UM–SJTU Joint Institute

Physics Laboratory
(Vp241)

Laboratory Report

Exercise 4
Polarization of Light

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Contents

1 Introduction

The objective of this exercise is to study the polarization of light. Specifically, Malus' law will be verified and the way how half- and quarter-wave plates work in optical systems will be explored.

1.1 Polarization of Light

Light in nature is electromagnetic wave. The electric field vector **E** in the electromagnetic wave is referred to as the light vector. In the plane perpendicular to the propagation direction of a light wave, the light vector may have different directions along which its magnitude oscillates. If the light vector oscillates in all possible transverse direction, the light is called *natural light*. Otherwise, the light is called *polarized*. The light, for which the light vector maintains a certain oscillation direction, is called *linearly polarized* and the axis defining the direction is called the polarization axis. The light with the light vector direction rotating about the propagation direction, so that its endpoint traces a circle, is called *circularly polarized* light. If the vector traces an ellipse, the light said to be *elliptically polarized*.

1.2 Polarizer

Ploarizer is a device that can produce polarized light. It polarizes the light by only allowing th light polarized in a certain direction to pass through. With such a property, it can also be used to detect and analyze the polarization state of light (it is then called an analyzer).

1.3 Malus' Law

A fundamental law that the polarization of light obeys is as follows. Suppose that we have two polarizers arranged so that their planes are parallel — the left one plays the role of a polarizer, the other one is an analyzer (see Figure 1). Let the angle between their transmission directions (polarization axes) be 0. The light is incident normally on the polarizer and then continues to the analizer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{\text{light}} = I_{\text{light},0} \cos^2 \theta, \tag{1}$$

where $I_{\text{light},0}$ is the intensity of the linearly polarized light incident on the analyzer. This is the so-called Malus' law.

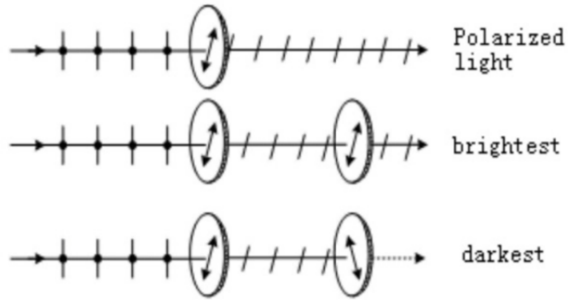

Figure 1: Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

1.4 Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

***This section is selected from [1], but the crucial points regarding this lab are highlighted here.**

Suppose that a linearly polarized light is incident morally on a crystal plate whose surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is α. Then the linearly polarized light is resolved into two waves: an e-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is

$$\Delta = (n_{\mathrm{e}} - n_{\mathrm{o}})d,$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda}(n_{\mathrm{e}} - n_{\mathrm{o}})d,$$

where λ is the wavelength, n_{e} is the refractive index for the extraordinary axis, and n_{o} is the refractive index for the ordinary axis. In a so-called positive crystal $\delta > 0$, whereas in a negative one $\delta < 0$.

As shown in Figure 4, when the light propagates through the crystal plate, the two components of the light vector are

$$\begin{aligned} E_x &= A_{\mathrm{o}} \cos \omega t \\ E_y &= A_{\mathrm{e}} \cos(\omega t + \delta), \end{aligned}$$

where $A_{\mathrm{e}} = A \cos \alpha$, $A_{\mathrm{o}} = A \sin \alpha$. Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_{\mathrm{o}}^2} + \frac{E_y^2}{A_{\mathrm{e}}^2} - 2\frac{E_x E_y}{A_{\mathrm{o}} A_{\mathrm{e}}} \cos \delta = \sin^2 \delta, \tag{2}$$

which is the equation of an ellipse.

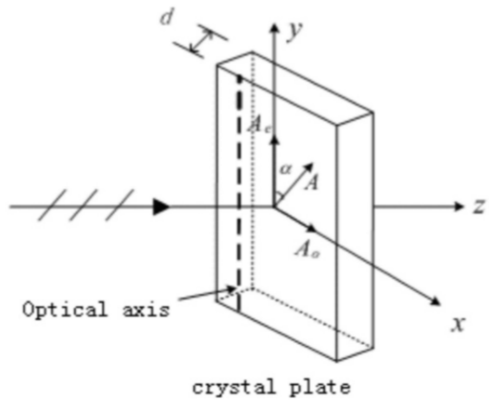

Figure 2: Linearly polarized light passing through a waveplate.

When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

- If $\Delta = k\lambda$, where $k = 0, 1, 2, ...$, the phase difference $\delta = 0$, and Eq. (2) reduces to

$$E_y = \frac{A_\mathrm{e}}{A_\mathrm{o}} E_x,$$

  which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a *full-wave* plate. The light goes through a full-wave plate without changing its polarization state.

- If $\Delta = (2k + 1)\lambda/2$, where $k = 0, 1, 2, ...$, the phase difference $\delta = \pi$, and Eq. (2) simplifies to

$$E_y = -\frac{A_\mathrm{e}}{A_\mathrm{o}} E_x.$$

  The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α**. A waveplate that satisfies the condition is called** ***1/2-wave plate*** **or** ***half-wave plate*****. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle** 2α**.** If $\alpha = \pi/4$, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

- Finally, if $\Delta = (2k + 1)\lambda/4$, where $k = 0, 1, 2, ...$, the phase difference $\delta = \pm\pi/2$, and Eq. (2) transforms into

$$\frac{E_x^2}{A_\mathrm{o}^2} \pm \frac{E_y^2}{A_\mathrm{e}^2} = 1.$$

  The transmitted light is elliptically polarized with a waveplate that satisfies the above condition is called a *1/4-wave plate* or a *quarter-waveplate* and is an important optical element in many polarization experiments.

If $A_\mathrm{e} = A_\mathrm{o} = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the *o*-wave and the *e*-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

- **if** $\alpha = 0$**, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate**;

- if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;
- **if $\alpha = \pi/4$, the transmitted light is circularly polarized**;
- **otherwise, the transmitted light is elliptically polarized**.

2 Experimental Setup

2.1 Apparatus

The experimental setup consists of a laser, a silicon photo-cell, a UT51 digital universal meter, two polarizers, a 1/2-wave plate, a 1/4-wave plate and an optical bench where the elements are placed.

The precision of the devices is shown in Table 1.

Instrument	Quantity	Precision
Scale on the element	Angle θ	2°
Universal meter	Current I	0.001 [μA]

Table 1: Precision of the measurement instruments.

3 Measurement Procedure

3.1 Apparatus Adjustment

1. Adjust the laser and the photo-cell so that the light can pass through the ϕ 6.0 aperture.
2. Placed the analyzer onto the optical bench, adjust its position so that the current detected is in the range $0.8 \sim 1.8$ μA.

3.2 Demonstration of Malus' Law

1. Place an analyzer between the polarizer and the photo-cell. Record this value as I_0.
2. Adjust the angle of the analyzer until the electric current reaches 0. Set this position as $\theta = 90^\circ$.
3. Rotate the analyzer from 90° to 0° and record the magnitude of the current I every 5° and record the values.
4. To verify Malus' Law, plot the graph I/I_0 vs. $\cos^2\theta$ and perform linear fitting.

3.3 Linearly Polarized Light and the Half-wave Plate

1. Place the 1/2-wave plate between the polarizer and the analyzer. Rotate it to make the light extinction appear again and set this position as the initial position.
2. Rotate the 1/2-wave plate for $\alpha = 10^\circ$ from the initial position and rotate the analyzer to make the light extinction appear again, record the angle of rotation $\Delta\theta$.
3. To explore the relationship, plot the graph $\Delta\theta$ vs. θ.

3.4 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

1. Replace the 1/2-wave plate by the 1/4-wave plate and rotate it to make the light extinction appear and set this position as the initial position. At this point $\alpha = 0°$. Then rotate the 1/4-wave plate and observe the change in the light intensity.
2. Rotate the analyzer for 360° and record the light intensity (which is indicated by the current I) for every 10° and record the data in a table.
3. Rotate the 1/4-wave plate for 20°, repeat Step 3.
4. Rotate the 1/4-wave plate for 45°, and repeat Step 3.
5. Rotate the 1/4-wave plate for 70°. Rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches a maximum.
6. To find out the relation, plot the relation between the rotation angle of the analyzer and the light amplitude in polar coordinates. Normalize the amplitude by its maximum value. Mark the position recorded in Step 6 and compare it with the data recorded.
7. To compare with the circular polarization, plot a linear fit to the data when the angle is 45°.

Cautions

1. Do not direct the laser beam into the eye.
2. Do not touch the surface of the polarizers or the wave plates.

4 Results

4.1 Demonstration of Malus' Law

The measurement data are presented in Table 2.

Maximum Electric Current I_0		1.037 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	1.037	50	0.470
5	1.033	55	0.373
10	1.014	60	0.294
15	0.986	65	0.206
20	0.947	70	0.147
25	0.896	75	0.085
30	0.822	80	0.042
35	0.730	85	0.015
40	0.643	90	0.003
45	0.560		

Table 2: Measurement data for Malus' law demonstration.

To find the relation between $\cos^2\theta$ and I/I_0, the two set of values need to be calculated first. Take the first set of data as an example.

$$\cos^2\theta = \cos^2(0 \times \frac{\pi}{180}) = 1 \pm 0,$$

$$\frac{I}{I_0} = \frac{1.037}{1.037} = 1.0000 \pm 0.0013.$$

Perform similar calculations to each of the other sets of data and the results are shown in Table 3.

θ [°] $\pm$ 2 [°]	I [μA] $\pm$ 0.001 [μA]	$\cos^2\theta$	I/I_0
0	1.037	1 $\pm$ 0	1.0000 $\pm$ 0.0013
5	1.033	0.992 $\pm$ 0.006	0.9961 $\pm$ 0.0013
10	1.014	0.970 $\pm$ 0.012	0.9778 $\pm$ 0.0013
15	0.986	0.933 $\pm$ 0.017	0.9508 $\pm$ 0.0013
20	0.947	0.88 $\pm$ 0.02	0.9132 $\pm$ 0.0013
25	0.896	0.82 $\pm$ 0.03	0.8640 $\pm$ 0.0013
30	0.822	0.75 $\pm$ 0.03	0.7927 $\pm$ 0.0012
35	0.730	0.67 $\pm$ 0.03	0.7040 $\pm$ 0.0012
40	0.643	0.59 $\pm$ 0.03	0.6201 $\pm$ 0.0011
45	0.560	0.50 $\pm$ 0.03	0.5400 $\pm$ 0.0011
50	0.470	0.41 $\pm$ 0.03	0.4532 $\pm$ 0.0011
55	0.373	0.33 $\pm$ 0.03	0.3597 $\pm$ 0.0010
60	0.294	0.25 $\pm$ 0.03	0.2835 $\pm$ 0.0010
65	0.206	0.18 $\pm$ 0.03	0.1986 $\pm$ 0.0010
70	0.147	0.12 $\pm$ 0.02	0.1418 $\pm$ 0.0010
75	0.085	0.067 $\pm$ 0.017	0.0820 $\pm$ 0.0010
80	0.042	0.030 $\pm$ 0.012	0.0405 $\pm$ 0.0010
85	0.015	0.008 $\pm$ 0.006	0.0145 $\pm$ 0.0010
90	0.003	0 $\pm$ 0	0.0029 $\pm$ 0.0010

Table 3: Result for $\cos^2\theta$ and I/I_0.

Then, a linear fit of the plot I/I_0 vs. θ are performed (Figure 3). The slope of the linear fitting is 1.01 $\pm$ 0.02.

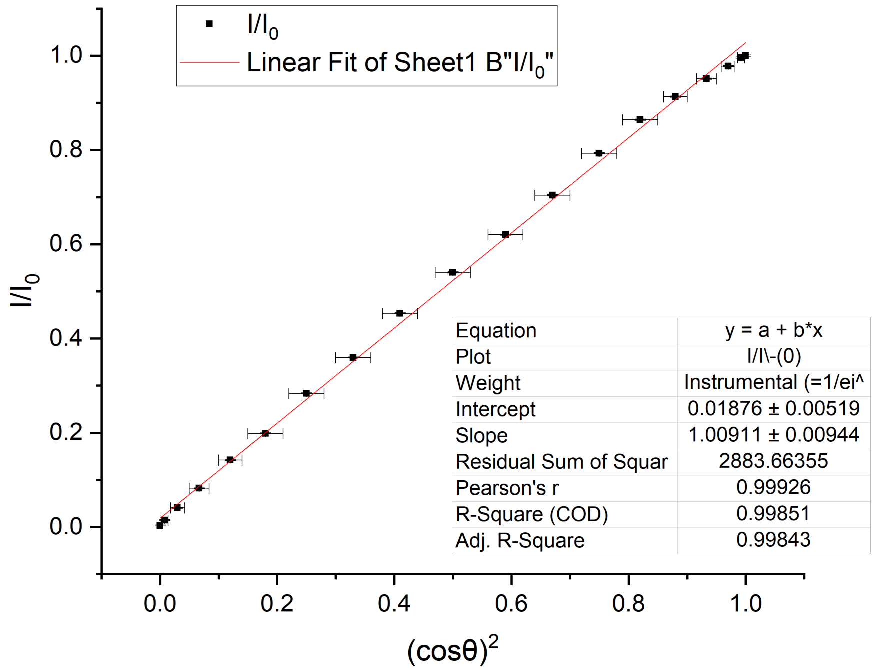

Figure 3: Linear fit of I/I_0 vs. $\cos^2\theta$ relation.

4.2 Linearly Polarized Light and the Half-wave Plate

The measurement data are presented in Table 4. Note that we have substracted each of the data by the initial angle in the report.

Rotation angle of the 1/2-wave plate $[°] \pm 2[°]$	Rotation angle of the analyzer $[°] \pm 2[°]$
initial	0
10	22
20	40
30	61
40	82
50	102
60	121
70	142
80	162
90	181

Table 4: Measurement data for the 1/2-wave plate.

To find the relation between $\Delta\theta$ and θ, the data are plotted in Figure 4 and linear fit is performed. The slope of the linear fit is 2.01 ± 0.02.

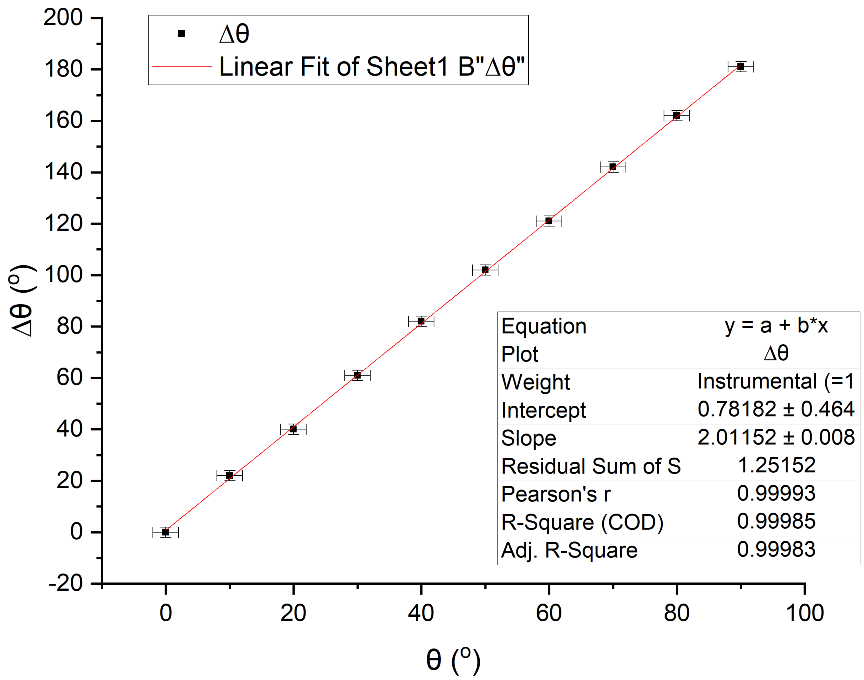

Figure 4: Linear fit of $\Delta\theta$ vs. θ.

Besides, in this part, when the half-wave plate rotates for 360°, **4** times of light extinction are observed. And when the analyzer rotates 360°, **2** times of light extinction during the analyzer rotating 360°. It can be concluded from the above result that **the polarization axis get rotated by twice of the origin angle** (2α) after the light passes through the 1/2-wave plate,

4.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

4.3.1 Rotation Angle: 0°

The measurement data for 0° rotation angle of 1/4-wave plate are presented in Table 5. Note that when filling the data sheet, 0° and 90 ° are mistaken and the mistake is corrected in the report.

Rotation angle of 1/4-wave plate: 0°

Maximum Electric Current I_0		0.805 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.729	180	0.805
10	0.716	190	0.779
20	0.668	200	0.706
30	0.570	210	0.599
40	0.446	220	0.470
50	0.316	230	0.342
60	0.196	240	0.210
70	0.093	250	0.101
80	0.029	260	0.031
90	0.003	270	0.003
100	0.025	280	0.023
110	0.091	290	0.090
120	0.195	300	0.195
130	0.317	310	0.310
140	0.460	320	0.421
150	0.596	330	0.542
160	0.705	340	0.649
170	0.779	350	0.713

Table 5: Measurement data for the 1/4-wave plate (rotation angle 0°).

As described in the procedure part, $\sqrt{I/I_0}$ is calculated. Take the first set of data as an example,

$$\sqrt{\frac{I}{I_0}} = \sqrt{\frac{0.729}{0.805}} = 0.9056 \pm 0.0009.$$

Perform similar calculations to each of the other sets of data and the results are presented in Table 6.

θ [°] ± 2[°]	$\sqrt{I/I_0}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$
0	0.9516 ± 0.0009	180	1.0000 ± 0.0009
10	0.9431 ± 0.0009	190	0.9837 ± 0.0009
20	0.9109 ± 0.0009	200	0.9365 ± 0.0009
30	0.8415 ± 0.0009	210	0.8626 ± 0.0009
40	0.7443 ± 0.0010	220	0.7641 ± 0.0009
50	0.6265 ± 0.0011	230	0.6518 ± 0.0010
60	0.4934 ± 0.0013	240	0.5108 ± 0.0013
70	0.3399 ± 0.0018	250	0.3542 ± 0.0018
80	0.190 ± 0.003	260	0.196 ± 0.003
90	0.061 ± 0.010	270	0.061 ± 0.010
100	0.176 ± 0.004	280	0.169 ± 0.004
110	0.3362 ± 0.0019	290	0.3344 ± 0.0019
120	0.4922 ± 0.0013	300	0.4922 ± 0.0013
130	0.6275 ± 0.0011	310	0.6206 ± 0.0011
140	0.7559 ± 0.0009	320	0.7232 ± 0.0010
150	0.8604 ± 0.0009	330	0.8205 ± 0.0009
160	0.9358 ± 0.0009	340	0.8979 ± 0.0009
170	0.9837 ± 0.0009	350	0.9411 ± 0.0009

Table 6: Results for $\sqrt{I/I_0}$ when rotation angle is 0°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 5).

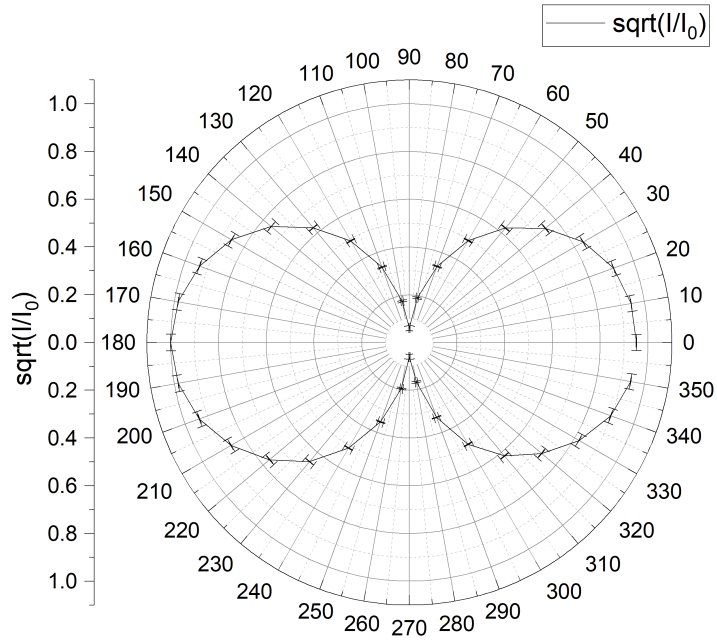

Figure 5: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is $0°$.

4.3.2 Rotation Angle: 20°

The measurement data for $20°$ rotation angle of 1/4-wave plate are shown in Table 7. Note that when filling the data sheet, $0°$ and $90\ °$ are mistaken and the mistake is corrected in the report.

Rotation angle of 1/4-wave plate: 20°			
Maximum Electric Current I_0		0.707 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.589	180	0.641
10	0.646	190	0.693
20	0.671	200	0.707
30	0.662	210	0.690
40	0.612	220	0.644
50	0.534	230	0.567
60	0.453	240	0.470
70	0.356	250	0.363
80	0.261	260	0.263
90	0.179	270	0.184
100	0.123	280	0.127
110	0.102	290	0.104
120	0.116	300	0.115
130	0.164	310	0.159
140	0.240	320	0.224
150	0.345	330	0.318
160	0.450	340	0.420
170	0.550	350	0.513

Table 7: Measurement data for the 1/4-wave plate (rotation angle 20°).

Similar to the previous section, $\sqrt{I/I_0}$ is calculated and the results are presented in Table 8 (For sample calculation please refer to section 4.3.1, which will not be repeated here).

θ [°] ± 2[°]	$\sqrt{I/I_0}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$
0	0.9127 ± 0.0010	180	0.9522 ± 0.0010
10	0.9559 ± 0.0010	190	0.9900 ± 0.0010
20	0.9742 ± 0.0010	200	1.0000 ± 0.0010
30	0.9677 ± 0.0010	210	0.9879 ± 0.0010
40	0.9304 ± 0.0010	220	0.9544 ± 0.0010
50	0.8691 ± 0.0010	230	0.8955 ± 0.0010
60	0.8005 ± 0.0010	240	0.8153 ± 0.0010
70	0.7096 ± 0.0011	250	0.7165 ± 0.0011
80	0.6076 ± 0.0012	260	0.6099 ± 0.0012
90	0.5032 ± 0.0014	270	0.5102 ± 0.0014
100	0.4171 ± 0.0017	280	0.4238 ± 0.0017
110	0.3798 ± 0.0019	290	0.3835 ± 0.0019
120	0.4051 ± 0.0018	300	0.4033 ± 0.0018
130	0.4816 ± 0.0015	310	0.4742 ± 0.0015
140	0.5826 ± 0.0013	320	0.5629 ± 0.0013
150	0.6986 ± 0.0011	330	0.6707 ± 0.0012
160	0.7978 ± 0.0011	340	0.7708 ± 0.0011
170	0.8820 ± 0.0010	350	0.8518 ± 0.0010

Table 8: Results for $\sqrt{I/I_0}$ when rotation angle is 20°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 6).

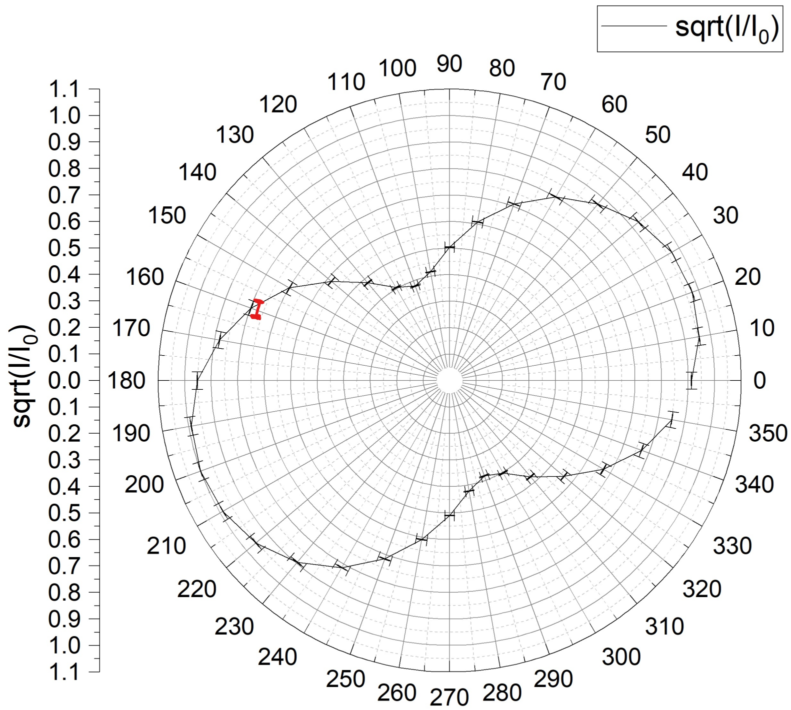

Figure 6: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is $20°$, together with the maximum point of rotation angle $70°$(marked in red).

4.3.3 Rotation Angle: 45°

The measurement data for $45°$ rotation angle of 1/4-wave plate are shown in Table 9. Note that when filling the data sheet, $0°$ and $90°$ are mistaken and the mistake is corrected in the report.

Rotation angle of 1/4-wave plate: 45°

Maximum Electric Current I_0		0.395 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.359	180	0.392
10	0.360	190	0.387
20	0.356	200	0.378
30	0.354	210	0.370
40	0.351	220	0.368
50	0.348	230	0.365
60	0.354	240	0.363
70	0.361	250	0.365
80	0.364	260	0.369
90	0.371	270	0.375
100	0.375	280	0.386
110	0.381	290	0.392
120	0.385	300	0.389
130	0.390	310	0.378
140	0.393	320	0.366
150	0.395	330	0.365
160	0.393	340	0.369
170	0.395	350	0.364

Table 9: Measurement data for the 1/4-wave plate (rotation angle 45°).

Similar to the previous section, $\sqrt{I/I_0}$ is calculated and the results are presented in Table 10 (For sample calculation please refer to section 4.3.1, which will not be repeated here).

θ [°] ± 2[°]	$\sqrt{I/I_0}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$
0	0.9533 ± 0.0018	180	0.9962 ± 0.0018
10	0.9547 ± 0.0018	190	0.9898 ± 0.0018
20	0.9494 ± 0.0018	200	0.9782 ± 0.0018
30	0.9467 ± 0.0018	210	0.9678 ± 0.0018
40	0.9427 ± 0.0018	220	0.9652 ± 0.0018
50	0.9386 ± 0.0018	230	0.9613 ± 0.0018
60	0.9467 ± 0.0018	240	0.9586 ± 0.0018
70	0.9560 ± 0.0018	250	0.9613 ± 0.0018
80	0.9600 ± 0.0018	260	0.9665 ± 0.0018
90	0.9691 ± 0.0018	270	0.9744 ± 0.0018
100	0.9744 ± 0.0018	280	0.9885 ± 0.0018
110	0.9821 ± 0.0018	290	0.9962 ± 0.0018
120	0.9873 ± 0.0018	300	0.9924 ± 0.0018
130	0.9937 ± 0.0018	310	0.9782 ± 0.0018
140	0.9975 ± 0.0018	320	0.9626 ± 0.0018
150	1.0000 ± 0.0018	330	0.9613 ± 0.0018
160	0.9975 ± 0.0018	340	0.9665 ± 0.0018
170	1.0000 ± 0.0018	350	0.9600 ± 0.0018

Table 10: Results for $\sqrt{I/I_0}$ when rotation angle is 45°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 7).

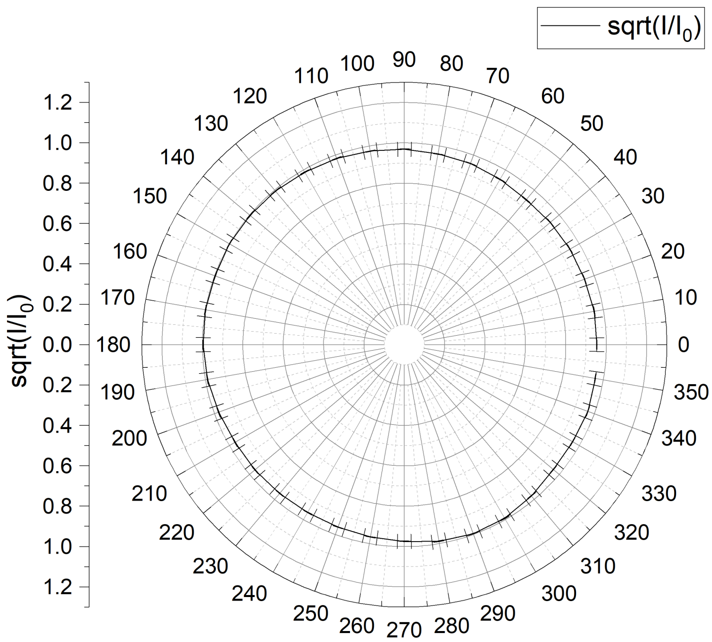

Figure 7: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 45°.

To compare the result with circular polarization, as described in the procedure part, the data is also plotted in Cartesian coordinate and linear fit is performed (Figure 8). The slope of the linear fitting is $6 \times 10^{-5} \pm 6 \times 10^{-5}$.

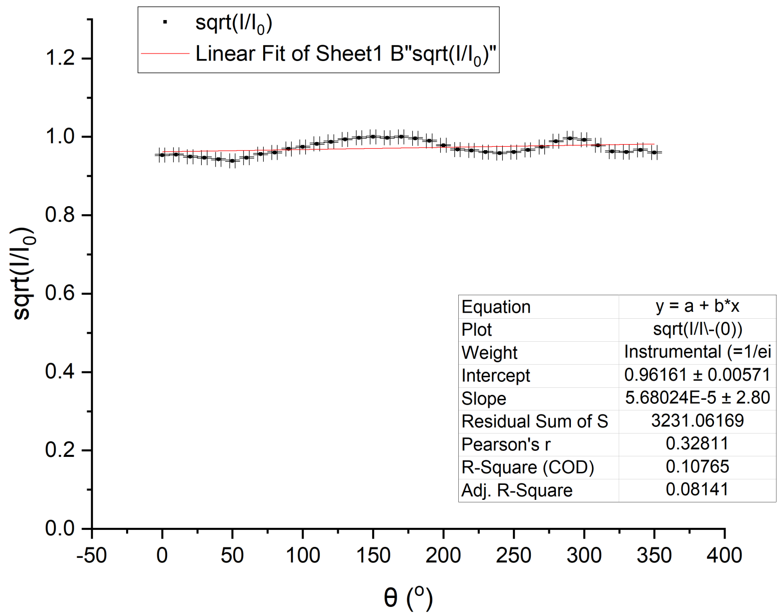

Figure 8: $\sqrt{I/I_0}$ vs. θ relation in the Cartesian coordinate.

4.3.4 Rotation Angle: 70°

In the condition of the rotation angle is 70°, the data when the light intensity reaches its maximum are presented in Table 11. Still, the data is subtracted by 90°. The position of this point is marked in red in Figure 6. It can seen that the value of θ when the light intensity reaches maximum in the case of 70° rotation angle is nearly symmetric about $\theta = 0$ axis with that ($\theta = 20°(200°)$) in the case of 20° rotation angle.

Rotation angle of the 1/4-wave plate: 70°	
θ [°] ± 2[°]	161
I [μA] ± 0.001 [μA]	0.682

Table 11: Measurement data for the 1/4-wave plate (rotation angle 70°).

5 Discussion and Conclusions

5.1 Demonstration of Malus' Law

The slope of the fitting is 1.01 ± 0.02 and the Pearson's r is 0.99916, which is very close to 1. This suggests that the value of I/I_0 is proportional to the value of $\cos^2\theta$ with the coefficient to be about 1.

Theoretically, by Eq. (1), the slope is

$$\frac{I/I_0}{\cos^2\theta} = 1,$$

Therefore, the relative error is

$$\varepsilon = \frac{1.01 - 1}{1} \times 100\% = 1\%.$$

This in an acceptable range of error verifies the Malus' Law.

5.2 Linearly Polarized Light and the Half-wave Plate

The slope of the linear fit is 2.01 ± 0.02. Theoretically, as introduced in the Introduction part, the rotation angle of the polarization axis is twice of the origin angle for a half-wave plate. Therefore the theoretical value of the slope of our linear fitting is 2. The relative error is therefore

$$\varepsilon = \frac{2.01 - 2}{2} \times 100\% = 0.5\%.$$

This conforms to the theoretical fact.

5.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

5.3.1 Rotation Angle: 0°

From Table 5 and Figure 5, it can be seen that the maximum of light intensity occurs at about $\theta = 0^\circ$. This suggests that polarizing axis is parallel to the optical axis of the plate, which is the theoretical conclusion stated in the Introduction part. The shape of the plot also indicates that it is linearly polarized.

5.3.2 Rotation Angle: 20°

From Table 7 and Figure 6, it can be seen that the maximum of light intensity occurs at about $\theta = 20^\circ$. This suggests that polarizing axis forms 20° to the optical axis of the plate, which conforms to the fact stated in [2]. The shape of the plot also indicates that it is elliptically polarized.

5.3.3 Rotation Angle: 45°

From Figure 7, it can be seen that the value of $\sqrt{I/I_0}$ changes little when θ changes. Besides, the slop of linear fit of the plot of $\sqrt{I/I_0}$ vs. θ in the Cartesian coordinate in Figure 8 is $6 \times 10^{-5} \pm 6 \times 10^{-5}$ and the Pearson's coefficient is only 0.328. All the results suggest that $\sqrt{I/I_0}$ is a constant, with no relation with the value of θ. Therefore it can be concluded that when rotation angle is 45°, the light is circularly polarized. This conforms to the fact stated in the Introduction part.

5.3.4 Rotation Angle: 70°

From section 4.3.4, we can state that the value of θ when the light intensity reaches maximum in the case of 70° rotation angle is nearly symmetric about $\theta = 0$ axis with that ($\theta = 20^\circ(200^\circ)$) in the case of 20° rotation angle. This conclusion conforms to the theoretical conclusion stated in [2].

5.4 Causes for errors and uncertainties

Possible causes for errors and uncrtainties are listed below:

1. The light of our torch may affect the light intensity detected by the photo-cell.
2. The precision of the scale on the optical elements us 2°, which results in a relatively large uncertainty.

3. It is really tough to adjust the center of all the optical elements to be in a line and we failed to do that in our experiment. This may lead to errors to the experiment.

5.5 Suggestions

Some suggestions for further improvement:

1. The experiment may be performed in an environment that is absolutely dark, where the readings are also available.
2. Improve the precision of the scale on the optical elements.

5.6 Conclusions

In this lab, we studied the polarization phenomenon and verify Malus' law. The way half- and quarter-wave plates work in optical systems is also explored. The experimental results we obtained is of relatively small uncertainties and conform to the theoretical facts in an acceptable range of uncertainty.

References

[1] VP241 Exercise 4: Polarization of Light, Department of Physics, Shanghai Jiaotong University.

[2] University Physics, Section 33.5, Young and Freedman.

A Measurement Uncertainty Analysis

A.1 Uncertainty for Data in Demonstration of Malus' Law

The uncertainty of $\cos^2\theta$ is

$$\begin{aligned} u_{\cos^2\theta} &= \sqrt{(\frac{\partial \cos^2\theta}{\partial\theta}u_\theta)^2} \\ &= |-2\cos\theta\sin\theta u_\theta| \\ &= |\sin 2\theta u_\theta|, \end{aligned}$$

where $u_\theta = 2[^\circ] = \pi/90$. Take $\theta = 0$ as an example,

$$u_{\cos^2\theta} = |\sin 2\theta u_\theta| = |\sin(2\times 5^\circ)\times\frac{\pi}{90}| = 0.006,$$

All the results are shown in Table 12.

The uncertainty of I/I_0 is

$$\begin{aligned} u_{I/I_0} &= \sqrt{(\frac{\partial I/I_0}{\partial I}u_I)^2 + (\frac{\partial I/I_0}{\partial I_0}u_{I_0})^2} \\ &= \sqrt{(\frac{u_I}{I_0})^2 + (-\frac{I}{I_0^2}u_{I_0})^2}, \end{aligned}$$

where $u_I = u_{I_0} = 0.001$ [μA], $I_0 = 1.037 \pm 0.001$ [μA]. Take the first set of data as an example,

$$u_{I/I_0} = \sqrt{(\frac{u_I}{I_0})^2 + (-\frac{I}{I_0^2}u_{I_0})^2} = \sqrt{(\frac{0.001}{1.037})^2 + (-\frac{1.037}{1.037^2}\times 0.001)^2} = 0.0013,$$

All the results are shown in Table 12.

$u_{\cos^2\theta}$	u_{I/I_0}	$u_{\cos^2\theta}$	u_{I/I_0}
0	0.0013	0.03	0.0011
0.006	0.0013	0.03	0.0010
0.012	0.0013	0.03	0.0010
0.017	0.0013	0.03	0.0010
0.02	0.0013	0.02	0.0010
0.03	0.0013	0.017	0.0010
0.03	0.0012	0.012	0.0010
0.03	0.0012	0.006	0.0010
0.03	0.0011	0	0.0010
0.03	0.0011		

Table 12: Uncertainty of $\cos^2\theta$ and I/I_0.

A.2 Uncertainty for Linearly Polarized Light and the Half-wave Plate

In this part, the uncertainty of the measurement depends on the device, which is

$$u_\theta = 2^\circ.$$

A.3 Uncertainty for Circularly and Elliptically Polarized Light and the 1/4-wave Plate

The uncertainty of $\sqrt{I/I_0}$ is

$$u_{\sqrt{I/I_0}} = \sqrt{(\frac{\partial\sqrt{I/I_0}}{\partial I}u_I)^2 + (\frac{\partial\sqrt{I/I_0}}{\partial I_0}u_{I_0})^2}$$
$$= \sqrt{\frac{1}{4II_0}u_I^2 + \frac{I}{4I_0^3}u_{I_0}^2},$$

where $u_I = u_{I_0} = 0.001$ [μA], $I_0 = 0.805 \pm 0.001$ [μA] for rotation angle of $0°$, $I_0 = 0.707 \pm 0.001$ [μA] for rotation angle of $20°$, $I_0 = 0.395 \pm 0.001$ [μA] for rotation angle of $45°$. Take the first set of data as an example,

$$u_{\sqrt{I/I_0}} = \sqrt{\frac{1}{4 \times 0.729 \times 0.805} \times 0.001^2 + \frac{0.729}{4 \times 0.805^3} \times 0.001^2} = 0.0009.$$

All the results are shown in Table 13.

Rotation angle: 0° $u_{\sqrt{I/I_0}}$	Rotation angle: 20° $u_{\sqrt{I/I_0}}$	Rotation angle: 45° $u_{\sqrt{I/I_0}}$
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0010	0.0010	0.0018
0.0011	0.0010	0.0018
0.0013	0.0010	0.0018
0.0018	0.0011	0.0018
0.003	0.0012	0.0018
0.010	0.0014	0.0018
0.004	0.0017	0.0018
0.0019	0.0019	0.0018
0.0013	0.0018	0.0018
0.0011	0.0015	0.0018
0.0009	0.0013	0.0018
0.0009	0.0011	0.0018
0.0009	0.0011	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0009	0.0010	0.0018
0.0010	0.0010	0.0018
0.0013	0.0010	0.0018
0.0018	0.0011	0.0018
0.003	0.0012	0.0018
0.010	0.0014	0.0018
0.004	0.0017	0.0018
0.0019	0.0019	0.0018
0.0013	0.0018	0.0018
0.0011	0.0015	0.0018
0.0010	0.0013	0.0018
0.0009	0.0012	0.0018
0.0009	0.0011	0.0018
0.0009	0.0010	0.0018

Table 13: Uncertainty for $\sqrt{I/I_0}$ when the rotation angle is 0°, 20° and 45°.

B Data Sheet

Please find the original data sheet attached at the end of the report.

UM-SJTU Physics Laboratory
Data Sheet (Exercise 4)

Name: [illegible]

Student ID: 518021911220

Name: [illegible] Hong Yichen

Student ID: 518370910011

Group: 17

Date: 8. Nov. 2019.

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with pencil or modified by correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used. You are required to hand in the original data with your lab report, so please keep the data sheet properly.

1.418.

Uncertainty of θ is [2]°.

Maximum Electric Current I_0		~~1.074~~ 1.037 ± 0.001 [uA]	
θ	I [uA] ± 0.001 [uA]	θ	I [uA] ± 0.001 [uA]
0°	1.037	50°	0.470
5°	1.033	55°	0.373
10°	1.017	60°	0.294
15°	0.986	65°	0.206
20°	0.947	70°	0.147
25°	0.896	75°	0.085
30°	0.832	80°	0.042
35°	0.730	85°	0.015
40°	0.643	90°	0.003
45°	0.560		

Table 1. Measurement data Malus' law demonstration.

Instructor's signature: ZXX

Rotation angle of the 1/2–wave plate	Rotation angle of the analyzer [°] ± [2]°
initial	210
10°	232
20°	250
30°	271
40°	292
50°	312
60°	331
70°	352
80°	12
90°	31

Table 2. Measurement data for the 1/2–wave plate.

Instructor's signature: ZXX

maximum

Rotation angle of 1/4–wave plate: 0°			
Maximum Electric Current I_0		0.805 ± 0.001 [µA]	
θ	I [µA] ± 0.001 [µA]	θ	I [µA] ± 0.001 [µA]
0°	0.003	180°	0.003
10°	0.023	190°	0.025
20°	0.090	200°	0.091
30°	0.195	210°	0.195
40°	0.310	220°	0.317
50°	0.421	230°	0.460
60°	0.542	240°	0.596
70°	0.649	250°	0.705
80°	0.713	260°	0.779
90°	0.729	270°	0.805
100°	0.716	280°	0.779
110°	0.668	290°	0.706
120°	0.570	300°	0.599
130°	0.446	310°	0.470
140°	0.316	320°	0.342
150°	~~0.9~~ 0.196	330°	0.210
160°	~~0.093~~	340°	0.101
170°	0.029	350°	0.031

Table 3. Measurement data for the 1/4–wave plate (rotation angle 0°).

Table 3~5의 0° [illegible]

Instructor's signature: [illegible]

5) [illegible] 20°

Rotation angle of the 1/4–wave plate: 20°			
Maximum Electric Current I_0		0.707 ± 0.001 μA	
θ	I [μA] ± 0.001 [μA]	θ	I [μA] ± 0.001 [μA]
0°	0.184	180°	0.179
10°	0.127	190°	0.123
20°	0.104	200°	0.102
30°	0.115	210°	0.116
40°	0.159	220°	0.164
50°	0.224	230°	0.240
60°	0.318	240°	0.345
70°	~~[illegible]~~ 0.420	250°	0.450
80°	0.513	260°	0.550
90°	0.589	270°	0.641
100°	0.646	280°	0.693
110°	0.671	290°	0.707
120°	0.662	300°	0.690
130°	0.612	310°	0.644
140°	0.534	320°	0.567
150°	0.453	330°	0.470
160°	0.356	340°	0.363
170°	0.261	350°	~~[illegible]~~ 0.263

Table 4. Measurement data for the 1/4–wave plate (rotation angle 20°).

Instructor's signature: ZXX

Rotation angle of the 1/4–wave plate: 45°			
Maximum Electric Current I_0		0.395 ± 0.001 [μA]	
θ	I [μA] ± 0.001 [μA]	θ	I [μA] ± 0.001 [μA]
0°	0.375	180°	0.371
10°	0.386	190°	0.375
20°	0.392	200°	0.381
30°	0.389	210°	0.385
40°	0.378	220°	0.390
50°	0.366	230°	0.393
60°	0.365	240°	0.395
70°	~~0.367~~ 0.369	250°	0.393
80°	~~0.359~~ 0.364	260°	0.395
90°	~~0.360~~ 0.359	270°	0.392
100°	~~0.356~~ 0.360	280°	0.387
110°	~~0.354~~ 0.356	290°	0.378
120°	0.354	300°	0.370
130°	0.351	310°	0.368
140°	0.348	320°	0.365
150°	0.354	330°	0.363
160°	0.361	340°	0.365
170°	0.364	350°	0.369

Table 5. Measurement data for the 1/4–wave plate (rotation angle 45°).

Rotation angle of the 1/4–wave plate: 70°	
θ [°] ± [2]°	251°
I [μA] ± 0.001 [μA]	0.682

29°
280

Table 6. Measurement data for the 1/4–wave plate (rotation angle 70°).

Instructor's signature: ________

rev. 2