RBC Model with Different Intertemporal Elasticities of Substitution

Description of Model

Utility Function	$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma}\right]$
TFP Evolution	$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1}$
Capital Evolution	$I_t = K_{t+1} - (1-\delta)K_t$
Production Function	$Y_t = A_t K_t^{\alpha}$
Market Clearing	$Y_t = C_t + I_t$

β	Household's subjective discount factor
γ	Household Preference Parameter
α	Cobb-Douglas production function parameter
δ	Capital depreciation rate
ρ	Autocorrelation of log TFP

Our Model VS. Prescott's RBC Model

- **Similarity**: Both models look at the effect of changing ρ (the autoregressive coefficient on log TFP) on the simulated impulse responses of model variables to a TFP shock
- **Key differences**: No labor supply and incorporation of one additional factor of household preference parameter

Our Model

$$\frac{1}{C_t^{\gamma}} = \beta E_t\left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} + 1 - \delta}{C_{t+1}^{\gamma}}\right]$$

$$log A_{t+1} = \rho \log A_t + \epsilon_{t+1}$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$Y_t = A_t K_t^{\alpha}$$

$$Y_t = C_t + I_t$$

Prescott's RBC Model

$$\frac{1}{C_t} = \beta E_t\left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}}\right]$$

$$\frac{\varphi}{1-L_t} = \frac{(1-\alpha)A_t K_t^{\alpha} L_t^{-\alpha}}{C_t}$$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$K_{t+1} = I_t + (1-\delta)K_t$$

$$Y_t = C_t + I_t$$

$$\log A_{t+1} = \rho \log A_t + \epsilon_{t+1}$$

Utility Function of Prescott	Utility Function of Our Model
$U0 = E0 \sum_{t=0}^{\infty} \beta^t (\log(C_t) + \varphi \log(1-L_t))$	$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma}\right]$

Utility Function

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} \right]$$

- γ(gamma): how willing the household is to substitute consumption
- 1/γ: household's intertemporal elasticity of substitution
- As γ increases, the household is less willing to substitute consumption today for consumption tomorrow

Solving Utility Function:

- In period 0, the household solves:

$$\max_{K1} \; E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t)^{1-\gamma} - 1}{1-\gamma}$$

$$s.t. \; C_t = A_t K_t^{\alpha} + (1-\delta)K_t - K_{t+1}$$

- The problem can be written as a choice of K1 only:

$$\max_{K1} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(A_t K_t^{\alpha} + (1-\delta)K_t - K_{t+1})^{1-\gamma} - 1}{1-\gamma}\right]$$

Solving Utility Function:

- Generalize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(A_t K_t^\alpha + (1-\delta)K_t - K_{t+1})^{1-\gamma} - 1}{1-\gamma}\right]$$

$$= \beta^0 \left[\frac{(A_0 K_0^\alpha + (1-\delta)K_0 - K_1)^{1-\gamma} - 1}{1-\gamma}\right]$$

$$+ \beta^1 E_0 \left[\frac{(A_1 K_1^\alpha + (1-\delta)K_1 - K_2)^{1-\gamma} - 1}{1-\gamma}\right]$$

$$+[terms\ independent\ of\ K_1]$$

- Derivative:

对K1求导

Euler equation.

$$\frac{\partial}{\partial K_1} U_0 = \frac{1}{1-\gamma} \left[(1-\gamma)(A_0 K_0^\alpha + (1-\delta)K_0 - K_1)^{-\gamma}(0 + 0 - \underline{1}) - 0\right]$$

$$+\beta E_0 \left[\frac{1}{1-\gamma}\left[(1-\gamma)(A_1 K_1^\alpha + (1-\delta)K_1 - K_2)^{-\gamma}(\alpha A_1 K_1^{\alpha-1} + (1-\delta) - 0) - 0\right]\right]$$

Solving Utility Function:

- The first-order condition for the optimal choice of K1 is

$$\frac{1}{(A_0K_0^{\alpha}+(1-\delta)K_0-K_1)^{\gamma}}=\beta E_0\,[\frac{\alpha A_1K_1^{\alpha-1}+1-\delta}{(A_1K_1^{\alpha}+(1-\delta)K_1-K_2)^{\gamma}}]$$

Euler Equation

$$\boxed{\frac{1}{C_t^{\gamma}}=\beta E_0\,[\frac{\alpha A_{t+1}K_{t+1}^{\alpha-1}+1-\delta}{C_{t+1}^{\gamma}}]}$$

Steady state values of At , Kt , Ct , Yt and It

```python
# Initialize the model into a variable named 'rbc_model'.
rbc_model = ls.model(equations = equilibrium_equations,
                     n_states=2,
                     n_exo_states=1,
                     var_names=var_names,
                     shock_names=shock_names,
                     parameters=parameters)
```

```python
# Compute the steady state numerically using .compute_ss() method of nk_model
parameters['gamma'] = 1
guess = [1, 1, 1, 1, 1]
rbc_model.compute_ss(guess)
```

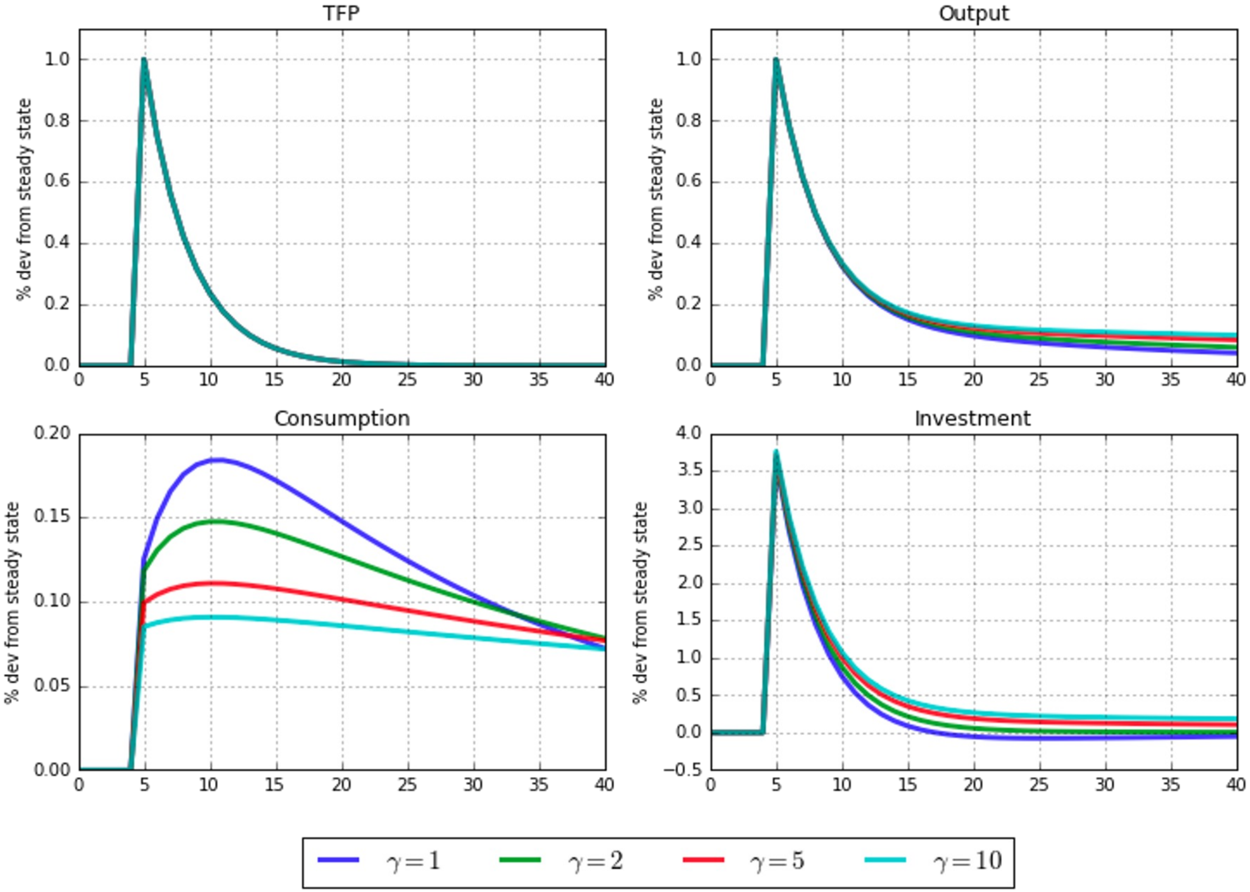
Steady State: Gamma=1

- Compute the steady state numerically using .compute_ss() method of nk_model by setting guess = [1, 1, 1, 1, 1]

a (TFP)	1.000000
k (capital)	34.398226
c (consumption)	2.589794
y (output)	3.449750
i (investment)	0.859956

41 period impulse response of the model's variables to a 0.01 unit shock to TFP in period 5 for gamma=1,2,5,10

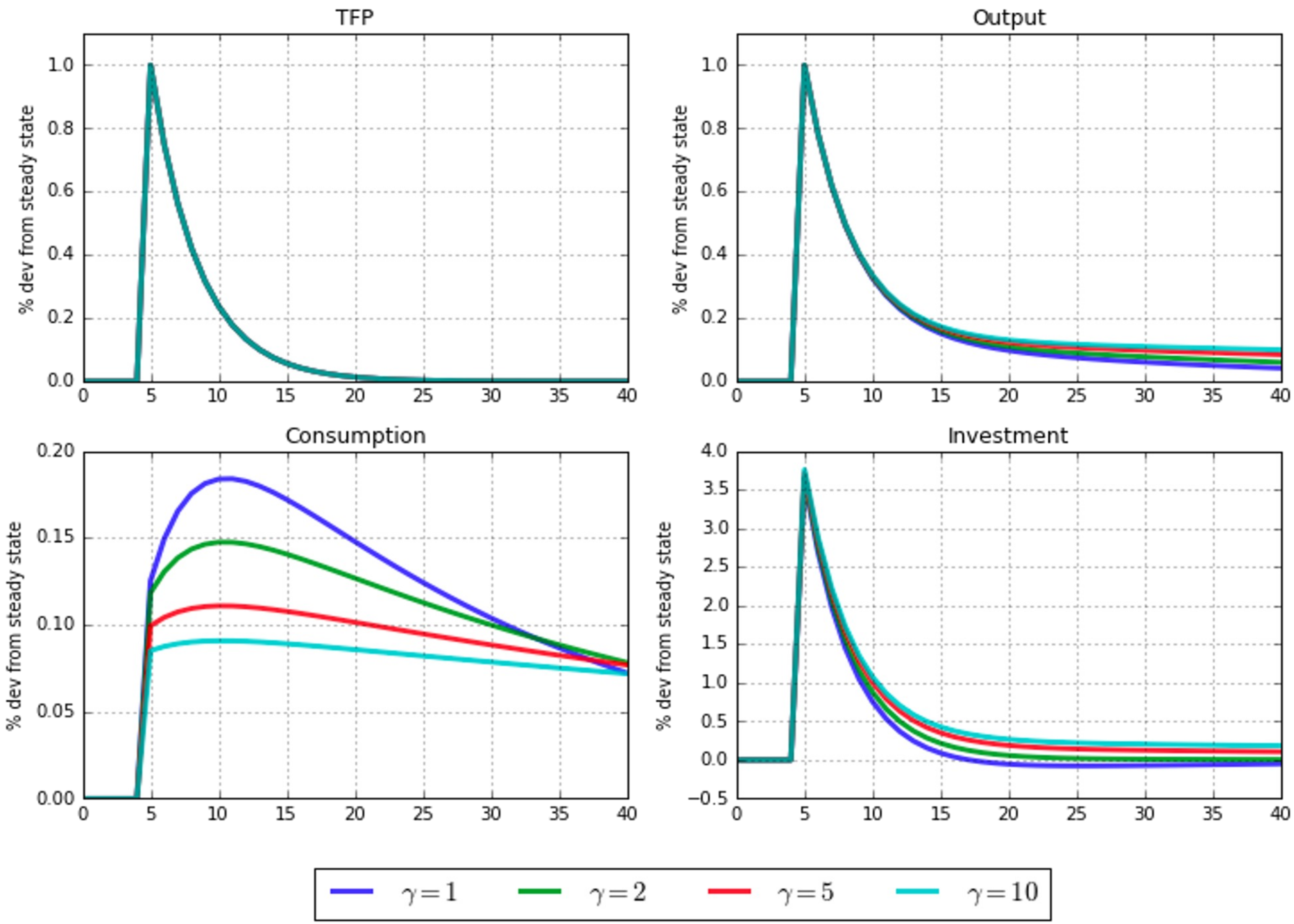
	e_a	a	k	c	y	i
0	0.00	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.00	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.00	0.000000	0.000000	0.000000	0.000000	0.000000
3	0.00	0.000000	0.000000	0.000000	0.000000	0.000000
4	0.00	0.000000	0.000000	0.000000	0.000000	0.000000
5	0.01	0.010000	0.000000	0.000851	0.010000	0.037553
6	0.00	0.007500	0.000939	0.000875	0.007829	0.028770
7	0.00	0.005625	0.001635	0.000891	0.006197	0.022177
8	0.00	0.004219	0.002148	0.000901	0.004971	0.017228
9	0.00	0.003164	0.002525	0.000906	0.004048	0.013510



- At period 5 when TFP starts, C,Y, I will increase for a certain amount,and subsequently K will start increase at the beginning of period 6
- With higher gamma, consumption will become smoother, in other words, if gamma is smaller you try to spend much aggressive and decrease at a higher rate subsequently

Impulse response of the model's variables for gamma=1,2,5,10

- People like to smooth their consumption over time as much as they can.
- When households have higher-than-average income, they save most for the future and so investment rises more than consumption when incomes are high.



401 period stochastic simulation-Comparison between different elasticity

γ=1

y	1.068278
c	0.603292
i	3.202199

γ=20

y	1.037653
c	0.277310
i	3.651613

- TFP shock correlation:

  Output- most correlated

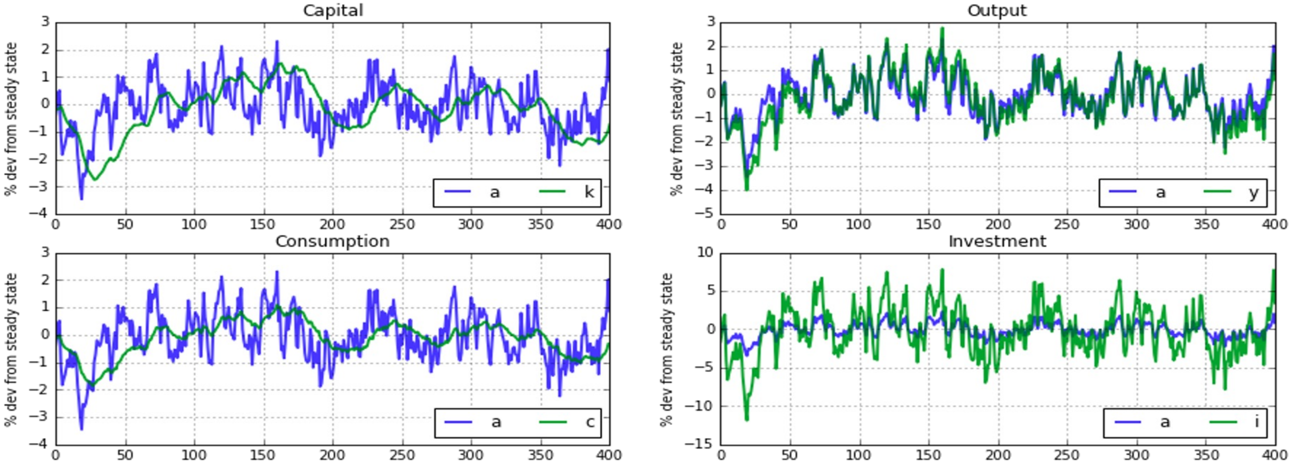
  Consumption & capital: slight similarity

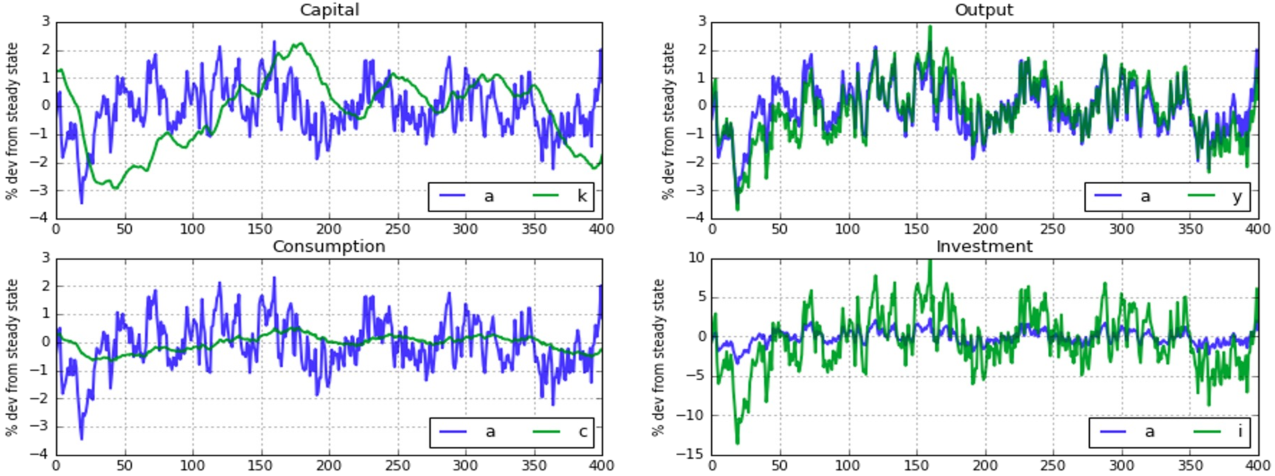
- Gamma increases:

  Output and consumption: less volatile

  Investment: more volatile

  Limitation: significance of the change





Conclusion

Results Interpretation

- When gamma increases, it will cause the household‘s willingness to substitute consumption across time less. Ultimately, consumption and output become less volatile, and investment more volatile

Thank you