Homework 1

Jialu Wang

September 7, 2022

Problem 1:

$$\boldsymbol{c}_{Katz} = \beta(\boldsymbol{I} - \alpha\boldsymbol{A})^{-1}\overrightarrow{\boldsymbol{1}} \tag{1}$$

The matrix $(\boldsymbol{I} - \alpha\boldsymbol{A})^{-1}$ diverges if det $(\boldsymbol{I} - \alpha\boldsymbol{A})^{-1}$ passes zero, i.e.:

$$\det(\boldsymbol{A} - \alpha^{-1}\boldsymbol{I}) = 0 \tag{2}$$

when $\alpha^{-1} = \lambda_1$, the determinant passes zero. λ_1 is the largest eigenvalue of $\boldsymbol{A}$. To ensure the convergence, $\alpha < \lambda_1^{-1}$

Problem 2:

The number of walks of length 1 between ν_i and ν_j is denoted by walk A_{ij}:

$$N_{ij}^{(1)} = A_{ij} \tag{3}$$

A common neighbour is a node that connects both ν_i and ν_j, i.e. it forms a walk of size 2, starting and ending at ν_i and ν_j, connecting by this neighbour. Since it's not a directed graph:

$$N_{ij}^{(2)} = \sum_{k=1}^{n} A_{ik}A_{kj} = [A^2]_{ij} \tag{4}$$

$[A^2]_{ij}$ is the number of common neighbours.

Problem 3: