IEOR E4004: Optimization Models and Methods
Instructor: Dr. Yaren Bilge Kaya
Deadline: 12/01/2023, 11:59 pm

Project Report Grading

Description	Grade
Submission of project report with names written and in the appropriate format	1 point
Quality of writing	1 point
Inclusion and standardized usage of necessary references	1 point
Strength of introduction, background, and motivation	2 points
Originality of the problem and the solution approach	2 points
Accuracy and sensibility of the mathematical model	5 points
Data cleanliness and meaningful formatting	2 points
Adequacy of discussion on results and insights	2 points
Effectiveness of project summarization	1 point

Table 1: Grading Criteria for Project Reports

Group Number:
Group Members:

Description	Grade	Out of
Names and appropriate format	1	1
Well written?	1	1
Standardized references?	1	1
Strong intro, background, and motivation?	1	2
Originality of problem and solution	2	2
Modeled correctly?	5	5
Data cleaned and formatted well?	2	2
Discussed results and insights?	2	2
Summarized well?	1	1
Total	16	17

IEOR4004 Team comments: Great job all of you! I really like that you chose to bring in predictive models to inform your optimization model! The model itself is presented in a clear format, which I appreciate. Left some notes throughout, please check them :)

IEOR 4004 Retail Price Optimization Report

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Abstract

This report presents a comprehensive study on retail price optimization, a critical aspect in the competitive retail industry. Utilizing a robust dataset, the study employs advanced analytical techniques to determine optimal pricing strategies that balance profitability and market competitiveness. The project encompasses data preparation and exploratory analysis, demand forecasting, price elasticity estimation, and multifaceted optimization models, each considering different constraints and market factors. The optimization models are centered around two decision variables: unit price and quantity sold, reflecting the intricate interplay between pricing, demand and market constraints. The findings offer valuable insights into pricing strategies, highlighting the importance of data-driven decision-making in the retail sector.

Introduction

reference here!

The retail industry is characterized by fierce competition and rapidly changing market dynamics, making price optimization a vital component for success. In this complex landscape, effective pricing strategies can significantly influence consumer behavior, drive sales, and enhance profitability. These strategies encompass a myriad of factors: internal aspects like product quality, historical sales trends, and production costs; and external elements including market demand, competitor positioning, and overall market performance. Particularly challenging is devising pricing strategies for a diverse range of products, each with distinct price attributes and category-specific considerations.

reference here!

Our project introduces a data-oriented strategy for optimizing retail pricing. Utilizing a comprehensive dataset encompassing sales figures, consumer feedback, rival pricing, and product features, we plan to create an adaptable model. The model will be sensitive to market changes and internal costs, offering retailers a means to make strategic and well-informed decisions about their pricing. We include the freight cost, production costs, and market constraints, such as competitor pricing and customer ratings, aiming to develop a model to provide retailers with a strategic tool to navigate the complexities of the market, ensuring competitiveness and profitability in a dynamic environment.

Background Information

really? then where are the references to those research papers?

The realm of retail price optimization is grounded in a rich history of both theoretical and applied research. Traditional approaches often relied on simplistic models, disregarding the multifaceted nature of market dynamics. However, recent advancements in computational power and data availability have enabled a more nuanced exploration of pricing strategies. Key to this evolution is the application of Linear Programming (LP).

LP is a mathematical method designed to find the optimal outcome in a model characterized by linear relationships. Its core strength lies in its proficiency in handling a multitude of linear constraints and a linear objective function, making it exceptionally suited for optimizing retail pricing strategies. In these scenarios, LP helps in balancing various variables and constraints, whether the goal is to maximize profitability or minimize costs.

Next, the report will detail the methods employed in this project, discussing the incorporation of LP into our optimization model and other statistical techniques used for demand prediction and price elasticity analysis.

Methods and Results

In our quest to optimize retail prices, the project adopted a multifaceted methodological approach, beginning with extensive data preparation, followed by sophisticated demand prediction and price elasticity analysis, and culminating in a detailed optimization process. Each step was meticulously crafted to ensure robustness and accuracy in our findings.

Data Cleaning and Preparation

The initial phase of the project involved data cleaning and preparation. This process involved removing redundant columns and transforming the data field for better analysis. Focusing on the top 10 products, we refined the dataset to concentrate on items with significant sales and market presence. This refined dataset underwent further analysis, including the encoding of categorical variables and computation of average competitor prices, thereby setting the stage for more detailed modeling.

Demand Prediction & Price Elasticity Analysis

✓ love it!

The demand prediction model was central to understanding the market dynamics. The model, constructed using linear regression, employed a range of numerical variables, excluding categorical ones, to predict the quantity of products sold. This demand estimation model was trained and evaluated using a split of 80% training and 20% testing data, ensuring a robust and reliable model. The model achieved R^2 scores of 0.846 on the test set and 0.883 on the full dataset, indicating a strong fit to the data.

Parallel to demand prediction, price elasticity analysis for each product was conducted using a log-log regression model. This analysis aimed to quantify the responsiveness of the quantity demanded to changes in price. However, the results indicated that the elasticity coefficients for the products were not statistically significant. This finding suggests that within the scope of the available data, price changes might not have a substantial impact on the quantity demanded for these products, or that the dataset might not capture all the variables influencing demand elasticity.

Original Profit Calculation

Before optimization, the original profit was calculated as a baseline. This involved aggregating the profit from each product, computed as the difference between unit price and the sum of unit cost and freight price, multiplied by the quantity sold. The total profit from this baseline calculation was $4369.03.

Optimization with Linear Programming (LP)

The optimization component of this project employed Linear Programming (LP) techniques to maximize profit under various constraints. Utilizing Gurobi, a powerful optimization solver, four distinct models were developed, each with its unique focus and constraints.

The LP model was constructed with the following specific parameters:

- Unit Costs (c_i): The average cost of producing product i.
- Freight Prices (f_i): The average freight cost associated with product i.
- Predicted Demand (d_i): The forecasted quantity demand for product i.
- Lag Prices (l_i): The average historical price from the previous month for product i.
- Product Scores (r_i): The average ratings for product i.
- Competitor Scores (x_i): The average competitor rating for product i
- Competitor Pricing (p_i): The average competitor price for product i

We focused on two main decision variables:

- Unit Prices (u_i): The prices at which product i is to be sold.
- Quantities Sold (q_i): The expected sales volume for each product i at the set price u_i.

The objective function in our LP model aimed to maximize profit, defined as the difference between its selling price and the sum of its unit production and freight costs, multiplied by the quantity sold. Formally, the objective function is expressed as:

$$Maximize \sum_i (u_i - c_i - f_i) \times q_i$$

Three essential constraints were uniformly applied across all models:

- Demand Constraints: $q_i \leq d_i \; \forall i$ The quantity sold does not exceed the predicted demand.
- Cost Constraints: $u_i \geq c_i \; \forall i$ The selling price must be at least equal to the unit cost.
- Non-Negativity Constraints: $u_i \geq 0 \; \forall i, \; q_i \geq 0 \; \forall i$

do we need non-negativity here?

With a well-defined framework in place, combining parameters, decision variables, and a profit-maximizing objective function, our LP model was primed to explore various retail pricing strategies. This setup laid the groundwork for investigating four distinct methods of manipulating constraints. Each method, distinguished by its unique constraint focus, allowed us to delve into the nuanced effects of different market and internal factors on optimal pricing.

Method 1: Lag-Prices Constraint

The first method in our optimization approach focused on Lag Price Constraints, aiming to gauge the influence of historical pricing on our current pricing strategy. This method was predicated on the idea that prices should not significantly deviate from those set in the previous month, thereby maintaining a level of consistency in the market and aligning with customer price expectations. Mathematically, this constraint was expressed as

$$0.9 \times l_i \leq u_i \leq 1.1 \times l_i \; for\ each\ product\ i$$

along with adhering to the standard demand and cost constraints.

The optimization under these constraints yielded an optimal objective value of 5671.63. This result indicated that staying close to historical prices could still offer a viable path to maximizing profits. The outcome suggested a moderate level of flexibility in pricing while adhering to market trends and customer familiarity with past prices.

Product ID	**Unit Price**	**Quantity Sold**
health9	24.64	15.0
health8	95.99	15.0
garden9	61.43	21.0
health5	385.04	7.0
health7	66.87	11.0
garden3	114.94	7.0
bed2	96.37	26.0
garden1	117.05	8.0
computers4	158.39	18.0
watches1	200.29	11.0

Method 2: Competitor Constraints

For the second method, we shifted our focus to Competitor Constraints, a crucial aspect in the retail landscape. This method was centered on the strategic positioning of our product prices in relation to those of our competitors, taking into account both our product ratings and competitor pricing. Here are the key aspects of competitor constraint:

- We implemented a comparative approach where our pricing was directly influenced by our competitors' pricing and ratings. Specifically, if our product rating was lower than a competitor's and our cost was less than their price, we set a cap on our price to be no more than the competitor's price. This was formulated as

  $$u_i \leq p_i \; if\ r_i < x_i \; and\ c_i \leq p_i \; for\ each\ product\ i$$

  Here, u_i is our unit price, r_i is our product rating, c_i is our unit cost, x_i and p_i denote the average of the three competitors' ratings and prices, respectively.

- In cases where our product's rating did not fall below the average competitor rating, we introduced a more flexible constraint, allowing our price to be up to four times the average competitor's price, indicated as
$$u_i \leq 4 \times p_i \, for\, each\, product\, i$$
This decision was driven by the need to keep the model feasible and competitive within the market context.
- Alongside these competitive pricing constraints, we sustained the standard demand and cost constraints. This was to ensure that our pricing covered production costs and that our projected sales volumes were in line with the anticipated market demand.

The optimization under these constraints resulted in an optimal objective value of 4463.41. When compared to the optimal value of 5671.63 from Method 1 (Lag Price Constraints), this result was lower. The decrease in profit underlines the impact of integrating competitive pricing strategies, which, while crucial for market competitiveness, can potentially lead to lower profit margins under certain market conditions.

Examining the specific product changes, we noticed varied adjustments in unit prices and quantities sold. For instance, products such as 'health9', 'health8', and 'garden1' experienced a reduction in their unit prices. Conversely, other products saw an increase in their prices. In terms of quantities sold, 'health7', 'garden3', and 'bed2' witnessed a significant uptick, while the sales volume for the rest of the products remained unchanged.

These specific changes highlight how different products respond to competitor-based pricing adjustments. The increase in quantities sold for certain products suggests that more competitive pricing can lead to higher sales volumes. However, this does not always translate into higher overall profitability, as seen in the decreased optimal profit. This nuanced result from the second method offers valuable insights into the complexities of balancing competitive pricing with overall profitability in the retail sector.

Product ID	**Unit Price**	**Quantity Sold**
health9	90.49	15.0
health8	105.23	15.0
garden9	55.19	21.0
health5	221.41	7.0
health7	24.67	0.0
garden3	58.50	0.0
bed2	43.80	0.0
garden1	239.61	8.0
computers4	150.67	18.0
watches1	148.15	11.0

Method 3: Integrated Constraints

Our third approach entailed an Integrated Constraints model, combining demand, lag price range, and competitor performance. This method offered a more comprehensive view of the pricing strategy by considering a wider range of market factors. Here are the constraints and strategy:

- Relaxed Price Change Constraint: We loosened the bounds on price fluctuations relative to historical prices, allowing unit prices (u_i) to vary between 80% and 120% of the previous month's prices (lag prices, l_i), This was represented by
  $$0.8 \times l_i \leq u_i \leq 1.2l_i \ for\ each\ product\ i$$
  This relaxation offered more flexibility in responding to current market conditions while still considering past pricing trends.
- Competitor Pricing Constraint: Similar to Method 2, we incorporated competitor pricing into our model. However, the constraints were adjusted to be more dynamic, considering our product ratings compared to competitors. If our rating was lower than the competitor's and our cost was less, we matched or undercut the competitor's price. Otherwise, the model was not restricted by these comparisons, offering more pricing freedom. This was expressed as:
  $$u_i \leq p_i\ if\ r_i < x_i\ and\ c_i \leq p_i$$
- Demand and Cost Constraints: As with the previous models, we maintained constraints ensuring that quantities sold did not exceed predicted demand and that selling prices covered unit costs.

The optimization in Method 3 yielded an optimal objective value of 7103.70, surpassing the outcomes of the previous two methods. This superior result highlights the effectiveness of a pricing strategy that comprehensively considers both internal and external market factors. It demonstrates that a more adaptable and nuanced approach to pricing, which accounts for a wider array of market influences, can lead to enhanced profitability. This method's success underlines the potential benefits for retailers in adopting flexible pricing strategies that are both responsive to market trends and grounded in a thorough understanding of the competitive landscape.

Product ID	**Unit Price**	**Quantity Sold**
health9	26.88	15.0
health8	104.72	15.0
garden9	67.02	21.0
health5	420.04	7.0
health7	72.95	11.0
garden3	125.39	7.0
bed2	105.13	26.0
garden1	127.69	8.0

computers4	173.88	18.0
watches1	218.50	11.0

Method 4: Sub-category Constraints

In this section, we shift our focus to previously unexplored factors critical for ensuring the rationality and practicality of our pricing strategy. Given the complexity of our product portfolio, which encompasses a diverse assortment of products with varying costs and market performances, it's essential to recognize that different products inherently have different price floors. This understanding led us to incorporate new subcategory-related constraints into our optimization model, while retaining the existing constraints on demand, cost, and competitive pricing.

With this approach, we aimed to respect the individual price floors inherent to different sub-categories of products. The objective was to maintain the perceived market value of each product within its specific sub-category, thereby ensuring a pricing strategy that is both competitive and realistic. Constraints include:

- Sub-category Minimum Price Constraint: We established a constraint for each product to prevent its price from falling below the historical minimum observed within its sub-category. This was mathematically formulated as
$$u_i \geq min\_price_{subcategory\ of\ i} \quad for\ each\ product\ i$$
ensuring that the unit price (u_i) of each product respects its sub-category's pricing history.
- Integration with the Constraints in Method 3: In addition to this new constraint, we continued to apply the relaxed price range, competitor pricing, demand, and cost constraints. This comprehensive approach sought to strike a balance between maintaining sub-category price integrity and aligning with broader market and internal cost factors.

The optimization of this method resulted in an optimal objective value of 7103.70, aligning with the outcome of Method 3. This similarity in results does not diminish the significance of sub-category constraints; rather, it reveals certain nuances:

- The dataset might be such that the lowest prices in each subcategory are already in line with optimal pricing based on other factors like demand and competitor pricing. If so, the subcategory constraints wouldn't impose additional restrictions that significantly alter the outcome.
- The model's sensitivity to subcategory pricing may be lower compared to other factors. While theoretically important, the practical impact of subcategory constraints on final pricing decisions could be limited in this context. This indicates that other factors like cost, demand, and competitor prices might play more dominant roles in determining optimal prices for our range of products.

By incorporating these subcategory constraints, we've enhanced our model's alignment with real-world market dynamics, where products are often segmented into various subcategories with distinct price expectations. This method underscores the importance of considering sub-category price floors in retail pricing strategies and reflects the multifaceted nature of pricing decisions in a competitive retail environment.

Product ID	Unit Price	Quantity Sold
health9	26.88	15.0
health8	104.72	15.0
garden9	67.02	21.0
health5	420.04	7.0
health7	72.95	11.0
garden3	125.39	7.0
bed2	105.13	26.0
garden1	127.69	8.0
computers4	173.88	18.0
watches1	218.50	11.0

Sensitivity Analysis & Model Improvement

Sensitivity analysis is a crucial technique in assessing how changes in key factors, such as predicted demand, affect product profitability and competitiveness. By adjusting the variables in our models, retailers can understand how sales and revenue respond to demand shifts, aiding in strategic planning for various market conditions.

In our analysis, adjusting the predicted demand by ±10% provided valuable insights into how demand changes affect retail operations. For instance, in Method 1, a 10% increase in predicted demand raises the profit value by 11.43% and increases quantity sold ranging from 6.67% to 14.29%. Conversely, a 10% decrease resulted in a 6.56% drop in optimal profit, with quantity reduction of up to -13.33%. This notable positive correlation between demand and profit indicates that retailers should invest in accurate demand forecasting methods and adopt flexible inventory strategies to adapt swiftly to demand fluctuations.

The observed stability in unit prices during our analysis can be attributed to several factors, including our pricing strategy and market positioning. However, a key factor was the statistically insignificant demand elasticity in our dataset. With a more comprehensive dataset that better captures market and consumer behavior complexities, we could calculate specific demand elasticities for each product. This would allow us to refine our model by modifying the objective function and the current demand constraint to include price elasticity effects. The proposed modified objective function, incorporating price elasticity, is:

$$Maximize \sum_i (u_i - c_i - f_i) \times q_i \times \left(\frac{u_i}{l_i}\right)^{\varepsilon_i}$$

Here, ε_i represents the price elasticity for product i. This formula is more realistic in discovering how change in price affects demand and, consequently, profit. Raising the price ratio to the power of elasticity models the demand response to price changes. For instance, if the price increases ($\frac{u_i}{l_i} > 1$) and demand is elastic ($\varepsilon_i < 0$), then demand decreases, indicated by $q_i \times (\frac{u_i}{l_i})^{\varepsilon_i} < 1$, thereby reducing profit.

Additionally, we propose adjusting the demand constraint to incorporate price elasticity factor:

$$q_i \leq d_i \times (1 + \varepsilon_i \times \frac{u_i - l_i}{l_i}) \ for\ each\ product\ i$$

This constraint dynamically adjusts the predicted demand based on the product's price sensitivity and the magnitude of the price change relative to historical prices. For instance, with elastic demand, minor price changes could lead to demand variations, and vice versa.

Incorporating elasticity into the model is theoretically significant, as it aligns the model more closely with market realities, where demand often responds variably to price changes. In practical terms, for datasets capturing significant elasticity, this enhancement would enable retailers to more accurately model market demand responses to different pricing strategies. Therefore, despite our dataset showing limited elasticity, recognizing and integrating this aspect is crucial for enhancing retail pricing models. Adapting these models for datasets with notable elasticity could provide more nuanced and effective pricing strategies, harmonizing market dynamics with internal cost factors.

Conclusion

Our project successfully bridges the theoretical aspects of optimization models with the practical challenges faced in retail pricing. We developed a robust model to enhance retail pricing strategies, focusing on competitiveness and profitability in a dynamic market. We conducted sensitivity analyses, adjusting quantity bounds in relation to predicted demand to determine each product's optimal quantity and unit price. Additionally, we explored the potential for enhancements by incorporating elasticity.

Our exploration revealed that including competitor and market considerations significantly impacts pricing strategy, as evidenced by the changes in optimal profits. Method 3, which integrates both internal (costs, demand, lag prices) and external factors (competitor prices and ratings), emerged as the most effective, achieving the highest profit compared to Methods 1 and 2. Interestingly, the introduction of subcategory-related constraints in Method 4 did not alter the optimal results obtained in Method 3. This suggests that, in our specific scenario, subcategory pricing was not a decisive factor in determining optimal prices.

Overall, this report offers valuable insights into the complexity of retail pricing strategies, demonstrating the effectiveness of LP models in navigating this multifaceted challenge. The findings provide a framework for retailers to develop data-driven, market-responsive pricing strategies, balancing profitability with competitive market positioning.

Bibliography

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