UM–SJTU Joint Institute

Physics Laboratory
(Vp241)

Laboratory Report

Exercise 2
The Hall Probe: Characteristics and Applications

Name: Kang Jiaming ID: 518021911220 Group: 17

26 Oct. 2017

Contents

1 Introduction

The objective of this exercise is basically to use a Hall probe to verify the Hall effect and apply it to measure magnetic field.

1.1 Hall Effect

Hall effect basically illustrates that when a conducting sheet with current I going through it is placed in a magnetic field which is perpendicular to the current, an electric potential difference will be generated. As shown in Figure 1, the electric potential difference, which is called the Hall voltage U_{H}, is calculated to be

$$U_{\mathrm{H}} = R_{\mathrm{H}}\frac{IB}{d} = K_{\mathrm{H}}IB, \tag{1}$$

where R_{H} is called the Hall coefficient and $K_{\mathrm{H}} = R_{\mathrm{H}}/d$ is the sensitivity of the Hall element.

With Eq.(1), ithe magnetic field can be found using a Hall element, provided that the sensitivity of it and the current through it are known.

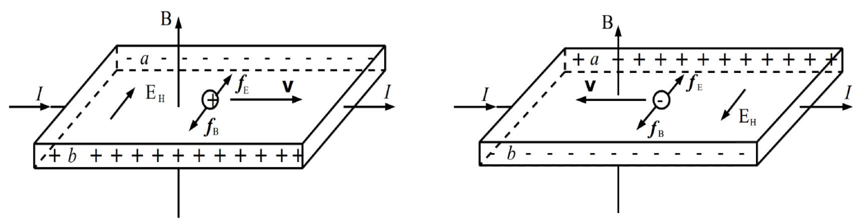

Figure 1: The principle of the Hall effect.

1.2 Integrated Hall Probe

However, the Hall voltage is usually very small, it should be amplified before the measurement. An integrated Hall probe, consisting of a Hall sensor, an amplifier, and a voltage compensator (Figure 2), can realize such functions. Hall probe satisfies the following equation:

$$B = \frac{U - U_0}{K_{\mathrm{H}}} \tag{2}$$

where U_0 is the output voltage when the magnetic field is zero.

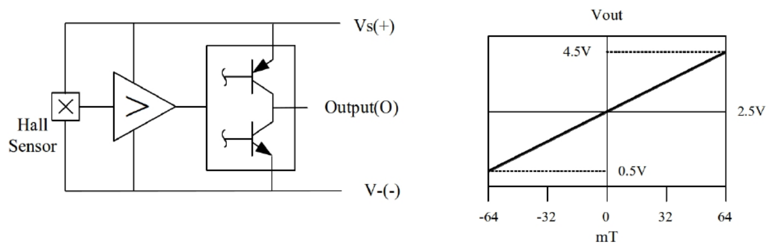

Figure 2: The integrated Hall probe SS495A (left). The relation between the output voltage U and the magnitude of the magnetic field B (right).

1.3 Magnetic Field Distribution Inside a Solenoid

Solenoid is a typical electromagnetic element. In this exercise we explore the magnetic field distribution of it with the Hall probe. The theoretical value of magnetic field distribution on the axis of a single layer solenoid can be calculated from the following formula:

$$B(x) = \mu_0 \frac{N}{L} I_{\mathrm{M}} \{ \frac{L+2x}{2[D^2+(L+2x)^2]^{\frac{1}{2}}} + \frac{L-2x}{2[D^2+(L-2x)^2]^{\frac{1}{2}}} \} = C(x) I_{\mathrm{M}}, \tag{3}$$

where N is the number of turns of the solenoid, L is its length, I_{M} is the current through the solenoid wire, and D is the solenoid's diameter. The magnetic permeability of vacuum is $\mu_0 = 4\pi \times 10^{-7}$H/m.

The solenoid used in this exercise has ten layers, and the magnetic field $B(x)$ for each layer can be calculated using Eq. (3). Then the net magnetic on the axis of the solenoid can be found by adding contributions due to all layers. The theoretical value of the magnetic field inside the solenoid with $I_{\mathrm{M}} = 0.1$ A is given in Table 1.

x [cm]	B [mT]	x [cm]	B [mT]
± 0.0	1.4366	± 8.0	1.4057
± 1.0	1.4363	± 9.0	1.3856
± 2.0	1.4356	± 10.0	1.3478
± 3.0	1.4343	± 11.0	1.2685
± 4.0	1.4323	± 11.5	1.1963
± 5.0	1.4292	± 12.0	1.0863
± 6.0	1.4245	± 12.5	0.9261
± 7.0	1.4173	± 13.0	0.7233

Table 1: Theoretical value of the magnetic field inside the solenoid.

2 Experimental Setup

The experimental setup (Figure 3) consists of an integrated Hall probe SS495A (Figure 4) with $K_{\mathrm{H}} = 31.25 \pm 1.25$ V/T (at the working voltage 5 V) or $K_{\mathrm{H}} = 3.125 \pm 0.125$ mV/G, a solenoid, a power supply, a voltmeter, a DC voltage divider, and a set of connecting wires.

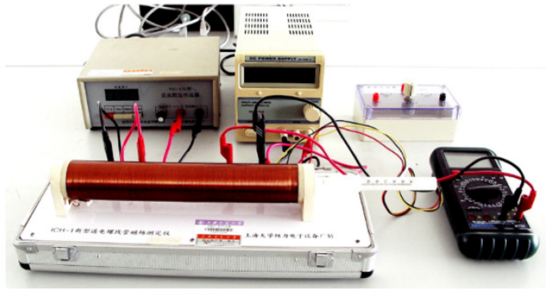

Figure 3: Measurement setup.

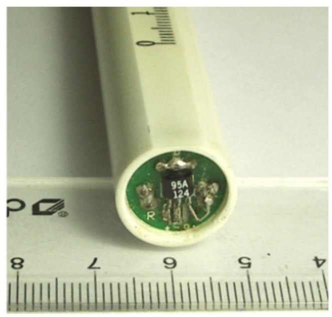

Figure 4: Measurement setup.

The precisions of the devices are shown in Table 2.

Instrument	Measured quantities	Uncertainties
Voltage source	Working voltage U_s	$0.5\%\,\mathrm{V}$
Multimeter	Output voltage U_0, U	$0.05\% + 6 \times 10^{-3}/10^{-4}\,\mathrm{V}$
Current source	Current I_0, I_M	$2\%\,\mathrm{mA}$
Graduated ruler	Distance	$0.05\,\mathrm{cm}$

Table 2: Information of measurement instruments.

3 Measurement Procedure

3.1 Relation Between Sensitivity $K_\mathbf{H}$ and Working Voltage $U_\mathbf{S}$

In this part, we explored the relation between sensitivity K_H and working voltage U_S by applying Eq.(2) and measuring the corresponding quantities.

First, we placed the integrated Hall probe at the center of the solenoid. Then we set the working voltage at 5 V and measured the output voltage U_0 ($I_\mathrm{M} = 0$) and U ($I_\mathrm{M} = 250$ mA). We also took the theoretical value of $B(x = 0)$ from Table 1 and calculated the sensitivity of the probe K_H by using Eq. (2). Then we measured K_H for different values of U_S (from 2.5 V to 10 V). Then we calculated $K_\mathrm{H}/U_\mathrm{S}$ and plotted the curve $K_\mathrm{H}/U_\mathrm{S}$ vs. U_S.

3.2 Relation Between Output Voltage U and Magnetic Field B

In this part, we explored the relation between the output voltage of the Hall probe and the magnetic field in which it is positioned by adjusting the current in the solenoid.

First, with $B = 0$, $U_\mathrm{S} = 5$ V, we connected the $2.4 \sim 2.6$ V output terminal of the DC voltage divider and the negative port of the voltmeter and adjusted the voltage until $U_0 = 0$. Next, we placed the integrated Hall probe at the center of the solenoid and measured the output voltage U for different values of I_M ranging from 0 to 500 mA, with intervals of 50 mA. Noticing the fact that the output voltage U is the amplified signal from U_H, I tried to explain the relation between $B(x = 0)$ and the Hall voltage U_H. Then the curve U vs. B is plotted and the sensitivity K_H is found out.The value is compared with the theoretical value given in the Apparatus section.

3.3 Magnetic Field Distribution Inside the Solenoid

In this part, we explore the magnetic field distribution inside a solenoid by adjusting the position of the Probe Hall and measuring the magnetic field at different positions.

The current in the solenoid is fixed to be $I_\mathrm{M} = 250$ mA, and thus the magnetic field distribution is fixed. Then the position of the Probe Hall is adjusted and the output voltage U and the corresponding position x are recorded. Then the curve $B = B(x)$ is plotted and compared with the theoretical value.

4 Results

4.1 Relation Between Sensitivity K_H and Working Voltage U_S

The measurement results of U_0 and U when $U_\mathrm{S} = 5.00$ V are shown in Table 3.

U_S [V] $\pm 0.5\%$ [V]	$U_0(I_\mathrm{M} = 0)$ [V] $\pm (0.05\% + 6 \times 10^{-3})$ [V]	$U(I_\mathrm{M} = 250\,\mathrm{mA})$ [V] $\pm (0.05\% + 6 \times 10^{-3})$ [V]
5.00 ± 0.03	2.476 ± 0.007	2.596 ± 0.007

Table 3: Data for U_0 and U with $U_\mathrm{S} = 5.00$ V.

According to the data in Table 1, we can obtain that when $I_\mathrm{M} = 100$ mA, $B(x = 0, I_\mathrm{M} = 100\,[\mathrm{mA}]) = 1.4366 \times 10^{-3}$ T. Eq. (3) implies that B is proportional to current I_M, and therefore,

$$B(x = 0, I_\mathrm{M} = 250\,[\mathrm{mA}]) = \frac{250}{100} \times 1.4366 \times 10^{-3} = 3.5915 \times 10^{-3}\ \mathrm{T}.$$

According to Eq. (2), the sensitivity of the probe K_H when $U_\mathrm{S} = 5.00$ V is then calculated as

$$K_\mathrm{H} = \frac{U - U_0}{B(x = 0, I_\mathrm{M} = 250\,[\mathrm{mA}])} = \frac{2.596 - 2.476}{3.5915 \times 10^{-3}} = 33.412 \pm 2.756\,[\mathrm{V/T}].$$

The measurement results of U_0 and U for different U_{S} are shown in Table 4. We calculate $K_{\mathrm{H}}/U_{\mathrm{S}}$ for each set of data to explore their relation. That is, for each set of data, we calculate

$$\frac{K_{\mathrm{H}}}{U_{\mathrm{S}}} = \frac{U - U_0}{BU_{\mathrm{S}}}.$$

Taking the first set of data as an example,

$$\frac{K_{\mathrm{H}}}{U_{\mathrm{S}}} = \frac{U - U_0}{BU_{\mathrm{S}}} = \frac{1.4519 - 1.3821}{3.5915 \times 10^{-3} \times 2.80} = 6.9 \pm 0.2\,[\mathrm{T}^{-1}].$$

The measurement results and the results of calculation of $K_{\mathrm{H}}/U_{\mathrm{S}}$ for each set of data are presented together in Table 4. A plot of the results $K_{\mathrm{H}}/U_{\mathrm{S}}$ vs. U_{S} using Origin is shown in Figure 5. The points in the plot indicates that the ratio of K_H to U_s decreases as U_s increases.

	U_{S} [V] $\pm 0.5\%$ [V]	U_0 [V] $\pm (0.05\% + 6 \times 10^{-3/-4}$ [V]	U [V] $\pm (0.05\% + 6 \times 10^{-3/-4}$ [V]	$K_{\mathrm{H}}/U_{\mathrm{S}}$ [T^{-1}]
1	2.80 ± 0.014	1.3821 ± 0.0013	1.4519 ± 0.0013	6.9 ± 0.2
2	3.20 ± 0.016	1.5817 ± 0.0014	1.6621 ± 0.0014	7.0 ± 0.2
3	3.60 ± 0.018	1.7811 ± 0.0015	1.8699 ± 0.0015	6.9 ± 0.2
4	4.00 ± 0.020	1.9795 ± 0.0016	2.0806 ± 0.0016	7.0 ± 0.2
5	4.40 ± 0.022	2.1793 ± 0.0017	2.286 ± 0.007	6.8 ± 0.5
6	4.80 ± 0.024	2.375 ± 0.007	2.491 ± 0.007	6.7 ± 0.6
7	5.20 ± 0.026	2.576 ± 0.007	2.697 ± 0.007	6.5 ± 0.5
8	5.60 ± 0.028	2.770 ± 0.007	2.900 ± 0.007	6.5 ± 0.5
9	6.00 ± 0.030	2.966 ± 0.007	3.100 ± 0.008	6.2 ± 0.5
10	6.40 ± 0.032	3.166 ± 0.008	3.306 ± 0.008	6.1 ± 0.5
11	6.80 ± 0.034	3.358 ± 0.008	3.501 ± 0.008	5.9 ± 0.5
12	7.20 ± 0.036	3.522 ± 0.008	3.699 ± 0.008	6.8 ± 0.4
13	7.60 ± 0.038	3.749 ± 0.008	3.895 ± 0.008	5.3 ± 0.4
14	8.00 ± 0.040	3.943 ± 0.008	4.092 ± 0.008	5.2 ± 0.4
15	8.40 ± 0.042	4.136 ± 0.008	4.286 ± 0.008	5.0 ± 0.4
16	8.80 ± 0.044	4.327 ± 0.008	4.480 ± 0.008	4.8 ± 0.4
17	9.20 ± 0.046	4.525 ± 0.008	4.682 ± 0.008	4.8 ± 0.3
18	9.60 ± 0.048	4.710 ± 0.008	4.868 ± 0.008	4.6 ± 0.3

Table 4: Data for U_0, U and $K_{\mathrm{H}}/U_{\mathrm{S}}$ for different U_{S}.

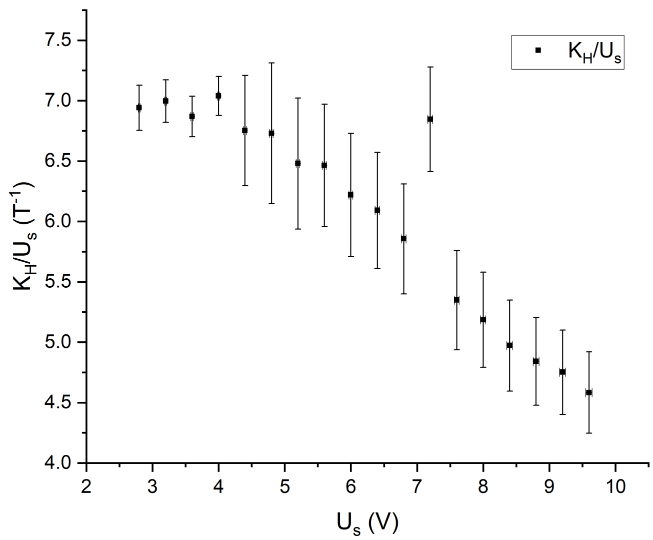

Figure 5: The $K_{\mathrm{H}}/U_{\mathrm{S}}$ vs. U_{S} relation.

4.2 Relation Between Output Voltage U and Magnetic Field B

According to Eq. (3), B is proportional to current I_{M}. We can obtain from Table 1 that when $I_{\mathrm{M}} = 100$ mA, $B(x=0, I_{\mathrm{M}} = 100\,[\mathrm{mA}]) = 1.4366 \times 10^{-3}$ T. Therefore, the theoretical value of the magnetic field is

$$B(x=0) = \frac{I_{\mathrm{M}}}{100} \times 1.4366 \times 10^{-3}.$$

Take the second set of data as an example,

$$B(x=0, I_{\mathrm{M}} = 50\,[\mathrm{mA}]) = \frac{1.4366 \times 10^{-3}}{50} \times I_{\mathrm{M}} = 0.718 \times 10^{-3}\,[\mathrm{T}].$$

It is noticed that the measured U is the amplified output of U_H and we supposed that $U = k \cdot U_H$. Then, theoretically, according to Eq. (2), we can derive that

$$B(x=0) = \frac{U - U_0}{K_{\mathrm{H}}} = \frac{U}{K_{\mathrm{H}}} = k \cdot \frac{U_{\mathrm{H}}}{K_{\mathrm{H}}},$$

where k is a constant. Therefore, $B(x=0)$ is supposed to be proportional to the Hall voltage U_{H}.

The experimental results are shown in Table 5. By applying linear fit to the I_{M} vs. U plot (Figure 6), the slope of the curve is then the measured sensitivity K_H, which is $0.03197/10^{-3} = 31.97\,\mathrm{V/T}$, with uncertainty $6 \times 10^{-4}/10^{-3} = 0.6\,\mathrm{V/T}$. The measurement result of the sensitivity is then $K_H = 32.0 \pm 0.6\,\mathrm{V/T}$.

	I_{M} [A] $\pm 2\%$ [A]	$B(x=0)$ $[10^{-3}\,\mathrm{T}]$	U [V] $\pm\,(0.05\% + 6\times 10^{-3/-4})$ [V]
1	0 ± 0	0 ± 0	0.000 ± 0.0006
2	0.050 ± 0.001	0.718 ± 0.014	0.02855 ± 0.0006
3	0.100 ± 0.02	1.44 ± 0.03	0.05004 ± 0.0006
4	0.150 ± 0.03	2.16 ± 0.04	0.07532 ± 0.0006
5	0.200 ± 0.04	2.87 ± 0.06	0.09897 ± 0.0006
6	0.250 ± 0.05	3.59 ± 0.07	0.1182 ± 0.0006
7	0.300 ± 0.06	4.31 ± 0.09	0.14301 ± 0.0006
8	0.350 ± 0.07	5.03 ± 0.10	0.16314 ± 0.0006
9	0.400 ± 0.08	5.75 ± 0.11	0.18792 ± 0.0006
10	0.450 ± 0.09	6.47 ± 0.13	0.21184 ± 0.0006
11	0.500 ± 0.10	7.18 ± 0.14	0.2319 ± 0.0006

Table 5: Measurement data for the I_{M} vs. U relation and the calculated data for $B(x=0)$.

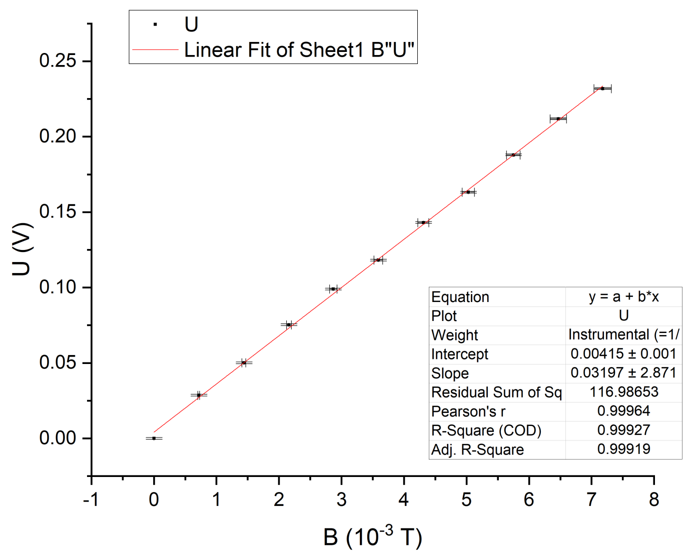

Figure 6: The linear fit of U vs. B relation.

4.3 Magnetic Field Distribution Inside the Solenoid

The measurement result of output voltage U and the corresponding position x are shown in Table 6. According to Eq.(2) and the measured value of K_H in section 4.2, $B(x)$ can be obtained from $B(x) = \frac{U}{K_{\mathrm{H}}} = \frac{U}{32.0}$. Take the first set of data as an example,

$$B(x) = \frac{U}{32.0} = \frac{0.00961}{32.0} = (0.30 \pm 0.02) \times 10^{-3} \text{ [T]}.$$

The $B(x)$ are calculated for each set of data and the results are shown in Table 7.

	x[cm]±0.05[cm]	U[V]±(0.05%+6 × 10^{-4})[V]		x[cm]±0.05[cm]	U[V]±(0.05%+6 × 10^{-4})[V]
1	0	0.0096 ± 0.0006	27	15.3	0.1195 ± 0.0006
2	0.2	0.0110 ± 0.0006	28	16.3	0.1196 ± 0.0006
3	0.4	0.0121 ± 0.0006	29	17.3	0.1196 ± 0.0006
4	0.6	0.0136 ± 0.0006	30	18.3	0.1195 ± 0.0006
5	0.8	0.0158 ± 0.0006	31	19.3	0.1194 ± 0.0006
6	1	0.0180 ± 0.0006	32	20.3	0.1190 ± 0.0006
7	1.3	0.0218 ± 0.0006	33	21.2	0.1187 ± 0.0006
8	1.6	0.0164 ± 0.0006	34	22.1	0.1183 ± 0.0006
9	2.1	0.0397 ± 0.0006	35	22.9	0.1178 ± 0.0006
10	2.5	0.0523 ± 0.0006	36	23.7	0.1169 ± 0.0006
11	3	0.0694 ± 0.0006	37	24.4	0.1159 ± 0.0006
12	3.6	0.0873 ± 0.0006	38	25.1	0.1140 ± 0.0006
13	1.2	0.0996 ± 0.0006	39	25.8	0.1117 ± 0.0006
14	1.9	0.1069 ± 0.0006	40	26.4	0.1084 ± 0.0006
15	5.6	0.1117 ± 0.0006	41	27	0.1026 ± 0.0006
16	6.3	0.1143 ± 0.0006	42	27.5	0.0951 ± 0.0006
17	7.1	0.1163 ± 0.0006	43	27.9	0.0863 ± 0.0006
18	7.9	0.1174 ± 0.0006	44	28.4	0.0709 ± 0.0006
19	8.8	0.1182 ± 0.0006	45	28.7	0.0590 ± 0.0006
20	9.7	0.1187 ± 0.0006	46	29	0.0496 ± 0.0006
21	10.7	0.1191 ± 0.0006	47	29.2	0.0432 ± 0.0006
22	11.7	0.1194 ± 0.0006	48	29.4	0.0374 ± 0.0006
23	12.7	0.1195 ± 0.0006	49	29.6	0.0322 ± 0.0006
24	13.7	0.1195 ± 0.0006	50	29.8	0.0284 ± 0.0006
25	14.7	0.1195 ± 0.0006	51	30	0.0244 ± 0.0006
26	15	0.1195 ± 0.0006			

Table 6: Data for the U vs. x relation

The theoretical curve of the magnetic field distribution inside the solenoid can be obtained from Eq.(3) and the data in Table 1, by multiplying the data in the table by $\frac{250}{100} = 2.5$, since we have set the current as 250 mA instead of 100 mA.

Then we plot the theoretical curve together with the measured value of the magnetic field distribution in Figure 7. The origin of the plot is set at the center of the solenoid, so 13 cm are subtracted from the x in the measurement data.

	x [cm] ± 0.05 [cm]	$B(x)$ $[10^{-3}$ T]		x [cm] ± 0.05 [cm]	$B(x)$ $[10^{-3}$ T]
1	0	0.30 ± 0.02	27	15.3	3.73 ± 0.07
2	0.2	0.34 ± 0.02	28	16.3	3.74 ± 0.07
3	0.4	0.38 ± 0.02	29	17.3	3.74 ± 0.07
4	0.6	0.42 ± 0.02	30	18.3	3.73 ± 0.07
5	0.8	0.49 ± 0.02	31	19.3	3.73 ± 0.07
6	1	0.56 ± 0.02	32	20.3	3.72 ± 0.07
7	1.3	0.68 ± 0.02	33	21.2	3.71 ± 0.07
8	1.6	0.51 ± 0.02	34	22.1	3.70 ± 0.07
9	2.1	1.24 ± 0.03	35	22.9	3.68 ± 0.07
10	2.5	1.63 ± 0.04	36	23.7	3.65 ± 0.07
11	3	2.17 ± 0.04	37	24.4	3.62 ± 0.07
12	3.6	2.73 ± 0.05	38	25.1	3.56 ± 0.07
13	1.2	3.11 ± 0.06	39	25.8	3.49 ± 0.07
14	1.9	3.34 ± 0.07	40	26.4	3.39 ± 0.07
15	5.6	3.49 ± 0.07	41	27	3.21 ± 0.06
16	6.3	3.57 ± 0.07	42	27.5	2.97 ± 0.06
17	7.1	3.63 ± 0.07	43	27.9	2.70 ± 0.05
18	7.9	3.67 ± 0.07	44	28.4	2.22 ± 0.05
19	8.8	3.69 ± 0.07	45	28.7	1.84 ± 0.04
20	9.7	3.71 ± 0.07	46	29	1.55 ± 0.03
21	10.7	3.72 ± 0.07	47	29.2	1.35 ± 0.03
22	11.7	3.73 ± 0.07	48	29.4	1.17 ± 0.03
23	12.7	3.73 ± 0.07	49	29.6	1.01 ± 0.03
24	13.7	3.73 ± 0.07	50	29.8	0.89 ± 0.03
25	14.7	3.73 ± 0.07	51	30	0.76 ± 0.02
26	15	3.73 ± 0.07			

Table 7: Data for the $B(x)$ vs. x relation.

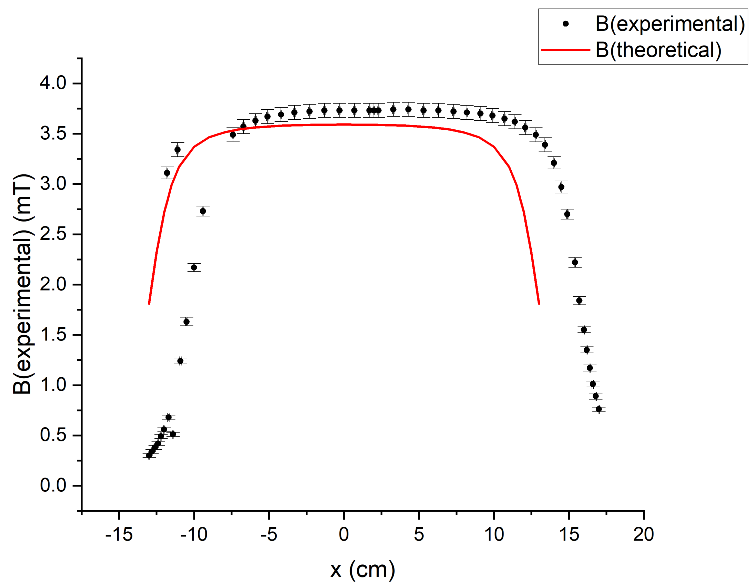

Figure 7: Measured and theoretical magnetic field distribution inside the solenoid.

5 Conclusions and Discussion

5.1 Relation Between Sensitivity $K_{\mathbf{H}}$ and Working Voltage $U_{\mathbf{S}}$

From Figure 5, the results of my experiment implies that the sensitivity decreases as working voltage increases.

However, theoretically, the sensitivity should increase as the working voltage does [1]. After searching for information, one reason that I come up with is that the temperature affects the sensitivity. As presented in [1], the sensitivity of SS495 increases significantly enough to affect the experimental result as temperature increases.

In view of this, I suggest that temperature should be controlled when conducting this experiment, that is, we turn on the current source for a short time and perform one or two set of measurement, and then cut the source, wait for a while and then turn on the source again to continue with the measurement.

5.2 Relation Between Output Voltage U and Magnetic Field B

In this part, the Pearson's r of the fitting is 0.99964, which implies that the output voltage is most likely to be linearly dependent on the magnetic field. Since the output voltage is 0 when magnetic field is zero, then we can deduce that the output voltage is proportional to the magnetic field.

Besides, the sensitivity we obtained from the plot is 32.0 V/T, this compares with the theoretical value 31.25 [1], with relative error

$$U_{rK_{\mathrm{H}}} = \frac{32.0 - 31.25}{31.25} \times 100\% = 2.4\,\%.$$

The experimental results agrees with the actual value.

One possible cause for the inaccuracy of the experiment is that Eq.(6) holds only "when the external magnetic field is not too strong" [2]. In this part, the current has been adjusted to up to 500 mA, which may result in non-proportional relation. Another cause may be that the input voltage of the amplifier exceeds the saturation value of the amplifier, which results in the inaccuracy of the output voltage.

5.3 Magnetic Field Distribution Inside the Solenoid

It can be seen from Figure 7 that the experimental value does not match the theoretical value in a satisfying range. However, it is observed that the theoretical curve can be obtained by moving the experimental curve to the left a little bit, ignoring the two obviously wrong sets of data. Inspired by this, I guess that one possible reason of the discrepancy is that the scale solenoid is inaccurate.

Despite the inaccuracy of our result, the variation trend of the curve agrees with the theoretical trend. The magnetic field increases rapidly as the distance from the center becomes smaller. After it reaches its maximum, it becomes nearly constant.

Some other comments and suggestions are also listed below:

1. The data displayed on the device is not stable, and we had to choose one data to record, which can lead to error. It is suggested that using apparatus of higher accuracy can reduce the uncertainty and inaccuracy of the experiment.

2. Generally, the magnetic field of the earth may affect the whole experiment.

6 Reference

[1] Specifications of SS495 provided by Honeywell Sensing and Control, posted on www.umjicanvas.com VP241 2019 Fall.
[2] VP241 Exercise 2: The hall probe: characteristics and applications, Shanghai Jiaotong University.

A Measurement Uncertainty Analysis

A.1 Uncertainty of Sensitivity $K_{\mathbf{H}}$ and Voltage Measurements

For Table 3, the uncertainties are calculated as

$$u_{U_{\mathrm{S}}} = 5.00 \times 0.5\% = 0.03 \text{ [V]},$$

$$u_{U_0} = 2.476 \times 0.05\% + 6 \times 10^{-3} = 0.007 \text{ [V]},$$
$$u_U = 2.596 \times 0.05\% + 6 \times 10^{-3} = 0.007 \text{ [V]}.$$

For $K_{\mathrm{H}} = \frac{U-U_0}{B}$, its uncertainty is

$$\begin{aligned} u_{K_{\mathrm{H}}} &= \sqrt{(\frac{\partial K_{\mathrm{H}}}{\partial U} u_U)^2 + (\frac{\partial K_{\mathrm{H}}}{\partial U_0} u_{U_0})^2} = \sqrt{(\frac{u_U}{B})^2 + (\frac{-u_{U_0}}{B})^2} \\ &= \sqrt{(\frac{0.007}{1.4366 \times 10^{-3} \times 250/100})^2 + (\frac{-0.007}{1.4366 \times 10^{-3} \times 250/100})^2} = 2.756 \text{ [V/T]}. \end{aligned}$$

For Table 4, the uncertainties of data for voltage measurements are calculated as follows. Take the first set of data as an example,

$$u_{U_{\mathrm{S}}} = 2.80 \times 0.5\% = 0.014 \text{ [V]},$$

$$u_{U_0} = 1.3821 \times 0.05\% + 6 \times 10^{-4} = 0.0013 \text{ [V]},$$
$$u_U = 1.4519 \times 0.05\% + 6 \times 10^{-4} = 0.0013 \text{ [V]}.$$

The uncertainty for $K_{\mathrm{H}}/U_{\mathrm{S}} = \dfrac{U - U_0}{BU_{\mathrm{S}}}$ is calculated as

$$\begin{aligned} u_{K_{\mathrm{H}}/U_{\mathrm{S}}} &= \sqrt{(\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial U} u_U)^2 + (\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial U_0} u_{U_0})^2 + (\frac{\partial K_{\mathrm{H}}/U_{\mathrm{S}}}{\partial U_{\mathrm{S}}} u_{U_{\mathrm{S}}})^2} \\ &= \sqrt{(\frac{u_U}{BU_{\mathrm{S}}})^2 + (\frac{-u_{U_0}}{BU_{\mathrm{S}}})^2 + (-\frac{U - U_0}{BU_{\mathrm{S}}^2} u_{U_{\mathrm{S}}})^2} \\ &= \sqrt{(\frac{u_U}{BU_{\mathrm{S}}})^2 + (\frac{-u_{U_0}}{BU_{\mathrm{S}}})^2 + (-\frac{U - U_0}{BU_{\mathrm{S}}^2} u_{U_{\mathrm{S}}})^2} \\ &= \sqrt{(\frac{0.0013}{1.4366 \times 10^{-3} \times 250/100 \times 2.80})^2 + (\frac{-0.0013}{1.4366 \times 10^{-3} \times 250/100 \times 2.80})^2 + (-\frac{1.4519 - 1.3821}{1.4366 \times 10^{-3} \times 250/100 \times 2}} \\ &= 0.2 \text{ [T}^{-1}\text{]}. \end{aligned}$$

The uncertainties of all other data in Table 4 are calculated in this way and the results are presented in Table 8.

	u_{U_S} [V]	u_{U_0} [V]	u_U [V]	$u_{K_\mathrm{H}/U_\mathrm{S}}$ [T^{-1}]
1	0.014	0.0013	0.0013	0.2
2	0.016	0.0014	0.0014	0.2
3	0.018	0.0015	0.0015	0.2
4	0.020	0.0016	0.0016	0.2
5	0.022	0.0017	0.007	0.5
6	0.024	0.007	0.007	0.6
7	0.026	0.007	0.007	0.5
8	0.028	0.007	0.007	0.5
9	0.030	0.007	0.008	0.5
10	0.032	0.008	0.008	0.5
11	0.034	0.008	0.008	0.5
12	0.036	0.008	0.008	0.4
13	0.038	0.008	0.008	0.4
14	0.040	0.008	0.008	0.4
15	0.042	0.008	0.008	0.4
16	0.044	0.008	0.008	0.4
17	0.046	0.008	0.008	0.3
18	0.048	0.008	0.008	0.3

Table 8: Uncertainties of data in Table 4.

A.2 Uncertainty of Input Current I_M, Output Voltage U and Magnetic Field B

Take the second set of data in Table 5 as an example.

The uncertainty for I_M is

$$u_{I_\mathrm{M}} = 50 \times 2\% = 1 \text{ [mA]} = 0.001 \text{ [A]}.$$

The uncertainty for U is

$$u_U = 0.02855 \times 0.05\% + 6 \times 10^{-4} = 0.0006 \text{ [V]}.$$

The uncertainty for $B = 0.014366 \times I_\mathrm{M}$ is

$$u_B = 0.014366 \times u_{I_\mathrm{M}} = 0.014366 \times 0.001 = 1.4 \times 10^{-5} \text{ [T]}.$$

The uncertainties of all other data in Table 5 are calculated in this way are the results are presented in Table 9.

	u_{I_M} [A]	u_B [T]	u_U [V]
1	0	0	0.0006
2	0.001	0.000014	0.0006
3	0.002	0.00003	0.0006
4	0.003	0.00004	0.0006
5	0.004	0.00006	0.0006
6	0.005	0.00007	0.0006
7	0.006	0.00009	0.0006
8	0.007	0.00010	0.0006
9	0.008	0.00011	0.0006
10	0.009	0.00013	0.0006
11	0.01	0.00014	0.0006

Table 9: Uncertainty of data in Table 5.

A.3 Uncertainty of Magnetic Field Inside the Solenoid Measurement

The uncertainty of position measurement is 0.05 cm.
As for the uncertainty of the output voltage, taking the first set of data as an example,

$$u_U = 0.00961 \times 0.05\% + 6 * 10^{-4} = 0.0006\ [V].$$

For the uncertainty of $B(x) = \dfrac{U}{K_\mathrm{H}}$,

$$u_B = \sqrt{(\frac{\partial B}{\partial U}u_U)^2 + (\frac{\partial B}{\partial K_\mathrm{H}}u_{K_\mathrm{H}})^2} = \sqrt{(\frac{u_U}{K_\mathrm{H}})^2 + (-\frac{U}{K_\mathrm{H}^2}u_{K_\mathrm{H}})^2}.$$

Taking the first set of data as an example,

$$u_B = \sqrt{(\frac{0.0006}{32.0})^2 + (-\frac{0.00961}{32.0} \times 0.6)^2} = 0.02 \times 10^{-3}\ [\mathrm{T}].$$

The uncertainties for all other sets of data are calculated and shown in Table 10.

	u_U [V]	$B(x)$ [10^{-3} T]		u_U [V]	$B(x)$ [10^{-3} T]
1	0.0006	0.02	27	0.0006	0.07
2	0.0006	0.02	28	0.0006	0.07
3	0.0006	0.02	29	0.0006	0.07
4	0.0006	0.02	30	0.0006	0.07
5	0.0006	0.02	31	0.0006	0.07
6	0.0006	0.02	32	0.0006	0.07
7	0.0006	0.02	33	0.0006	0.07
8	0.0006	0.02	34	0.0006	0.07
9	0.0006	0.03	35	0.0006	0.07
10	0.0006	0.04	36	0.0006	0.07
11	0.0006	0.04	37	0.0006	0.07
12	0.0006	0.05	38	0.0006	0.07
13	0.0006	0.06	39	0.0006	0.07
14	0.0006	0.07	40	0.0006	0.07
15	0.0006	0.07	41	0.0006	0.06
16	0.0006	0.07	42	0.0006	0.06
17	0.0006	0.07	43	0.0006	0.05
18	0.0006	0.07	44	0.0006	0.05
19	0.0006	0.07	45	0.0006	0.04
20	0.0006	0.07	46	0.0006	0.03
21	0.0006	0.07	47	0.0006	0.03
22	0.0006	0.07	48	0.0006	0.03
23	0.0006	0.07	49	0.0006	0.03
24	0.0006	0.07	50	0.0006	0.03
25	0.0006	0.07	51	0.0006	0.02
26	0.0006	0.07			

Table 10: The uncertainties of U and B.

B Data Sheet

Please find the original data sheet at the end of this report.

UM-SJTU Physics Laboratory Vp241
Data Sheet (Exercise 2)

Name: [illegible] Student ID: 518021911220

Name: ________ Student ID: ________

Group: 17 Date: Oct.25.2019

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with a pencil or modified with a correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used.
You are required to hand in the original data with your lab report, so please keep the data sheet properly.

5V $+6\times10^{-3}/6\times10^{-4}$ $+6\times10^{-3}/6\times10^{-4}$

U_S [V] ± 0.5% [V]	$U_0(I_M = 0)$ [V] ± 0.05% [V]	$U(I_M = 250$ mA) [V] ± 0.05% [V]
5V	2.4786	2.596

Table 1. Data for U_0 and U with $U_S = 5$ V.

中心；
双≈13cm

2.8V ~ ~~10~~V，必须<10!!
不能行10!!

$+6\times10^{-3}/6\times10^{-4}$ $+6\times10^{-3}/6\times10^{-4}$

	U_S [V] ± 0.5% [V]	U_0 [V] ± 0.05% [V]	U [V] ± 0.05% [V]
1	2.80	1.3821	1.4519
2	3.20	1.5817	1.6621
3	3.60	1.7811	1.8699
4	4.00	~~1.980~~ 1.9795	2.0806
5	4.40	2.1793	2.286
6	4.80	2.375	2.491
7	5.20	2.576	2.697
8	5.60	2.770	2.900
9	6.00	2.966	3.100
10	6.40	3.166	3.3056
11	6.80	3.358	3.501
12	7.20	3.552	3.699
13	7.60	3.749	3.895
14	8.00	3.943	4.092
15	8.40	4.136	4.286
16	8.80	4.327	4.480
17	9.20	4.525	4.682
18	9.60	4.710	4.868

Table 2. Data for U_0 and U with different U_S.

Instructor's signature: B

$+6\times10^{-3}/6\times10^{-4}$

	I_M [mA] ± 2% [mA]	U [V] ± 0.05% [V]
1	0	2.869 0
2	50	28.55 ~~2.869~~ $\times10^{-3}$
3	100	50.04×10^{-3}
4	150	75.32×10^{-3}
5	200	98.97×10^{-3}
6	250	118.20×10^{-3}
7	300	143.01×10^{-3}
8	350	163.14×10^{-3}
9	400	187.92×10^{-3}
10	450	211.84×10^{-3}
11	500	0.2319

Table 3. Measurement data for the I_M vs. U relation.

Instructor's signature: B

Uncertainty: ① 地磁场
，周围磁场（可去除的）
② U_s: 4.99~5.00~5.01 对实验不确定
③ 导线电阻 $+6\times10^{-3}/6\times10^{-4}$

$+6\times10^{-3}/6\times10^{-4}$

	x [cm] ± 0.05 [cm]	U [mV] ± 0.05% [mV]		x [cm] ± 0.05 [cm]	U [mV] ± 0.05% [mV]
1	0	9.671	27	15.3	119.47
2	0.2	11.00	28	16.3	119.56
3	0.4	12.12	29	17.3	119.64
4	0.6	13.58	30	18.3	119.48
5	0.8	15.80	31	19.3	119.36
6	1.0	18.02	32	20.3	119.03
7	1.3	21.80	33	21.2	118.67
8	1.6	26.44	34	22.1	118.33
9	2.1 ~~2.9~~ ~~2.0~~ ~~1.9~~	39.74	35	22.9	117.77
10	2.5 ~~2.5~~ ~~2.3~~	52.27	36	23.7	116.90
11	3.0 ~~3.0~~ ~~2.7~~	69.40	37	24.4	~~116.98~~ 115.88
12	3.6 ~~3.2~~ ~~3.1~~	87.33	38	25.1	114.03
13	4.2 ~~3.9~~ ~~3.5~~	99.59	39	25.8	111.72
14	4.9 ~~4.3~~ ~~4.0~~	106.94	40	26.4	108.39
15	5.6 ~~4.9~~ ~~4.5~~	111.68	41	27.0	102.59
16	6.3 ~~5.6~~ ~~5.0~~	114.30	42	27.5	95.05
17	7.1 ~~6.3~~ ~~5.6~~	116.27	43	27.9	86.29
18	7.9 ~~7.0~~ ~~7.1~~ ~~6.2~~	117.39	44	28.4	70.90
19	8.8 ~~7.9~~ ~~6.8~~ ~~6.9~~	118.18	45	28.7	~~59.0~~ 58.97
20	9.7 ~~8.8~~ ~~7.6~~	118.69	46	29.0	49.60
21	10.7 ~~9.7~~	119.13	47	29.2	43.23
22	11.7 ~~10.7~~	119.44	48	29.4	37.40
23	12.7 ~~11.8~~	119.48	49	29.6	32.20
24	13.7 ~~12.9~~	119.50	50	29.8	28.42
25	14.7	119.46	51	30.0	24.43
26	15	119.48	52		

Table 4. Data for the U vs. x relation.

x: 0~13 cm

[illegible]

Instructor's signature: B

rev. 3-1