}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)} dt$$

At stage 3, during $t \in [t_{r2}, t_{r3})$. Similar to the method during Section A, the changed speed in yaw direction is same to the derivations in stage 4 of Section A.

$$\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ -\frac{(m_b m_t (\omega_t l_t^2 - d_b \omega_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2})l_t - d_b v_t sin(\theta_{bar} + \omega_t t - \omega_t t_{r2})))}{(m_b m_t d_b^2 - 2m_b m_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2})d_b l_t + m_b m_t l_t^2 + I_{b,y}^B (m_b + m_t))} \end{bmatrix},$$

The changed angle in yaw direction at $t = t_{r3}$ is:

$$\Delta\varphi_{b,y} = \int_{t_{r1}}^{t_{r2}} -\frac{(m_b m_t (\omega_t l_t^2 - d_b \omega_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2})l_t - d_b v_t sin(\theta_{bar} + \omega_t t - \omega_t t_{r2})))}{(m_b m_t d_b^2 - 2m_b m_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2})d_b l_t + m_b m_t l_t^2 + I_{b,y}^B (m_b + m_t))} dt + \omega_t \int_{t_{r1}}^{t_{r2}} \frac{l_{t,\min} m_b m_t \omega_t (l_{t,\min} + d_b cos(\omega_t t))}{(m_b m_t d_b^2 + 2m_b m_t cos(\omega_t t) d_b l_{t,\min} + m_b m_t l_{t,\min}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)} dt$$

At stage 4, during $t \in [t_{r3}, t_{r4}]$

$$
\boldsymbol{\omega}_b^B = \begin{bmatrix} 0 \\ 0 \\ -\dfrac{(m_b m_t(\omega_t l_{t,\max}^2 - d_b \omega_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2}) l_{t,\max}}{\begin{array}{l}(m_b m_t d_b^2 - 2 m_b m_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2}) d_b l_{t,\max} \\ + m_b m_t l_{t,\max}^2 + I_{b,y}^B (m_b + m_t))\end{array}} \end{bmatrix},
$$

The changed angle in yaw direction at $t = t_{r4}$ is:

$$
\begin{aligned}
\Delta \varphi_{b,y} = & \int_{t_{r1}}^{t_{r2}} - \frac{\begin{array}{l}(m_b m_t(\omega_t l_t^2 - d_b \omega_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2}) l_t \\ - d_b v_t sin(\theta_{bar} + \omega_t t - \omega_t t_{r2})))\end{array}}{\begin{array}{l}(m_b m_t d_b^2 - 2 m_b m_t cos(\theta_{bar} + \omega_t t - \omega_t t_{r2}) d_b l_t \\ + m_b m_t l_t^2 + I_{b,y}^B (m_b + m_t))\end{array}} dt \\
& + \omega_t \int_{t_{r1}}^{t_{r2}} \frac{l_{t,\min} m_b m_t \omega_t (l_{t,\min} + d_b cos(\omega_t t))}{\begin{array}{l}(m_b m_t d_b^2 + 2 m_b m_t cos(\omega_t t) d_b l_{t,\min} \\ + m_b m_t l_{t,\min}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)\end{array}} dt \\
& - \omega_t \int_{t_{r1}}^{t_{r2}} \frac{l_{t,\max} m_b m_t \omega_t (l_{t,\max} + d_b cos(\omega_t t))}{\begin{array}{l}(m_b m_t d_b^2 + 2 m_b m_t cos(\omega_t t) d_b l_{t,\max} \\ + m_b m_t l_{t,\min}{}^2 + I_{b,y}^B m_b + I_{b,y}^B m_t)\end{array}} dt
\end{aligned}
$$

The final changed angle after multiple TRMP is presented in the paper.