Continuous Algorithms for Optimization and Sampling

Course Description

Traditional algorithms in computer science are designed in a discrete manner. Nonetheless, recent years have witnessed great advances from a continuous perspective, particularly in the design of optimization and sampling algorithms. There is a deep connection between optimization and sampling, either through optimization as the limit of sampling, or through sampling as optimization in the space of probability measures. Motivated by this viewpoint, this course aims to develop a systematic way to design and analyze algorithms for both areas from the continuous perspective. More particularly, this course starts from continuous optimization, discusses stochastic optimization in detail, introduces optimal transport as a bridge connecting optimization and sampling, and finally delves into sampling.

Prerequisites

Students should be familiar with multivariate calculus, linear algebra, basic probability and statistics, and have good MATLAB or Python programming skills. Prior knowledge of optimization and sampling is helpful but not required.

Topics (subject to change)

- Introduction
- Continuous optimization
    - Subgradient method
    - Proximal gradient method
    - Accelerated gradient method
    - Inexact proximal point methods
- Stochastic optimization
    - Stochastic approximation and sample average approximation
    - Dynamic programming and reinforcement learning
    - Risk averse optimization
    - Distributionally robust optimization
    - Statistical efficiency
- Optimal transport
    - Monge-Kantorovich problem and Kantorovich duality
    - Wasserstein space and Otto's calculus
    - Transport inequalities
    - Entropic regularization and gradient flows
    - Sinkhorn algorithm
    - Economics applications
- Sampling
    - Rejection sampling and Metropolis–Hastings filter
    - Stochastic calculus
    - Langevin Monte Carlo
    - Proximal sampling
    - Generative models

Textbooks

1. Amir Beck. *First-Order Methods in Optimization*. SIAM, 2017.
2. Alexander Shapiro, Darinka Dentcheva, and Andrzej Ruszczynski. *Lectures on Stochastic Programming: Modeling and Theory*. SIAM, 2021.
3. Cédric Villani. *Topics in Optimal Transportation*. American Mathematical Society, 2021.
4. Sinho Chewi. *Log-Concave Sampling*. Draft, 2023.

Recommended Readings

1. Yin Tat Lee and Santosh Vempala. *Techniques in Optimization and Sampling*. Draft, 2023.
2. Sébastien Bubeck*. Convex Optimization: Algorithms and Complexity.* Foundations and Trends® in Machine Learning. 2015.
3. Alexander Shapiro*. Tutorial on Risk Neutral, Distributionally Robust and Risk Averse Multistage Stochastic Programming.* EJOR, 2020.
4. Alexander Shapiro. *Topics in Stochastic Programming*.
5. Gabriel Peyré and Marco Cuturi*. Computational Optimal Transport.* Foundations and Trends® in Machine Learning. 2019.
6. Marcel Nutz*. Introduction to Entropic Optimal Transport.* Lecture notes, 2022.
7. Alfred Galichon. *Optimal Transport Methods in Economics.* Princeton University Press, 2018.
8. John Duchi. *Introductory Lectures on Stochastic Optimization*.
9. John Duchi. *Introductory Lectures on Information Theory and Statistics*.
10. William Haskell. *Introduction to Optimization*. Lecture notes, 2018.
11. William Haskell. *Introduction to Dynamic Programming*. Lecture notes, 2018.
12. Aaron Sidford. *Optimization Algorithms*. Lecture notes, 2023.