Line-Art and Its Mathematical Models

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Abstract

In this paper, the authors describe the principles of *Straight Line Strokes* illustration, present the mathematical model of the principles, and show how a great number of lines can be implemented as main part of an automated drawing system named *Line-Art*. Different from traditional drawing art, Line-Art generates pictures without curves, colors, ink marks, brushes, and oil paint, but only with Straight Line Strokes. Generated pictures are composed, clipped, and plotted. The paper also introduces how to use the initial value problem of the ordinary differential equation to describe a drawing art, e.g. Line-Art.

Keywords: Computer art, mathematical model, Line-Art, ordinary differential equations.

1 Introduction

In 1980s, Hans Dehlinger presented a special method of generating complex pictures with computers named Line-Art. Different from traditional drawing art, Line-Art generates pictures without curves, brushes, colors, ink marks, and oil paint, but only with straight line strokes. Complex pictures are generated by composing, clipping, and plotting a large number of broken-lines in which every broken-line has a special definition and connotation. Line-Arts have been exhibited in many countries on SIGGRAPH Traveling Art Show 1989. The authors of this paper have cooperated to develop the Line-Art software on Apollo Workstation and set up its mathematical models since 1990.

2 Importance and Fascination of the Line

It is well known that the line is one of the basic elements in drawing art, the strokes of a pen-equipped hand which add up to form a drawing are elementary expressions of human will to design. Drawing has existed since man first began to depict his environment. Stone age document of multiply scratched stone board may date back 30,000 years in history. The lines in the drawing have composed the beautiful shapes in the ancient art objects. Lines are very important elements in Chinese traditional painting too. The persons, the mountains and rivers, the flowers and birds, all of them have been expressed with the line as the composition language in Chinese painting. The Chinese calligraphy, in fact, is the activity of abstract lines, and they can strongly express the author's idea, emotion, and

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character. In the western painting, the lines are also very important, the lines that come from the great masters' hand have very distinctive characters. We know that the lines have their special connotation in the model design as follows.

- Line is the skeleton of the forming.
- Line moves to generate the surface.
- Line is the connection of points, and the edge of the surface.
- Line produces a cushioning effect on color conflict.
- Line can be used to express the emotion by shapes (curve, straight, slanting, ...).
- Line is an abstract language in painting.
- Line can be used to express a rhythm in drawing.

3 Hand-drawing and Machine-drawing

It is interesting to compare the production processes of both the hand-drawing and the machine-drawing. People can create an unbelievable multitude of lines by hand. But obviously the abilities of the human hand are limited. More lines, more difficulty, specially for lines requiring some distinct features. If we use computer to create a large number of lines with special features it would be much easier.

Some characteristics of hand-drawing are the following:

- Usually no line equals another. Only through enormous concentration is it possible to draw identical lines.
- The position of pen to paper, the speed of drawing, pushing, rubbing etc., the pressure differences, the mechanics and the motion of hand generate wide spheres of expression, indescribable in geometrical terms.
- The hand that draws lines operates under the constant control of the eye, thereby allowing a direct feedback. The spectrum of this feedback may begin with the rational control over every single stroke and may end with a totally subdued perception of all activities.
- The hand gets tired.
- The lines drawn by hand are always with more or less faint deviation.
- The hand draws slowly.

Some characteristics of machine-drawing are the following:

- The machine is indefatigable, exact, and fast.
- The machine works with constant regularity. Thousands of lines are performed under absolutely the same conditions.
- Irregularities, if they occur, are due to mechanical shortcomings like the tearing up of or the lack of absorptive capacity of the paper, dried-up pen and so forth.
- Pressure, drawing speed, the position of pen to paper, and pushing are always the same.
- The pen's activity can be controlled by an algorithm program.

This is, of course, exactly what we expect from a machine when we work in CAD. An artist, however, may find it more interesting to use the machine's stronger ability to create and compose some nice drawings. When painting, there are some artists who resort to a kind of program, a set of temporarily adopted restrictions to carry through a visual concept. Such a program can, for example, order the hand to act as follows: "make short, powerful strokes", or "draw in small, circling movements", or "go to and fro in the direction of the developing line". A computer program is a sequence of orders, that are followed by the machine. So, it is possible to develop a computer-drawing system.

4 Describing and Generating Lines in "Line-Art"

In Dehlinger's Line-Art, the lines are broken-line, named as D-line. The characteristics of D-line are as follows:

- D-line starts from a start-point given by the designer.
- D-line's position is stochastic and is placed inside an angle area, and the start-point is the vertex of the angle area.
- The length of every segment of the D-line is stochastic, it is between A and B. A and B are given by the designer.
- The D-lines can be composed by clipping.

4.1 Generating Starting-points

A line starts from and ends in the point(s). Described from a physical point of view both points have the same properties. From an artist's view they don't. The beginning point seems to be important; it will be placed with more consciousness. From it the line evolves, and where it ends may not yet decided. The decision where to start seems to be easier. While the line is evolving, the decision space is expanding. For the algorithmic generation of lines it makes sense to pay special attention to their starting-points. Such points may be lined up alone familiar line-shapes like: straight line, zigzag line, curve line, circular line, etc.

4.2 Experimental Restriction to D-lines

The D-lines as such may assume a multitude of possible shapes. The most simple case is a straight line. All of the experiments so far are based on polygons. Despite the fact, that it constitutes a severe constraint on what a D-line is allowed to be, it is an interesting assumption from an experimental point of view. It is always possible to relax constraints at later points, and many of the findings observed using polygonal lines can be extrapolated to other types of lines.

4.3 Parameters

The analysis of a D-line will very quickly reveal some of its basic parameters: the number of segments, its total length, the relative length of segments, the variations of the angle at turning points, the spread of the entire line, and so on. The generating program is designed to accept these parameters and draw D-lines accordingly which originate from such point-sets as described above.

4.4 Clipping and Other Specific Operations

Through "clipping", arbitrary shapes can be created. Clipping is one of the well established computer-specific operations such as scaling, moving, rotating, and so on. Some of those operations have been used by artists long before computers came into being.

5 Mathematical model

Consider a D-line as the trace of the specific motion from a start-point. Generating a D-line can be reduced to solving the initial value problem of the ordinary differential

equation

$$\frac{du}{dt} = f, \quad u(0) = P_0 \tag{1}$$

where u, f, P_0 are vectors on the plane, that is,

$$u = (x(t), y(t)),\ f = f(f_1(t), f_2(t)),\ P_0 = (x_0, y_0)$$

We solve (1) by Euler scheme. The difference scheme is the following:

$$u_{n+1} = u_n + h_n f_n \tag{2}$$

or

$$u_{n+1} = u_n + h_n \frac{f_n + f_{n+1}}{2}, \qquad h_n = t_{n+1} - t_n, \quad n = 0, 1, 2, \ldots \tag{3}$$

where $u_j = u(t_j)$.

Let $u(t)$ be a D-line that satisfies (1), in which function f must be chosen carefully. Firstly, we consider a simple case. Let $f_1(t), f_2(t)$ be piecewise constant functions:

$$f_1 = C_{1j} = random(a_{1j} - h_{1j}, a_{1j} + h_{1j}),$$

$$f_2 = C_{2j} = random(a_{2j} - h_{2j}, a_{2j} + h_{2j}), \qquad t \in [t_j, tt_{j+1}], \qquad j = 0, 1, 2, \ldots, n$$

In this case, $f_1(t), f_2(t)$ control the derivative of the trace of D-line. In order to generate the D-line well, we can also use the many-knot cardinal spline interpolation to generate D-line. Let $r_0, r_1, \ldots, r_n$ be the starting-points, $r_j = (x_j, y_j), j = 0, 1, 2, \ldots, n$.

$$r(t) = \sum_i r_i q_i(t), \qquad t \in [0, n] \tag{4}$$

where $q_i(t)$ are the second order fundamental functions. $q_i(t) = q(t - i)$, and

$$q(t) = 2\Omega_2(t) - \frac{1}{2}[\Omega_2(t + \frac{1}{2}) + \Omega_2(t - \frac{1}{2})] \tag{5}$$

$$\Omega_2(t) = \frac{1}{2}[(\frac{3}{2} - |t|)_+^2 - 3(\frac{1}{2} - |t|)_+^2] \tag{6}$$

Let

$$\Omega_k'(t) = \bar{\Delta}\Omega_{k-1}(t) \tag{7}$$

$$q'(t) = 2[\Omega_1(t + \frac{1}{2}) - \Omega_1(t - \frac{1}{2})] - \frac{1}{2}[\Omega_1(t + 1) - \Omega_1(t - 1)] \tag{8}$$

where

$$\Omega_1(t) = (1 - |t|)_+ \tag{9}$$

And the notation

$$u(t)_+ = \begin{cases} u(t) & u(t) > 0 \\ 0 & u(t) \le 0 \end{cases} \tag{10}$$

Now, we obtain the formula of f as follows,

$$f(t) = r'(t) = \sum_i r_i q'(t - i) \tag{11}$$

It is necessary to add two points $r_{-1} = r_0$ and $r_{n+1} = r_n$ in (11), the sum for i from -1 to $n + 1$.

6 Conclusions

We have introduced a new kind of drawing art system, Line-Art. It is different from all other traditional and model drawing arts. The beautiful and complex pictures created by Line-Art are without curves, brushes, colors, ink marks, and oil paint, but only with a large number of broken-lines. We have set up the mathematical model of Line-Art. It is an experiment of combining science with art. Line-Art can be used in model design and model drawing, and some examples have been showed in this paper (see Fig.1–3).

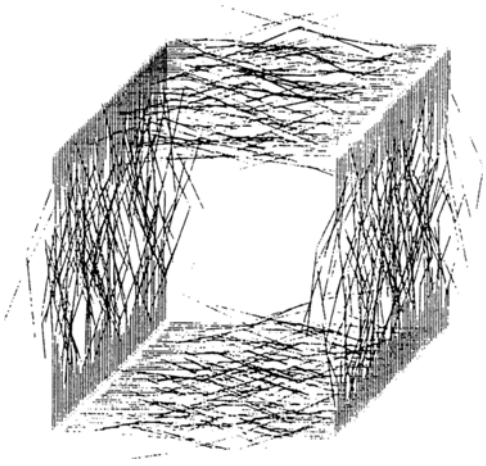

Fig.1. Example of Line-Art No.1.

Fig.2. Example of Line-Art No.2.

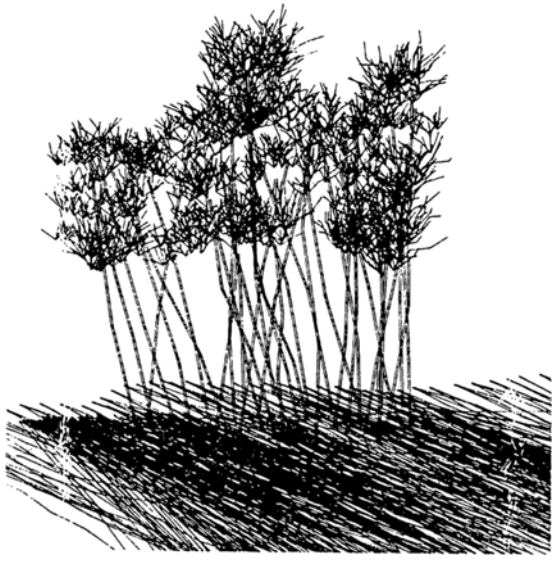

Fig.3. Example of Line-Art No.3.

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