COUNTING UNIPOTENT REPRESENTATIONS OF REAL REUDCTIVE GROUPS

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1. Counting unipotent representations

In this section, let $G_{\mathbb{C}}$ be a connected complex reductive group and $\mathfrak{g}$ is its Lie algebra. Fix a antiholomorphic involution σ on $G_{\mathbb{C}}$ and a corresponding Cartan involution θ of $G_{\mathbb{C}}$. Let G be a finite central extension of a open subgroup os $G_{\mathbb{C}}^{\sigma}$ and

$$\mathrm{pr}\colon G \to G_{\mathbb{C}}^{\sigma}$$

be the canonical projection. Let $K = \mathrm{pr}^{-1}(G_{\mathbb{C}}^{\sigma})$.

Let ${}^{a}\mathfrak{h}$ be the abstract Cartan subalgebra of $\mathfrak{g}$ and ${}^{a}X$ be the lattice of abstarct weight spaces. Let ${}^{a}R \subseteq {}^{a}X$, ${}^{a}R^{+}$ and ${}^{a}Q$ be the abstract root system, the set of positive roots and the root lattice. Let

$$\mathsf{C} = \left\{ \mu \in {}^{a}\mathfrak{h}^{*} \;\middle|\; \begin{array}{l} \text{either } \langle \mathrm{Re}(\mu), \check{\alpha}\rangle > 0 \text{ or} \\ \langle \mathrm{Re}(\mu), \check{\alpha}\rangle = 0 \text{ and } \sqrt{-1}\,\langle \mathrm{Im}(\mu), \check{\alpha}\rangle > 0 \end{array} \right\}$$

and $\overline{\mathsf{C}}$ be the closure of C in ${}^{a}\mathfrak{h}$.

1.1. Coherent family.

For each finite dimensional $\mathfrak{g}$-module or $G_{\mathbb{C}}$-module F, let F^{*} be its contragredient representation and let $\Delta(F) \subseteq {}^{a}X$ denote the multi-set of weights in F.

Let $\Pi_{\Lambda_0}(G_{\mathbb{C}})$ be the set of irreducible finite dimensional representations of $G_{\mathbb{C}}$ with extermal weight in Λ_0 and $\mathcal{G}_{\Lambda}(G_{\mathbb{C}})$ be the subgroup generated by $\Pi_{\Lambda_0}(G_{\mathbb{C}})$. Let

$${}^{a}P := \{\, \mu \in {}^{a}X \mid \mu \text{ is a } {}^{a}\mathfrak{h}\text{-weight of an } F \in \Pi_{\mathrm{fin}}(G_{\mathbb{C}}) \,\}.$$

Via the highest weight theory, every $W(G_{\mathbb{C}})$-orbit $W \cdot \mu$ in ${}^{a}P$ corresponds with the irreducible finite dimensional representation $F \in \Pi_{\mathrm{fin}}(G_{\mathbb{C}})$ with extermal weight μ.

Now the Grothendieck group $\mathcal{G}(G_{\mathbb{C}})$ of finite dimensional representation of $G_{\mathbb{C}}$ is identified with $\mathbb{Z}[{}^{a}P/W]$. In fact $\mathcal{G}(G_{\mathbb{C}})$ is a $\mathbb{Z}$-algebra under the tensor product and equiped with the involution $F \mapsto F^{*}$.

Fix a W-invariant sub-lattice $\Lambda_0 \subset {}^{a}X$ containing ${}^{a}Q$.

Take a lattice $\Lambda = \lambda + \Lambda_0 \in \mathfrak{h}^{*}/\Lambda$ with $\lambda \in \overline{\mathsf{C}}$. Let

$$R(\lambda) := \{\, \alpha \in {}^{a}R \mid \langle \lambda, \check{\alpha}\rangle \in \mathbb{Z} \,\} \quad \text{and} \quad W(\lambda) := \langle s_{\alpha} | \alpha \in R(\lambda)\rangle .$$

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Definition 1.1. *Suppose $\mathcal{M}$ is an abelian group with $\mathcal{G}_\Lambda(G_\mathbb{C})$-action*

$$\mathcal{G}_\Lambda(G_\mathbb{C}) \times \mathcal{M} \ni (F, m) \mapsto F \otimes m.$$

In addition, we fix a subgroup $\mathcal{M}_\mu$ of $\mathcal{M}$ for each for each $\mu \in \Lambda$.

A function $f\colon \Lambda \to \mathcal{M}$ is called a coherent family based on Λ if it satisfies $f(\mu) \in \mathcal{M}_\mu$ and

$$F \otimes f(\mu) = \sum_{\nu \in \Delta(F)} f(\mu + \nu) \qquad \forall \mu \in \Lambda, F \in \Pi_{\Lambda_0}(G_\mathbb{C}).$$

Let $\mathrm{Coh}_\Lambda(\mathcal{M})$ be the abelian group of all coherent families based on Λ and value in $\mathcal{M}$.

In this paper, we will consider the following cases.

Suppose $\mathcal{M} = \mathbb{Q}$ and $F \otimes m = \dim(F) \cdot m$ for $F \in \Pi_{\Lambda_0}(G_\mathbb{C})$ and $m \in \mathcal{M}$. We let $\mathcal{M}_\mu = \mathcal{M}$ for every $\mu \in \Lambda$. When $\Lambda = \Lambda_0$, the set of $W(G_\mathbb{C})$-harmonic polynomials on ${}^a\mathfrak{h}^*$ is natrually identified with $\mathrm{Coh}_\Lambda(\mathcal{M})$ via restriction (Vogan's result)

Let $\mathcal{G}(\mathfrak{g}, K)$ be the Grothendieck group of finite length $(\mathfrak{g}, K)$-modules and $\mathcal{G}_\mu(\mathfrak{g}, K)$ be the subgroup of $\mathcal{G}(\mathfrak{g}, K)$ generated by the set of irreducible $(\mathfrak{g}, K)$-modules with infinitesimal character μ.

Then $\mathrm{Coh}_\Lambda(\mathcal{G}(\mathfrak{g}, K))$ is the group of coherent families of Harish-Chandra modules.

Fixing a Borel subalgebra $\mathfrak{b} \subset \mathfrak{g}$, let $\mathcal{G}(\mathfrak{g}, \mathfrak{b})$ be the Gorthendieck group of the category $\mathscr{O}^\mathfrak{b}$. The space $\mathrm{Coh}_\Lambda(\mathcal{G}(\mathfrak{g}, \mathfrak{b}))$ is defined similarly.

Note that the lattice Λ is stable under the $W(\lambda)$ action. We can define $W(\lambda)$ action on $\mathrm{Coh}_\Lambda(\mathcal{M})$ by

$$w \cdot f(\mu) = f(w^{-1}\mu) \qquad \forall \mu \in \Lambda, w \in W(\lambda).$$

Lemma 1.2 (Translation principal)**.** *Suppose $\mathcal{M} = \mathcal{G}(\mathfrak{g}, K)$ or $\mathcal{G}(\mathfrak{g}, \mathfrak{b})$. When $\mu \in \Lambda \cap \overline{\mathsf{C}}$, the evaluation map*

$$\mathrm{ev}_\mu \mathrm{Coh}_\Lambda(\mathcal{M}) \to \mathcal{M}_\mu \qquad f \mapsto f(\mu)$$

is surjective. Moreover

2. Parameterize of Unipotent representations

We fix an abstract complex Cartan subgroup $\mathbf{H}_a$ and $\mathfrak{h}_a$ in $\mathbf{G}$ and a set of simple roots Π_a. Let $\mathcal{P}(\mathbf{G})$ be the set of all Langlands parameters of G-modules with character ρ (i.e. the infinitesimal character of the trivial representation). For $\gamma \in \mathcal{P}(\mathbf{G})$, let $\mathcal{L}(\gamma)$, $\mathcal{S}(\gamma)$ and Φ_γ be the corresponding Langlands quotient, standard module and coherent family such that $\Phi_\gamma(\rho) = \mathcal{L}(\gamma)$. Let $\mathcal{M}(\mathbf{G})$ be the span of $\mathcal{L}(\gamma)$. Let $\{\mathbb{B}\}$ be the set of all blocks. Then $\mathcal{P}(\mathbf{G}) = \bigsqcup_\mathcal{B} \mathcal{B}$. The Weyl group $W = W(G)$ acts on $\mathcal{M}(\mathbf{G})$ by coherent continuation. Let $\mathcal{M}_\mathcal{B}$ be the submodule of $\mathcal{M}(\mathbf{G})$ spand by $\gamma \in \mathcal{B}$, then

$$\mathcal{M}(\mathbf{G}) = \bigoplus_\mathcal{B} \mathcal{M}_\mathcal{B}$$

Let $\tau(\gamma) \subset \Pi_a$ be the τ-invariant of γ.

Let $\check{\mathcal{O}}$ be even orbit. $\lambda = \frac{1}{2}\check{h}$. Define

$$S(\lambda) = \{\, \alpha \in \Pi_a \mid \langle \alpha, \lambda \rangle = 0 \,\}.$$

Let $\mathcal{P}_\lambda(\mathbf{G})$ be the set of all Langlands parameters with infinitesimal character λ. Let $T_{\lambda,\rho}$ be the translation functor. Let

$$\mathcal{B}(S) = \{\, \gamma \in \mathcal{B} \mid S \cap \tau(\gamma) = \varnothing \,\}$$

and

$$\mathcal{P}(\mathbf{G}, S) = \bigsqcup_\mathcal{B} \mathcal{B}(S)$$

Then

$$\begin{aligned} \mathcal{P}(\mathbf{G}, S) &\longrightarrow \mathcal{P}_\lambda(\mathbf{G}) \\ \gamma &\longmapsto T_{\lambda,\rho}(\gamma) \end{aligned}$$

Let $\mathcal{O}$ be a complex nilpotent orbit in $\mathfrak{g}$. Let

$$\mathcal{B}(S, \mathcal{O}) = \{ \gamma \in \mathcal{B}(S) \mid \mathrm{AV}_{\mathbb{C}}(\mathcal{L}(\gamma)) \subset \overline{\mathcal{O}} \}$$

Let

$$\begin{aligned} m_S(\sigma) &= [\sigma : \mathrm{Ind}_{W(S)}^W \mathbf{1}] \\ m_{\mathcal{B}}(\sigma) &= [\sigma : \mathcal{M}_{\mathcal{B}}] \end{aligned}$$

Barbasch [10, Theorem 9.1] established the following theorem.

Theorem 2.1.

$$|\mathcal{B}(S, \mathcal{O})| = \sum_\sigma m_{\mathcal{B}}(\sigma) m_S(\sigma)$$

Here $\sigma \times \sigma$ running over the $W \times W$ appears in the double cell $\mathcal{C}(\mathcal{O})$.

Proof. We need to take the graded module of $\mathcal{M}(\mathbf{G})$ with respect to the $\overset{LR}{\leqslant}$. By abuse of notation, we identify the basis $\mathcal{P}(\mathbf{G})$ with its image in the graded module. Note that $S \cap \tau(\lambda) = \varnothing$ if and only if $W(S)$ acts on γ trivially by [70, Lemma 14.7]. On the other hand, by [70, Theorem 14.10, and page 58], $\mathrm{AV}_{\mathbb{C}}(\mathcal{L}(\gamma)) \subset \overline{\mathcal{O}}$ only if γ generate a W-module in the double cell of $\mathcal{O}$. $\square$

Now assume $S = S(\lambda)$. By [12, Cor 5.30 b) and c)], $[\sigma : \mathrm{Ind}_{W(S)}^W \mathbf{1}] = [\mathbf{1}|_{W(S)} : \sigma] \leqslant 1$.

References

[1] J. Adams, *Discrete spectrum of the reductive dual pair $(O(p,q), Sp(2m))$*, Invent. Math. **74** (1983), no. 3, 449–475.

[2] J. Adams, D. Barbasch, and D. A. Vogan, *The Langlands classification and irreducible characters for real reductive groups*, Progress in Math., vol. 104, Birkhauser, 1991.

[3] Jeffrey Adams and Fokko du Cloux, *Algorithms for representation theory of real reductive groups*, Journal of the Institute of Mathematics of Jussieu **8** (2009), no. 2, 209-259, DOI 10.1017/S1474748008000352.

[4] J. Arthur, *On some problems suggested by the trace formula*, Lie group representations, II (College Park, Md.), Lecture Notes in Math. 1041 (1984), 1–49.

[5] ———, *Unipotent automorphic representations: conjectures*, Orbites unipotentes et représentations, II, Astérisque **171-172** (1989), 13–71.

[6] L. Auslander and B. Kostant, *Polarizations and unitary representations of solvable Lie groups*, Invent. Math. **14** (1971), 255–354.

[7] D. Barbasch, *Unipotent representations for real reductive groups*, Proceedings of ICM (1990), Kyoto (2000), 769–777.

[8] Dan Barbasch and David Vogan, *Weyl Group Representations and Nilpotent Orbits*, Representation Theory of Reductive Groups: Proceedings of the University of Utah Conference 1982, 1983, pp. 21–33.

[9] D. Barbasch, *Orbital integrals of nilpotent orbits*, The mathematical legacy of Harish-Chandra, Proc. Sympos. Pure Math. **68** (2000), 97–110.

[10] ———, *The unitary spherical spectrum for split classical groups*, J. Inst. Math. Jussieu **9** (2010), 265–356.

[11] ———, *Unipotent representations and the dual pair correspondence*, J. Cogdell et al. (eds.), Representation Theory, Number Theory, and Invariant Theory, In Honor of Roger Howe. Progress in Math. **323** (2017), 47–85.

[12] D. Barbasch and D. A. Vogan, *Unipotent representations of complex semisimple groups*, Annals of Math. **121** (1985), no. 1, 41–110.

[13] R. Brylinski, *Dixmier algebras for classical complex nilpotent orbits via Kraft-Procesi models. I*, The orbit method in geometry and physics (Marseille, 2000). Progress in Math. **213** (2003), 49–67.

[14] Roger W. Carter, *Finite groups of Lie type*, Wiley Classics Library, John Wiley & Sons, Ltd., Chichester, 1993.

[15] W. Casselman, *Canonical extensions of Harish-Chandra modules to representations of G*, Canad. J. Math. **41** (1989), 385–438.

[16] F. Du Cloux, *Sur les représentations différentiables des groupes de Lie algébriques*, Ann. Sci. École Norm. Sup. **24** (1991), no. 3, 257–318.

[17] D. H. Collingwood and W. M. McGovern, *Nilpotent orbits in semisimple Lie algebra: an introduction*, Van Nostrand Reinhold Co., 1993.

[18] A. Daszkiewicz, W. Kraśkiewicz, and T. Przebinda, *Nilpotent orbits and complex dual pairs*, J. Algebra **190** (1997), no. 2, 518 - 539.

[19] ———, *Dual pairs and Kostant-Sekiguchi correspondence. II. Classification of nilpotent elements*, Central European J. Math. **3** (2005), 430–474.

[20] J. Dixmier and P. Malliavin, *Factorisations de fonctions et de vecteurs indéfiniment différentiables*, Bull. Sci. Math. (2) **102** (1978), 307–330.

[21] M. Duflo, *Théorie de Mackey pour les groupes de Lie algébriques*, Acta Math. **149** (1982), no. 3-4, 153–213.

[22] R. Gomez and C.-B. Zhu, *Local theta lifting of generalized Whittaker models associated to nilpotent orbits*, Geom. Funct. Anal. **24** (2014), no. 3, 796–853.

[23] A. Grothendieck and J. Dieudonné, *Éléments de géométrie algbrique IV: Étude locale des schémas et des morphismes de schémas. II*, Inst. Hautes Études Sci. Publ. Math. **24** (1965).

[24] ———, *Éléments de géométrie algbrique IV: Étude locale des schémas et des morphismes de schémas. III*, Inst. Hautes Études Sci. Publ. Math. **28** (1966).

[25] M. Harris, J.-S. Li, and B. Sun, *Theta correspondences for close unitary groups*, Arithmetic Geometry and Automorphic Forms, Adv. Lect. Math. (ALM) **19** (2011), 265–307.

[26] R. Hartshorne, *Algebraic Geometry*, Graduate Texts in Mathematics, 52. New York-Heidelberg-Berlin: Springer-Verlag, 1983.

[27] H. He, *Unipotent representations and quantum induction*, arXiv:math/0210372 (2002).

[28] J.-S. Huang and J.-S. Li, *Unipotent representations attached to spherical nilpotent orbits*, Amer. J. Math. **121** (1999), no. 3, 497–517.

[29] J.-S. Huang and C.-B. Zhu, *On certain small representations of indefinite orthogonal groups*, Represent. Theory **1** (1997), 190–206.

[30] R. Howe, *θ-series and invariant theory*, Automorphic Forms, Representations and L-functions, Proc. Sympos. Pure Math, vol. 33, 1979, pp. 275-285.

[31] ———, *On a notion of rank for unitary representations of the classical groups*, Harmonic analysis and group representations, Liguori, Naples (1982), 223-331.

[32] ———, *Transcending classical invariant theory*, J. Amer. Math. Soc. **2** (1989), 535–552.

[33] ———, *Perspectives on invariant theory: Schur duality, multiplicity-free actions and beyond*, Piatetski-Shapiro, I. et al. (eds.), The Schur lectures (1992). Ramat-Gan: Bar-Ilan University, Isr. Math. Conf. Proc. 8, (1995), 1-182.

[34] D. Jiang, B. Liu, and G. Savin, *Raising nilpotent orbits in wave-front sets*, Represent. Theory **20** (2016), 419–450.

[35] A. A. Kirillov, *Unitary representations of nilpotent Lie groups*, Uspehi Mat. Nauk **17** (1962), 57–110.

[36] B. Kostant, *Quantization and unitary representations*, Lectures in Modern Analysis and Applications III, Lecture Notes in Math. **170** (1970), 87–208.

[37] H. Kraft and C. Procesi, *On the geometry of conjugacy classes in classical groups*, Comment. Math. Helv. **57** (1982), 539–602.

[38] S. S. Kudla and S. Rallis, *Degenerate principal series and invariant distributions*, Israel J. Math. **69** (1990), 25–45.

[39] S. S. Kudla, *Some extensions of the Siegel-Weil formula*, In: Gan W., Kudla S., Tschinkel Y. (eds) Eisenstein Series and Applications. Progress in Mathematics, vol 258. Birkhäuser Boston (2008), 205–237.

[40] S. T. Lee and C.-B. Zhu, *Degenerate principal series and local theta correspondence II*, Israel J. Math. **100** (1997), 29–59.

[41] ———, *Degenerate principal series of metaplectic groups and Howe correspondence*, D. Prasad at al. (eds.), Automorphic Representations and L-Functions, Tata Institute of Fundamental Research, India, (2013), 379–408.

[42] J.-S. Li, *Singular unitary representations of classical groups*, Invent. Math. **97** (1989), no. 2, 237–255.

[43] Q. Liu, *Algebraic Geometry and Arithmetic Curves*, Oxford University Press, 2006.

[44] H. Y. Loke and J. Ma, *Invariants and K-spectrums of local theta lifts*, Compositio Math. **151** (2015), 179–206.
[45] G. Lusztig and N. Spaltenstein, *Induced unipotent classes*, j. London Math. Soc. **19** (1979), 41–52.
[46] G. Lusztig, *Intersection cohomology complexes on a reductive group*, Invent. Math. **75** (1984), no. 2, 205–272, DOI 10.1007/BF01388564. MR732546
[47] G. W. Mackey, *Unitary representations of group extentions*, Acta Math. **99** (1958), 265–311.
[48] W. M McGovern, *Cells of Harish-Chandra modules for real classical groups*, Amer. J. of Math. **120** (1998), 211–228.
[49] C. Mœglin, *Front d'onde des représentations des groupes classiques p-adiques*, Amer. J. Math. **118** (1996), 1313–1346.
[50] ———, *Paquets d'Arthur Spéciaux Unipotents aux Places Archimédiennes et Correspondance de Howe*, J. Cogdell et al. (eds.), Representation Theory, Number Theory, and Invariant Theory, In Honor of Roger Howe. Progress in Math. **323** (2017), 469–502.
[51] C. Mœglin, M.-F. Vignéras, and J.-L. Waldspurger, *Correspondances de Howe sur un corps p-adique*, Lecture Notes in Mathematics, vol. 1291, Springer, 1987.
[52] K. Nishiyama, H. Ochiai, K. Taniguchi, H. Yamashita, and S. Kato, *Nilpotent orbits, associated cycles and Whittaker models for highest weight representations*, Astérisque **273** (2001), 1–163.
[53] K. Nishiyama, H. Ochiai, and C.-B. Zhu, *Theta lifting of nilpotent orbits for symmetric pairs*, Trans. Amer. Math. Soc. **358** (2006), 2713–2734.
[54] K. Nishiyama and C.-B. Zhu, *Theta lifting of unitary lowest weight modules and their associated cycles*, Duke Math. J. **125** (2004), no. 03, 415–465.
[55] T. Ohta, *The closures of nilpotent orbits in the classical symmetric pairs and their singularities*, Tohoku Math. J. **43** (1991), no. 2, 161–211.
[56] ———, *Induction of nilpotent orbits for real reductive groups and associated varieties of standard representations*, Hiroshima Math. J. **29** (1999), no. 2, 347–360.
[57] ———, *Nilpotent orbits of $\mathbb{Z}_4$-graded Lie algebra and geometry of moment maps associated to the dual pair* $(\mathrm{U}(p,q),\mathrm{U}(r,s))$, Publ. RIMS **41** (2005), no. 3, 723–756.
[58] A. Paul and P. Trapa, *Some small unipotent representations of indefinite orthogonal groups and the theta correspondence*, University of Aarhus Publ. Series **48** (2007), 103–125.
[59] V. L. Popov and E. B. Vinberg, *Invariant Theory*, Algebraic Geometry IV: Linear Algebraic Groups, Invariant Theory, Encyclopedia of Mathematical Sciences, vol. 55, Springer, 1994.
[60] T. Przebinda, *The duality correspondence of infinitesimal characters*, Colloq. Math. **70** (1996), 93–102.
[61] ———, *Characters, dual pairs, and unitary representations*, Duke Math. J. **69** (1993), no. 3, 547–592.
[62] S. Rallis, *On the Howe duality conjecture*, Compositio Math. **51** (1984), 333–399.
[63] S. Sahi, *Explicit Hilbert spaces for certain unipotent representations*, Invent. Math. **110** (1992), no. 2, 409–418.
[64] J. Sekiguchi, *Remarks on real nilpotent orbits of a symmetric pair*, J. Math. Soc. Japan **39** (1987), no. 1, 127–138.
[65] W. Schmid and K. Vilonen, *Characteristic cycles and wave front cycles of representations of reductive Lie groups*, Annals of Math. **151** (2000), no. 3, 1071–1118.
[66] E. Sommers, *Lusztig's canonical quotient and generalized duality*, J. Algebra **243** (2001), no. 2, 790–812.
[67] T. A. Springer and R. Steinberg, *Seminar on algebraic groups and related finite groups; Conjugate classes*, Lecture Notes in Math., vol. 131, Springer, 1970.
[68] B. Sun and C.-B. Zhu, *A general form of Gelfand-Kazhdan criterion*, Manuscripta Math. **136** (2011), 185–197.
[69] P. Trapa, *Special unipotent representations and the Howe correspondence*, University of Aarhus Publication Series **47** (2004), 210–230.
[70] D. A. Vogan, *Irreducible characters of semisimple Lie groups. IV. Character-multiplicity duality*, Duke Math. J. **49** (1982), no. 4, 943–1073. MR683010
[71] ———, *Unitary representations of reductive Lie groups*, Ann. of Math. Stud., vol. 118, Princeton University Press, 1987.
[72] ———, *Associated varieties and unipotent representations*, Harmonic analysis on reductive groups, Proc. Conf., Brunswick/ME (USA) 1989, Prog. Math. **101** (1991), 315–388.
[73] ———, *The method of coadjoint orbits for real reductive groups*, Representation theory of Lie groups (Park City, UT, 1998). IAS/Park City Math. Ser. **8** (2000), 179–238.

[74] ———, *Unitary representations of reductive Lie groups*, Mathematics towards the Third Millennium (Rome, 1999). Accademia Nazionale dei Lincei, (2000), 147–167.
[75] N. R. Wallach, *Real reductive groups I*, Academic Press Inc., 1988.
[76] ———, *Real reductive groups II*, Academic Press Inc., 1992.
[77] H. Weyl, *The classical groups: their invariants and representations*, Princeton University Press, 1947.
[78] S. Yamana, *Degenerate principal series representations for quaternionic unitary groups*, Israel J. Math. **185** (2011), 77–124.

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