Q1:

(i) (Empirical) Get the singular value vector $(\sigma_1, \sigma_2, ...)$ of SVD for this dataset, sorting them in descending order.

(ii) (Empirical) What is the likely value of k (degree of freedom) of the data?

(iii) (Theory) Let

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,40} \\ x_{2,1} & \cdots & x_{2,40} \\ \vdots & \vdots & \vdots \\ x_{1000,1} & \cdots & x_{1000,40} \end{bmatrix}$$

and suppose we want to compute the SVD decomposition of XX^T. Note that XX^T is a $1000x1000$ matrix. Show a computationally efficient way to compute the SVD decomposition of XX^T.

A:

(i) The singular value vector of SVD for this dataset in descending order is [1089.67547396, 1027.89906723, 909.56529034, 885.82803953, 759.97260629, 683.9141783, 663.09093336, 565.85700372, 509.46586879, 461.29400539, 9.15827494, 9.00480151, 8.90682002 , 8.80650525, 8.66727228, 8.58844142, 8.51717892, 8.41727672, 8.28530571, 8.26504108, 8.17096672 8.13002631, 8.06764902, 7.98587734, 7.91909172, 7.8586083, 7.80691957, 7.68862159, 7.62347627, 7.54280159, 7.43411144, 7.33798541, 7.26026044, 7.1937195, 6.97768789, 6.93967099, 6.8187177, 6.72712347, 6.68866903, 6.59213939].

(ii) We can tell that the number of distinguished large singular value is 10. So the degree of freedom is 10.

(iii) $X = USV^T$, $XX^T = USV^TVSU^T = US^2U^T$. This is a convenient way of SVD of XX^T.

Q2:

A:

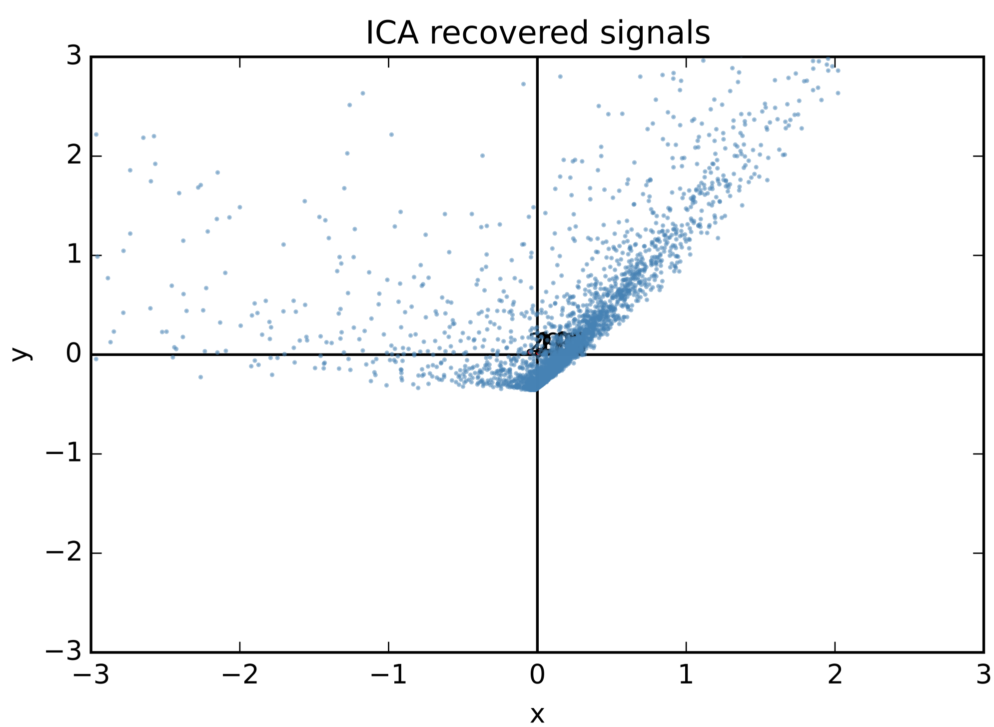
(i) $X = USV^T$. The SVD of X gives us top 10 singular values as [1121.90788958, 477.38365262, 401.11318769, 330.58629337, 317.60437782, 275.32441062, 270.7558153, 248.85843676, 239.1103667, 220.42486397].

(ii) A big sparse matrix is a waste of memory storage and consumption.

(iii) After subtracting $X_{i,j}$ by α_j and SVD decomposition, we get matrix U out of $Y = USV^T$. The entries of U with respect to the position of X implies whether user i will like movie j more than average user that has seen j. If the entry in U is larger than 0, it implies that the user i will like this movie more than average user that has seen it.

(iv)

$W = A^{-1}$, $s^i = A^{-1}x^i = Wx^i$



(v)