A constant envelope variation of OFDM waveform

- Energy Efficient Radio waveform research by Jiamin Xie

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I. Introduction

Due to a high demand for fast and rigid wireless connection nowadays in the society, an increasing amount of researches and investigations discuss the variation of the carrier waveforms and their practical implementations in the past few years. Since radio frequency bandwidth is a limited resource, it invokes great research interest towards the methods (i) to intelligently share the bandwidth between primary and secondary user and (ii) to find radio waveform that is efficient in both in terms of spectrum usage and energy consumption.

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier scheme that is widely adopted in broadband digital communication, systems such as the 4th generation (4G) mobile communication. As an example, OFDM is used in long-term evolution (LTE) thanks to its frequency spectrum efficiency and inter-symbol interference (ISI) immunity to a multipath channel. However, one major drawback that OFDM has is peak to average power ratio (PAPR), which puts a limitation on practical amplifiers. This problem naturally has become a key research topic.

In fact, the wireless industry has grown exponentially ever since the burst in the semiconductor electronics field, which provides a compact realization of various theoretically optimal waveforms. As such, various multi-carrier communications systems are being discussed for future wireless communications systems such as the 5th generation (5G) mobile network. CE-OFDM waveform is one of the candidate methods that enables the 5G. The core of the technique is to provide a constant envelope variation to the OFDM waveform and effectively reduce the PAPR to a constant 0 dB of all the time.

In this report, a comparative study is provided between CE-OFDM and typical OFDM with respect to (i) PAPR and (ii) bit error rate (BER). Specifically, the theoretic analyzes for both techniques are given as well. Then, the PAPR and BER are evaluated through MATLAB simulation over an AWGN channel. The CE-OFDM reveals a perfect PAPR elimination. Also, A tradeoff between bit error rate (BER) and spectral efficiency of using a CE-OFDM will come to the surface as we compare the plot results from the model.

II. OFDM waveform

In digital communications, information is expressed into 0s and 1s, namely bits. The term symbol represents the various combinations of bits in different sizes. [1] For example, in a quadrature amplitude modulation (QAM), the 0s and 1s are packed into the 4 different symbols as shown in the constellation map in Figure 1. These types of digital modulation schemes convey the symbols serially into a subcarrier waveform by changing the carrier amplitudes and phases. However, OFDM is a multi-carrier modulation scheme that conveys symbols into parallel sets and load them to the subcarriers with a minimum frequency spacing to maximize the spectral efficiency.

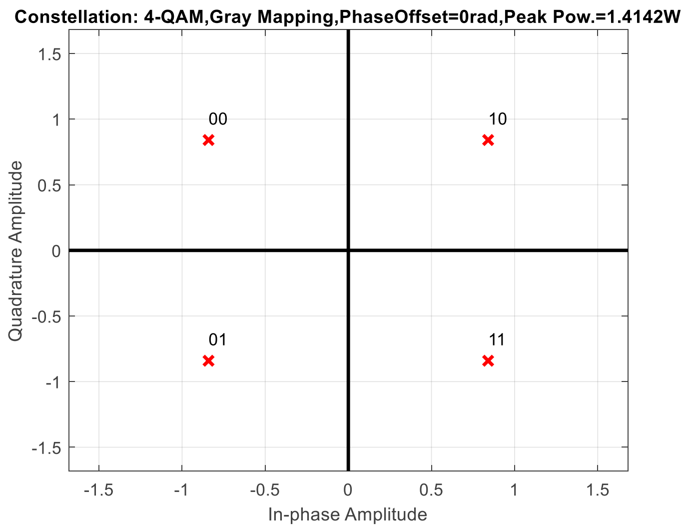

Figure 1. 4-QAM constellation map

A. OFDM Concept

As indicated in the name, the OFDM breaks a bandwidth into small orthogonal sub-bands, each of which is separated by a subcarrier spacing $\Delta f = \frac{W}{N}$. Let W denote the bandwidth, and N is the numbers of subcarriers as shown in Figure 2.

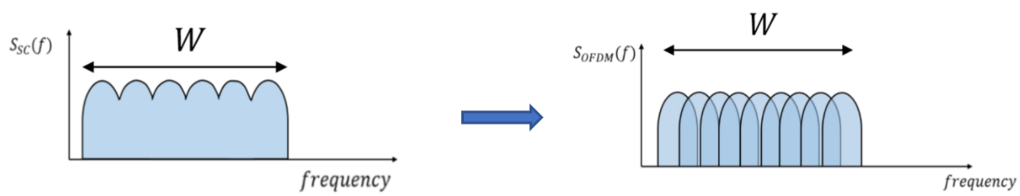

Figure 2 Single carrier vs. Multi-Carrier in frequency domain [2]

Figure 3 gives a more intuitive representation of OFDM in the time domain. Each subcarrier is transmitted parallelly and simultaneously. The symbol duration of each OFDM subcarrier is related to its frequency separation by, $T_s = \frac{1}{\Delta f}$ to have an orthogonality between subcarriers. In this way, $T_s = \frac{N}{W}$, which means the symbol duration is N times greater than a single carrier scheme with $T_s = \frac{1}{W}$. Additionally, since one OFDM symbol is composed of data symbols with each relevant subcarrier, the effective symbol rate of the two schemes remain the same. [2]

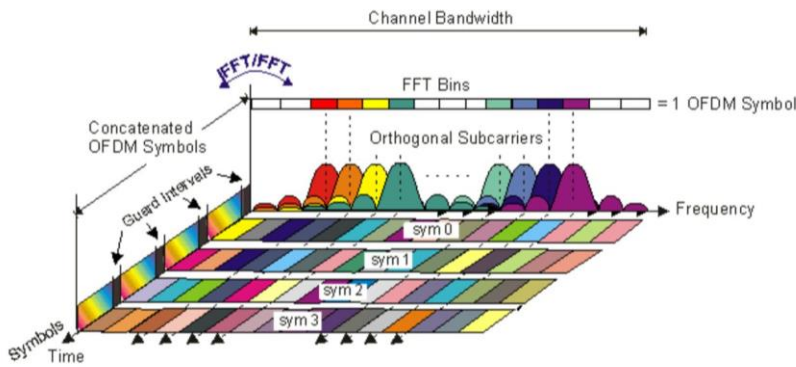

Figure 3 OFDM in time and frequency domain

The longer symbol duration leads to robustness against ISI in a multi-path channel. Even though subcarriers may suffer from a time delay caused by multi path, this delay will not severely impact the integrity of the whole data set since the symbol duration will be designed much greater than the time delay, τ_{max}. Moreover, by a cyclic prefix, a copy of the waveform tail attached to the front, an OFDM system can further eliminate the ISI interference entirely as shown simply in the Figure 4.

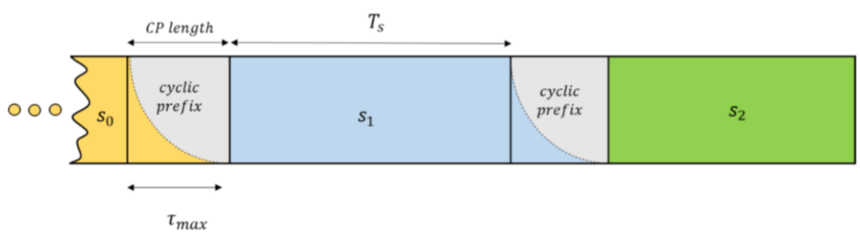

Figure 4 OFDM immunity to ISI with cyclic prefix [2]

In all, the OFDM multi-carrier scheme multiplexes in the frequency domain to maximize the data rate and rigidity of the system compare to a single-carrier modulation. However, a cost is that this also increases the requirements for a more complex circuitry and digital signal processing techniques such as FFT and IFFT.

B. OFDM Mathematical Representation

The mathematical expression for a baseband OFDM signal is given by:

$$\mathrm{S(t)} = \sum_{i}\sum_{k=0}^{N-1} I_{i,k}e^{j2\pi k\Delta f(t-iT_s)}\text{, where } \Delta\mathrm{f} = \frac{1}{T_s}$$

$I_{i,k}$ is the symbol value where at ith row and kth column of a complex matrix. N is the number of subcarriers.

Therefore, we can show theoretically the subcarriers are mutually orthogonal to each other,

$$\frac{1}{T_s}\int_0^{T_s}\left(e^{j2\pi k_i \cdot \Delta f \cdot t}\cdot e^{-j2\pi k_{i+1}\Delta f t}\right)dt = \begin{cases} 0, & k_i \neq k_{i+1} \\ 1, & k_i = k_{i+1} \end{cases}$$

Each data symbol takes up one subcarrier separated by Δf, and the orthogonality keeps the overlapping subcarriers from inter-carrier interferences. Thus, an OFDM system provides a higher spectral efficiency compared to a traditional single-carrier scheme. Additionally, the authors in [3] provide a more intuitive reasoning on the spectral efficiency consideration of an OFDM system.

The OFDM in the mathematical form is essentially the same as the algorithm for a FFT/IFFT processing in the digital domain. A discrete-time version of OFDM is therefore given by,

$$\mathrm{s[n]} = s(t)|_{t=\frac{nT_s}{N}} = \sum_{k=0}^{N-1} I_{0,k} e^{\frac{j2\pi kn}{N}}, n = 0,1,\ldots N-1$$

We can also show that the IDFT of the symbols $I_{0,k}$ equals to,

$$\mathrm{g[m]} = \frac{1}{N}\cdot\sum_{k=0}^{N-1} I_{0,k} e^{\frac{j2\pi km}{N}}, m = 0,1,\ldots N-1,$$

where the two representations differ only from a scale of the IDFT size, *N*. Moreover, the IDFT/DFT is equivalently processed with a greater computation efficiency by the (inverse) Fast-Fourier transform using a FFT size with a value of power of 2. Upon this similarity, a digital domain OFDM system model is readily to be built up.

C. OFDM System Representation

The OFDM transmitter system blocks are shown below,

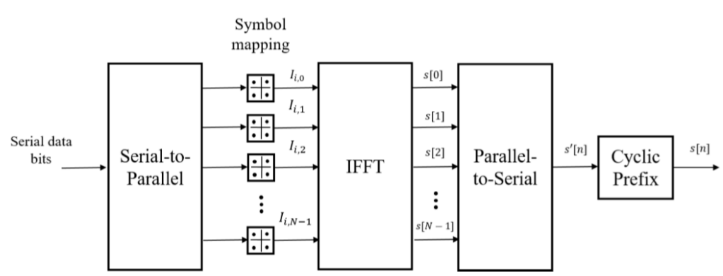

Figure 5 OFDM transmitter block diagram [2]

In Figure 5, the serial to parallel block is to convert a serial binary input into a parallel combination, or a data matrix for assigning the symbols to subcarriers. As will be shown in the appendix, this can be simulated by simply reshaping a row vector to a column vector in MTALAB. Additionally, since the size of the Fast Fourier transform is mathematically equivalent to the number of the subcarriers. The parallel data or the column vector should have a fixed same column size as the FFT size.

The symbol mapping blocks are there for aligning the binary data bits into different symbols by a specified digital bandpass modulation schemes. Popular modulation schemes are such as QPSK, 16-QAM or 64-QAM are good examples. They also yield different performance of the OFDM system as we discussed beyond. This system block can be usually simulated by the built-in powerful MATLAB system modulators or coding manually through the definition of the scheme.

The IFFT block represent the heart of the OFDM transmitter system. As we have investigated, this system block assigns each symbol to each subcarrier and therefore transforms the symbol into the time domain by performing an IFFT algorithm. In a MATLAB simulation, it becomes simply a one-line code. However, since the IFFT algorithm scales the digital OFDM representation by a scale of the FFT size, one needs to pay attention on normalizing the transmitted signal energy by a square root of the FFT size. A fail to perform such normalization will potentially lead to a less signal power and yield wrong results for waveform analysis.

Finally, the parallel to serial block is to set up for a transmission through a channel in the time domain. A cyclic prefix block is added after the P/S process for ensuring the mitigation of ISI over a multipath channel. The cyclic prefix size is determined by a physical layer or standard usually. A longer CP leads to higher BER, however, it expands the bandwidth of the transmit signal gradually. In [4], one can also find a thorough method of determining a CP length. In a simulation, CP insertion is achieved by appending an amount of the elements at the end of a transmitting signal vector to the front.

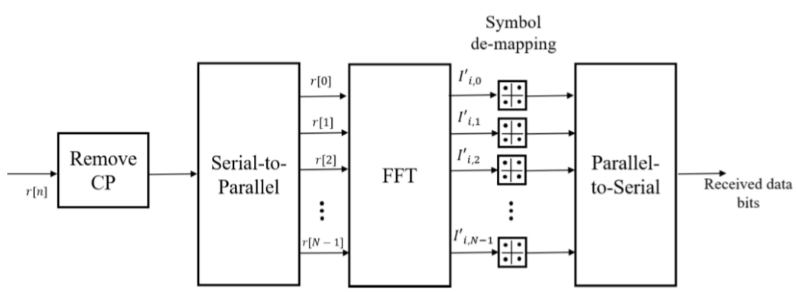

Figure 6 OFDM receiver block diagram [2]

Additionally, an OFDM receiver block diagram is shown above in Figure 6. One simply needs to reverse the operations as done in the transmitter to model the receiver. After integrating codes of the transmitter and receiver, a whole OFDM system can then be simulated in MATLAB.

D. OFDM PAPR Consideration

As aforementioned, one major drawback of an OFDM system is PAPR issue. The PAPR can be formulated as

$$PAPR\{s(t)\} = \frac{\max\{s^2(t)\}}{E\{s^2(t)\}}$$

The reason it becomes a concern of the OFDM system is because the digital symbols are assigned to the amplitudes of the subcarriers. Therefore, a wider variation between the amplitudes of symbols yields higher possibilities that the amplitudes of the OFDM envelope combine constructively and destructively.

As shown in Figure 7 is my simulation plot of the instantaneous power of an OFDM system against time,

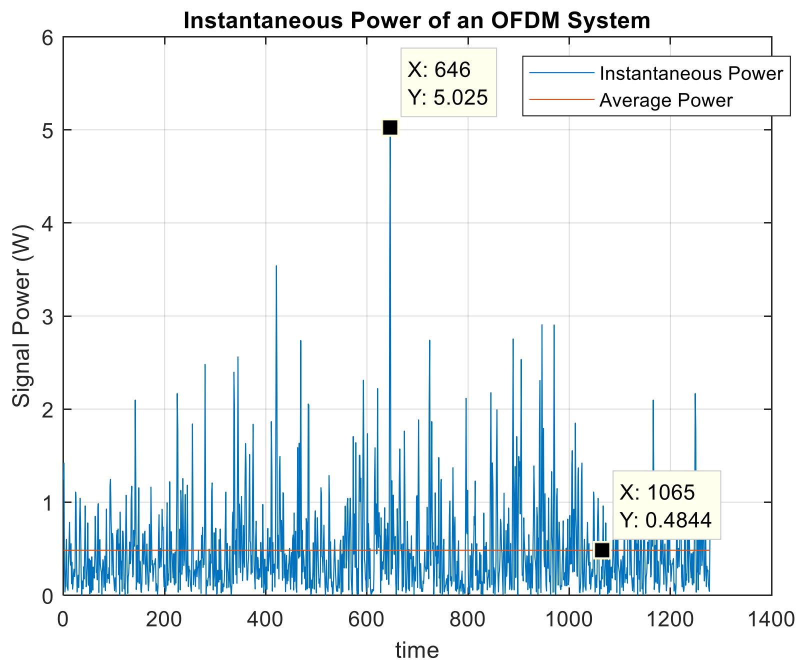

Figure 7 Signal Power analysis of an OFDM System (data size = 1026, FFT size = 1024)

Note that: For different random data inputs, the PAPR values may vary. In this case, as one can calculated from the data tips shown in Figure 6, $PAPR = 10.37$ or 20.32 dB. This unpredicted data input element also introduces an issue to the real implementation of the OFDM system, especially for an amplifier design.

In such a system, this requires an amplifier with a stable linearity at a high gain, which leads to a high power efficiency. This can be seen straightly from the plot as most of the time the power needed is lower than 2.5 W; however, for maintaining an integrity of the message, the amplifier must accommodate to deliver the highest power element, which leads to a less power efficiency of the system due to input power backoff.

Since the OFDM system can provide a huge benefit towards a multi-path channel than traditional schemes' do, researchers have developed several PAPR reduction techniques toward OFDM in years. This includes iterative clipping and filtering (ICF), selected mapping (SLM), commanding technique, and partial transmit sequence (PTS), Cross-Entropy-based Tone injection, etc. [5]

III. CE-OFDM waveform

Although the various PAPR reduction and/or power amplifier linearization techniques are available in the literature, the OFDM technique itself cannot be completely free from PAPR issues. [6]

One technique that fundamentally addresses such a PAPR problem is Constant-Envelope (CE-OFDM), wherein a phase modulation of OFDM symbols is performed after the IDFT. The main benefit of CE-OFDM is that it is a way to solve the PAPR issue without changing the system structure significantly. In this report, the theoretical and simulation results show that the PAPR can be even reduced to a constant 0 dB effectively of all the time.

The analysis below will continue to examine its characteristics as a waveform and a system.

A. CE-OFDM analytical representation

The constant variation of the OFDM waveform is performed by using a phase modulator essentially. In other words, the OFDM symbols after the IFFT algorithm will be used to phase modulate another carrier signal. Generally, the process of conveying the message into the carrier phase instead of into the amplitude is analogous to the development from AM to PM scheme in the analogue communication systems.

To see this, consider the baseband OFDM waveform,

$$\mathrm{m(t)} = \sum_{i} \sum_{k=1}^{N} I_{i,k} \cdot p_k(t - iT_B),$$

The $\{I_{i,k}\}$ term is the data symbol and $\{p_k(t - iT_B)\}$ are the mutually orthogonal subcarriers.

For a CE-OFDM baseband transmission, m(t) needs to be passed through a phase modulator. One can view the representation as:

$$\mathrm{s(t)} = e^{j\gamma m(t)}, where\ \gamma\ is\ a\ constant$$

If we let $A_m(t), \varphi_m(t)$ denote the amplitude and phase of m(t), respectively, the bandpass CE-OFDM signal will then be given by, [6]

$$\mathrm{y(t)} = Re\{s(t)e^{j2\pi f_c t}\}$$

$$= Re\{e^{j\gamma A_m(t)\exp[j\varphi_m(t)]} \cdot e^{j2\pi f_c t}\}$$

$$= e^{-\gamma A_m(t)\sin(\varphi_m(t))} \cdot \cos[2\pi f_c t + \gamma A_m(t) \cdot \cos(\varphi_m(t))]$$

If we assume $m(t)$ is a real-valued signal,

$$y(t) = \cos(2\pi f_c t + \gamma m(t))$$

We can see that $y(t)$ becomes the same transmitted signal as in a single carrier phase modulation scheme.

In fact, limiting the message signal to the real domain puts a major constraint of using a CE-OFDM system, which leads to a higher bit error rate compared to an OFDM system as we will discover later.

B. CE-OFDM system parameters

The constant γ denotes a multiplication of two important parameters, a design index and a statistical constant. It is referred to as $2\pi h \cdot C_n$.

h is the modulation index, which gives the designer a freedom of setting the signal energy as we will see later. The *2π* term converts the values into radians for phase modulation.

C_n is a statistical consideration, namely an normalization constant. It normalizes the variance of modulated subcarriers by having it times with the OFDM symbol.

By definition,

$$C_n \equiv \sqrt{\frac{2}{N \cdot \sigma_I^2}},$$

where *N* is the number of subcarriers and σ_I^2 is the variance of the data symbols.

Additionally, for a complete CE-OFDM baseband signal, there is another memory phase term θ_i that is designed to make the modulation phase-continuous. It smoothens the phase discontinuity occurs at the junction between two symbol periods.

In a mathematical form, it is given as,

$$\theta_i \equiv \theta_{i-1} + 2\pi h C_n \sum_{k=1}^{N} \left[I_{i-1,k} p_k(T_b - \varepsilon) - I_{i,k} p_k(0) \right], \varepsilon \to 0$$

As the name suggests, it memorizes the last CE-OFDM phase and corrects the current one by adding the discrimination between them to the current phase. In such way, the supplying power will be saved greatly from operating a CE-OFDM system.

In all, the composed phase signal of a CE-OFDM waveform is shown below,

$$\varphi(t) = \theta_i + 2\pi h C_n \sum_{k=1}^{N} I_{i,k} q_k(t - iT_B), iT_B \le t \le (i+1)T_B$$

where T_B is a bit period.

With all CE-OFDM parameters designs, the phase signal variance will be equaled to $2\pi h^2$ as a result. The modulation index, h will allow us to manipulate the CE-OFDM system performance as indicated in the later section.

C. CE-OFDM System Representation

A simplified baseband CE-OFDM transmitting system under consideration is represented by a diagram shown in Figure 8 below,

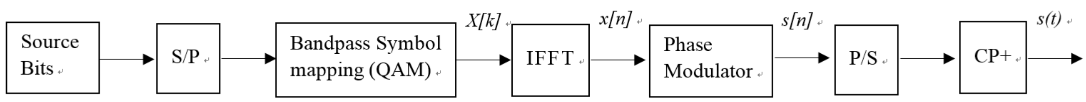

Figure 8 CE-OFDM transmitter diagram [7]

Compare to figure 5, we see this CE-OFDM transmitting system provides a minor modification to the conventional OFDM system.

The simplicity of the variation is given by a conjugate, symmetric, zero-padded input data vector [8, p. 719],

$$X[k]|_{k=(0\ to\ 2N_{QAM}+N_{zp}+1)} = \left\{0, X[1], X[2], \dots, X\left[N_{QAM}\right], 0_{1\times N_{zp}}, 0, X^*\left[N_{QAM}\right], \dots X^*[2], X^*[1]\right\}$$

Note that $\{X[k]\}_{k=1}{}^{N_{QAM}}$ are M_{QAM}-*QAM* data symbols, and $0_{1\times N_{zp}}$ is an array of N_{zp} zero paddings. Thus, the IDFT size can be determined by $N_{DFT} = 2N_{QAM} + N_{zp} + 2$. Importantly, the two zeros at the index $k = 0$ and $k = N_{QAM} + N_{zp} + 1$ maintain the symmetry of this data vector and cannot be ignored.

Moreover, one can find in [6, pp.124-126] that this representation can effectively take advantage of the IFFT algorithm to convert any complex valued input to a real valued $x\,[n]$ for phase modulation. This essentially provides a variety to the methods of bandpass symbol mapping since it can still be a complex valued process.

By keeping the input vector conjugated and symmetric, the OFDM system adds a phase modulator after the IFFT block and is readily to be used as an CE-OFDM system.

In a MATLAB simulation, one can create such a conjugate, symmetric data vector by matrix operations. The phase modulator is done by taking the OFDM symbol as a phase argument of an exponential. Note the CE-OFDM and OFDM system MATLAB script is attached to the appendix.

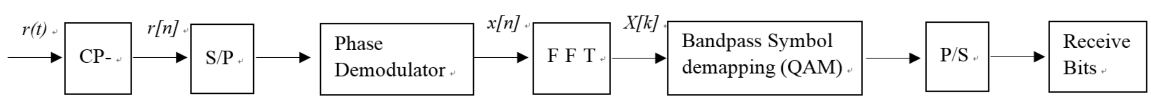

Figure 9 CE-OFDM receiver diagram [7]

Additionally, the baseband CE-OFDM receiver system diagram shown in Figure 8.

D. CE-OFDM PAPR Performance

The CE-OFDM system transmits the information by modulating the phase of the carrier signal. Therefore, theoretically, the instantaneous power will be the same as the mean power, which is 1 in magnitude, 0 in dB by Euler's Identity. This should effectively yield a PAPR value of 1.

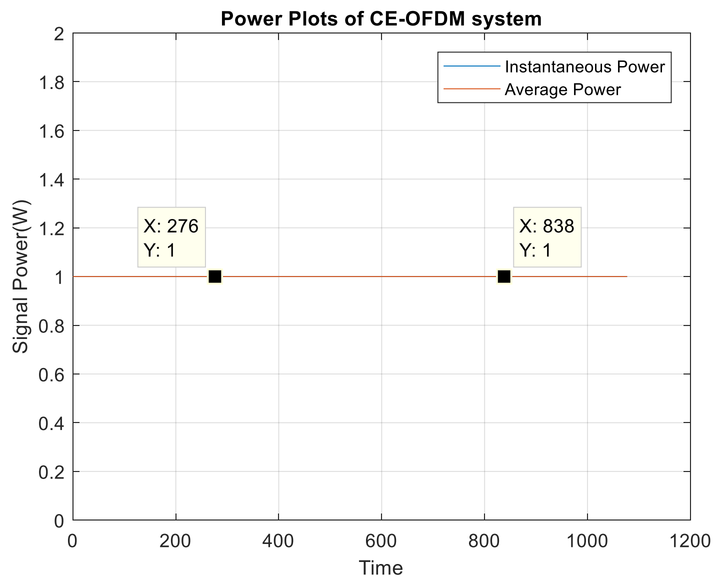

Figure 10 Signal Power analysis of an CE-OFDM System (data size = 1026, FFT size = 1078)

As shown in figure 9, my simulation result matches with the theoretical anticipation of the PAPR ratio as a value of 1.

Although CE-OFDM is a rigorous solution to the PAPR issue of the OFDM system, it has a minor flaw of poor BER performance at some design choices as we will discover in the next section.

IV. Systems BER performance over an AWGN channel

In this section, we will show and compare the BER simulation results from the systems that have been described above.

A. OFDM BER performance

We will examine the OFDM BER performance under different digital symbol modulation schemes using simulation results.

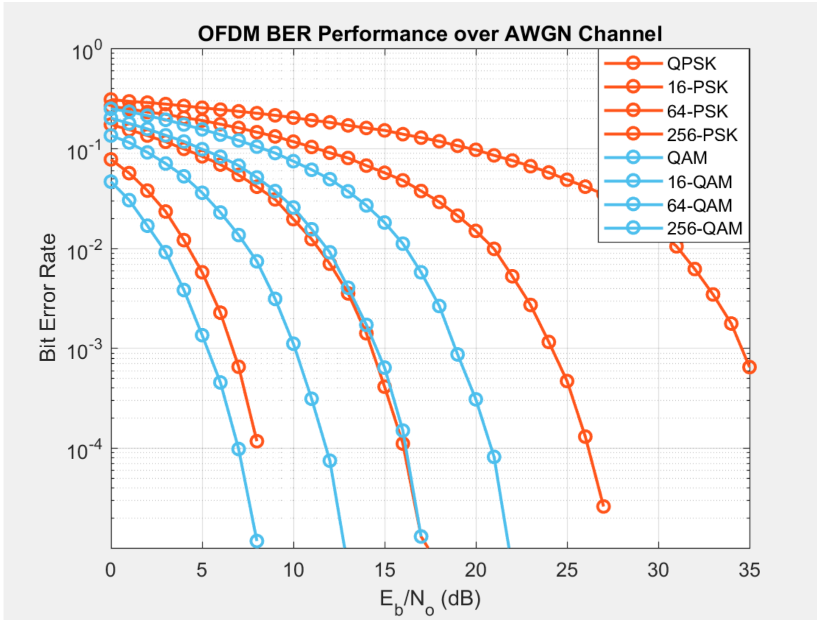

Figure 11 OFDM systems using PSK and QAM BER comparison

As shown in Figure 11, we can first see that each digital modulation scheme trades off performance and symbol rate by its nature. Secondly, when they are used in an OFDM system, PSK symbol mapping techniques clearly yields a higher bit error rate. Even further, the 64-QAM gives a similar BER performance as the 16-PSK mapping. However, 64-QAM maps 64 data into one symbol, which means a 4 times data rate over 16-PSK. This essentially indicates a choice of 64-QAM over 16-PSK when we are designing for a high data rate system.

B. CE-OFDM vs. OFDM

We analyzed and compared the bit error rate performance of an OFDM and CE-OFDM system with both using the 8-PAM digital modulation scheme. Note that for a simplicity, we will refer to the value of $2\pi h$ in the figure 11 as modulation index of a CE-OFDM system. However, one should keep in mind that the 2π term is a conversion to radians. The modulation index solely refers to constant h except for this section.

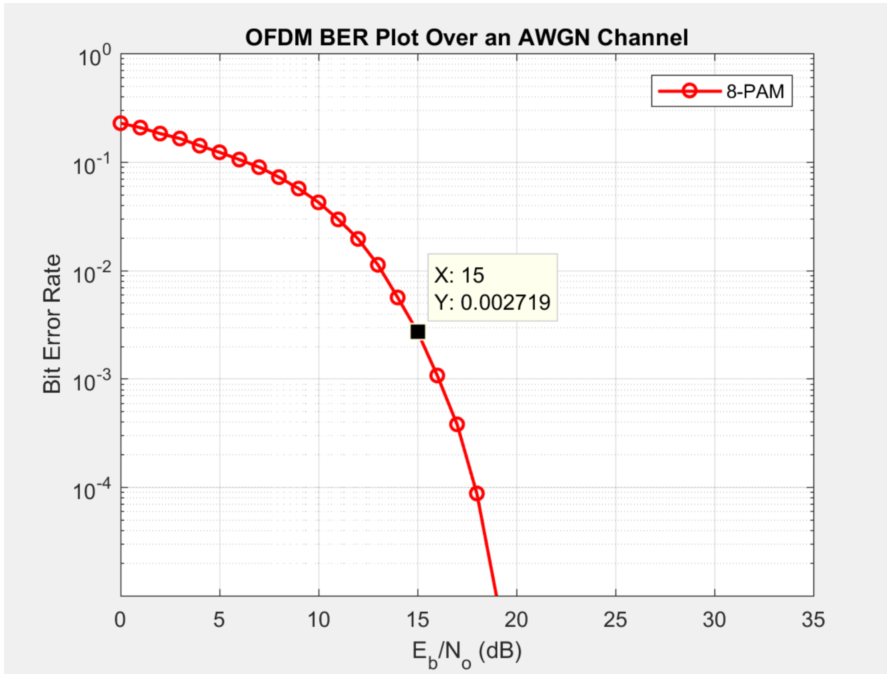

Figure 12. 8-PAM OFDM BER Plot (data size = 1026, FFT size = 1024)

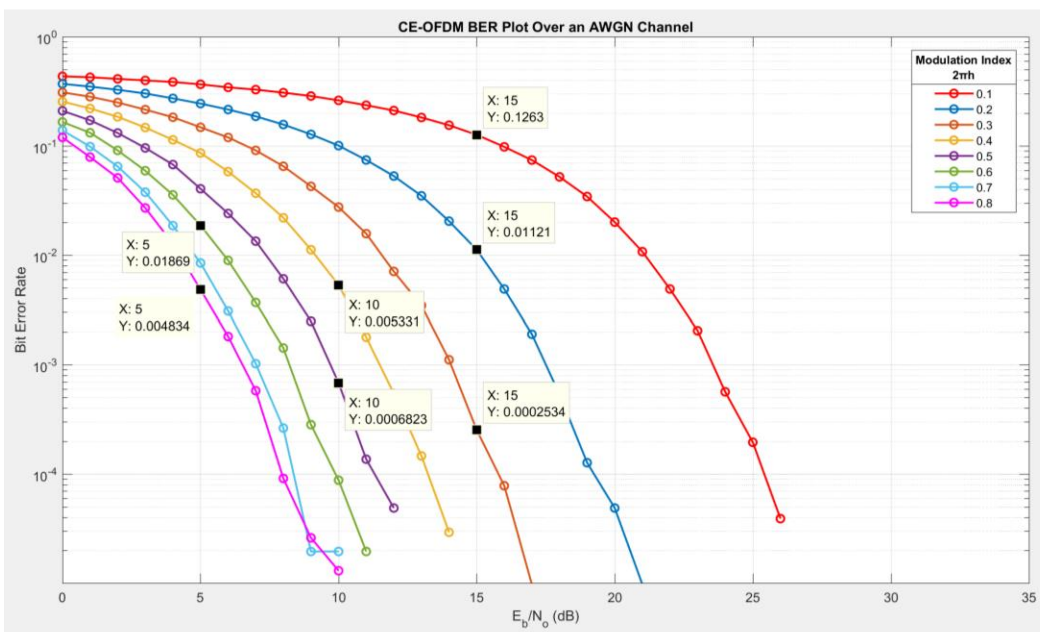

Figure 13 CE-OFDM BER Plot in an AWGN Channel (8-PAM, data size = 1026, FFT size = 1078)

As shown in figure 10 and 11, we picked the energy per bit to noise ratio as 15 dB and compared the data.

In an OFDM system, it shows that the bit error rate maintains a value of 0.00272. However, in an CE-OFDM system, the bit error rate decreases as we increase the modulation index constant. At a modulation index value of 0.3, we can clearly see the bit error rate drops down to 0.000253, which is 10 times smaller than the BER of a conventional OFDM system at a 15 dB EbNo.

Moreover, as we shall see a more rigorous analysis in [6], the CE-OFDM system performs just as the phase modulation scheme in the analogue communication systems. The CE-OFDM system not only provides a better BER performance at a higher modulation index, but also gives designer a freedom to tradeoff between the spectrum efficiency and the performance.

V. Conclusions

In this report, we have shown that the CE-OFDM provides a perfect solution to the PAPR issue of OFDM system and effectively reduces PAPR to 1 of all time. We thoroughly went through the concepts of multicarrier modulation schemes, especially for the OFDM system. Furthermore, a comparison between the performance of CE-OFDM system and OFDM system has been developed.

The OFDM system shows a significant mitigation to the inter-symbol interference in a multi-path channel. By a cyclic prefix insertion, the OFDM system can even be immune to the ISI, however, this requires a tradeoff to wider bandwidth. Simulation wise, this can be proven by changing the AWGN to a Rayleigh fading channel in the OFDM system script appears in the appendix. The feature of a multicarrier modulation technique in the OFDM system also reveals a way to further approach the least spectral efficiency of systems. Moreover, we see the OFDM system results in a high PAPR value, which limits the choices of amplifiers and increases the cost. This leads our interest toward finding a more energy-efficient waveform.

The Constant Envelope variation of the OFDM system creates a perfect solution to the PAPR issue. Both theoretical and observational results have revealed that CE-OFDM has a PAPR of 0 dB. Even better is the minor modification from an OFDM system to a CE-OFDM system, however, this requires a doubled input data size. Utilizing a CE-OFDM system also sacrifices a whole imaginary axis from modulation for creating a real phase of an exponential. This indicates the CE-OFDM system is less spectral efficient than the OFDM system in its nature. Although less spectral efficiency is usually a poor design, CE-OFDM do create a freedom to the system designer by trading off the spectral efficiency with performance. On the other hand, since CE-OFDM is a more complex system compared to OFDM system, the results of investigation towards certain techniques can become enormous and beneficial.

Acknowledgements

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Thanks to Seungmo Kim on helping me of the research process and the patience on mentoring all my questions. I have learned a lot from you.

VI. References

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Appendix

1. Simplified OFDM System MATLAB Script

```
%% Initialization
clear all; clc; close all;
%% Random Signal Generation
L = 1026; %num of data 1.6M %516
t = 0:1:L-1;
%generate random signal data
p_data = round(rand(L,1)); %parallel data
s_data = reshape(p_data,1,L); %serial data

%% Transmitter - Symbol Mapping using Digital Modulation
modulationMethod = 'PAM';
modulationOrder = 8;
if(strcmp('PSK',modulationMethod))
    H = comm.PSKModulator();
    H.BitInput = true;
    H.ModulationOrder = modulationOrder;
    H.PhaseOffset = 0;

    iH = comm.PSKDemodulator();
    iH.BitOutput = true;
    iH.ModulationOrder = modulationOrder;
    iH.PhaseOffset = 0;

elseif (strcmp('QAM',modulationMethod))
    H = comm.RectangularQAMModulator();
    H.BitInput = true;
    H.ModulationOrder = modulationOrder;
    H.NormalizationMethod = 'Peak power';
    H.PeakPower = sqrt(log2(modulationOrder));

    iH = comm.RectangularQAMDemodulator();
    iH.BitOutput = true;
    iH.ModulationOrder = modulationOrder;
    iH.NormalizationMethod = 'Peak power';
    iH.PeakPower = sqrt(log2(modulationOrder));

elseif (strcmp('PAM',modulationMethod))
    H = comm.PAMModulator();
    H.BitInput = true;
    H.ModulationOrder = modulationOrder;
    H.NormalizationMethod = 'Peak power';
    H.PeakPower = sqrt(log2(modulationOrder));
```

```
    iH = comm.PAMDemodulator();
    iH.BitOutput = true;
    iH.ModulationOrder = modulationOrder;
    iH.NormalizationMethod = 'Peak power';
    iH.PeakPower = sqrt(log2(modulationOrder));
end
p_mod = step(H,p_data);

%% OFDM parameters
numFFT = 1024;                                  % FFT Size
scSpacing = 10.416e3;                              % subcarrier
spacing (Hz): initial = 1.5 KHz
Ts = 1/scSpacing;                               % (sec): symbol time
Fs = numFFT*scSpacing;                          % (Hz): sampling rate
CPLengthT = 24e-6;                             % CP Length (unit: sec)
numCP = floor((CPLengthT/Ts)*numFFT);           % (unit: samples)

%% Transmitter - OFDM transmitted signal
pts = length(p_mod);
if pts>= numFFT
    a = pts/numFFT;
    pts = ceil(a)*numFFT;
    p_td = zeros(pts,1);
    for i = 1:a
        if(i == 1)
            p_td(1:numFFT) = ifft(p_mod(1:numFFT),numFFT);
        else
            p_td(numFFT*(i-1)+1:numFFT*i) = ...
            ifft(p_mod(numFFT*(i-1)+1:numFFT*i),numFFT);
        end
    end
    if(p_td(end)~= p_td(numFFT*i))
            p_td(numFFT*i+1:end) = ...
            ifft(p_mod(numFFT*i+1:end),numFFT); % ifft to time domain
    end
else
    pts = numFFT;
    p_td = ifft(p_mod,numFFT);
end

p_td = sqrt(numFFT) .* p_td;
s_td = reshape(p_td, 1,pts); % parallel to serial conversion
s_cyc = [s_td(end-numCP+1:end),s_td]; % adding cyclic prefix in serial
time domain

%% Noise Model - AWGN
% Received SNR
EbNodBStep = 1;                                    % (dB)
EbNodBMax = 35;                                    % (dB)
EbNodB = 0:EbNodBStep:EbNodBMax;
```

```
% BER initialization
BER = ones(1,length(EbNodB));
itmax = 100;
for ii = 1:length(EbNodB)
    BER_tmp = 0;
    for it = 1:itmax
N = comm.AWGNChannel;
N.NoiseMethod = 'Signal to noise ratio (Eb/No)';
N.BitsPerSymbol = log2(modulationOrder);
N.EbNo = EbNodB(ii);
n_s_cyc = step(N,s_cyc);

%% Receiver - OFDM demodulated signal
r_cyc = n_s_cyc;
rs_td = r_cyc(1+numCP:end); %remove cyclic prefix from serial time
data
rp_td = reshape(rs_td, length(rs_td),1); %serial to parallel
conversion
rpts = length(rp_td);
rp_mod = zeros(rpts,1);
for iter = 0:numFFT:rpts-1
    rp_mod(iter+1:iter+numFFT) =
fft(rp_td(iter+1:iter+numFFT),numFFT);%FFT to frequency domain
end
rp_mod = rp_mod./sqrt(numFFT);

%% Receiver - Symbol Demapping using Digital Demodulation
rp_data = step(iH,rp_mod); %demapped parallel data

%% Receiver - Extract original signal
r_data = reshape(rp_data,1,length(rp_data)); %Parallel to serial
conversion

%% Analysis - BER,PAPR Plot
errorRate = comm.ErrorRate();
errors = errorRate(p_data,rp_data(1:length(p_data)));
% errors = errorRate(p_data,n_p_data);
BER_tmp = BER_tmp+errors(1);
    end
    BER(ii) = BER_tmp/ itmax;
end
% BER Plot
figure;
semilogy(EbNodB,BER,'o-r', 'Linewidth', 1.5);
grid on
hold on
xlabel('E_b/N_o (dB)')
ylabel('Bit Error Rate')
xlim([0 EbNodBMax])
ylim([1e-5 1e0]);
%PAPR Plot
```

```
mean = s_cyc*s_cyc'/length(s_cyc);
t =0:1:length(s_cyc)-1;
meanSquareValue = zeros(1,length(t));
for i = 1:length(t)
    meanSquareValue(i)= mean;
end
figure;
plot(t,s_cyc.*conj(s_cyc));
grid on;
hold on;
plot(t,meanSquareValue);
title('PAPR Plot of an OFDM System');
xlabel('time');
ylabel('Signal Power (W)');
legend('PAPR','Average Power');
```

2. *Simplified CE-OFDM System MATLAB Script*

```
%% Initialization
clear all; clc; close all;
%% Random Signal Generation
L = 1026; %num of data 1.6M %516
t = 0:1:L-1;
%generate random signal data
p_data = round(rand(L,1)); %parallel data
s_data = reshape(p_data,1,L); %serial data

%% Transmitter - Symbol Mapping using Digital Modulation
modulationOrder = 4;
modulationMethod = 'QAM';
if strcmpi(modulationMethod, 'QAM')
    H = comm.RectangularQAMModulator();
    H.BitInput = true;
    H.ModulationOrder = modulationOrder;
    H.NormalizationMethod = 'Peak power';
    H.PeakPower = sqrt(log2(modulationOrder));

    iH = comm.RectangularQAMDemodulator();
    iH.BitOutput = true;
    iH.ModulationOrder = modulationOrder;
    iH.NormalizationMethod = 'Peak power';
    iH.PeakPower = sqrt(log2(modulationOrder));

elseif strcmpi(modulationMethod,'PAM')
    H = comm.PAMModulator();
    H.BitInput = true;
```

```
    H.ModulationOrder = modulationOrder;
    H.NormalizationMethod = 'Peak power';
    H.PeakPower = sqrt(log2(modulationOrder));

    iH = comm.PAMDemodulator();
    iH.BitOutput = true;
    iH.ModulationOrder = modulationOrder;
    iH.NormalizationMethod = 'Peak power';
    iH.PeakPower = sqrt(log2(modulationOrder));
end
p_mod = step(H,p_data);
%% Creating complex conjuagte, symmetrix data vector
numCP = 0;
N_ = 2*length(p_mod);      %Number of subcarriers
Nzp = 50;      %number of zero padding
if(Nzp >0)
    zp = zeros(Nzp,1);       %create vector for zero padding
    p_mod_sym = [0;p_mod;zp;0;conj(flipud(p_mod))];   %symmetry vector
for ifft
else
    p_mod_sym = [0;p_mod;0;conj(flipud(p_mod))];
end
Nfft = length(p_mod_sym);                 %number of FFT
p_td = sqrt(Nfft).*ifft(p_mod_sym,Nfft); %Time domain signal
s_td = reshape(p_td, 1, length(p_td));  %S/P

%% CE-OFDM parameters
Cnorm = sqrt(2/(N_*var(p_mod)));%(modulationOrder^2-1)/3)); %Cnorm =
sqrt(2/(N*SymbolVariance^2))
% for KK = 1.0:0.1:1.5
K = 0.4;    %2pi*h

%% CE-OFDM transmitter - Phase modulator and CP+
syms a
b = double(solve(K^2 == var(K*Cnorm*s_td*a),a));
ce_td = exp(1i*K*Cnorm*s_td*b);  %CE-OFDM transmitted signal
% figure;
% t  = 0:1:length(ce_td)-1;
% plot(t,ce_td);
% xlim([0,20]);
s_cyc = [ce_td(end-numCP+1:end),ce_td];  %add cp

%% Noise Model - AWGN
% Received SNR
EbNodBStep = 1;                                    % (dB)
EbNodBMax = 35;                                    % (dB)
EbNodB = 0:EbNodBStep:EbNodBMax;
% BER initialization
BER = ones(1,length(EbNodB));
itmax = 150;
for ii = 1:length(EbNodB)
```

```
    BER_tmp = 0;
    for it = 1:itmax
        N = comm.AWGNChannel;
        N.NoiseMethod = 'Signal to noise ratio (Eb/No)';
        N.BitsPerSymbol = log2(modulationOrder);
        N.EbNo = EbNodB(ii);
        n_s_cyc = step(N,s_cyc);

%% CE-OFDM Receiver
        r_cyc = n_s_cyc;
        rs_td = r_cyc(1+numCP:end);                 %%remove cp
        rs_msg = zeros(1,length(rs_td));
        ss_msg = zeros(1,length(rs_td));
        for jj = 1:1:length(rs_msg)
            re = real(rs_td(jj));
            im = imag(rs_td(jj));
            if(im>0 && re<0)
                rs_msg(jj) = (atan(im/re)+pi)/K/Cnorm/b;
            elseif(im<0 && re<0)
                rs_msg(jj) = (atan(im/re)-pi)/K/Cnorm/b;
            else
                rs_msg(jj) = (atan(im/re))/K/Cnorm/b;
            end
        end
        rp_msg = reshape(rs_msg,length(rs_msg),1);
        p_msg_sym = fft(rp_msg,Nfft)./sqrt(Nfft);
        Lmsg = length(p_msg_sym)-Nzp-2;
        p_msg_mod = p_msg_sym(2:1+Lmsg/2);
        p_msg = step(iH,p_msg_mod);

%% Analysis - BER, Plot
        errorRate = comm.ErrorRate();
        errors = errorRate(p_data,p_msg);
        BER_tmp = BER_tmp+errors(1);
        it;
    end
    BER(ii) = BER_tmp/ itmax;
end
%BER Plot
figure;
semilogy(EbNodB,BER,'o-r', 'Linewidth', 1.5);
grid on
hold on
xlabel('E_b/N_o (dB)')
ylabel('Bit Error Rate')
xlim([0 EbNodBMax])
ylim([1e-5 1e0]);

mean = s_cyc*s_cyc'/length(s_cyc);
t =0:1:length(s_cyc)-1;
meanSquareValue = zeros(1,length(t));
```

```
for i = 1:length(t)
    meanSquareValue(i)= mean;
end

%PAPR Plot
figure;
plot(t,s_cyc.*conj(s_cyc));
grid on;
hold on;
title('Power Plots of CE-OFDM system');
xlabel('Time');
ylabel('Signal Power(W)');
legend('Instantaneous Power','Average Power');
plot(t,meanSquareValue);
ylim([0,2]);
```