CS57300 Homework 2
Student.Solution.

Q1

(i): I agree with Ronald that the argument made by Jimmy based on only the average donation is not reasonable. If we consider the claims on the scope of many years, the data from 2012 donation is just one sample. If we want to make any statistical inference based on this sample, we need to consider the variance in the sample and perform suitable statistical test. Jimmy is claiming that x1>x2 (sample mean) => u1>u2 (population mean), which is not true. Also, even after performing the statistical test and we reject the null hypothesis, we can only say within a certain confidence level that the data support the claim. There is always some probability that we could make false positive and false negative errors. If Jimmy is only arguing about the fact on 2012 and the data is for all population, then the argument has no problem.

(ii): In this case, since the sample size is big, we can use t-test based on central limit theorem. After removing negative and zero donations in the data set, perform a two-sample one sided t-test. H0: u1-u2=0. Ha: u1-u2<0.

T-test results from R:

Welch Two Sample t-test
data: demdonations and gopdonations
t = -4.7121, df = 160400, p-value = 1.227e-06
alternative hypothesis: true difference in means is less than 0

Since the p-value is much smaller than 0.05 (95% confidence level), we can claim that the data support Democrats' claim.

(iii): There are two states that have missing data, VT only has donations for DEM and WY only has donations for GOP. So I put VT and WY as states where H0 can't be rejected. The Type I error rate is 0.05.

States support the claim:
"AL" "MA" "ME" "NC" "NM" "TX" "UT" "VT"
States don't support the claim:
"AK" "AR" "AZ" "CA" "CO" "CT" "DE" "FL" "GA" "HI" "IA" "ID" "IL" "IN" "KS" "KY" "LA" "MD" "MI" "MN" "MO" "MS" "MT" "ND" "NE" "NH" "NJ" "NV" "NY" "OH" "OK" "OR" "PA" "RI" "SC" "SD" "TN" "VA"
"WA" "WI" "WV" "WY"

The method is to use the data points of each states and perform a two-sample one sided t-test between DEM and GOP. The null hypothesis is that u1=u2, and alternative hypothesis is u1<u2. When performing the t-test, the threshold for significance is adjusted for multiple tests using the Bonferoni's correction method. So the threshold becomes to 0.05/48, since we are performing 48 tests. Tests with p-values that are less than the threshold are taken as significant states that support the claim.

(iv): The method used t-test assume the averages have a normal distribution. When the sample size is big, this is a reasonable assumption based on central limit theorem. However, when the sample size is small, this assumption may not be close to the real distribution and thus the test might not be appropriate. Also the Bonferoni's correction is conservative, we might miss some sates that actually support the claim. Using this method, the contribution of each state to the support can not be quantified.

(v): H0: u1-u2=0. Ha: u1-u2>0.
With type I error rate of 0.05, we can claim that the following states support the claim:
"FL" "GA" "IL" "MN" "NJ" "SC" "WA" "WV" "WY"
and the following states do not support the claim:
[1] "AK" "AL" "AR" "AZ" "CA" "CO" "CT" "DE" "HI" "IA" "ID" "IN" "KS" "KY" "LA" "MA" "MD" "ME" "MI"
[20] "MO" "MS" "MT" "NC" "ND" "NE" "NH" "NM" "NV" "NY" "OH" "OK" "OR" "PA" "RI" "SD" "TN" "TX" "UT"
[39] "VA" "VT" "WI"
Similar multiple t-tests were performed but with opposite direction.

(vi): For (i), the answer is the same, we could not just conclude on population means based on sample means. We need to do statistical inference.
For (ii), since the sample size is large, we can apply t-test. The t-test results from R are as follows:

```
        Welch Two Sample t-test
data:  dem_id[, 2] and gop_id[, 2]
t = -1.9104, df = 1331.1, p-value = 0.02815
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
    -Inf -6826.216
sample estimates:
mean of x mean of y
 250664.2  299988.6
```

Because the p-value is less than 0.05, so with confidence level of 95%, we can say that the data support the democrat's claim.
For (iii), with type I error rate of 0.05, the states that support the democrats' claim are:
"IL" "VT"
States that do not support the claim are:
[1] "AK" "AL" "AR" "AZ" "CA" "CO" "CT" "DE" "FL" "GA" "HI" "IA" "ID" "IN" "KS" "KY" "LA" "MA" "MD"
[20] "ME" "MI" "MN" "MO" "MS" "MT" "NC" "ND" "NE" "NH" "NJ" "NM" "NV" "NY" "OH" "OK" "OR" "PA" "RI"
[39] "SC" "SD" "TN" "TX" "UT" "VA" "WA" "WI" "WV" "WY"

For each state, we perform the following t-test:
H0: u1-u2=0

Ha: u1-u2<0, which u1 is the average donation per candidate. We adjust the threshold by Bonferoni's correction for 48 tests. For state "AK", only one ID in DEM, so I used one sample t-test.

Q2:

(i): Since the data is discrete, I choose to use chi-square test. From the data, make a 2X2 table

	donation	zero
Population1b	4008	5992
Population2b	4557	5443

Sample means are (average donation in each population) x1= 0.4008, x2=0.4557

H0: u1-u2=0

Ha: u1-u2!=0

Perform a chi-square test, p-value = 4.842e-15, so we reject the null hypothesis. We say that they have different averages.

(ii):

	donation	zero
Population3b	360	640
Population4b	450	550

Average donation in each population are x1=0.36, x2=0.45

H0: u1-u2=0

Ha: u1-u2!=0

Perform a chi-square test, p-value = 5.034e-05, so we reject the null hypothesis. We say that they have different averages.

(iii): The data is continuous and the sample size is large. Based on central limit theorem, I choose to use t-test.

Average donation in each population are x1=2.647086, x2=2.245484

H0: u1-u2=0

Ha: u1-u2!=0

The t-test results are:

```
        Welch Two Sample t-test
data:  pop1p[, 2] and pop2p[, 2]
t = 3.1035, df = 15345, p-value = 0.001916
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.1479554 0.6552486
sample estimates:
mean of x mean of y
 2.647086  2.245484
```

The p-value is less than 0.05, so we reject the null hypothesis and claim the average is different.

(iv)

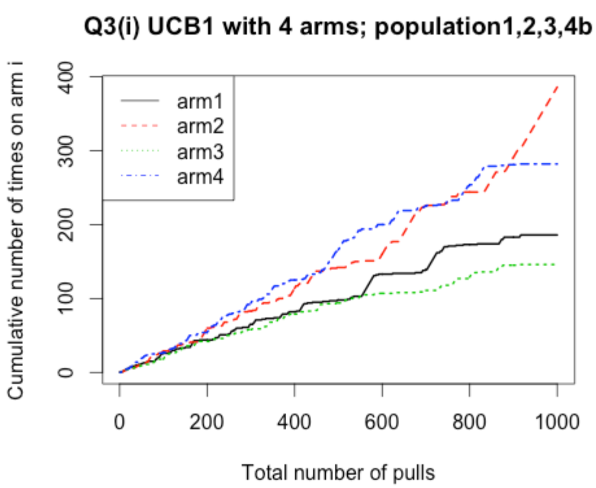
Average donation in each population are x1=2.0704, x2=2.5688

H0: u1-u2=0
Ha: u1-u2!=0
The t-test results are:

Welch Two Sample t-test
data: pop3p[, 2] and pop4p[, 2]
t = -1.5458, df = 1353.6, p-value = 0.1224
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.1309008 0.1341008
sample estimates:
mean of x mean of y
  2.0704   2.5688

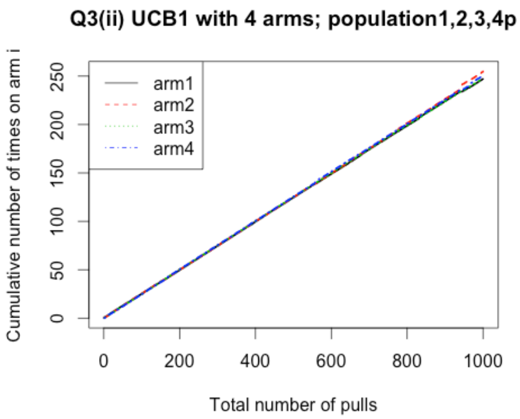
The p-value is greater than 0.05, we could not reject the null hypothesis. Thus we claim that their averages are not statistically different.

Q3:
(i)(a): The total reward is 440. For the first 4 times, I pull arm1,2,3,4 once each and thereafter use UCB1 to determine which arm to pull.
(b):

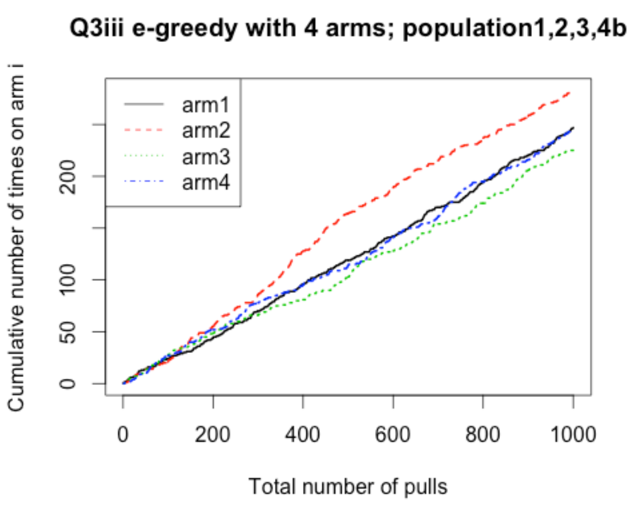


(ii)(a): To make the donations fall into the range of [0,1], I normalized the donations using the maximum donation. Total donation with the MAB is 2629.8. First, pull arm1,2,3,4 each once and then use the UCB1 to determine which arm to pull.

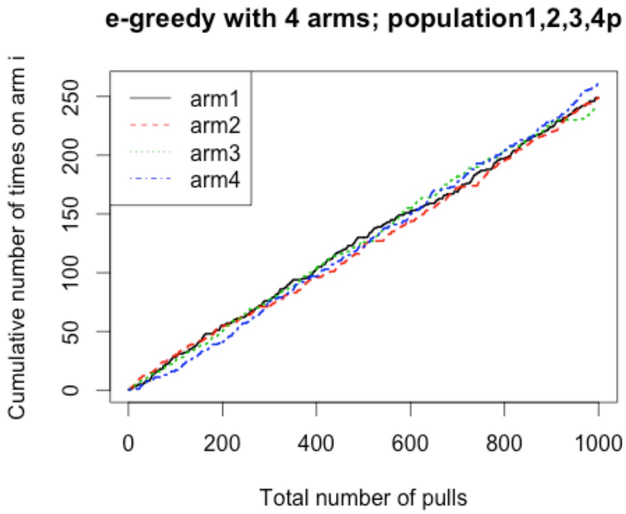
(b):



(iii): Redo MAB using ε-greedy using the population_b datasets. Repeat the calculation three times, the donations are: 423, 421, 422. To test if the ε-greedy is worse than the UCB1 (440), I performed a one-sample t-test with H0: u1=440, Ha: u1<440. It is significant with p-value= 0.0005136. So for these donation datasets, ε-greedy is worse than UCB1. There is one potential problem when using the ε-greedy algorithm. The ε was 1 for many times, so there was lots of randomization during the choosing process. In this case, we may try to fix ε as a relative small value.
The plot with rewards 422 is:

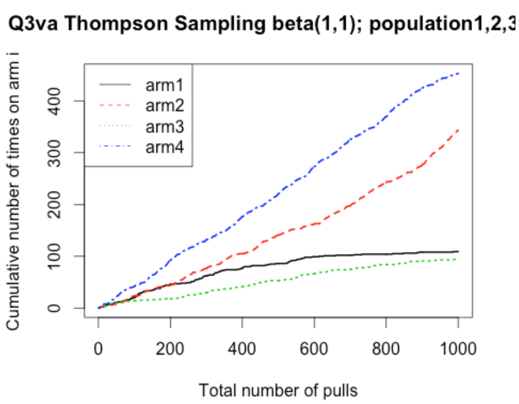


Redo MAB with ε-greedy using the population_p datasets. Repeat the calculation 100 times and the mean is 2818.819. To test if this is statistically better than UCB1 (2629.8), I performed t-test with H0: u1=2629.8 and Ha: u1>2629.8 and got p-value is 2.033e-14, so we reject the null hypothesis and claim that for this sets of data, ε-greedy method is better.
One plot is like:

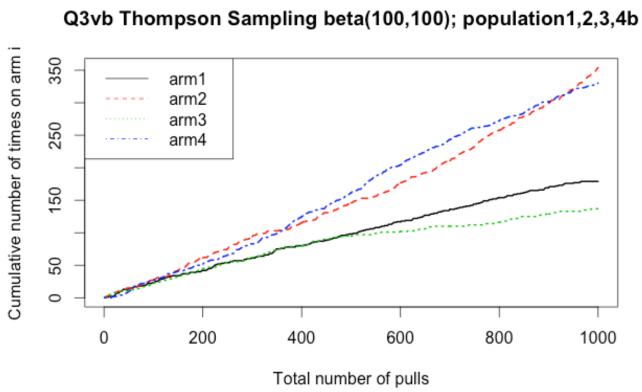


(iv): For Q3(i) the average donations for arm1,2,3,4 are: x1= 0.4008, x2=0.4557, x3=0.36, x4=0.45 and their variances are: 0.2401834, 0.2480623, 0.2306306, 0.2477477. Since the variances are small, so the upper bound is close to the mean and the choices of arms reflects the trend of averages.
However, for Q3(ii) the average donations are: x1=2.647086, x2=2.245484, x3=2.0704, x4=2.5688 and their variances are: 129.829, 37.62412, 16.11351, 87.84236. Since the choice is determined by the upper bound, it almost evenly chooses arms.

(v):(a) Reward is 450 (from one run).

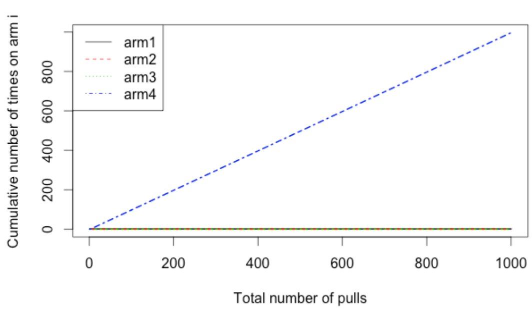


(b) Reward is 446 (one run).



(c): Q3vb explored arms more times than Q3va. For beta(100,100) prior, the choices will keep explore different arms more times than beta(1,1) prior. So when the distribution of rewards has larger variance, beta(100,100) prior is better. In this case, we give more chance to explore different arms.

(d): When Beta(ε,ε),ε->0, the Thompson sampling degenerate into "play the winner" MAB approach. Because from beginning, the sampling distribution is beta(success,failure), and the mean is the current average. It tends to choose the arm with maximum average so far and keeps choosing the winner. Since the sampling process is random, there is still small change to try other arms (like the e-greedy strategy).
Most of the times like:



Few times like: