The Application of NOMA on High-Speed Railway With Partial CSI

Jingyi Fan*, Jiayi Zhang*, Shuaifei Chen*, Jiakang Zheng*, and Bo Ai†
*School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China
†State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China

***Abstract*—High-speed railway (HSR) wireless communications are required to support high data rate with seamless connectivity. In this paper, we investigate the outage performance of a downlink single-cell non-orthogonal multiple access (NOMA) based wireless network in HSR scenarios, where it is challenging to derive the perfect channel state information (CSI) and the distribution of the users are quite different from the traditional cellular scenarios. More specifically, the performance of NOMA over composite large-scale and Rician fading channel is studied with two kinds of partial CSI, e.g., imperfect small-scale CSI and no small-scale CSI. We derive the exact closed-form expression of the outage probability based on partial CSI, respectively, by using the Gauss-Chebyshev quadrature method. Finally, simulation results evidence the validity of the derived results and show that the average outage probability of NOMA systems outperforms conventional orthogonal multiple access systems.**

I. Introduction

Non-orthogonal multiple access (NOMA) has attracted significant attention because of its promising application in the fifth generation (5G) wireless communication networks, and it is recognized as an effective approach to improve the spectral efficiency [1]–[4]. In [5], the outage performance for downlink NOMA was analysed with statistical channel state information (CSI). For a downlink single-cell NOMA system network, most research works about NOMA systems have assumed that the BS knows the perfect knowledge of CSI, and assumed that the users are uniformly distributed in a disc modeled as homogeneous possion point process [6]. However, in practice, it is challenging to obtain the perfect CSI due to high mobility or imperfect channel estimation scheme.

On the other hand, with the rapid development of high-speed railway (HSR) around the world, high quality and data rate wireless communications are required for onboard realtime HD video surveillance and passenger services, railway mobile ticketing, and the Internet of Things for railways [7]. Because of the convenience and comfort, people prefer to taking HSR for travelling and business trip, railway transportation communication becomes a more and more significant topic [8]. As a key technology to significantly improve the data rate, NOMA is a feasible approach to employ in HSR wireless communications. However, as far as we know, there are few work on employing NOMA scheme in HSR scenarios, where the perfect CSI is very difficult to derive.

This work was supported in part by the Beijing Natural Haidian Joint Fund under Grant L172020, the National Key Research and Development Program under Grant 2016YFE0200900, the National Natural Science Foundation of China under Grant 61601020, Grant 61725101, and Grant U1834210, the Beijing Natural Science Foundation under Grant 4182049 and Grant L171005, the Major projects of Beijing Municipal Science and Technology Commission under Grant Z181100003218010, the Royal Society Newton Advanced Fellowship under Grant NA191006, and State Key Lab of Rail Traffic Control and Safety under Grant RCS2018ZZ007 and Grant RCS2019ZZ007.

To fill this gap, we investigate the impact of partial CSI on the performance of the downlink NOMA network for HSR. First, we focus on the minimum mean square error (MMSE) channel estimation error model for the small-scale fading channel, and derive a closed-form approximation for the outage probability. Second, we assume that the no small-scale CSI of the channels is known. Since the channels are sorted accordingly by distances, order statistics of the distance is applied to yield closed-form expressions for the outage probability. Finally, we present a comparison between the NOMA scheme based on two kinds of partial CSI (imperfect small-scale CSI and no small-scale CSI) with conventional OMA scheme, respectively, and it is shown that the derived results and Monte Carlo simulation results are matched well.

II. System Model

We consider a single-cell downlink NOMA-based network in the HSR scenario, in which the base station (BS) is located at the track side, and M users can be modeled as uniformly distributed on aboard the high-speed train. We assume that all users and the BS are equipped with single antenna, and they communicates directly without a relay. The composite small-scale and large-scale fading channel between the k-th user and the BS is denoted by g_k as [6], [9], [10]

$$g_k = h_k \beta_k^{1/2}, \tag{1}$$

where h_k represents the small-scale fading, and β_k represents the large-scale fading. Motivated by [8], [11]–[13], we utilize the classical Rician channel model. Moreover, we consider the distribution of HSR users, which is quite different from traditional cellular scenarios. The large-scale fading coefficient β_k includes both path loss and shadow fading.

Theoretically, the optimal system performance is achieved with perfect CSI, while it is difficult to obtain the perfect CSI in practical HSR scenarios. Therefore, for NOMA systems, we take two kinds of partial CSI models into account, e.g., imperfect small-scale CSI and no small-scale CSI. Furthermore, the NOMA systems based on the different partial CSI models are described as follows.

978-1-7281-1220-6/19/$31.00 ©2019 IEEE

A. NOMA With Imperfect CSI

First, we assume that the BS estimates the small-scale fading channel by using the minimum mean square error (MMSE) channel estimation error model [14]. Since the large-scale fading factors vary slowly, we suppose that the large-scale fading efficient β_k can be perfectly estimated by the BS. The small-scale channel response h_k is given as $h_k = \hat{h}_k + e$, where e is the channel estimation error with mean zero and variance σ_e^2 , denoted by $e \sim \mathcal{CN}(0, \sigma_e^2)$. Due to using MMSE, the channel estimation $\hat{h}_k$ and the channel estimation error e are uncorrelated. Therefore, the composite fading channel is expressed as

$$g_k = \left(\hat{h}_k + e\right) \beta_k^{1/2} = \hat{h}_k \beta_k^{1/2} + e\beta_k^{1/2} = \hat{g}_k + e\beta_k^{1/2}. \quad (2)$$

Without loss of generality, we suppose that the estimated channel gains are sorted as $|\hat{g}_1|^2 \geq |\hat{g}_2|^2 \geq \ \cdots \geq |\hat{g}_M|^2$ and define the user set as $\{U_1, U_2, \ldots, U_M\}$. On the basis of the NOMA principle, the signal transmitted by the BS to all users can be expressed as

$$x = \sum_{l=1}^{M} \sqrt{\alpha_l P_t} s_l, \quad (3)$$

where P_t is the transmitting power of the BS, s_l is the signal of the l-th user, and α_l is the normalized power allocation factor with $\alpha_1 < \alpha_2 < \cdots < \alpha_M$ and $\sum_{l=1}^{M} \alpha_l = 1$. Therefore the received signal at U_k is given by

$$y_k = \hat{g}_k \sum_{l=1}^{M} \sqrt{\alpha_l P_t} s_l + e\beta_k^{1/2} \sum_{l=1}^{M} \sqrt{\alpha_l P_t} s_l + w_k, \quad (4)$$

where w_k is additive white Gaussian noise (AWGN) at U_k with zero mean and variance σ^2.

According to (4), in order to employ SIC at U_k correctly, U_k needs to first detect the signal s_l sent to the user U_l $(k+1 \leq l \leq M)$ and subtract it from the received signal, then U_k decode its own signal without the interference from the user with worse channel condition. Therefore, the rate for U_k to detect the signal s_l can be formulated as

$$R^{I}_{l\to k} = \log_2 \left(1 + \frac{P_t \alpha_l |\hat{g}_k|^2}{P_t \left(|\hat{g}_k|^2 \sum_{j=1}^{l-1} \alpha_j + \beta_k \sigma_e^2 \right) + \sigma^2} \right). \quad (5)$$

Furthermore, U_k can decode the message of U_l $(k+1 \leq l \leq M)$, the rate of U_k is given by

$$R_k = \log_2 \left(1 + \frac{\alpha_k |\hat{g}_k|^2}{|\hat{g}_k|^2 b_{k-1} + \beta_k \sigma_e^2 + \frac{1}{\rho}} \right), \quad k \geq 2, \quad (6)$$

where $b_{k-1} = \sum_{l=1}^{k-1} \alpha_l$, $\rho = \frac{P_t}{\sigma^2}$ is the transmit SNR. Moreover, we have $R_1 = \log_2 \left(1 + \frac{\alpha_1 |\hat{g}_1|^2}{\beta_1 \sigma_e^2 + \frac{1}{\rho}} \right)$ for the first user.

B. NOMA Without CSI

Furthermore, we have the user set as $\{U_1, U_2, \ldots, U_M\}$, but distinct in the sorting principle. In this part, we sort the user only based on the distance between the BS and the user without considering the channel estimation error, and the distances are ordered as $d_1 \leq d_2 \leq \cdots \leq d_M$. Since the distance determines the large-scale fading and the channel gain, and the distance varies much slower compared to the channel in small-scale fading conditions, the BS is reasonable to obtain the knowledge of the distance of the wireless channels. The channel gain is given as $|g_k|^2 = |h_k|^2 \beta_k$, where β_k is the function of d_k. It is clear to see that the joint channel gains $|g_k|^2$ are not indispensably ordered, i.e, $|g_k|^2$ might be smaller than $|g_j|^2$ when $k > j$. According to the NOMA scheme, the received signal at U_k is formulated as

$$y_k = g_k \sum_{l=1}^{M} \sqrt{\alpha_l P_t} s_l + w_k. \quad (7)$$

Similar to (5), we define $R^{II}_{l\to k}$ as the rate of U_k to detect the message from U_l. If $k < l$, U_k would decode the signal of U_l successfully, and $R^{II}_{l\to k}$ is given by

$$R^{II}_{l\to k} = \log_2 \left(1 + \frac{P_t \alpha_l |\hat{g}_k|^2}{P_t |g_k|^2 \sum_{j=1}^{l-1} \alpha_j + \sigma^2} \right). \quad (8)$$

We assume that $R^{II}_{l\to k} \geq R_l^*$ always holds for all $k < l$, where R_l^* is the targeted rate of user U_l, then the rate of the k-th user U_k is evaluated as

$$R_k = \log_2 \left(1 + \frac{\alpha_k |g_k|^2}{|g_k|^2 b_{k-1} + \frac{1}{\rho}} \right), \quad k \geq 2, \quad (9)$$

and $R_1 = \log_2 \left(1 + \rho \alpha_1 |g_1|^2 \right)$ for the first user.

III. Performance Analysis

The outage probability is an index of the event that the data rate achieved by the instantaneous channel is less than the targeted rate. In this section, we study the average outage probability of the users in HSR scenarios, and derive closed-form expressions for the outage performance based on partial CSI in the following.

A. Outage Probability With Imperfect CSI

The k-th user can detect to other users with worse channel gain than its own. The event that the k-th user successfully decodes the i-th user's message is given by [15, Eq. (30)]

$$\hat{E}_{k,i} = \left\{ |\hat{g}_k|^2 > \frac{\varepsilon_i (\rho \sigma_e^2 \beta_k + 1)}{\rho \left(\alpha_i - \varepsilon_i \sum_{l=1}^{i-1} \alpha_l \right)} \right\}, \quad (10)$$

where $\varepsilon_i = 2^{R_i^*} - 1$, R_i^* denotes the targeted rate of U_i. Note that $\alpha_i > \varepsilon_i \sum_{l=1}^{i-1} \alpha_l$. Accordingly, the outage probability of the k-th user can be expressed as

$$P_k^I = F_{|\hat{g}_k|^2} \left(\vec{\eta} \left(\rho \sigma_e^2 \beta_k + 1 \right) \right), \quad (11)$$

where $\eta_i = \frac{\varepsilon_i}{\rho \left(\alpha_i - \varepsilon_i \sum_{l=1}^{i-1} \alpha_l \right)}$, and $\vec{\eta} = \max_{k+1 \leq i \leq M} \eta_i$.

Theorem 1. *With imperfect small-scale CSI, the outage probability of the k-th user can be evaluated as* (12)*, shown at the* *bottom of this page, where n is a parameter to balance the accuracy and complexity and $x_i = \frac{L_t}{2}\left(\cos\frac{2i-1}{2n}\pi+1\right)$.*

Proof: Please see Appendix A. ∎

By using Gauss-Chebyshev quadrature, the accurate approximation can be achieved with a small value of n, as shown by in Section IV. Moreover, the condition, $\alpha_i > \varepsilon_i \sum_{l=1}^{i-1}\alpha_l$ is required in (12), i.e., with improper α_i and R_i^*, the outage performance is poor.

B. Outage Probability Without CSI

With only the distance information known at the BS and d_k is smaller than d_i, the k-th nearest user should decode the i-th $(k+1 \le i \le M)$ user's signal firstly and then, detect its own signal. The event that the k-th user can successfully decode the i-th user's signal is defined as

$$E_{k,i} = \left\{ |g_k|^2 > \frac{\varepsilon_i}{\rho\left(\alpha_i - \varepsilon_i \sum_{l=1}^{i-1}\alpha_l\right)} \right\}, \tag{13}$$

where the condition, $\alpha_i > \varepsilon_i \sum_{l=1}^{i-1}\alpha_l$, should be satisfied.

Consequently, the outage probability of the k-th user can be expressed as

$$P_k^I = 1 - \Pr\left\{ \bigcap_{i=k+1}^{M} \left(|g_k|^2 > \eta_i\right) \right\} = F_{|g_k|^2}\left(\vec{\eta}\right). \tag{14}$$

where $\eta_i = \frac{\varepsilon_i}{\rho\left(\alpha_i - \varepsilon_i \sum_{l=1}^{i-1}\alpha_l\right)}$ and $\vec{\eta} = \max\limits_{k+1\le i\le M} \eta_i$.

In the following Theorem, an exact closed-form expression for the outage probability in NOMA system is provided, assuming that only the distance information is known and no small-scale CSI at the BS.

Theorem 2. *In NOMA HSR system with no small-scale CSI, the outage probability of the k-th user is given by* (15)*, shown at the bottom of next page, where n is the number of Gauss-Chebyshev quadrature approximation terms, $x_i = \frac{L_t}{2}\left(\cos\frac{2i-1}{2n}\pi+1\right)$.*

Proof: Please see Appendix B. ∎

Note that the final results of (12) and (15) contain integrals if the approximate forms are not derived, which will significantly increase the computational complexity. By using Gauss-Chebychev quadrature, the outage probability only depends on the Marcum Q-function and the finite-sum of the Gauss-Chebychev quadrature terms [15]. Furthermore, we can intuitively observe the variation of the outage performance when the system parameter changed.

Furthermore, we give the diversity order of the users in NOMA system with no small-scale in the following Proposition.

Proposition 1. *The diversity order gain of the k-th user in NOMA HSR systems based on distance can be expressed as*

$$d = -\lim_{\rho\to\infty} \frac{\log P_k^{II}}{\log\rho} = 1. \tag{16}$$

Proof: When $\rho\to\infty$, then $\vec{\eta}\to 0$, the value of Marcum Q function is given by

$$Q_1(\sqrt{2K}, 0) = 1. \tag{17}$$

Substituting (17) into (15), the proof is completed. ∎

Note that all users in the NOMA scheme have the diversity gain equal to one, which is consistent with the results in [15].

C. Outage Probability of OMA HSR System

For comparison, we investigate the outage performance for OMA based HSR systems with partial CSI. The rate of U_k with imperfect small-scale CSI can be formulated as

$$R_k = \frac{1}{M}\log_2\left(1 + \frac{|\hat{g}_k|^2}{\beta_k\sigma_e^2 + \frac{1}{\rho}}\right), \quad k = 1,2,...,M, \tag{18}$$

and the rate of the k-th user U_k with no small-scale CSI can be expressed as

$$R_k = \frac{1}{M}\log_2\left(1 + \rho|g_k|^2\right), \quad k = 1,2,\ldots,M. \tag{19}$$

Therefore, we can obtain the outage probability expressions by using (18) and (19). Note that the results will be used as benchmarks for purpose of comparing with the NOMA systems in Section IV.

IV. Numerical Results

In this section, we provide Monte Carlo simulations compared with numerical results to validate the accuracy of the analytical results obtained in Section III. We consider the same setup and parameters as in [8], [10]. The Monte Carlo simulation results are obtained through 10^6 independent trials.

In Fig. 1, we can observe the reduction of outage probability with the increase of transmitting power. Obviously, the outage performance of U_1 based on the NOMA scheme is much better than the one based on the OMA scheme by sacrificing only small portion of the performance of U_2, i.e. in terms of the average outage probability for all users. NOMA outperforms compared with conventional OMA, shown as the dashed line without symbols. It is clear to see that the outage performance is improved with the increase of Rician K-factor. Furthermore, if the constraint, $\alpha_i > \varepsilon_i \sum_{l=1}^{i-1}\alpha_l$, is not satisfied, the outage probability will always be 1, shown as the curves with $R_1^* = R_2^* = 2$ bits/s/Hz. In this case, we can improve

$$P_k^I = k\binom{M}{k}\frac{\sum_{r=0}^{k-1}\binom{k-1}{r}(-1)^r}{r+M-k+1}\left(\frac{\pi}{2n}\sum_{i=1}^{n}\left|\sin\frac{2i-1}{2n}\pi\right|\left(1 - Q_1\left(\frac{\sqrt{2K}}{\sqrt{1-\sigma_e^2}}, \frac{\sqrt{\left(\vec{\eta}\left(\rho\sigma_e^2\beta\left(\sqrt{x_i^2+h_z^2+l_z^2}\right)+1\right)\right)}}{\sqrt{(1-\sigma_e^2)\left(\beta\left(\sqrt{x_i^2+h_z^2+l_z^2}\right)\right)}}\right)\right)\right)^{r+M-k+1}, \tag{12}$$

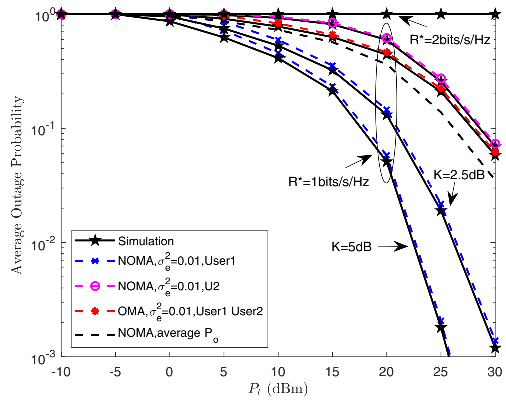

Fig. 1. Average outage probability of NOMA with imperfect small-scale CSI (M=2, and $\sigma_e^2 = 0.01$).

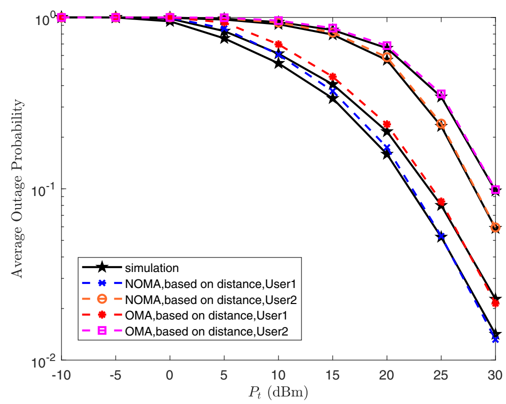

Fig. 2. Average outage probability in NOMA with no small-scale CSI ($M = 2$ and $R_1^* = R_2^* = 1$ bits/s/Hz).

the outage performance by adjusting the power allocation coefficients. Since we finally use the approximation expression of the unordered composite channel gain as show in (23), the expression in (24) is more accurate for high SNR.

Fig. 2 shows the outage probability of the no small-scale CSI based NOMA scheme, by comparing Monte Carlo simulations and analytical results in Theorem 2. Note that the analytical results match well with the Monte Carlo simulations. The outage probability decreases with the increase of the transmit power. Both NOMA and OMA systems with channels based on distance, achieves the same diversity gain of 1. However, the NOMA performs better in outage probability performance than the the OMA. In HSR scenario, the high speed mobility leads to significant Doppler effect, which brings a larger channel estimation error, resulting in the deterioration of the outage performance and inter-channel interference.

V. Conclusions

In this paper, we have investigated the outage probability of NOMA systems with partial CSI in HSR scenarios, including imperfect small-scale CSI and no small-scale CSI. It shows that, in practical HSR scenarios, NOMA systems with different partial CSI can always achieve better outage performance than conventional OMA systems with the specific distribution of the users. Moreover, the outage performance is improved with the increase of Rician K-factor.

Appendix A
Proof of Theorem 1

In order to derive the approximate expression for outage probability given in (12), we first employ order statistics [16]. The cumulative distribution function (CDF) of the estimation channel gain $|\hat{g}_k|^2$ can be expressed as

$$\begin{aligned} F_{|\hat{g}_k|^2}(x) &= k\binom{M}{k}\int_0^{F_{|\bar{g}_k|^2}(x)} t^{M-k}(1-t)^{k-1}dt \\ &= k\binom{M}{k}\sum_{r=0}^{k-1}\binom{k-1}{r}(-1)^r\frac{\left(F_{|\bar{g}_k|^2}(x)\right)^{r+M-k+1}}{r+M-k+1}, \end{aligned} \quad (20)$$

where $F_{|\bar{g}_k|^2}(x)$ is the CDF of the unordered estimation channel gain. Since the large-scale fading coefficient β_k is the function of d_k, and the small-scale fading is modeled as Rician fading with the unordered estimation channel $h_k = \bar{h}_k + e$. Thus the conditional CDF $F_{|\bar{g}_k|^2|d_k}(z|d_k)$ can be written as

$$F_{|\bar{g}_k|^2|d_k}(z|d_k) = 1-Q_1\left(\frac{\sqrt{2K}}{\sqrt{1-\sigma_e^2}}, \frac{\sqrt{\beta^{-1}(d_k)z}}{\sqrt{1-\sigma_e^2}}\right), \quad (21)$$

where $Q_1(a,b) = \exp\left(-\frac{a^2+b^2}{2}\right)\sum\limits_{k=0}^{\infty}\left(\frac{a}{b}\right)^k I_k(ab)$ is the first-order Marcum Q-function and $I_k(ab)$ is the first kind k-th order modified Bessel function. The probability distribution function (PDF) of the user's position x_k is $f_{x_k}(x) = \frac{1}{L_t}$, therefore the PDF of the distance d_k from U_k to the BS is given by

$$f_{d_k}(x) = \frac{x}{L_t\sqrt{x^2 - h_z^2 - l_z^2}} \quad (22)$$

Then,

$$\begin{aligned} F_{|\bar{g}_k|^2}(z) &= \int_{\sqrt{h_z^2+l_z^2}}^{\sqrt{{L_t}^2+h_z^2+l_z^2}} F_{|\bar{g}_k|^2|d_k}(z|x)f_{d_k}(x)\,dx \\ &= \int_{\sqrt{h_z^2+l_z^2}}^{\sqrt{{L_t}^2+h_a^2+l_z^2}} \frac{x}{L_t\sqrt{x^2-h_z^2-l_z^2}} \\ &\quad \times\left(1-Q_1\left(\frac{\sqrt{2K}}{\sqrt{1-\sigma_e^2}}, \frac{\sqrt{\beta^{-1}(x)z}}{\sqrt{1-\sigma_e^2}}\right)\right)dx. \end{aligned} \quad (23)$$

$$P_k^{II} = k\binom{M}{k}\sum_{j=0}^{M-k}\binom{M-k}{j}\frac{(-1)^j}{{L_t}^{k+j-1}}\frac{\pi}{2n}\sum_{i=1}^{n}\left|\sin\frac{2i-1}{2n}\pi\right| {x_i}^{k+j-1}\left(1-Q_1\left(\sqrt{2K}, \sqrt{\left(\beta^{-1}\left(\sqrt{x_i^2+h_z^2+l_z^2}\right)\right)\vec{\eta}}\right)\right), \quad (15)$$

$$F_{|\bar{g}_k|^2}(z)=\frac{\pi}{2n}\sum_{i=1}^{n}\left|\sin\frac{2i-1}{2n}\pi\right|\left(1-Q_1\left(\frac{\sqrt{2K}}{\sqrt{1-\sigma_e^2}},\frac{\sqrt{\left(\beta^{-1}\left(\sqrt{x_i^2+h_z^2+l_z^2}\right)\right)z}}{\sqrt{1-\sigma_e^2}}\right)\right). \tag{24}$$

$$F_{|g_k|^2}(z)=k\binom{M}{k}\sum_{j=0}^{M-k}\binom{M-k}{j}\frac{(-1)^j}{{L_t}^{k+j-1}}\frac{\pi}{2n}\sum_{i=1}^{n}\left|\sin\frac{2i-1}{2n}\pi\right|{x_i}^{k+j-1}\left(1-Q_1\left(\sqrt{2K},\sqrt{\left(\beta^{-1}\left(\sqrt{x_i^2+h_z^2+l_z^2}\right)\right)z}\right)\right). \tag{28}$$

It is difficult to solve the integral in (23), while we can derive the approximate formula as (24) at the top of this page by using Gauss-Chebyshev quadrature [17]. Note that $x_i=\frac{L_t}{2}(\phi_i+1)$, and $\phi_i=\cos\frac{2i-1}{2n}\pi$. Moreover, n is a parameter to balance the accuracy and complexity.

Following similar steps in [18], (24) can be derived and is more accurate for high SNRs. This is because we use the approximation of the unordered composite channel gain as show in (23). The outage probability of the k-th user based on NOMA scheme with imperfect small-scale CSI is $P_{out,k}^{I}=F_{|\hat{g}_k|^2}\left(\vec{\eta}\left(\rho\sigma_e^2\beta_k+1\right)\right)$. Consequently substituting (24) into (20), the proof is completed.

Appendix B
Proof of Theorem 2

The PDF and CDF of the distance d are given as

$$f_d(x)=\frac{x}{L_t\sqrt{x^2-h_z^2-l_z^2}}\ ,F_d(x)=\frac{\sqrt{x^2-h_z^2-l_z^2}}{L_t}, \tag{25}$$

where $\sqrt{h_z^2+l_z^2}<x\le\sqrt{{L_t}^2+h_z^2+l_z^2}$. Let $d_1\le d_2\le\cdots\le d_M$, according to order statistics [16], the PDF of the distance d_k from the BS to the k-th nearest user can be expressed as

$$\begin{aligned}f_{d_k}(x)&=k\binom{M}{k}(F_d(x))^{k-1}(1-F_d(x))^{M-k}f_d(x)\\&=k\binom{M}{k}\sum_{j=0}^{M-k}(-1)^j\binom{M-k}{j}\frac{x(\sqrt{x^2-h_z^2-l_z^2})^{k+j-2}}{{L_t}^{k+j}},\end{aligned} \tag{26}$$

where the Taylor expansion is used in the second to third steps.

Since the small-scale fading and the large-scale fading are independent, the CDF of the channel gain $|g_k|^2$ is given by

$$\begin{aligned}F_{|g_k|^2}(z)=&\int_{\sqrt{h_z^2+l_z^2}}^{\sqrt{{L_t}^2+h_z^2+l_z^2}}\left(1-Q_1\left(\sqrt{2K},\sqrt{\beta^{-1}(x)z}\right)\right)k\binom{M}{k}\\&\times\sum_{j=0}^{M-k}\binom{M-k}{j}(-1)^j\frac{x\left(\sqrt{x^2-h_z^2-l_z^2}\right)^{k+j-2}}{{L_t}^{k+j}}dx.\end{aligned} \tag{27}$$

Applying Gauss-Chebyshev quadrature again, (27) can be written as (28) at the top of this page, where $x_i=\frac{L_t}{2}(\phi_i+1)$, and $\phi_i=\cos\frac{2i-1}{2n}\pi$. Substituting $\vec{\eta}$ into (28), we can derive the outage probability expression (15) to complete the proof.

References

[1] V. W. Wong, R. Schober, D. W. K. Ng, and L.-C. Wang, *Key technologies for 5G wireless systems*. Cambridge university press, 2017.

[2] Z. Ding, Y. Liu, J. Choi, Q. Sun, M. Elkashlan, H. V. Poor *et al.*, "Application of non-orthogonal multiple access in LTE and 5G networks," *IEEE Commun. Mag.*, vol. 55, no. 2, pp. 185–191, Feb. 2017.

[3] Y. Liang, X. Li, J. Zhang, and Z. Ding, "Non-orthogonal random access for 5G networks," *IEEE Trans. Wireless Commun.*, vol. 16, no. 7, pp. 4817–4831, Jul. 2017.

[4] J. Zhang, L. Dai, X. Li, Y. Liu, and L. Hanzo, "On low-resolution ADCs in practical 5G millimeter-wave massive MIMO systems," *IEEE Commun. Mag.*, vol. 56, no. 7, pp. 205–211, Jul. 2018.

[5] X. Wang, J. Wang, L. He, and J. Song, "Outage analysis for downlink NOMA with statistical channel state information," *IEEE Wireless Commun. Lett.*, vol. 7, no. 2, pp. 142–145, Apr. 2017.

[6] Y. Liu, Z. Ding, M. Elkashlan, and H. V. Poor, "Cooperative non-orthogonal multiple access with simultaneous wireless information and power transfer," *IEEE J. on Sel. Areas Commun.*, vol. 34, no. 4, pp. 938–953, Apr. 2016.

[7] W. Zeng, J. Zhang, K. P. Peppas, B. Ar, and Z. Zhong, "UAV-aided wireless information and power transmission for high-speed train communications," in *Proc. IEEE ITSC*, 2018, pp. 3409–3414.

[8] F. Hasegawa, A. Taira, G. Noh, B. Hui, H. Nishimoto, A. Okazaki, A. Okamura, J. Lee, and I. Kim, "High-speed train communications standardization in 3GPP 5G NR," *IEEE Commun. Stand. Mag.*, vol. 2, no. 1, pp. 44–52, Mar. 2018.

[9] F. Fang, H. Zhang, J. Cheng, S. Roy, and V. C. Leung, "Joint user scheduling and power allocation optimization for energy-efficient NOMA systems with imperfect CSI," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 12, pp. 2874–2885, Dec. 2017.

[10] Ö. Özdogan, E. Björnson, and J. Zhang, "Performance of cell-free massive MIMO with Rician fading and phase shifts," *arXiv*, 2019.

[11] J. Zhang, L. Dai, Z. He, S. Jin, and X. Li, "Performance analysis of mixed-ADC massive MIMO systems over Rician fading channels," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1327–1338, Jun. 2017.

[12] J. Zhang, L. Dai, S. Sun, and Z. Wang, "On the spectral efficiency of massive MIMO systems with low-resolution ADCs," *IEEE Commun. Lett.*, vol. 20, no. 5, pp. 842–845, May 2016.

[13] J. Zhang, L. Dai, Z. He, B. Ai, and O. A. Dobre, "Mixed-ADC/DAC multipair massive MIMO relaying systems: Performance analysis and power optimization," *IEEE Trans. Commun.*, vol. 67, no. 1, pp. 140–153, Jan. 2019.

[14] S. S. Ikki and S. Aissa, "Two-way amplify-and-forward relaying with gaussian imperfect channel estimations," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 956–959, Jul. 2012.

[15] Z. Yang, Z. Ding, P. Fan, and G. K. Karagiannidis, "On the performance of non-orthogonal multiple access systems with partial channel information," *IEEE Trans. Commun*, vol. 64, no. 2, pp. 654–667, Feb. 2016.

[16] H. A. David and H. N. Nagaraja, "Order statistics," *Encyclopedia of Statistical Sciences*, 2004.

[17] F. B. Hildebrand, *Introduction to numerical analysis*. Courier Corporation, 1987.

[18] Z. Ding, Z. Yang, P. Fan, and H. V. Poor, "On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users," *IEEE signal processing letters*, vol. 21, no. 12, pp. 1501–1505, Dec. 2014.