Vp160 Recitation Class

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Overview

1. Before We Start
2. Review
   - Notations and Units
   - Uncertainty and Significant Figures
   - Back-of-the-envelope Calculation
   - Vectors and Basic Vector Operations
   - Coordinate Systems
   - 1D Kinematics
3. Exercises

Content of a Recitation Class

- A quick review (some derivation will be repeated in class)
- Some exercises
- The key points in exercises and exams (important!)

Notations and Units

Scientific Notation

- In the form of $a \times 10^n (1 \leq |a| < 10)$
- Used to represent an extremely large or small number with the required significant figures
- e.g. $123456 = 1.2 \times 10^6$

Unit Prefixes and Conversion

- Add a prefix to the given unit to measure in a different scale.

p	n	μ	m	c	k	M
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^{3}	10^{6}

- The procedure of a unit conversion is as follows:

$$1000 \text{ m} = 1000 \text{ m} \times \frac{1 \text{ km}}{1000 \text{ m}} = 1 \text{ km}.$$

Uncertainty and Significant Figures

Uncertainty

- Because of limitations of measurement devices, imperfect measurement procedures and randomness of environmental conditions, as well as human factors related to the experimenter himself, no measurement can ever be perfect. Its result may therefore only be treated as an estimate of what we call the "exact value" of a physical quantity. The experiment may both overestimate and underestimate the value of the physical quantity, and it is crucial to provide a measure of the error, or better uncertainty, that a result of the experiment carries (cited from *Introduction to Measurement Data Analysis* in Vp141).
- The detailed calculation will be encountered in Vp141. The principles of uncertainty analysis will be explained in Ve401.

Uncertainty and Significant Figures

Significant Figures

- The general rules follow what you have learned in Vc211.
- e.g. 0.0109 (3) 5.2×10^6 (2)
- A crucial thing in writing Vp141 reports is the significant figures of the uncertainty that you calculate! Experimental uncertainty should almost always **be rounded to one significant figure**. The only exception is when the uncertainty (if written in scientific notation) **has a leading digit of 1 and a second digit should be kept**.
- e.g. the average of 4 masses is 1.2345 g and the standard deviation is 0.323 g, so the uncertainty should be written as 0.3 g. The value should be written as (1.2 ± 0.3) g. The exception is when the uncertainty (if written in scientific notation) has a leading digit of 1 and a second digit should be kept. For example, (1.234 ± 0.172) g should be written as (1.23 ± 0.17) g (cited from the same source as in the previous slide).

Back-of-the-envelope Calculation

Definition

A quick estimation of some physical quantities

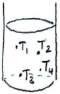
Comment

You should have a basic idea of approximating something such as the order of magnitude when doing exercises. It may serve as a sanity check.

Vectors and Basic Vector Operations

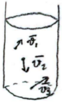
Vectors

Example



local temperature of a liquid

SCALAR



velocities of objects floating in a liquid

VECTOR

number (magnitude) + direction

Notation: $\vec{u}$, $\bar{u}$, $\underline{u}$

length of $\bar{u}$: $|\bar{u}| = u$
(magnitude)

Examples: velocity, force, momentum, electric / magnetic field, angular velocity

Vector Operations

Addition and Subtraction

"Parallelogram Rule"

Multiplication by Scalar

With the same or the opposite direction

Scalar Product

$\mathbf{u} \cdot \mathbf{w} = |\mathbf{u}||\mathbf{w}| \cos\theta$, θ is the smaller angle between the two vectors with their **starting points overlapped**.

Vector Product

- $\mathbf{u} \times \mathbf{w} = |\mathbf{u}||\mathbf{w}| \sin\theta$
- The product is a vector so we should judge its direction by "Right Hand Rule."
- Also take a look at the properties of vector product.

Coordinate Systems

Cartesian Coordinates

Dot product

$$\bar{u} = u_x \hat{n}_x + u_y \hat{n}_y + u_z \hat{n}_z = (u_x, u_y, u_z)$$

$$\bar{w} = w_x \hat{n}_x + w_y \hat{n}_y + w_z \hat{n}_z = (w_x, w_y, w_z)$$

$$\bar{u} \circ \bar{w} = u_x w_x + u_y w_y + u_z w_z$$

(components with dot products of different versors vanish)

the 1st property of unit vectors

$\hat{n}_x \circ \hat{n}_y = 0, \ldots$

$\hat{n}_x \circ \hat{n}_x = 1, \ldots$

Note: $\bar{u} \circ \bar{u} = u_x^2 + u_y^2 + u_z^2 = u^2$

Vector product (cross product)

(*)

$$\bar{u} \times \bar{w} = (u_y w_z - u_z w_y)\hat{n}_x + (u_z w_x - u_x w_z)\hat{n}_y + (u_x w_y - u_y w_x)\hat{n}_z =$$

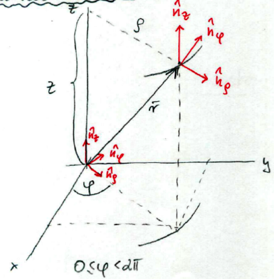
$$= \begin{vmatrix} \hat{n}_x & \hat{n}_y & \hat{n}_z \\ u_x & u_y & u_z \\ w_x & w_y & w_z \end{vmatrix}$$

→ "determinant" (will learn in math class)

Coordinate Systems

Cylindrical Coordinates

Cylindrical coordinates



coordinates: ϱ, φ, z

unit vectors: $\hat{n}_\varrho, \hat{n}_\varphi, \hat{n}_z$ (mutually orthogonal)

here versors are not fixed

unlike in Cartesian Coord. System

$$\begin{cases} \varrho = \sqrt{x^2+y^2} \\ \varphi = \arctan y/x \ ! \\ z = z \end{cases} \qquad \begin{cases} x = \varrho \cos\varphi \\ y = \varrho \sin\varphi \\ z = z \end{cases}$$

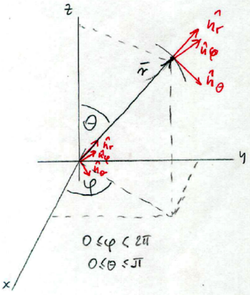
Position vector

$$\bar{r} = \varrho \hat{n}_\varrho + z \hat{n}_z$$

↳ carries information about the angle φ

Coordinate Systems

Spherical Coordinates



Coordinates : r, φ, θ

unit vectors : $\hat{n}_r, \hat{n}_\varphi, \hat{n}_\theta$ (mutually orthogonal)

$$\begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \varphi = \arctan y/x \quad ! \\ \theta = \arctan \dfrac{\sqrt{x^2+y^2}}{z} \quad ! \end{cases} \qquad \begin{cases} x = r \sin\theta \cos\varphi \\ y = r \sin\theta \sin\varphi \\ z = r \cos\theta \end{cases}$$

Position vector

$$\bar{r} = r\,\hat{n}_r$$

↳ carries info about angles φ, θ

Coordinate Systems

Comments

The formulae above are easy to derive. However, it's better to memorize the special 2D case of cylindrical and spherical coordinates, i.e. polar coordinates. It will be of great use in the future.

Basic Concepts

Positive Direction

Before solving the problem, please indicate the positive direction on the paper or remind yourselves of it by heart.

Average and Instantaneous Quantities

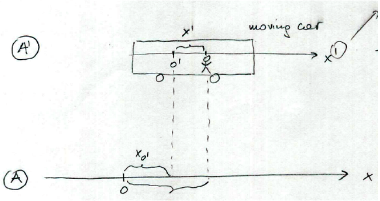
- $v_x(t) = \dfrac{x(t+\Delta t) - x(t)}{\Delta t}$ vs. $v_x(t) = \dfrac{\mathrm{d}x(t)}{\mathrm{d}t}$
- $a_x(t) = \dfrac{v(t+\Delta t) - v(t)}{\Delta t}$ vs. $a_x(t) = \dfrac{\mathrm{d}v(t)}{\mathrm{d}t}$
- Instantaneous case is when Δt approaches 0.
- **The derivative and anti-derivative relationship is very useful in calculation.**
- Notice the two kinds of notation of differentiation.

Basic Concepts

Speed and Velocity

Speed corresponds to the distance travelled, while velocity corresponds to the displacement.

Relative Motion



$$x = x_{o'} + x'$$

$$v_x = v_{o'x} + v'_x$$

$$a_x = a_{o'x} + a'_x$$

Dimension Analysis

Question

A simple pendulum consists of a light inextensible string AB with length L, with the end A fixed, and a point mass M attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of M, L and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

Answer

$[\mathrm{s}]=[\mathrm{kg}]^a[\mathrm{m}]^b[\mathrm{m}\cdot\mathrm{s}^{-2}]^c$, so we can obtain $a = 0,\ b = 1/2,\ c = -1/2$, which agrees with the knowledge that the period of a pendulum is only related to gravity and length.

Cross Product

Question

Find a vector $\mathbf{u}$ such that $(2, -3, 4) \times \mathbf{u} = (4, 3, -1)$.

Answer

$$\begin{cases} 2u_y + 3u_x = -1 \\ -3u_z - 4u_y = 4 \\ 4u_x - 2u_z = 3 \end{cases}$$

Unfortunately, there is no solution to the above equations. Why?
Here we use the property of cross product. Since $(2, -3, 4) \cdot (4, 3, -1) \neq 0$, we definitely can't find such a vector $\mathbf{u}$.

Underdamped Oscillation

Question

A particle moves along a straight line with velocity $v_x(t) = -\beta A\omega e^{-\beta t}\cos\omega t$, where A and ω are positive constants.

- What are the units of these constants?
- Find $a_x(t)$ and $x(t)$, assuming that $x(0) = 5$ [m].

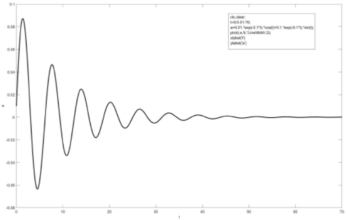
Answer

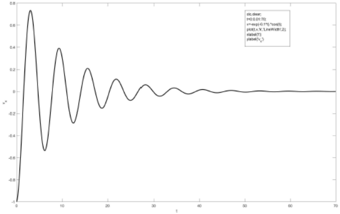
β and ω have a unit [s^{-1}], A has a unit [m·s].
$a_x(t) = \beta^2 A\omega e^{-\beta t}\cos\omega t + \beta A\omega^2 e^{-\beta t}\sin\omega t$.

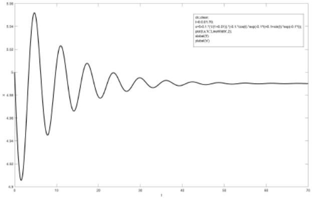
$$x(t) = 5 - \frac{\beta A\omega}{\beta^2+\omega^2}[\beta(1 - e^{-\beta t}\cos\omega t) + \omega e^{-\beta t}\sin\omega t].$$

Underdamped Oscillation

You will learn more about it in the near future. Now you can just have a general impression on these figures.







A Similar Question

Question

A particle moves along a straight line with acceleration $a_x = kv^3$, where v is the magnitude of velocity. At the instant of time $t = 0$, it has velocity $v_x(0) = v0$ and position $x(0) = 0$. Find $v(t), x(t)$, and $v(x)$.

Comment

How can we derive $v(x)$ without calculating $v(t)$ and $x(t)$?