

# UCI 2021 SPRING

## MSP Project Deliverable

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## Challenges and Assumptions

The weekly firefighter personnel requirements are:

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
5a-5p	11	13	15	19	15	13	9
5p-5a	17	13	15	17	19	21	14

Big fire faces the challenges of how to distribute firefighters at either day or night that satisfy the working requirement in a minimum cost.

Assumption: The town runs two schedules per day, 12 hours each. Firefighters are compensated hourly at \$30 per hour plus 33% for benefits, normal shift for a firefighter is 4 days 12 hours a day followed by 3 consecutive days off, working on either the day or the night shift (not both).

## Optimal schedule for firefighters and the associated cost

	A	B	C	D	E	F	G	H
1	<b>Full time employees start</b>							
2		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
3	5a-5p	6	4	4	5	2	2	1
4	5p-5a	5	3	7	2	7	5	0
5	Full time employee working	>=	Employees required working					
6		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
7	5a-5p	11	13	15	19	15	13	10
8	5p-5a	17	13	15	17	19	21	14
9		>=	>=	>=	>=	>=	>=	>=
10		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
11	5a-5p	11	13	15	19	15	13	9
12	5p-5a	17	13	15	17	19	21	14
13	<b>Cost per hour person</b>							
14		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
15	5a-5p	39.90	39.90	39.90	39.90	39.90	39.90	39.90
16	5p-5a	46.55	46.55	46.55	46.55	46.55	46.55	46.55
17	total cost per day	14763	13486.2	15561	18593.4	17795.4	17955	12608.4
18	total cost	110762.4						

In full time employees' start, We first set up a decision variable from both 5a-5p and 5p-5a, and next we create a blank working space to set up the formula for each day. From 5a-5p, people work 4 days 12 hours a day followed by 3 consecutive days off, and from 5p-5a, it follows the same separate trend. Next, we added each day's cost to reach a final total cost minimization as the objective. Finally, we use solver by setting a number of constraints to get the final solution

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$3:\$H\$4 = integer

\$B\$7:\$H\$7 >= \$B\$11:\$H\$11

\$B\$8:\$H\$8 >= \$B\$12:\$H\$12

Add  
Change  
Delete  
Reset All  
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

## Solution change with increases in the needed number of firefighters on the weekend night shifts (Friday, Saturday, and Sunday) of 5%

	A	B	C	D	E	F	G	H
1	5%							
2	Full time employees start							
3		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
4	5a-5p	6	4	4	5	2	2	1
5	5p-5a	4	3	8	2	7	6	0
6	Full time employee working	>=	Employees required working					
7		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8	5a-5p	11	13	15	19	15	13	10
9	5p-5a	17	13	15	17	20	23	15
10		>=	>=	>=	>=	>=	>=	>=
11		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
12	5a-5p	11	13	15	19	15	13	9
13	5p-5a	17	13	15	17	19.95	22.05	14.7
14	Cost per hour person							
15		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
16	5a-5p	39.90	39.90	39.90	39.90	39.90	39.90	39.90
17	5p-5a	46.55	46.55	46.55	46.55	46.55	46.55	46.55
18	total cost per day	14763	13486.2	15561	18593.4	18354	19072.2	13167
19	total cost	112996.8						

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$4:\$H\$5 = integer

\$B\$8:\$H\$9 >= \$B\$12:\$H\$13

Add  
Change  
Delete  
Reset All  
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close Solve

Using the same solution process as solving the optimal schedule with 5% increase respectively in Friday, Saturday, and Sunday's night shift. When number of firefighters increase 5% respectively in Friday, Saturday, and Sunday, the total number of full time employees increase from 212 to 216. The total cost increases overall with 112996.8.

## Solution change with increases in the needed number of firefighters on the weekend night shifts (Friday, Saturday, and Sunday) of **10%**

	A	B	C	D	E	F	G	H
20	10%							
21	Full time employees start							
22		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
23	5a-5p	6	4	4	5	2	2	1
24	5p-5a	4	3	8	2	8	6	0
25	Full time employee working	>= Employees required working						
26		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
27	5a-5p	11	13	15	19	15	13	10
28	5p-5a	18	13	15	17	21	24	16
29		>=	>=	>=	>=	>=	>=	>=
30		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
31	5a-5p	11	13	15	19	15	13	9
32	5p-5a	17	13	15	17	20.9	23.1	15.4
33	Cost per hour person							
34		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
35	5a-5p	39.90	39.90	39.90	39.90	39.90	39.90	39.90
36	5p-5a	46.55	46.55	46.55	46.55	51.205	51.205	51.205
37	total cost per day	15321.6	13486.2	15561	18593.4	20085.66	20971.44	14619.36
38	total cost	118638.66						

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$23:\$H\$24 = integer

\$B\$27:\$H\$28 >= \$B\$31:\$H\$32

Add  
Change  
Delete  
Reset All  
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

Using the same solution process as solving the optimal schedule with **10%** increase respectively in Friday, Saturday, and Sunday's night shift. When number of firefighters requirement increase **10%** respectively in Friday, Saturday, and Sunday, the full time employee's working increases from 212 to 220. The total cost increases overall with 118638.66

## Solution change with increases in the needed number of firefighters on the weekend night shifts (Friday, Saturday, and Sunday) of **20%**?

	A	B	C	D	E	F	G	H
39	<b>20%</b>							
40	<b>Full time employees start</b>							
41		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
42	5a-5p	6	4	4	5	2	2	1
43	5p-5a	2	4	9	2	8	7	0
44	Full time employee working	>=	Employees required working					
45		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
46	5a-5p	11	13	15	19	15	13	10
47	5p-5a	17	13	15	17	23	26	17
48		>=	>=	>=	>=	>=	>=	>=
49		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
50	5a-5p	11	13	15	19	15	13	9
51	5p-5a	17	13	15	17	22.8	25.2	16.8
52	<b>Cost per hour person</b>							
53		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
54	5a-5p	39.90	39.90	39.90	39.90	39.90	39.90	39.90
55	5p-5a	46.55	46.55	46.55	46.55	46.55	46.55	46.55
56	total cost per day	14763	13486.2	15561	18593.4	20029.8	20748	14284.2
57	total cost	117465.6						

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$42:\$H\$43 = integer

\$B\$46:\$H\$47 >= \$B\$50:\$H\$51

Add
Change
Delete
Reset All
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close
Solve

Using the same solution process as solving the optimal schedule with **20%** increase respectively in Friday, Saturday, and Sunday's night shift. When number of firefighters increase **20%** respectively in Friday, Saturday, and Sunday, the total number of full time employees increase from 212 to 224. The total cost increases overall with 117465.6

## Solution change when part-time employees work two consecutive days with 5 days off either day or night but not both day and night?

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Full time employees start								Part time start						
2		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
3	5a-5p	5	5	3	4	3	3	0	0	0	2	0	0	0	0
4	5p-5a	5	3	7	2	7	5	0	0	0	0	0	0	0	0
5		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
6	5a-5p	11	13	13	17	15	13	10	0	0	2	2	0	0	0
7	5p-5a	17	13	15	17	19	21	14	0	0	0	0	0	0	0
8	Sum of all employees														
9		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
10	5a-5p	11	13	15	19	15	13	10							
11	5p-5a	17	13	15	17	19	21	14							
12		>=	>=	>=	>=	>=	>=	>=							
13	Employees' requirement														
14		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
15	5a-5p	11	13	15	19	15	13	9							
16	5p-5a	17	13	15	17	19	21	14							
17	Cost per hour person														
18		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
19	5a-5p	39.90	39.90	39.90	39.90	39.90	39.90	39.90							
20	5p-5a	46.55	46.55	46.55	46.55	46.55	46.55	46.55							
21	total cost	14763	13486.2	15561	18593.4	17795.4	17955	12608.4							
22	total cost	110762.4													

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$10:\$H\$11 >= \$B\$15:\$H\$16

\$B\$3:\$O\$4 = integer

Add
Change
Delete
Reset All
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close Solve

Assumption: part time cost day shift and night shift cost is equal to that of full time employees

In this questions, we have the same minimum cost as in benchmark solution in question B. we set two decisions variables: full time and part time employee's start. In full time employees, they work for 4 days 12 hours a day followed by 3 consecutive days off while part time employees work two consecutive days with 5 days off. Then we sum their working days satisfied by the employee's requirement, using solver to get the final minimum cost of 110762.4

## Solution for upper limits of 0% of part time firefighters

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	0%														
2	Full time employees start								Part time start						
3		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
4	5a-5p	6	4		4	5	2	2	1	0	0	0	0	0	0
5	5p-5a	5	3		7	2	7	5	0	0	0	0	0	0	0
6		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
7	5a-5p	11	13		15	19	15	13	10	0	0	0	0	0	0
8	5p-5a	17	13		15	17	19	21	14	0	0	0	0	0	0
9	Sum of all employees														
10		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
11	5a-5p	11	13		15	19	15	13	10						
12	5p-5a	17	13		15	17	19	21	14						
13		>=		>=		>=		>=							
14	Employees' requirement														
15		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
16	5a-5p	11	13		15	19	15	13	9						
17	5p-5a	17	13		15	17	19	21	14						
18	Full time employee	Part time employee		Part time employee requirement	Part time percentage requirement										
19	212	0	<=		0	0%									
20	Cost per hour person														
21		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
22	5a-5p	39.90	39.90		39.90	39.90	39.90	39.90							
23	5p-5a	46.55	46.55		46.55	46.55	46.55	46.55							
24	total cost	14763	13486.2		15561	18593.4	17795.4	17955	12608.4						
25	total cost	110762.4													

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$11:\$H\$12 >= \$B\$41:\$H\$42

\$B\$19 <= \$D\$19

\$B\$4:\$O\$5 = integer

Add  
Change  
Delete  
Reset All  
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

Assumption: part time cost day shift and night shift cost is equal to that of full time employees

In this questions, we have the same minimum cost as in benchmark solution in question B. we set two decisions variables: full time and part time employee's start and additional requirement of sum of part time employees working as 0% of total employees working. We got 212 Full time employees working and 0 part time employees working. We get the same minimum cost of 110762.4



## Solution for upper limits of 20% of part time firefighters

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
26	20%														
27	Full time employees start								Part time start						
28		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
29	5a-5p		5	5	3	4	3	3	0	0	0	2	0	0	0
30	5p-5a		5	3	7	2	7	5	0	0	0	0	0	0	0
31		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
32	5a-5p		11	13	13	17	15	13	10	0	0	2	2	0	0
33	5p-5a		17	13	15	17	19	21	14	0	0	0	0	0	0
34	Sum of all employees														
35		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
36	5a-5p		11	13	15	19	15	13	10						
37	5p-5a		17	13	15	17	19	21	14						
38	>=	>=	>=	>=	>=	>=	>=	>=							
39	Employees' requirement														
40		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
41	5a-5p		11	13	15	19	15	13	9						
42	5p-5a		17	13	15	17	19	21	14						
43	Full time employee	Part time employee		Part time employee requirement	Part time percentage requirement										
44	208	4	<=	42.4	20%										
45	Cost per hour person														
46		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
47	5a-5p		39.90	39.90	39.90	39.90	39.90	39.90	39.90						
48	5p-5a		46.55	46.55	46.55	46.55	46.55	46.55	46.55						
49	total cost		14763	13486.2	15561	18593.4	17795.4	17955	12608.4						
50	total cost		110762.4												

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$29:\$O\$30 = integer

\$B\$36:\$H\$37 >= \$B\$41:\$H\$42

\$B\$44 <= \$D\$44

Add
Change
Delete
Reset All
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close Solve

Assumption: part time cost day shift and night shift cost is equal to that of full time employees

In this questions, we have the same minimum cost as in benchmark solution in question B. we set two decisions variables: full time and part time employee's start and additional requirement of sum of part time employees working as 20% of total employees working. Part time employees is served as a more flexible solution whose number should be restricted to the relative minimum level and full time employees is chosen as the first priority. Thus, We got 208 Full time employees and 4 part time employees working with the same total cost of 110762.4

# Solution for upper limits of 40% of part time firefighters

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
51	40%														
52	Full time employees start								Part time start						
53		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
54	5a-5p		5	5	3	4	3	3	0	0	0	2	0	0	0
55	5p-5a		5	3	7	2	7	5	0	0	0	0	0	0	0
56		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
57	5a-5p		11	13	13	17	15	13	10	0	0	2	2	0	0
58	5p-5a		17	13	15	17	19	21	14	0	0	0	0	0	0
59	Sum of all employees														
60		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
61	5a-5p		11	13	15	19	15	13	10						
62	5p-5a		17	13	15	17	19	21	14						
63	>=	>=	>=	>=	>=	>=	>=	>=							
64	Employees' requirement														
65		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
66	5a-5p		11	13	15	19	15	13	9						
67	5p-5a		17	13	15	17	19	21	14						
68	Full time employee	Part time employee	Part time employee requirement	Part time percentage requirement											
69	208	4	<=	84.8	40%										
70	Cost per hour person														
71		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday							
72	5a-5p		39.90	39.90	39.90	39.90	39.90	39.90	39.90						
73	5p-5a		46.55	46.55	46.55	46.55	46.55	46.55	46.55						
74	total cost		14763	13486.2	15561	18593.4	17795.4	17955	12608.4						
75	total cost		110762.4												

Solver Parameters

Set Objective:

To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

\$B\$54:\$O\$55 = integer

\$B\$61:\$H\$62 >= \$B\$66:\$H\$67

\$B\$69 <= \$D\$69

Add  
Change  
Delete  
Reset All  
Load/Save

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:  Options

Solving Method


Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Close

Solve

Assumption: part time cost day shift and night shift cost is equal to that of full time employees

In this questions, we have the same minimum cost as in benchmark solution in question B. we set two decisions variables: full time and part time employee's start and additional requirement of sum of part time employees working as 40% of total employees working. Part time employees is served as a more flexible solution whose number should be restricted to the relative minimum level and full time employees is chosen as the first priority. Thus, We got 208 Full time employees and 4 part time employees working with the same total cost of 110762.4



Decrease number of employees starting requirement	Total Cost	Benchmark cost	Difference
1	111799.8	110762.4	1037.4
2	112837.2	110762.4	2074.8
3	113874.6	110762.4	3112.2
4	114912.0	110762.4	4149.6
5	115949.4	110762.4	5187.0
6	116986.8	110762.4	6224.4
7	118024.2	110762.4	7261.8
8	119061.6	110762.4	8299.2
9	120099.0	110762.4	9336.6
10	121136.4	110762.4	10374.0

Assumptions: No part time employees participate in, Every worker for working overtime is permitted to work overtime for 96 hours to satisfy double-shift worker requirement(4days working day and night), Within 48 hours, Worker could work either day or night with sum of day and night shift to be compensated with a total of  $24 * (\text{dayshift} + \text{nightshift}) \text{cost} = 24 * (39.9 + 46.55) = 2074.8$ , and over 48 hours, double shift workers are compensated with additional benefits  $1.5 * 24 * (39.9 + 46.55) = 3112.2$ , total cost compensation is calculated by summing the cost of full time employees working and double shift workers= total employees dayshift working  $* 39.9 +$  total employees nightshift working  $* 46.6 +$  double shift worker total starting  $* 24 * (\text{normal compensation within 48 hours} = (39.90 + 46.55) * 24) +$  double shift worker total starting  $* (\text{ additionally compensation for working over 48 hours} = (39.90 + 46.55) * 24 * 1.5)$ , and then plus double shift workers to both normal full time workers who work both day and night shift for the both satisfaction of day and night shift requirement. The overall trend is that, when we decrease the number of total employees starting, the total cost display an upward increasing trend.

In summary, 4 days overtime is not recommended since it will increase the total cost with additional cost of 1037.4 when decreasing the requirement of one employees starting compared with the benchmark solution for firefighters.



## Appendix

Question B:	Page3
Question C:	Page4-6
Question D:	Page7
Question E:	Page8-10
Question F:	Page11

## Conclusion

❖ In the total full time employees' setting, we get the minimum cost of 110762.4 and with the increase of percentage of total firefighters working requirement, the cost increase as well

❖ When we add part time employees with the same day and night cost and benefits as that of full time employees, Part time employees is served as a more flexible solution whose number should be restricted to the relative minimum level and full time employees is chosen as the first priority and the total cost is the same as the benchmark solutions of Question B. With the more part time employees percentage required in the total employees working requirement, the part time employees' number is always restricted to a low level because full time employees is more economical that work more days with the same expense spent.

❖ When adding the double shift firefighters and at the same time decreases the number employees starting number by 1 to 10, we have additional cost of 1037.4, and therefore it is not wise to adding the double shift firefighters for the overtime of 4 days that works for both day and night