STAC67 A1

2026-01-14

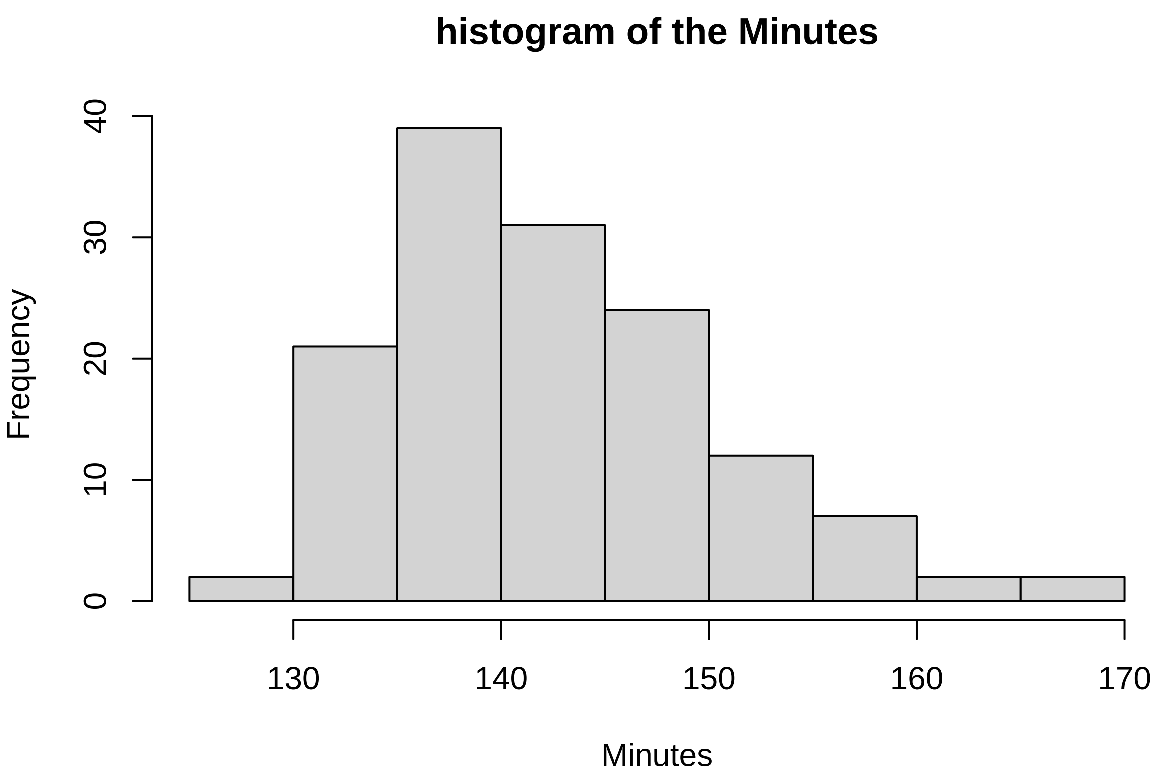
Q1

(a)

```
csvFile = "OlympicMarathon2016.csv"
df = read.csv(csvFile)
mu = mean(df[["Minutes"]])
sigma2 = var(df[["Minutes"]])
sprintf("Minutes has mean %f and variance %f", mu, sigma2)
```

```
## [1] "Minutes has mean 142.367143 and variance 59.638923"
```

```
hist(df[["Minutes"]], xlab="Minutes", main="histogram of the Minutes")
```



(b)

```
set.seed(1008494620)
population = df[["Minutes"]]
sampleCnt = 10000
n = 30
```

```r
yBar = numeric(sampleCnt)
stats = numeric(sampleCnt)
for (i in 1:sampleCnt) {
  y = sample(population, size=n, replace=TRUE)
  yBar[i] = mean(y)
  s2 = var(y)
  stats[i] = (n-1) * s2 / sigma2
}
```

(b)(i)

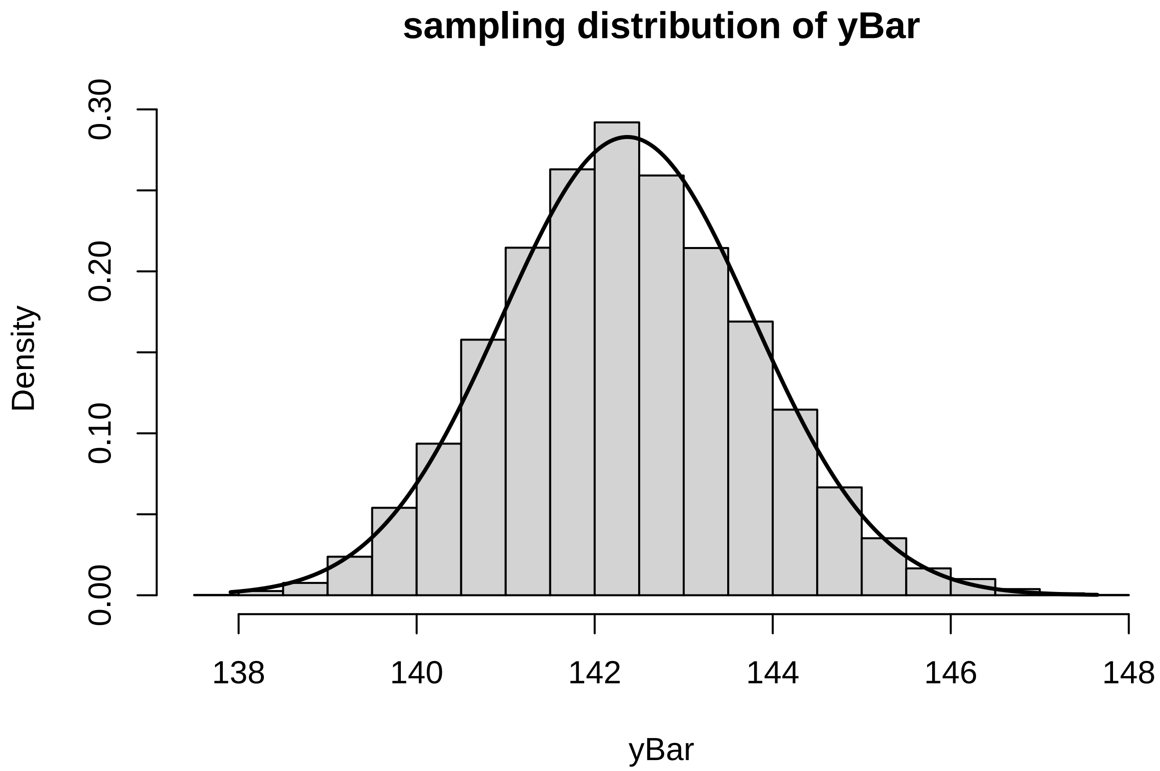
```r
mean_yBar = mean(yBar)
var_yBar = var(yBar)
mean_stats = mean(stats)
var_stats = var(stats)

percentiles = c(0.025, 0.25, 0.5, 0.75, 0.975)
percentiles_pop = mapply(
  function(p) {
    quantile(df[["Minutes"]], p)
  },
  percentiles
)
percentiles_yBar = mapply(
  function(p) {
    quantile(yBar, p)
  },
  percentiles
)
percentiles_stats = mapply(
  function(p) {
    quantile(stats, p)
  },
  percentiles
)

cat(
  sprintf("theoretical values\n"),
  sprintf("==================\n"),
  sprintf("mean = %f, var = %f\n", mu, sigma2),
  sprintf("percentiles:\n"),
  percentiles_pop
)

cat(
  sprintf("\n\nresult of sample means\n"),
  sprintf("======================\n"),
  sprintf("mean = %f, var = %f\n", mean_yBar, var_yBar),
  sprintf("percentiles:\n"),
  percentiles_yBar
)

cat(
```

```r
  sprintf("\n\nresult of scaled sample var\n"),
  sprintf("===========================\n"),
  sprintf("mean = %f, var = %f\n", mean_stats, var_stats),
  sprintf("percentiles:\n"),
  percentiles_stats
)
```

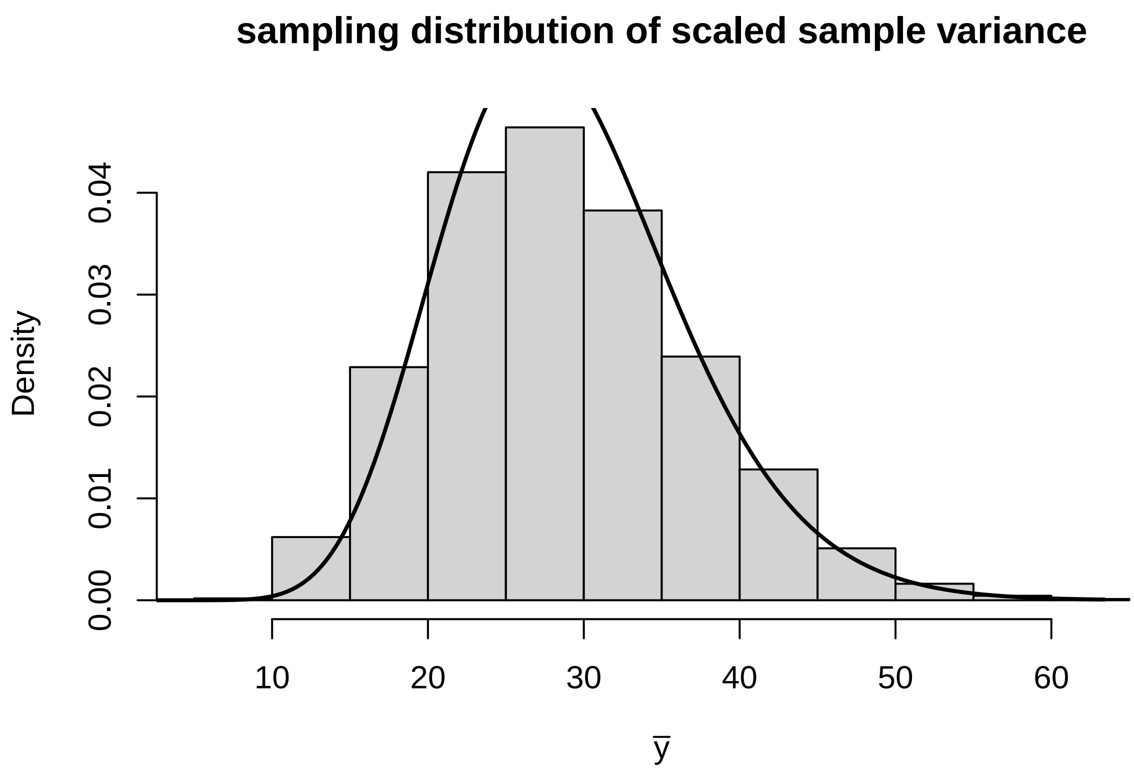
```
## theoretical values
##  ==================
##  mean = 142.367143, var = 59.638923
##  percentiles:
##  131.1555 137.1225 140.765 147.035 159.8852
##
## result of sample means
##  ======================
##  mean = 142.354664, var = 1.984513
##  percentiles:
##  139.7113 141.3853 142.3157 143.2964 145.1947
##
## result of scaled sample var
##  ===========================
##  mean = 28.747533, var = 69.091112
##  percentiles:
##  14.39847 22.80234 28.02941 34.012 46.61924
```

(b) (ii)

```r
hist(yBar,
  probability = TRUE,
  main="sampling distribution of yBar",
  xlab="yBar",
)
x1 = seq(min(yBar), max(yBar), length.out = 500)
lines(x1, dnorm(x1, mean=mu, sd=sqrt(sigma2/n)), lwd=2)
```




```r
hist(stats,
  probability=TRUE,
  main="sampling distribution of scaled sample variance",
  xlab=expression(bar(y)),
)
x2 = seq(0, max(stats), length.out = 500)
lines(x2, dchisq(x2, df=n - 1), lwd=2)
```



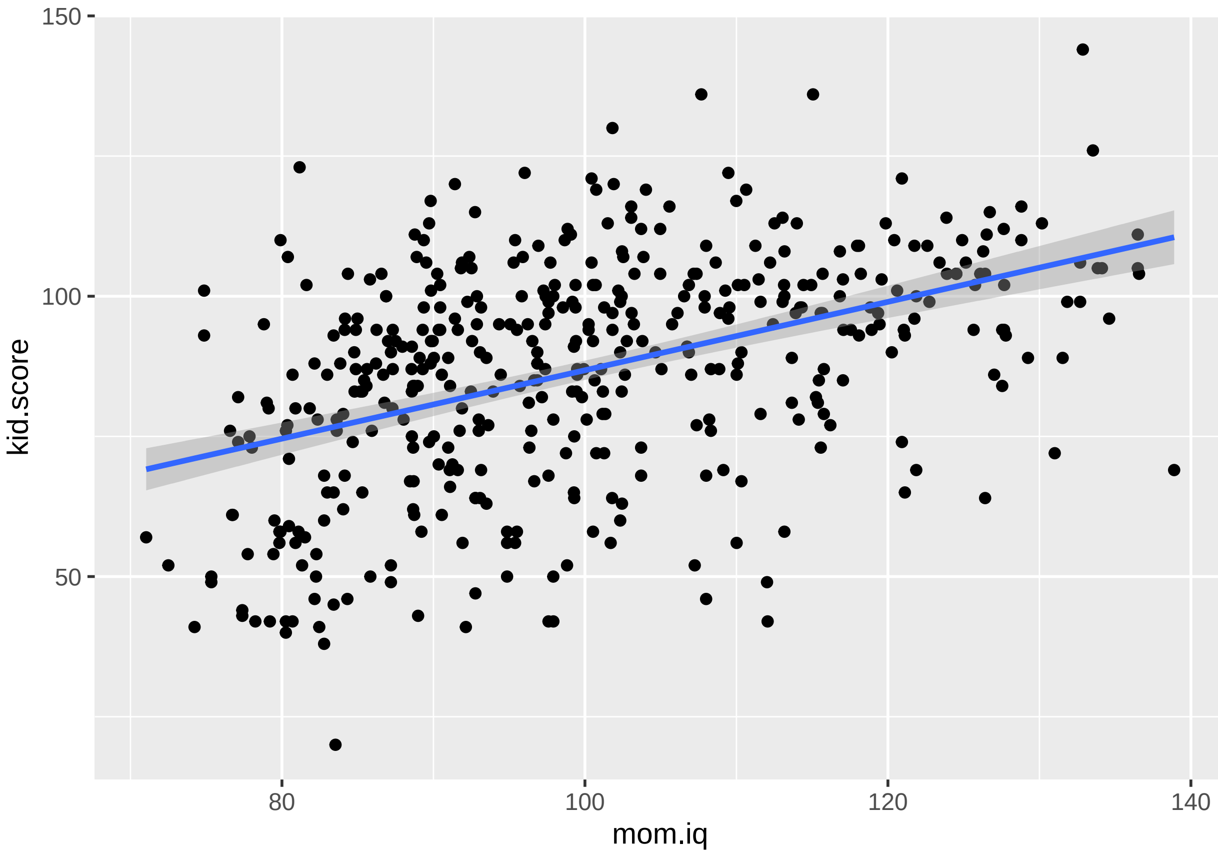
Q7

(a)

```r
library(ggplot2)
```

```
## Warning: package 'ggplot2' was built under R version 4.5.2
```

```r
csvFile = "kidiq.csv"
df = read.csv(csvFile)
ggplot(
  df,
  aes(
    x = mom.iq,
    y = kid.score,
  )
) + geom_point() + geom_smooth(formula = y~x, method = "lm")
```



(b)

```r
y = df[["kid.score"]]
x = df[["mom.iq"]]
fit = lm(y~x, data = df)
beta0 = fit$coefficients[["(Intercept)"]]
beta1 = fit$coefficients[["x"]]
cat(sprintf("beta0 = %f, beta1 = %f", beta0, beta1))
```

```
## beta0 = 25.799778, beta1 = 0.609975
```

(c)

```r
x_bar = mean(x)
y_bar = mean(y)

SS_yy = sum((y - y_bar)^2)
SS_xy = sum((y - y_bar) * (x - x_bar))
SS_xx = sum((x - x_bar)^2)

beta1_hat = SS_xy / SS_xx
beta0_hat = y_bar - beta1_hat * x_bar

cat(
  sprintf("R built-in function\n"),
  sprintf("==================\n"),
```

```r
  sprintf("intercept = %f, slope = %f\n", beta0, beta1),

  sprintf("\nComputed result\n"),
  sprintf("===============\n"),
  sprintf("beta0_hat = %f, beta1_hat = %f", beta0_hat, beta1_hat)
)
```

```
## R built-in function
##  ==================
##  intercept = 25.799778, slope = 0.609975
##
## Computed result
##  ===============
##  beta0_hat = 25.799778, beta1_hat = 0.609975
```

Interpretation:

- $\hat{\beta}_0 = 25.799778$ implies, in theoretical speaking, the children's test scores is 25.799778 when their mother's IQ scores is 0.
- $\hat{\beta}_1 = 0.609975$ implies that children's test scores are positively correlated with their mother's IQ scores – every increment in the one unit of their mother's IQ scores will result in a raise of 0.6009975 on children's test scores.

(c)

```r
alpha = 0.05
n = length(x)

sse = SS_yy - beta1_hat * SS_xy
ss = sse / (n - 2)
se_beta1 = sqrt(ss / SS_xx)
se_beta0 = sqrt((1/n + x_bar^2/SS_xx) * ss)

# two-sided CI
tcrit = qt(1 - alpha/2, df = n - 2)

beta0_ci_upper = beta0_hat + tcrit * se_beta0
beta0_ci_lower = beta0_hat - tcrit * se_beta0

beta1_ci_upper = beta1_hat + tcrit * se_beta1
beta1_ci_lower = beta1_hat - tcrit * se_beta1

cat(
  sprintf("handcrafted CIs\n"),
  sprintf("===============\n"),
  sprintf("            2.5%%      97.5%%\n"),
  sprintf("beta0: %f, %f\n", beta0_ci_lower, beta0_ci_upper),
  sprintf("beta1: %f, %f\n", beta1_ci_lower, beta1_ci_upper),

  sprintf("\n\nbuilt-in function\n"),
  sprintf("=================\n")
)
```

```
## handcrafted CIs
```

```
## ===============
##              2.5%      97.5%
## beta0: 14.169279, 37.430277
## beta1: 0.494953, 0.724996
##
##
## built-in function
## =================
```

```r
confint(fit)
```

```
##                 2.5 %     97.5 %
## (Intercept) 14.1692789 37.4302768
## x            0.4949534  0.7249957
```

Interpretation:

- For β_1: On every increment of the mother's IQ scores, the average response of the children's test scores is increased by between 0.4949534 and 0.7249957 on average. However, in the real world practice, having a IQ score of 0 is meaningless, the interpretation is therefore a theoretical baseline under the scene of statistics study, instead of an important quantity.
- For β_0: When the mother's IQ scores is zero, the children's test scores are expected to between 14.1692789 and 37.4302768.
- For the constructed confidence intervals: When the procedure of random sampling is performed under the same population, 95% of the constructed intervals are going to contains the true parameter β_0 and β_1.

(d)

```r
t = beta1_hat / se_beta1
tcrit = qt(0.975, df = n-2)
cat(sprintf("test stat: %f, the one-sided critical t is given by: %f\n", t, tcrit))
if (t > tcrit) {
  cat("We reject H0. Evidence suggests a positive association.\n")
} else {
  cat("We fail to reject H0. Insufficient evidence of a positive association at this alpha.\n")
}
p_val <- 1 - pt(t, df = n-2)
cat(sprintf("p-value is given by: %f\n", p_val))
```

```
## test stat: 10.423188, the one-sided critical t is given by: 1.965471
## We reject H0. Evidence suggests a positive association.
## p-value is given by: 0.000000
```

(e)

```r
tt = 0.1 / se_beta1
p = 2 * (1 - pt(tt, df = n-2))
sprintf("the probability is: %f", p)
```

```
## [1] "the probability is: 0.088208"
```