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## Introduction

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This page shows how to perform a number of statistical tests using SAS. Each section gives a brief description of the aim of the statistical test, when it is used, an example showing the SAS commands and SAS output (often excerpted to save space) with a brief interpretation of the output. You can see the page [Choosing the Correct Statistical Test \(https://stats.idre.ucla.edu/other/mult-pkg/whatstat/\)](https://stats.idre.ucla.edu/other/mult-pkg/whatstat/) for a table that shows an overview of when each test is appropriate to use. In deciding which test is appropriate to use, it is important to consider the type of variables that you have (i.e., whether your variables are categorical, ordinal or interval and whether they are normally distributed), see [What is the difference between categorical, ordinal and interval variables? \(https://stats.idre.ucla.edu/other/mult-pkg/whatstat/what-is-the-difference-between-categorical-ordinal-and-interval-variables/\)](https://stats.idre.ucla.edu/other/mult-pkg/whatstat/what-is-the-difference-between-categorical-ordinal-and-interval-variables/) for more information on this.

Please note that the information on this page is intended only as a very brief introduction to each analysis. This page may be a useful guide to suggest which statistical techniques you should further investigate as part of the analysis of your data. This page does not include necessary and important information on many topics, such as the assumptions of the statistical techniques, under what conditions the results may be questionable, etc. Such information may be obtained from a statistics text or journal article. Also, the interpretation of the results given on this page is very minimal and should not be used as a guide for writing about the results. Rather, the intent is to orient you to a few key points. For many analyses, the output has been abbreviated to save space, and potentially important information is not presented here.

## About the hsb data file

here (<https://stats.idre.ucla.edu/wp-content/uploads/2016/02/hsb2.sas7bdat>) . You can store this file anywhere on your computer, but in the examples we show, we will assume the file is stored in a folder named **c:/mydata/sas/notes/hsb2.sas7bdat**. If you store the file in a different location, change **c:/mydata** to the location where you stored the file on your computer.

## One sample t-test

```
proc ttest data = "c:/mydata/hsb2" h0 = 50;
  var write;
run;
```

The TTEST Procedure

#### Statistics

|          |     | Lower CL |        | Upper CL | Lower CL |         | Upper CL |         |
|----------|-----|----------|--------|----------|----------|---------|----------|---------|
| Variable | N   | Mean     | Mean   | Mean     | Std Dev  | Std Dev | Std Dev  | Std Err |
| write    | 200 | 51.453   | 52.775 | 54.097   | 8.6318   | 9.4786  | 10.511   | 0.6702  |

#### T-Tests

| Variable | DF  | t Value | Pr >  t |
|----------|-----|---------|---------|
| write    | 199 | 4.14    | <.0001  |

The mean of the variable **write** for this particular sample of students is 52.775, which is statistically significantly different from the test value of 50. We would conclude that this group of students has a significantly higher mean on the writing test than 50.

## One sample median test

provides the location counts of the data shown at the bottom of the output.

```
proc univariate data = "c:/mydata/hsb2" loccount mu0 = 50;  
    var write;  
run;
```

|        |          |                     |          |
|--------|----------|---------------------|----------|
| Mean   | 52.77500 | Std Deviation       | 9.47859  |
| Median | 54.00000 | Variance            | 89.84359 |
| Mode   | 59.00000 | Range               | 36.00000 |
|        |          | Interquartile Range | 14.50000 |

Tests for Location: Mu0=50

| Test        | -Statistic- | -----p Value----- |
|-------------|-------------|-------------------|
| Student's t | t 4.140325  | Pr >  t  <.0001   |
| Sign        | M 27        | Pr >=  M  0.0002  |
| Signed Rank | S 3326.5    | Pr >=  S  <.0001  |

Location Counts: Mu0=50.00

| Count          | Value |
|----------------|-------|
| Num Obs > Mu0  | 12    |
| Num Obs ^= Mu0 | 198   |
| Num Obs < Mu0  | 72    |

You can use either the sign test or the signed rank test. The difference between these two tests is that the signed rank requires that the variable be from a symmetric distribution. The results indicate that the median of the variable **write** for this group is statistically significantly different from 50.

## Binomial test

from SAS, SAS, however, we will use the SAS statements to produce the exact p-value.

```
proc freq data = "c:/mydata/hsb2";  
  tables female / binomial(p=.5);  
  exact binomial;  
run;
```

---

|   |     |       |     |        |
|---|-----|-------|-----|--------|
| 0 | 91  | 45.50 | 91  | 45.50  |
| 1 | 109 | 54.50 | 200 | 100.00 |

Binomial Proportion for female = 0

---

|                      |        |
|----------------------|--------|
| Proportion (P)       | 0.4550 |
| ASE                  | 0.0352 |
| 95% Lower Conf Limit | 0.3860 |
| 95% Upper Conf Limit | 0.5240 |

Exact Conf Limits

|                      |        |
|----------------------|--------|
| 95% Lower Conf Limit | 0.3846 |
| 95% Upper Conf Limit | 0.5267 |

Test of H0: Proportion = 0.5

|                   |         |
|-------------------|---------|
| ASE under H0      | 0.0354  |
| Z                 | -1.2728 |
| One-sided Pr < Z  | 0.1015  |
| Two-sided Pr >  Z | 0.2031  |

Exact Test

|                   |        |
|-------------------|--------|
| One-sided Pr <= P | 0.1146 |
|-------------------|--------|



The results indicate that there is no statistically significant difference ( $p = .2292$ ). In other words, the proportion of females in this sample does not significantly differ from the hypothesized value of 50%.

### See also

- [Recent Advances in Categorical Data Analysis \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/categorical.pdf\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/categorical.pdf)

## Chi-square goodness of fit

**testp=** option on the **tables** statement.

```
proc freq data = "c:/mydata/hsb2";
  tables race / chisq testp=(10 10 10 70);
run;
```

The FREQ Procedure

| race | Frequency | Percent | Test<br>Percent | Cumulative<br>Frequency | Cumulative<br>Percent |
|------|-----------|---------|-----------------|-------------------------|-----------------------|
| 1    | 24        | 12.00   | 10.00           | 24                      | 12.00                 |
| 2    | 11        | 5.50    | 10.00           | 35                      | 17.50                 |
| 3    | 20        | 10.00   | 10.00           | 55                      | 27.50                 |
| 4    | 145       | 72.50   | 70.00           | 200                     | 100.00                |

Chi-Square Test  
for Specified Proportions

```
-----
Chi-Square          5.0286
DF                  3
Pr > ChiSq          0.1697
```

Sample Size = 200

## Two independent samples t-test

```
proc ttest data = "c:/mydata/hsb2";
  class female;
  var write;
run;
```

# The TTEST Procedure

## Statistics

|          |            |     | Lower CL |        | Upper CL |         | Lower CL |         | Upper CL |  |
|----------|------------|-----|----------|--------|----------|---------|----------|---------|----------|--|
| Variable | female     | N   | Mean     | Mean   | Mean     | Std Dev | Std Dev  | Std Dev | Std Err  |  |
| write    | 0          | 91  | 47.975   | 50.121 | 52.267   | 8.9947  | 10.305   | 12.066  | 1.0803   |  |
| write    | 1          | 109 | 53.447   | 54.991 | 56.535   | 7.1786  | 8.1337   | 9.3843  | 0.7791   |  |
| write    | Diff (1-2) |     | -7.442   | -4.87  | -2.298   | 8.3622  | 9.1846   | 10.188  | 1.3042   |  |

## T-Tests

| Variable | Method        | Variances | DF  | t Value | Pr >  t |
|----------|---------------|-----------|-----|---------|---------|
| write    | Pooled        | Equal     | 198 | -3.73   | 0.0002  |
| write    | Satterthwaite | Unequal   | 170 | -3.66   | 0.0003  |

## Equality of Variances

| Variable | Method   | Num DF | Den DF | F Value | Pr > F |
|----------|----------|--------|--------|---------|--------|
| write    | Folded F | 90     | 108    | 1.61    | 0.0187 |

**See also**

- [Annotated Output of Proc TTest \(/sas/output/proc-ttest/\)](/sas/output/proc-ttest/)

## Wilcoxon-Mann-Whitney test

The Wilcoxon-Mann-Whitney test is a non-parametric analog to the independent samples t-test and can be used when you do not assume that the dependent variable is a normally distributed interval variable (you need only assume that the variable is at least ordinal). We will use the same data file (the [hsb2 data file \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/hsb2.sas7bdat\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/hsb2.sas7bdat)) and the same variables in this example as we did in the [independent t-test example \(/sas/whatstat/#2ittest\)](/sas/whatstat/#2ittest) above and will not assume that **write**, our dependent variable, is normally distributed.

```
run ;
```

| female | N   | Sum of<br>Scores | Expected<br>Under H0 | Std Dev<br>Under H0 | Mean<br>Score |
|--------|-----|------------------|----------------------|---------------------|---------------|
| -----  |     |                  |                      |                     |               |
| 0      | 91  | 7792.0           | 9145.50              | 406.559086          | 85.626374     |
| 1      | 109 | 12308.0          | 10954.50             | 406.559086          | 112.917431    |

Average scores were used for ties.

#### Wilcoxon Two-Sample Test

Statistic                      7792.0000

#### Normal Approximation

Z                                -3.3279

One-Sided Pr < Z              0.0004

Two-Sided Pr > |Z|            0.0009

#### t Approximation

One-Sided Pr < Z              0.0005

Two-Sided Pr > |Z|            0.0010

Z includes a continuity correction of 0.5.

- FAQ: Why is the Mann-Whitney significant when the medians are equal?
- [Exact Methods in the NPAR1WAY Procedure \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/exact.pdf\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/exact.pdf)
- [Sample Size Computations and Power Analysis with the SAS System \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/powersamplesize.pdf\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/powersamplesize.pdf)

## Chi-square test



schtyp\*female / chisq; Remember that the chi-square test assumes that the expected count for each cell is five or higher. This assumption is easily met in the examples below. However, if this assumption is not met in your data, please see the section on Fisher's exact test below.

```
proc freq data = "c:/mydata/hsb2";  
  tables schtyp*female / chisq;  
run;
```

```
schtyp(type of school)
```

```
      female
```

```
Frequency|
```

```
Percent  |
```

```
Row Pct  |
```

```
Col Pct  |          0|          1|   Total
```

```
-----+-----+-----+
      1 |      77 |      91 |      168
      |  38.50 |  45.50 |      84.00
      |  45.83 |  54.17 |
      |  84.62 |  83.49 |
```

```
-----+-----+-----+
      2 |      14 |      18 |       32
      |   7.00 |   9.00 |      16.00
      |  43.75 |  56.25 |
      |  15.38 |  16.51 |
```

```
-----+-----+-----+
Total          91          109          200
              45.50       54.50      100.00
```

```
Statistics for Table of schtyp by female
```

```
Statistic                      DF          Value          Prob
```

```
-----
```

|                         |        |
|-------------------------|--------|
| Phi Coefficient         | 0.0153 |
| Contingency Coefficient | 0.0153 |
| Cramer's V              | 0.0153 |

Sample Size = 200

These results indicate that there is no statistically significant relationship between the type of school attended and gender (chi-square with one degree of freedom = 0.0470,  $p = 0.8283$ ).

Let's look at another example, this time looking at the relationship between gender (**female**) and socio-economic status (**ses**). The point of this example is that one (or both) variables may have more than two levels, and that the variables do not have to have the same number of levels. In this example, **female** has two levels (male and female) and **ses** has three levels (low, medium and high).

```
proc freq data = "c:/mydata/hsb2";  
    tables female*ses / chisq;  
run;
```

| female                   |  | ses   |   |       |       |       |  |        |
|--------------------------|--|-------|---|-------|-------|-------|--|--------|
| Frequency                |  |       |   |       |       |       |  |        |
| Percent                  |  |       |   |       |       |       |  |        |
| Row Pct                  |  |       |   |       |       |       |  |        |
| Col Pct                  |  | 1     | 2 | 3     | Total |       |  |        |
| -----+-----+-----+-----+ |  |       |   |       |       |       |  |        |
| 0                        |  | 15    |   | 47    |       | 29    |  | 91     |
|                          |  | 7.50  |   | 23.50 |       | 14.50 |  | 45.50  |
|                          |  | 16.48 |   | 51.65 |       | 31.87 |  |        |
|                          |  | 31.91 |   | 49.47 |       | 50.00 |  |        |
| -----+-----+-----+-----+ |  |       |   |       |       |       |  |        |
| 1                        |  | 32    |   | 48    |       | 29    |  | 109    |
|                          |  | 16.00 |   | 24.00 |       | 14.50 |  | 54.50  |
|                          |  | 29.36 |   | 44.04 |       | 26.61 |  |        |
|                          |  | 68.09 |   | 50.53 |       | 50.00 |  |        |
| -----+-----+-----+-----+ |  |       |   |       |       |       |  |        |
| Total                    |  | 47    |   | 95    |       | 58    |  | 200    |
|                          |  | 23.50 |   | 47.50 |       | 29.00 |  | 100.00 |

Statistics for Table of female by ses

| Statistic | DF | Value | Prob |
|-----------|----|-------|------|
| -----     |    |       |      |

|                         |        |
|-------------------------|--------|
| Contingency Coefficient | 0.1496 |
| Cramer's V              | 0.1513 |

Sample Size = 200

Again we find that there is no statistically significant relationship between the variables (chi-square with two degrees of freedom = 4.5765,  $p = 0.1014$ ).

### See also

- [Annotated Output of Proc Freq \(/sas/output/proc-freq/\)](/sas/output/proc-freq/)
- [Sample Size Computations and Power Analysis with the SAS System \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/powersamplesize.pdf\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/powersamplesize.pdf)
- [SAS Learning Module: An Overview of Statistical Tests in SAS \(/sas/modules/an-overview-of-statistical-tests-in-sas/\)](/sas/modules/an-overview-of-statistical-tests-in-sas/)

## Fisher's exact test

could be below five, so we will use Fisher's exact test with the **fisher** option on the **tables** statement.

```
proc freq data = "c:/mydata/hsb2";  
  tables schtyp*race / fisher;  
run;
```

| schtyp (type of school)        |       |       |       |       | race   |
|--------------------------------|-------|-------|-------|-------|--------|
| Frequency                      |       |       |       |       |        |
| Percent                        |       |       |       |       |        |
| Row Pct                        |       |       |       |       |        |
| Col Pct                        | 1     | 2     | 3     | 4     | Total  |
| -----+-----+-----+-----+-----+ |       |       |       |       |        |
| 1                              | 22    | 10    | 18    | 118   | 168    |
|                                | 11.00 | 5.00  | 9.00  | 59.00 | 84.00  |
|                                | 13.10 | 5.95  | 10.71 | 70.24 |        |
|                                | 91.67 | 90.91 | 90.00 | 81.38 |        |
| -----+-----+-----+-----+-----+ |       |       |       |       |        |
| 2                              | 2     | 1     | 2     | 27    | 32     |
|                                | 1.00  | 0.50  | 1.00  | 13.50 | 16.00  |
|                                | 6.25  | 3.13  | 6.25  | 84.38 |        |
|                                | 8.33  | 9.09  | 10.00 | 18.62 |        |
| -----+-----+-----+-----+-----+ |       |       |       |       |        |
| Total                          | 24    | 11    | 20    | 145   | 200    |
|                                | 12.00 | 5.50  | 10.00 | 72.50 | 100.00 |

Statistics for Table of schtyp by race

| Statistic | DF | Value | Prob |
|-----------|----|-------|------|
| -----     |    |       |      |

|                         |        |
|-------------------------|--------|
| Contingency Coefficient | 0.1158 |
| Cramer's V              | 0.1166 |

WARNING: 38% of the cells have expected counts less than 5. Chi-Square may not be a valid test.

#### Fisher's Exact Test

---

|                       |        |
|-----------------------|--------|
| Table Probability (P) | 0.0077 |
| Pr <= P               | 0.5975 |

Sample Size = 200

These results suggest that there is not a statistically significant relationship between race and type of school ( $p = 0.5975$ ). Note that the Fisher's exact test does not have a "test statistic", but computes the p-value directly.

### See also

- [Sample Size Computations and Power Analysis with the SAS System \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/powersamplesize.pdf\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/powersamplesize.pdf)
- [SAS Learning Module: An Overview of Statistical Tests in SAS \(/sas/modules/an-overview-of-statistical-tests-in-sas/\)](/sas/modules/an-overview-of-statistical-tests-in-sas/)





between the three program types (**prog**). We will also use the means statement to output the mean of write for each level of program type. Note that this will not tell you if there is a statistically significant difference between any two sets of means.

```
proc glm data = "c:/mydata/hsb2";  
  class prog;  
  model write = prog;  
  means prog;  
run;  
quit;
```

```

Class          Levels    Values
prog              3      1 2 3

Number of observations      200

Dependent Variable: write    writing score

                                Sum of
Source                DF        Squares    Mean Square    F Value    Pr > F
Model                  2        3175.69786    1587.84893     21.27     <.0001
Error                 197        14703.17714      74.63542
Corrected Total       199        17878.87500

R-Square      Coeff Var      Root MSE    write Mean
0.177623      16.36983      8.639179      52.77500

Source                DF        Type I SS    Mean Square    F Value    Pr > F
prog                  2        3175.697857    1587.848929     21.27     <.0001

Source                DF        Type III SS    Mean Square    F Value    Pr > F
prog                  2        3175.697857    1587.848929     21.27     <.0001

Level of      -----write-----
prog          N          Mean          Std Dev

```

The mean of the dependent variable differs significantly among the levels of program type. However, we do not know if the difference is between only two of the levels or all three of the levels. (The F test for the model is the same as the F test for **prog** because **prog** was the only variable entered into the model. If other variables had also been entered, the F test for the **Model** would have been different from **prog**.) We can also see that the students in the academic program have the highest mean writing score, while students in the vocational program have the lowest.

## See also

- [SAS FAQ: How can I do ANOVA contrasts? \(/sas/faq/how-can-i-do-anova-contrasts/\)](/sas/faq/how-can-i-do-anova-contrasts/)
- [SAS Textbook Examples: Design and Analysis \(/sas/examples/da/design-and-analysis-a-researchers-handbook-by-geoffrey-keppel-download-sas-data-files/\)](/sas/examples/da/design-and-analysis-a-researchers-handbook-by-geoffrey-keppel-download-sas-data-files/)

## Kruskal Wallis test

variables as in the example above, but we will not assume that **write** is a normally distributed interval variable.

```
proc npar1way data = "c:/mydata/hsb2";  
  class prog;  
  var write;  
run;
```

| prog | N   | Sum of<br>Scores | Expected<br>Under H0 | Std Dev<br>Under H0 | Mean<br>Score |
|------|-----|------------------|----------------------|---------------------|---------------|
| 1    | 45  | 4079.0           | 4522.50              | 340.927342          | 90.644444     |
| 3    | 50  | 3257.0           | 5025.00              | 353.525185          | 65.140000     |
| 2    | 105 | 12764.0          | 10552.50             | 407.705133          | 121.561905    |

Average scores were used for ties.

#### Kruskal-Wallis Test

Chi-Square            34.0452  
 DF                      2  
 Pr > Chi-Square      <.0001

The results indicate that there is a statistically significant difference among the three type of programs (chi-square with two degrees of freedom = 34.0452,  $p = 0.0001$ ).

### See also

- Exact Methods in the NPAR1WAY Procedure (<https://stats.idre.ucla.edu/wp-content/uploads/2016/02/exact.pdf>)



```
proc ttest data = "c:/mydata/hsb2";  
  paired write*read;  
run;
```

#### The TTEST Procedure

##### Statistics

|              |     | Lower CL |       | Upper CL | Lower CL |         | Upper CL |         |
|--------------|-----|----------|-------|----------|----------|---------|----------|---------|
| Difference   | N   | Mean     | Mean  | Mean     | Std Dev  | Std Dev | Std Dev  | Std Err |
| write - read | 200 | -0.694   | 0.545 | 1.7841   | 8.0928   | 8.8867  | 9.8546   | 0.6284  |

##### T-Tests

| Difference   | DF  | t Value | Pr >  t |
|--------------|-----|---------|---------|
| write - read | 199 | 0.87    | 0.3868  |

These results indicate that the mean of **read** is not statistically significantly different from the mean of **write** (t = 0.87, p = 0.3868).

## See also

- [Annotated Output of Proc TTest \(/sas/output/proc-ttest/\)](/sas/output/proc-ttest/)





scores for each subject. This is necessary because SAS will not calculate the difference for you in **proc univariate**.

```
data hsb2a;  
  set 'c:/mydata/hsb2';  
  diff = read - write;  
run;  
  
proc univariate data = hsb2a;  
  var diff;  
run;
```

| Location                  |             | Variability         |          |
|---------------------------|-------------|---------------------|----------|
| Mean                      | -0.54500    | Std Deviation       | 8.88667  |
| Median                    | 0.00000     | Variance            | 78.97284 |
| Mode                      | 6.00000     | Range               | 45.00000 |
|                           |             | Interquartile Range | 13.00000 |
| Tests for Location: Mu0=0 |             |                     |          |
| Test                      | -Statistic- | -----p Value-----   |          |
| Student's t               | t -0.86731  | Pr >  t             | 0.3868   |
| Sign                      | M -4.5      | Pr >=  M            | 0.5565   |
| Signed Rank               | S -658.5    | Pr >=  S            | 0.3677   |

The results suggest that there is not a statistically significant difference between **read** and **write**.

If you believe the differences between **read** and **write** were not ordinal but could merely be classified as positive and negative, then you may want to consider a sign test in lieu of sign rank test. Note that the SAS output gives you the results for both the Wilcoxon signed rank test and the sign test without having to use any options. Using the sign test, we again conclude that there is no statistically significant difference between **read** and **write** (p=.5565).



answered Q2 correctly and Q1 incorrectly. These counts can be considered in a two-way contingency table. The null hypothesis is that the two questions are answered correctly or incorrectly at the same rate (or that the contingency table is symmetric).

```
data set1;
  input Q1correct Q2correct students;
  datalines;
  1 1 172
  0 1 6
  1 0 7
  0 0 15
run;

proc freq data=set1;
  table Q1correct*Q2correct;
  exact mcnem;
  weight students;
run;
```

| Q1correct          |       | Q2correct |        |  |
|--------------------|-------|-----------|--------|--|
| Frequency          |       |           |        |  |
| Percent            |       |           |        |  |
| Row Pct            |       |           |        |  |
| Col Pct            | 0     | 1         | Total  |  |
| -----+-----+-----+ |       |           |        |  |
| 0                  | 15    | 6         | 21     |  |
|                    | 7.50  | 3.00      | 10.50  |  |
|                    | 71.43 | 28.57     |        |  |
|                    | 68.18 | 3.37      |        |  |
| -----+-----+-----+ |       |           |        |  |
| 1                  | 7     | 172       | 179    |  |
|                    | 3.50  | 86.00     | 89.50  |  |
|                    | 3.91  | 96.09     |        |  |
|                    | 31.82 | 96.63     |        |  |
| -----+-----+-----+ |       |           |        |  |
| Total              | 22    | 178       | 200    |  |
|                    | 11.00 | 89.00     | 100.00 |  |

Statistics for Table of Q1correct by Q2correct

McNemar's Test

```
Exact      Pr >= S      1.0000
```

#### Simple Kappa Coefficient

```
-----  
Kappa      0.6613  
ASE         0.0873  
95% Lower Conf Limit  0.4901  
95% Upper Conf Limit  0.8324
```

```
Sample Size = 200
```

McNemar's test statistic suggests that there is not a statistically significant difference in the proportions of correct/incorrect answers to these two questions.

## See also

## One-way repeated measures ANOVA



rb4wide (<https://stats.idre.ucla.edu/wp-content/uploads/2016/02/rb4wide.sas7bdat>), which is used in Kirk's book Experimental Design. In this data set, **y1 y2 y3** and **y4** represent the dependent variable measured at the 4 levels of **a**, the repeated measures independent variable.

```
proc glm data = 'c:/mydata/rb4wide';  
  model y1 y2 y3 y4 = ;  
  repeated a ;  
run;  
quit;
```

|                    |    |    |    |    |
|--------------------|----|----|----|----|
| Dependent Variable | Y1 | Y2 | Y3 | Y4 |
|--------------------|----|----|----|----|

|            |   |   |   |   |
|------------|---|---|---|---|
| Level of a | 1 | 2 | 3 | 4 |
|------------|---|---|---|---|

Manova Test Criteria and Exact F Statistics for the Hypothesis of no a Effect

H = Type III SSCP Matrix for a

E = Error SSCP Matrix

S=1      M=0.5      N=1.5

| Statistic              | Value      | F Value | Num DF | Den DF | Pr > F |
|------------------------|------------|---------|--------|--------|--------|
| Wilks' Lambda          | 0.24580793 | 5.11    | 3      | 5      | 0.0554 |
| Pillai's Trace         | 0.75419207 | 5.11    | 3      | 5      | 0.0554 |
| Hotelling-Lawley Trace | 3.06821705 | 5.11    | 3      | 5      | 0.0554 |
| Roy's Greatest Root    | 3.06821705 | 5.11    | 3      | 5      | 0.0554 |

Repeated Measures Analysis of Variance

Univariate Tests of Hypotheses for Within Subject Effects

| Source | DF | Type III SS | Mean Square | F Value | Pr > F | Adj G - G | Pr > F | Adj H - F |
|--------|----|-------------|-------------|---------|--------|-----------|--------|-----------|
| a      | 3  | 49.00000000 | 16.33333333 | 11.63   | 0.0001 | 0.0015    | 0.0003 |           |

The results indicate that the model as well as both factors (**a** and **s**) are statistically significant. The p-value given in this output for **a** (0.0001) is the “regular” p-value and is the p-value that you would get if you assumed compound symmetry in the variance-covariance matrix.

## See also

- [SAS Library: Analysis of Variance in SAS \(/sas/library/sas-libraryproc-anova-and-proc-glm-summary-for-analysis-of-variance/\)](/sas/library/sas-libraryproc-anova-and-proc-glm-summary-for-analysis-of-variance/)
- [SAS Library: Repeated Measures Anova \(/sas/library/sas-libraryrepeated-measures-anova-using-sas-proc-glm/\)](/sas/library/sas-libraryrepeated-measures-anova-using-sas-proc-glm/)
- [SAS Library: Comparing Methods of Analyzing Repeated Measures Data \(/sas/library/sas-library-comparing-methods-of-analyzing-repeated-measures-data/\)](/sas/library/sas-library-comparing-methods-of-analyzing-repeated-measures-data/)
- [SAS Textbook Examples: Design and Analysis, Chapter 16 \(/sas/examples/da/design-and-analysis-by-keppelchapter-16/\)](/sas/examples/da/design-and-analysis-by-keppelchapter-16/)

## Repeated measures logistic regression

contains three pulse measurements from each of 30 people assigned to two different diet regiments and three different exercise regiments. If we define a “high” pulse as being over 100, we can then predict the probability of a high pulse using diet regiment.

```
proc genmod data='c:/mydata/exercise' descending;  
  class id diet / descending;  
  model highpulse = diet / dist = bin link = logit;  
  repeated subject = id / type = exch;  
run;
```

|   |   |    |
|---|---|----|
| 1 | 1 | 27 |
| 2 | 0 | 63 |

PROC GENMOD is modeling the probability that highpulse='1'.

#### Parameter Information

| Parameter | Effect    | diet |
|-----------|-----------|------|
| Prm1      | Intercept |      |
| Prm2      | diet      | 2    |
| Prm3      | diet      | 1    |

Algorithm converged.

#### GEE Model Information

|                       |                |
|-----------------------|----------------|
| Correlation Structure | Exchangeable   |
| Subject Effect        | id (30 levels) |
| Number of Clusters    | 30             |

The GENMOD Procedure

```
Maximum Cluster Size          3
Minimum Cluster Size          3

Algorithm converged.

      Exchangeable Working
      Correlation

Correlation      0.3306722695

      GEE Fit Criteria

QIC      113.9859
QICu     111.3405

      Analysis Of GEE Parameter Estimates
      Empirical Standard Error Estimates

      Standard   95% Confidence
Parameter  Estimate   Error      Limits          Z Pr > |Z|

Intercept  -1.2528    0.4328   -2.1011   -0.4044   -2.89    0.0038
diet       2  0.7538    0.6031   -0.4283    1.9358    1.25    0.2114
```

## Factorial ANOVA

... economic status (ses), as independent variables, and we will include an interaction of female by ses. Note that in this case, you do not need to have the interaction term(s) in your data set. Rather, you can have SAS create it/them temporarily by placing an asterisk between the variables that will make up the interaction term(s).

```
proc glm data = "c:/mydata/hsb2";  
  class female ses;  
  model write = female ses female*ses;  
run;  
quit;
```



|                 |           | Sum of      |             |         |        |  |
|-----------------|-----------|-------------|-------------|---------|--------|--|
| Source          | DF        | Squares     | Mean Square | F Value | Pr > F |  |
| Model           | 5         | 2278.24419  | 455.64884   | 5.67    | <.0001 |  |
| Error           | 194       | 15600.63081 | 80.41562    |         |        |  |
| Corrected Total | 199       | 17878.87500 |             |         |        |  |
| R-Square        | Coeff Var | Root MSE    | write Mean  |         |        |  |
| 0.127427        | 16.99190  | 8.967476    | 52.77500    |         |        |  |
| Source          | DF        | Type I SS   | Mean Square | F Value | Pr > F |  |
| female          | 1         | 1176.213845 | 1176.213845 | 14.63   | 0.0002 |  |
| ses             | 2         | 1080.599437 | 540.299718  | 6.72    | 0.0015 |  |
| female*ses      | 2         | 21.430904   | 10.715452   | 0.13    | 0.8753 |  |
| Source          | DF        | Type III SS | Mean Square | F Value | Pr > F |  |
| female          | 1         | 1334.493311 | 1334.493311 | 16.59   | <.0001 |  |
| ses             | 2         | 1063.252697 | 531.626349  | 6.61    | 0.0017 |  |
| female*ses      | 2         | 21.430904   | 10.715452   | 0.13    | 0.8753 |  |

These results indicate that the overall model is statistically significant (F = 5.67, p = 0.001). The variables **female** and **ses** are also statistically significant (F = 16.59, p = 0.0001 and F = 6.61, p = 0.0017, respectively). However, that interaction between **female** and **ses** is not statistically significant (F = 0.13, p = 0.8753).

- [SAS Textbook Examples: Applied Linear Statistical Models \(/examples/alsm/\)](/examples/alsm/)
- [SAS Textbook Examples: Fox, Applied Regression Analysis, Chapter 8 \(/sas/examples/ara/applied-regression-analysis-by-john-foxchapter-8-analysis-of-variance/\)](/sas/examples/ara/applied-regression-analysis-by-john-foxchapter-8-analysis-of-variance/)
- [SAS Learning Module: Comparing SAS and Stata Side by Side \(/sas/modules/comparing-sas-and-stata-side-by-side/\)](/sas/modules/comparing-sas-and-stata-side-by-side/)
- [SAS Textbook Examples from Design and Analysis: Chapter 10 \(/sas/examples/da/design-and-analysis-by-keppelchapter-10/\)](/sas/examples/da/design-and-analysis-by-keppelchapter-10/)

## Friedman test

format; we will use **proc transpose** to change our data from the wide format that they are currently in to a long format. We create a variable to code for the type of score, which we will call **rwm** (for read, write, math), and **col1** that contains the score on the dependent variable, that is the reading, writing or math score. To obtain the Friedman test, you need to use the **cmh2** option on the **tables** statement in **proc freq**.

```
proc sort data = "c:/mydata/hsb2" out=hsbsort;
  by id;
run;

proc transpose data=hsbsort out=hsblong name=rwm;
  by id;
  var read write math;
run;

proc freq data=hsblong;
  tables id*rwm*col1 / cmh2 scores=rank noprint;
run;
```

| Cochran-Mantel-Haenszel Statistics (Based on Rank Scores)  |                        |    |        |        |
|--|------------------------|----|--------|--------|
| Statistic  | Alternative Hypothesis | DF | Value  | Prob   |
| 1  | Nonzero Correlation    | 1  | 0.0790 | 0.7787 |
| 2  | Row Mean Scores Differ | 2  | 0.6449 | 0.7244 |
| Total Sample Size = 600  |                        |    |        |        |
| The Row Mean Scores Differ is the same as the Friedman's chi-square, and we see that with a value of 0.6449 and a p-value of 0.7244, it is not statistically significant. Hence, there is no evidence that the distributions of the three types of scores are different. |                        |    |        |        |

## Ordered logistic regression

...commented, categorizing a continuous variable in this way, we are simply creating a variable to use for the example. We will use gender (**female**), reading score (**read**) and social studies score (**socst**) as predictor variables in this model. The **desc** option on the **proc logistic** statement is used so that SAS models the odds of being in the lower category. The Response Profile table in the output shows the value that SAS used when conducting the analysis (given in the Ordered Value column), the value of the original variable, and the number of cases in each level of the outcome variable. (If you want SAS to use the values that you have assigned the outcome variable, then you would want to use the **order = data** option on the **proc logistic** statement.) The note below this table reminds us that the “Probabilities modeled are cumulated over the lower Ordered Values.” It is helpful to remember this when interpreting the output. The **expb** option on the model statement tells SAS to show the exponentiated coefficients (i.e., the proportional odds ratios).

```
data hsb2_ordered;
  set "c:/mydata/hsb2";
  if 30 <= write <=48 then write3 = 1;
  if 49 <= write <=57 then write3 = 2;
  if 58 <= write <=70 then write3 = 3;
run;

proc logistic data = hsb2_ordered desc;
  model write3 = female read socst / expb;
run;
```

|                           |                   |
|---------------------------|-------------------|
| Data Set                  | WORK.HSB2_ORDERED |
| Response Variable         | write3            |
| Number of Response Levels | 3                 |
| Model                     | cumulative logit  |
| Optimization Technique    | Fisher's scoring  |

|                             |     |
|-----------------------------|-----|
| Number of Observations Read | 200 |
| Number of Observations Used | 200 |

#### Response Profile

| Ordered<br>Value | write3 | Total<br>Frequency |
|------------------|--------|--------------------|
| 1                | 3      | 78                 |
| 2                | 2      | 61                 |
| 3                | 1      | 61                 |

Probabilities modeled are cumulated over the lower Ordered Values.

#### Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

2.1211            3            0.5477

Model Fit Statistics

| Criterion | Intercept | Intercept         |
|-----------|-----------|-------------------|
|           | Only      | and<br>Covariates |
| AIC       | 440.627   | 322.553           |
| SC        | 447.224   | 339.044           |
| -2 Log L  | 436.627   | 312.553           |

Testing Global Null Hypothesis: BETA=0

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 124.0745   | 3  | <.0001     |
| Score            | 93.1890    | 3  | <.0001     |
| Wald             | 76.6752    | 3  | <.0001     |

Analysis of Maximum Likelihood Estimates

Standard            Wald

|        |   |        |        |         |        |       |
|--------|---|--------|--------|---------|--------|-------|
| FEMALE | 1 | 1.2856 | 0.3225 | 15.8901 | <.0001 | 3.617 |
| READ   | 1 | 0.1177 | 0.0215 | 29.8689 | <.0001 | 1.125 |
| SOCST  | 1 | 0.0802 | 0.0190 | 17.7817 | <.0001 | 1.083 |

#### Odds Ratio Estimates

| Effect | Point Estimate | 95% Wald Confidence Limits |       |
|--------|----------------|----------------------------|-------|
| FEMALE | 3.617          | 1.922                      | 6.805 |
| READ   | 1.125          | 1.078                      | 1.173 |
| SOCST  | 1.083          | 1.044                      | 1.125 |

#### Association of Predicted Probabilities and Observed Responses

|                    |       |           |       |
|--------------------|-------|-----------|-------|
| Percent Concordant | 83.8  | Somers' D | 0.681 |
| Percent Discordant | 15.7  | Gamma     | 0.685 |
| Percent Tied       | 0.6   | Tau-a     | 0.453 |
| Pairs              | 13237 | c         | 0.840 |

The results indicate that the overall model is statistically significant ( $p < .0001$ ), as are each of the predictor variables ( $p < .0001$ ). There are two intercepts for this model because there are three levels of the outcome variable. We also see that the test of the proportional odds assumption is non-significant ( $p = .5477$ ). One of the assumptions underlying ordinal logistic (and ordinal probit) regression is that the relationship between each pair of outcome groups is the same. In other words, ordinal



case, we would need different models (such as a generalized ordered logit model) to describe the relationship between each pair of outcome groups.

### See also

[SAS Annotated Output: Ordered logistic regression \(/sas/output/ordered-logistic-regression/\)](#)

## Factorial logistic regression

...erally, it is a common practice to use gender as an outcome variable. The two predictor variables are **prog** (program type) and **schtyp** (school type) as our predictor variables. Because neither **prog** nor **schtyp** are continuous variables, we need to include them on the **class** statement. The **desc** option on the **proc logistic** statement is necessary so that SAS models the odds of being female (i.e., female = 1). The **expb** option on the model statement tells SAS to show the exponentiated coefficients (i.e., the odds ratios).

```
proc logistic data = "c:/mydata/hsb2" desc;  
  class prog schtyp;  
  model female = prog schtyp prog*schtyp / expb;  
run;
```

|           | Intercept | Intercept<br>and<br>Covariates |
|-----------|-----------|--------------------------------|
| Criterion | Only      |                                |
| AIC       | 277.637   | 284.490                        |
| SC        | 280.935   | 304.280                        |
| -2 Log L  | 275.637   | 272.490                        |

Testing Global Null Hypothesis: BETA=0

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 3.1467     | 5  | 0.6774     |
| Score            | 2.9231     | 5  | 0.7118     |
| Wald             | 2.6036     | 5  | 0.7608     |

Type III Analysis of Effects

| Effect      | DF | Wald<br>Chi-Square | Pr > ChiSq |
|-------------|----|--------------------|------------|
| prog        | 2  | 1.1232             | 0.5703     |
| schtyp      | 1  | 0.4132             | 0.5203     |
| prog*schtyp | 2  | 2.4740             | 0.2903     |

Analysis of Maximum Likelihood Estimates

|             |     |   |         |        |        |        |       |
|-------------|-----|---|---------|--------|--------|--------|-------|
| prog        | 2   | 1 | -0.1964 | 0.3438 | 0.3264 | 0.5678 | 0.822 |
| schtyp      | 1   | 1 | -0.2034 | 0.3164 | 0.4132 | 0.5203 | 0.816 |
| prog*schtyp | 1 1 | 1 | -0.6269 | 0.4568 | 1.8838 | 0.1699 | 0.534 |
| prog*schtyp | 2 1 | 1 | 0.3400  | 0.3438 | 0.9783 | 0.3226 | 1.405 |

The results indicate that the overall model is not statistically significant (LR chi2 = 3.1467, p = 0.6774). Furthermore, none of the coefficients are statistically significant either. In addition, there is no statistically significant effect of program (p = 0.5703), school type (p = 0.5203) or of the interaction (p = 0.2903).

## Correlation

```
proc corr data = "c:/mydata/hsb2";  
  var read write;  
run;
```

The CORR Procedure

2 Variables:      read      write

Pearson Correlation Coefficients, N = 200

Prob > |r| under H0: Rho=0

|               | read    | write   |
|---------------|---------|---------|
| read          | 1.00000 | 0.59678 |
| reading score |         | <.0001  |
| write         | 0.59678 | 1.00000 |
| writing score | <.0001  |         |

In the second example below, we will run a correlation between a dichotomous variable, **female**, and a continuous variable, **write**. Although it is assumed that the variables are interval and normally distributed, we can include dummy variables when performing correlations.

## The CORR Procedure

2 Variables:      female      write

Pearson Correlation Coefficients, N = 200

Prob > |r| under H0: Rho=0

|               | female  | write             |
|---------------|---------|-------------------|
| female        | 1.00000 | 0.25649<br>0.0002 |
| write         | 0.25649 | 1.00000           |
| writing score | 0.0002  |                   |

In the first example above, we see that the correlation between **read** and **write** is 0.59678. By squaring the correlation and then multiplying by 100, you can determine what percentage of the variability is shared. Let's round 0.59678 to be 0.6, which when squared would be .36, multiplied by 100 would be 36%. Hence **read** shares about 36% of its variability with **write**. In the output for the second example, we can see the correlation between **write** and **female** is 0.25649. Squaring this number yields .0657871201, meaning that **female** shares approximately 6.5% of its variability with **write**.

## See also

- [Annotated SAS Output: Correlation \(/sas/output/proc-corr/\)](/sas/output/proc-corr/)

## Simple linear regression

proc reg data = "c:/mydata/hsb2";

model write = read / stb;

run;

quit;



## Analysis of Variance

| Source          | DF  | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|-----|----------------|-------------|---------|--------|
| Model           | 1   | 6367.42127     | 6367.42127  | 109.52  | <.0001 |
| Error           | 198 | 11511          | 58.13866    |         |        |
| Corrected Total | 199 | 17879          |             |         |        |

|                |          |          |        |
|----------------|----------|----------|--------|
| Root MSE       | 7.62487  | R-Square | 0.3561 |
| Dependent Mean | 52.77500 | Adj R-Sq | 0.3529 |
| Coeff Var      | 14.44788 |          |        |

## Parameter Estimates

| Variable  | Label         | DF | Parameter Estimate | Standard Error | t Value | Pr >  t | Standardized Estimate |
|-----------|---------------|----|--------------------|----------------|---------|---------|-----------------------|
| Intercept | Intercept     | 1  | 23.95944           | 2.80574        | 8.54    | <.0001  | 0                     |
| read      | reading score | 1  | 0.55171            | 0.05272        | 10.47   | <.0001  | 0.59678               |

We see that the relationship between **write** and **read** is positive (.55171) and based on the t-value (10.47) and p-value (0.000), we conclude this relationship is statistically significant. Hence, there is a statistically significant positive linear relationship between reading and writing.

- [SAS Library: Overview of SAS PROC REG \(/sas/library/sas-libraryoverview-of-sas-proc-reg/\)](/sas/library/sas-libraryoverview-of-sas-proc-reg/)
- [SAS Textbook Examples: Applied Linear Statistical Models \(/examples/alsm/\)](/examples/alsm/)
- [SAS Textbook Examples: Regression Analysis by Example, Chapter 2 \(/sas/examples/chp/regression-analysis-by-example-by-chatterjee-hadi-and-pricechapter-2-simple-linear-regression/\)](/sas/examples/chp/regression-analysis-by-example-by-chatterjee-hadi-and-pricechapter-2-simple-linear-regression/)

## Non-parametric correlation

correlation.

```
proc corr data = "c:/mydata/hsb2" spearman;  
  var read write;  
run;
```

The CORR Procedure

2 Variables:      read      write

Spearman Correlation Coefficients, N = 200

Prob > |r| under H0: Rho=0

|               | read    | write   |
|---------------|---------|---------|
| read          | 1.00000 | 0.61675 |
| reading score |         | <.0001  |
| write         | 0.61675 | 1.00000 |
| writing score | <.0001  |         |

The results suggest that the relationship between **read** and **write** ( $\rho = 0.61675$ ,  $p = 0.000$ ) is statistically significant.



female as the outcome variable to indicate how the code for this command is structured and how to interpret the output. The first variable listed on the model statement is the outcome (or dependent) variable, and all of the rest of the variables are listed after the equals sign and are predictor (or independent) variables. You can use the **expb** option on the **model** statement if you want to see the odds ratios. In our example, **female** will be the outcome variable, and **read** will be the predictor variable. As with OLS regression, the predictor variables must be either dichotomous or continuous; they cannot be categorical.

```
proc logistic data = "c:/mydata/hsb2" desc;  
  model female = read / expb;  
run;
```

| Parameter | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq | Exp (Est) |
|-----------|----|----------|----------------|-----------------|------------|-----------|
| Intercept | 1  | 0.7261   | 0.7420         | 0.9577          | 0.3278     | 2.067     |
| read      | 1  | -0.0104  | 0.0139         | 0.5623          | 0.4533     | 0.990     |

#### Odds Ratio Estimates

| Effect | Point Estimate | 95% Wald Confidence Limits |
|--------|----------------|----------------------------|
| read   | 0.990          | 0.963 1.017                |

#### Association of Predicted Probabilities and Observed Responses

|                    |      |           |       |
|--------------------|------|-----------|-------|
| Percent Concordant | 50.3 | Somers' D | 0.069 |
| Percent Discordant | 43.4 | Gamma     | 0.073 |
| Percent Tied       | 6.3  | Tau-a     | 0.034 |
| Pairs              | 9919 | c         | 0.534 |

The results indicate that reading score (**read**) is not a statistically significant predictor of gender (i.e., being female), Wald chi-square = 0.5623, p = 0.4533.

See also

- [SAS Textbook Examples: Applied Logistic Regression, Chapter 1 \(/sas/examples/alr2/applied-logistic-regression-](#)

with-a-labeled-outcome-variable/

- Some Issues Using PROC LOGISTIC for Binary Logistic Regression (<https://stats.idre.ucla.edu/wp-content/uploads/2016/02/ts274.pdf>)

## Multiple regression

the `outest` option on the model statement tells SAS to display the standardized regression coefficients (seen on the far right of the output).

```
proc reg data = "c:/mydata/hsb2";  
  model write = female read math science socst / stb;  
run;  
quit;
```



## Analysis of Variance

| Source          | DF  | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|-----|----------------|-------------|---------|--------|
| Model           | 5   | 10757          | 2151.38488  | 58.60   | <.0001 |
| Error           | 194 | 7121.95060     | 36.71109    |         |        |
| Corrected Total | 199 | 17879          |             |         |        |

|                |          |          |        |
|----------------|----------|----------|--------|
| Root MSE       | 6.05897  | R-Square | 0.6017 |
| Dependent Mean | 52.77500 | Adj R-Sq | 0.5914 |
| Coeff Var      | 11.48075 |          |        |

## Parameter Estimates

| Variable  | Label                | DF | Parameter Estimate | Standard Error | t Value | Pr >  t | Standardized Estimate |
|-----------|----------------------|----|--------------------|----------------|---------|---------|-----------------------|
| Intercept | Intercept            | 1  | 6.13876            | 2.80842        | 2.19    | 0.0300  | 0                     |
| female    |                      | 1  | 5.49250            | 0.87542        | 6.27    | <.0001  | 0.28928               |
| read      | reading score        | 1  | 0.12541            | 0.06496        | 1.93    | 0.0550  | 0.13566               |
| math      | math score           | 1  | 0.23807            | 0.06713        | 3.55    | 0.0005  | 0.23531               |
| science   | science score        | 1  | 0.24194            | 0.06070        | 3.99    | <.0001  | 0.25272               |
| socst     | social studies score | 1  | 0.22926            | 0.05284        | 4.34    | <.0001  | 0.25967               |

- [Annotated Output of PROC REG \(https://stats.idre.ucla.edu/sas/output/regression-analysis/\)](https://stats.idre.ucla.edu/sas/output/regression-analysis/)
- [Regression with SAS: Chapter 1 – Simple and Multiple Regression \(/sas/webbooks/reg/chapter1/regressionwith-saschapter-1-simple-and-multiple-regression/\)](/sas/webbooks/reg/chapter1/regressionwith-saschapter-1-simple-and-multiple-regression/)
- [Example 3d Graphs from Multiple Regression: Testing and Interpreting Interactions \(/sas/examples/aw/example-3d-graphs-from-multiple-regression-testing-and-interpreting-interactionsby-leona-aiken-and-stephen-west/\)](/sas/examples/aw/example-3d-graphs-from-multiple-regression-testing-and-interpreting-interactionsby-leona-aiken-and-stephen-west/)
- [SAS Textbook Examples: Applied Linear Statistical Models \(/examples/alsm/\)](/examples/alsm/)
- [SAS Frequently Asked Questions \(/sas/faq/\)](/sas/faq/)

## Analysis of covariance

```
proc glm data = "c:/mydata/hsb2";  
  class prog;  
  model write = prog read;  
run;  
quit;
```

| Source          | DF  | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|-----|----------------|-------------|---------|--------|
| Model           | 3   | 7017.68123     | 2339.22708  | 42.21   | <.0001 |
| Error           | 196 | 10861.19377    | 55.41425    |         |        |
| Corrected Total | 199 | 17878.87500    |             |         |        |

|          |           |          |            |
|----------|-----------|----------|------------|
| R-Square | Coeff Var | Root MSE | write Mean |
| 0.392512 | 14.10531  | 7.444075 | 52.77500   |

| Source | DF | Type I SS   | Mean Square | F Value | Pr > F |
|--------|----|-------------|-------------|---------|--------|
| prog   | 2  | 3175.697857 | 1587.848929 | 28.65   | <.0001 |
| read   | 1  | 3841.983376 | 3841.983376 | 69.33   | <.0001 |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|--------|----|-------------|-------------|---------|--------|
| prog   | 2  | 650.259965  | 325.129983  | 5.87    | 0.0034 |
| read   | 1  | 3841.983376 | 3841.983376 | 69.33   | <.0001 |

The results indicate that even after adjusting for reading score (**read**), writing scores still significantly differ by program type (**prog**)  $F = 5.87$ ,  $p = 0.0034$ .

See also

- [SAS Textbook Examples Design and Analysis, Chapter 14 \(/sas/examples/da/design-and-analysis-by-](/sas/examples/da/design-and-analysis-by-)

- [SAS Code for Some Advanced Experimental Designs \(/sas/library/sas-librarysas-code-for-some-advanced-experimental-designs/\)](/sas/library/sas-librarysas-code-for-some-advanced-experimental-designs/)

## Multiple logistic regression

we can use **female** as the outcome variable to illustrate how the code for this command is structured and how to interpret the output. In our example, **female** will be the outcome variable, and **read** and **write** will be the predictor variables. The **desc** option on the **proc logistic** statement is necessary so that SAS models the probability of being female (i.e., female = 1). The **expb** option on the **model** statement tells SAS to display the exponentiated coefficients (i.e., the odds ratios).

```
proc logistic data = "c:/mydata/hsb2" desc;  
  model female = read write / expb;  
run;
```

|                           |                  |
|---------------------------|------------------|
| Data Set                  | WORK.HSB2        |
| Response Variable         | female           |
| Number of Response Levels | 2                |
| Number of Observations    | 200              |
| Model                     | binary logit     |
| Optimization Technique    | Fisher's scoring |

#### Response Profile

| Ordered<br>Value | female | Total<br>Frequency |
|------------------|--------|--------------------|
| 1                | 1      | 109                |
| 2                | 0      | 91                 |

Probability modeled is female=1.

#### Model Convergence Status

Convergence criterion (GCONV=1E-8) satisfied.

#### Model Fit Statistics

|           |                  |
|-----------|------------------|
| Intercept | Intercept<br>and |
|-----------|------------------|

Testing Global Null Hypothesis: BETA=0

| Test             | Chi-Square | DF | Pr > ChiSq |
|------------------|------------|----|------------|
| Likelihood Ratio | 27.8186    | 2  | <.0001     |
| Score            | 26.3588    | 2  | <.0001     |
| Wald             | 23.4135    | 2  | <.0001     |



|           |   |         |        |         |        |       |
|-----------|---|---------|--------|---------|--------|-------|
| Intercept | 1 | -1.7061 | 0.9234 | 3.4137  | 0.0647 | 0.182 |
| read      | 1 | -0.0710 | 0.0196 | 13.1251 | 0.0003 | 0.931 |
| write     | 1 | 0.1064  | 0.0221 | 23.0748 | <.0001 | 1.112 |

#### Odds Ratio Estimates

| Effect | Point Estimate | 95% Wald Confidence Limits |       |
|--------|----------------|----------------------------|-------|
| read   | 0.931          | 0.896                      | 0.968 |
| write  | 1.112          | 1.065                      | 1.162 |

#### Association of Predicted Probabilities and Observed Responses

|                    |      |           |       |
|--------------------|------|-----------|-------|
| Percent Concordant | 69.3 | Somers' D | 0.396 |
| Percent Discordant | 29.7 | Gamma     | 0.400 |
| Percent Tied       | 1.0  | Tau-a     | 0.198 |
| Pairs              | 9919 | c         | 0.698 |

These results show that both **read** (Wald chi-square = 13.1251, p = 0.0003) and **write** (Wald chi-square = 23.0748, p = 0.0001) are significant predictors of **female**.

### See also

- [SAS Textbook Examples: Applied Logistic Regression, Chapter 2 \(/sas/examples/alr2/applied-logistic-regression-](/sas/examples/alr2/applied-logistic-regression-)

## Discriminant analysis

predict the type of program (**prog**) to which a student belongs.

```
proc discrim data = "c:/mydata/hsb2" can;  
  class prog;  
  var read write math;  
run;
```

|           |   |                    |     |
|-----------|---|--------------------|-----|
| Variables | 3 | DF Within Classes  | 197 |
| Classes   | 3 | DF Between Classes | 2   |

### Class Level Information

|      | Variable |           |          |            | Prior       |
|------|----------|-----------|----------|------------|-------------|
| prog | Name     | Frequency | Weight   | Proportion | Probability |
| 1    | _1       | 45        | 45.0000  | 0.225000   | 0.333333    |
| 2    | _2       | 105       | 105.0000 | 0.525000   | 0.333333    |
| 3    | _3       | 50        | 50.0000  | 0.250000   | 0.333333    |

### Pooled Covariance Matrix Information

|             | Natural Log of the |
|-------------|--------------------|
| Covariance  | Determinant of the |
| Matrix Rank | Covariance Matrix  |
| 3           | 12.18440           |

### Pairwise Generalized Squared Distances Between Groups

$$D^2(i|j) = (\bar{X}_i - \bar{X}_j)' \text{COV}^{-1} (\bar{X}_i - \bar{X}_j)$$

|      |         |         |         |
|------|---------|---------|---------|
| prog | 1       | 2       | 3       |
| 1    | 0       | 0.73810 | 0.31771 |
| 2    | 0.73810 | 0       | 1.90746 |
| 3    | 0.31771 | 1.90746 | 0       |

### Canonical Discriminant Analysis

|   | Canonical<br>Correlation | Adjusted<br>Canonical<br>Correlation | Approximate<br>Standard<br>Error | Squared<br>Canonical<br>Correlation |
|---|--------------------------|--------------------------------------|----------------------------------|-------------------------------------|
| 1 | 0.512534                 | 0.502546                             | 0.052266                         | 0.262691                            |
| 2 | 0.067247                 | 0.031181                             | 0.070568                         | 0.004522                            |

Test of H0: The canonical correlations in  
the current row and all  
that follow are zero

Eigenvalues of  $\text{Inv}(\mathbf{E}) * \mathbf{H}$   
=  $\text{CanRsqr} / (1 - \text{CanRsqr})$

|   | Eigenvalue | Difference | Proportion | Cumulative | Likelihood Approximate<br>Ratio | F Value | Num DF | Den DF | Pr > F |
|---|------------|------------|------------|------------|---------------------------------|---------|--------|--------|--------|
| 1 | 0.3563     | 0.3517     | 0.9874     | 0.9874     | 0.73397507                      | 10.87   | 6      | 390    | <.0001 |
| 2 | 0.0045     |            | 0.0126     | 1.0000     | 0.99547788                      | 0.45    | 2      | 196    | 0.6414 |

Total Canonical Structure

## Between Canonical Structure

| Variable | Label         | Can1     | Can2      |
|----------|---------------|----------|-----------|
| read     | reading score | 0.999644 | -0.026693 |
| write    | writing score | 0.995813 | 0.091410  |
| math     | math score    | 0.999433 | -0.033682 |

## Pooled Within Canonical Structure

| Variable | Label         | Can1     | Can2      |
|----------|---------------|----------|-----------|
| read     | reading score | 0.778465 | -0.184093 |
| write    | writing score | 0.775344 | 0.630310  |
| math     | math score    | 0.912889 | -0.272463 |

## Total-Sample Standardized Canonical Coefficients

| Variable | Label         | Can1        | Can2         |
|----------|---------------|-------------|--------------|
| read     | reading score | 0.299373057 | -0.449624188 |
| write    | writing score | 0.363246854 | 1.298397979  |
| math     | math score    | 0.659035164 | -0.743012325 |

## Pooled Within-Class Standardized Canonical Coefficients

## Raw Canonical Coefficients

| Variable | Label         | Can1         | Can2         |
|----------|---------------|--------------|--------------|
| read     | reading score | 0.0291987615 | -.0438532096 |
| write    | writing score | 0.0383228947 | 0.1369822435 |
| math     | math score    | 0.0703462492 | -.0793100780 |

## Class Means on Canonical Variables

| prog | Can1         | Can2         |
|------|--------------|--------------|
| 1    | -.3120021323 | 0.1190423066 |
| 2    | 0.5358514591 | -.0196809384 |
| 3    | -.8444861449 | -.0658081053 |

## Linear Discriminant Function

$$\text{Constant} = -\frac{1}{2} \sum_j \bar{X}' \text{COV}^{-1} \bar{X}_j \quad \text{Coefficient Vector} = \text{COV}^{-1} \sum_j \bar{X}_j$$

## Linear Discriminant Function for prog

| Variable | Label | 1 | 2 | 3 |
|----------|-------|---|---|---|
|----------|-------|---|---|---|

Generalized Squared Distance Function

$$D_j^2(X) = (X - \bar{X}_j)' \text{COV}_j^{-1} (X - \bar{X}_j)$$

Posterior Probability of Membership in Each prog

$$\text{Pr}(j|X) = \frac{\exp(-.5 D_j^2(X))}{\sum_k \exp(-.5 D_k^2(X))}$$

Number of Observations and Percent Classified into prog

| From<br>prog | 1           | 2           | 3           | Total         |
|--------------|-------------|-------------|-------------|---------------|
| 1            | 11<br>24.44 | 17<br>37.78 | 17<br>37.78 | 45<br>100.00  |
| 2            | 18<br>17.14 | 68<br>64.76 | 19<br>18.10 | 105<br>100.00 |



|                                |         |         |         |        |
|--------------------------------|---------|---------|---------|--------|
|                                | 21.50   | 46.00   | 32.50   | 100.00 |
| Priors                         | 0.33333 | 0.33333 | 0.33333 |        |
| Error Count Estimates for prog |         |         |         |        |
|                                | 1       | 2       | 3       | Total  |
| Rate                           | 0.7556  | 0.3524  | 0.4200  | 0.5093 |
| Priors                         | 0.3333  | 0.3333  | 0.3333  |        |

Clearly, the SAS output for this procedure is quite lengthy, and it is beyond the scope of this page to explain all of it. However, the main point is that two canonical variables are identified by the analysis, the first of which seems to be more related to program type than the second.

## See also

- [New Features in SAS/INSIGHT in Version 7 \(https://stats.idre.ucla.edu/wp-content/uploads/2016/02/insight7.pdf\)](https://stats.idre.ucla.edu/wp-content/uploads/2016/02/insight7.pdf)
- [discriminant function analysis \(http://faculty.chass.ncsu.edu/garson/PA765/discrim.htm\)](http://faculty.chass.ncsu.edu/garson/PA765/discrim.htm)

## One-way MANOVA

to conduct a MANOVA. The **h=** on the **manova** statement is used to specify the hypothesized effect.

```
proc glm data = "c:/mydata/hsb2";  
  class prog;  
  model read write math = prog;  
  manova h=prog;  
run;  
quit;
```

|                 |           | Sum of      |             |         |        |
|-----------------|-----------|-------------|-------------|---------|--------|
| Source          | DF        | Squares     | Mean Square | F Value | Pr > F |
| Model           | 2         | 3716.86127  | 1858.43063  | 21.28   | <.0001 |
| Error           | 197       | 17202.55873 | 87.32263    |         |        |
| Corrected Total | 199       | 20919.42000 |             |         |        |
|                 |           |             |             |         |        |
| R-Square        | Coeff Var | Root MSE    | read Mean   |         |        |
| 0.177675        | 17.89136  | 9.344658    | 52.23000    |         |        |
|                 |           |             |             |         |        |
| Source          | DF        | Type I SS   | Mean Square | F Value | Pr > F |
| prog            | 2         | 3716.861270 | 1858.430635 | 21.28   | <.0001 |
|                 |           |             |             |         |        |
| Source          | DF        | Type III SS | Mean Square | F Value | Pr > F |
| prog            | 2         | 3716.861270 | 1858.430635 | 21.28   | <.0001 |

|                 |     |             |            |       |        |
|-----------------|-----|-------------|------------|-------|--------|
| Model           | 2   | 3175.69786  | 1587.84893 | 21.27 | <.0001 |
| Error           | 197 | 14703.17714 | 74.63542   |       |        |
| Corrected Total | 199 | 17878.87500 |            |       |        |

|          |           |          |            |
|----------|-----------|----------|------------|
| R-Square | Coeff Var | Root MSE | write Mean |
| 0.177623 | 16.36983  | 8.639179 | 52.77500   |

|        |    |             |             |         |        |
|--------|----|-------------|-------------|---------|--------|
| Source | DF | Type I SS   | Mean Square | F Value | Pr > F |
| prog   | 2  | 3175.697857 | 1587.848929 | 21.27   | <.0001 |

|        |    |             |             |         |        |
|--------|----|-------------|-------------|---------|--------|
| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
| prog   | 2  | 3175.697857 | 1587.848929 | 21.27   | <.0001 |

|                 |     |             |          |
|-----------------|-----|-------------|----------|
| Error           | 197 | 13463.69111 | 68.34361 |
| Corrected Total | 199 | 17465.79500 |          |

|          |           |          |           |
|----------|-----------|----------|-----------|
| R-Square | Coeff Var | Root MSE | math Mean |
| 0.229140 | 15.70333  | 8.267019 | 52.64500  |

|        |    |             |             |         |        |
|--------|----|-------------|-------------|---------|--------|
| Source | DF | Type I SS   | Mean Square | F Value | Pr > F |
| prog   | 2  | 4002.103889 | 2001.051944 | 29.28   | <.0001 |

|        |    |             |             |         |        |
|--------|----|-------------|-------------|---------|--------|
| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
| prog   | 2  | 4002.103889 | 2001.051944 | 29.28   | <.0001 |

E = Error SSCP Matrix

| Characteristic |         | Characteristic Vector | V'EV=1     |             |
|----------------|---------|-----------------------|------------|-------------|
| Root           | Percent | read                  | write      | math        |
| 0.35628297     | 98.74   | 0.00208033            | 0.00273039 | 0.00501196  |
| 0.00454266     | 1.26    | -0.00312441           | 0.00975958 | -0.00565061 |
| 0.00000000     | 0.00    | -0.00904826           | 0.00054800 | 0.00823531  |

MANOVA Test Criteria and F Approximations for the Hypothesis of No Overall prog Effect

H = Type III SSCP Matrix for prog

E = Error SSCP Matrix

S=2 M=0 N=96.5

| Statistic              | Value      | F Value | Num DF | Den DF | Pr > F |
|------------------------|------------|---------|--------|--------|--------|
| Wilks' Lambda          | 0.73397507 | 10.87   | 6      | 390    | <.0001 |
| Pillai's Trace         | 0.26721285 | 10.08   | 6      | 392    | <.0001 |
| Hotelling-Lawley Trace | 0.36082563 | 11.70   | 6      | 258.23 | <.0001 |
| Roy's Greatest Root    | 0.35628297 | 23.28   | 3      | 196    | <.0001 |

NOTE: F Statistic for Roy's Greatest Root is an upper bound.

NOTE: F Statistic for Wilks' Lambda is exact.

This command produces four different test statistics that are used to evaluate the statistical significance of the relationship

- [SAS Textbook Examples: Design and Analysis \(/sas/examples/da/design-and-analysis-a-researchers-handbook-by-geoffrey-keppel-download-sas-data-files/\)](/sas/examples/da/design-and-analysis-a-researchers-handbook-by-geoffrey-keppel-download-sas-data-files/)
- [SAS Learning Module: Missing Data in SAS \(/sas/modules/missing-data-in-sas/\)](/sas/modules/missing-data-in-sas/)

## Multivariate multiple regression

statement tests the hypothesis that all estimated parameters except the intercept are zero. In other words, the multivariate tests test whether the independent variable specified predicts the dependent variables together, holding all of the other independent variables constant. You can put a label in front of the **mtest** statement to aid in the interpretation of the output (this is particularly useful when you have multiple **mtest** statements).

```
proc reg data = "c:/mydata/hsb2";  
  model write read = female math science socst;  
  female: mtest female;  
  math:   mtest math;  
  science: mtest science;  
  socst:  mtest socst;  
run;  
quit;
```



## Analysis of Variance

| Source          | DF  | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|-----|----------------|-------------|---------|--------|
| Model           | 4   | 10620          | 2655.02312  | 71.32   | <.0001 |
| Error           | 195 | 7258.78251     | 37.22453    |         |        |
| Corrected Total | 199 | 17879          |             |         |        |

|                |          |          |        |
|----------------|----------|----------|--------|
| Root MSE       | 6.10119  | R-Square | 0.5940 |
| Dependent Mean | 52.77500 | Adj R-Sq | 0.5857 |
| Coeff Var      | 11.56076 |          |        |

## Parameter Estimates

| Variable  | Label                | DF | Parameter Estimate | Standard Error | t Value | Pr >  t |
|-----------|----------------------|----|--------------------|----------------|---------|---------|
| Intercept | Intercept            | 1  | 6.56892            | 2.81908        | 2.33    | 0.0208  |
| female    |                      | 1  | 5.42822            | 0.88089        | 6.16    | <.0001  |
| math      | math score           | 1  | 0.28016            | 0.06393        | 4.38    | <.0001  |
| science   | science score        | 1  | 0.27865            | 0.05805        | 4.80    | <.0001  |
| socst     | social studies score | 1  | 0.26811            | 0.04919        | 5.45    | <.0001  |

Model: MODEL1

|                           |                      | Sum of     | Mean       |          |         |         |
|---------------------------|----------------------|------------|------------|----------|---------|---------|
| Source                    | DF                   | Squares    | Square     | F Value  | Pr > F  |         |
| Model                     | 4                    | 12220      | 3054.91459 | 68.47    | <.0001  |         |
| Error                     | 195                  | 8699.76166 | 44.61416   |          |         |         |
| Corrected Total           | 199                  | 20919      |            |          |         |         |
|                           |                      |            |            |          |         |         |
| Root MSE                  | 6.67938              | R-Square   | 0.5841     |          |         |         |
| Dependent Mean            | 52.23000             | Adj R-Sq   | 0.5756     |          |         |         |
| Coeff Var                 | 12.78840             |            |            |          |         |         |
| Parameter Estimates       |                      |            |            |          |         |         |
|                           |                      |            | Parameter  | Standard |         |         |
| Variable                  | Label                | DF         | Estimate   | Error    | t Value | Pr >  t |
| Intercept                 | Intercept            | 1          | 3.43000    | 3.08624  | 1.11    | 0.2678  |
| female                    |                      | 1          | -0.51261   | 0.96436  | -0.53   | 0.5956  |
| math                      | math score           | 1          | 0.33558    | 0.06999  | 4.79    | <.0001  |
| science                   | science score        | 1          | 0.29276    | 0.06355  | 4.61    | <.0001  |
| socst                     | social studies score | 1          | 0.30976    | 0.05386  | 5.75    | <.0001  |
|                           |                      |            |            |          |         |         |
| Model: MODEL1             |                      |            |            |          |         |         |
| Multivariate Test: female |                      |            |            |          |         |         |

| Statistic              | Value      | F Value | Num DF | Den DF | Pr > F |
|------------------------|------------|---------|--------|--------|--------|
| Wilks' Lambda          | 0.83011470 | 19.85   | 2      | 194    | <.0001 |
| Pillai's Trace         | 0.16988530 | 19.85   | 2      | 194    | <.0001 |
| Hotelling-Lawley Trace | 0.20465280 | 19.85   | 2      | 194    | <.0001 |
| Roy's Greatest Root    | 0.20465280 | 19.85   | 2      | 194    | <.0001 |

Model: MODEL1

Multivariate Test: math

Multivariate Statistics and Exact F Statistics

S=1 M=0 N=96

| Statistic              | Value      | F Value | Num DF | Den DF | Pr > F |
|------------------------|------------|---------|--------|--------|--------|
| Wilks' Lambda          | 0.84006791 | 18.47   | 2      | 194    | <.0001 |
| Pillai's Trace         | 0.15993209 | 18.47   | 2      | 194    | <.0001 |
| Hotelling-Lawley Trace | 0.19037995 | 18.47   | 2      | 194    | <.0001 |
| Roy's Greatest Root    | 0.19037995 | 18.47   | 2      | 194    | <.0001 |

Model: MODEL1

Multivariate Test: science

Multivariate Statistics and Exact F Statistics

|                        |            |       |   |     |        |
|------------------------|------------|-------|---|-----|--------|
| Pillai's Trace         | 0.16642538 | 19.37 | 2 | 194 | <.0001 |
| Hotelling-Lawley Trace | 0.19965265 | 19.37 | 2 | 194 | <.0001 |
| Roy's Greatest Root    | 0.19965265 | 19.37 | 2 | 194 | <.0001 |

Model: MODEL1

Multivariate Test: socst

#### Multivariate Statistics and Exact F Statistics

S=1 M=0 N=96

| Statistic              | Value      | F Value | Num DF | Den DF | Pr > F |
|------------------------|------------|---------|--------|--------|--------|
| Wilks' Lambda          | 0.77932902 | 27.47   | 2      | 194    | <.0001 |
| Pillai's Trace         | 0.22067098 | 27.47   | 2      | 194    | <.0001 |
| Hotelling-Lawley Trace | 0.28315509 | 27.47   | 2      | 194    | <.0001 |
| Roy's Greatest Root    | 0.28315509 | 27.47   | 2      | 194    | <.0001 |

With regard to the univariate tests, each of the independent variables is statistically significant predictor for writing. All of the independent variables are also statistically significant predictors for reading except **female** ( $t = -0.53$ ,  $p = 0.5956$ ). All of the multivariate tests are also statistically significant.

## Canonical correlation

**cancorr** statement provides additional output that many researchers might find useful.

```
proc cancorr data = "c:/mydata/hsb2" all;  
  var read write;  
  with math science;  
run;
```

The CANCERR Procedure

|                |     |
|----------------|-----|
| VAR Variables  | 2   |
| WITH Variables | 2   |
| Observations   | 200 |

Means and Standard Deviations

| Variable | Mean      | Standard<br>Deviation | Label         |
|----------|-----------|-----------------------|---------------|
| read     | 52.230000 | 10.252937             | reading score |
| write    | 52.775000 | 9.478586              | writing score |
| math     | 52.645000 | 9.368448              | math score    |
| science  | 51.850000 | 9.900891              | science score |

|       | read   | write  |
|-------|--------|--------|
| read  | 1.0000 | 0.5968 |
| write | 0.5968 | 1.0000 |

Correlations Among the WITH Variables

|         | math   | science |
|---------|--------|---------|
| math    | 1.0000 | 0.6307  |
| science | 0.6307 | 1.0000  |

Correlations Between the VAR Variables and the WITH Variables

|       | math   | science |
|-------|--------|---------|
| read  | 0.6623 | 0.6302  |
| write | 0.6174 | 0.5704  |

|   | Correlation | Correlation | Error    | Correlation |
|---|-------------|-------------|----------|-------------|
| 1 | 0.772841    | 0.771003    | 0.028548 | 0.597283    |
| 2 | 0.023478    | .           | 0.070849 | 0.000551    |

Test of H0: The canonical correlations in  
the current row and all  
that follow are zero

Eigenvalues of  $\text{Inv}(E) * H$   
=  $\text{CanRsqr} / (1 - \text{CanRsqr})$

Likelihood Approximate

|   | Eigenvalue | Difference | Proportion | Cumulative | Ratio      | F Value | Num DF | Den DF | Pr > F |
|---|------------|------------|------------|------------|------------|---------|--------|--------|--------|
| 1 | 1.4831     | 1.4826     | 0.9996     | 0.9996     | 0.40249498 | 56.47   | 4      | 392    | <.0001 |
| 2 | 0.0006     |            | 0.0004     | 1.0000     | 0.99944876 | 0.11    | 1      | 197    | 0.7420 |

Multivariate Statistics and F Approximations

S=2      M=-0.5      N=97

| Statistic              | Value      | F Value | Num DF | Den DF | Pr > F |
|------------------------|------------|---------|--------|--------|--------|
| Wilks' Lambda          | 0.40249498 | 56.47   | 4      | 392    | <.0001 |
| Pillai's Trace         | 0.59783426 | 42.00   | 4      | 394    | <.0001 |
| Hotelling-Lawley Trace | 1.48368501 | 72.58   | 4      | 234.16 | <.0001 |
| Roy's Greatest Root    | 1.48313347 | 146.09  | 2      | 197    | <.0001 |

NOTE: F Statistic for Roy's Greatest Root is an upper bound.

|       |               | V1           | V2           |
|-------|---------------|--------------|--------------|
| read  | reading score | 0.063261313  | 0.1037907932 |
| write | writing score | 0.0492491834 | -0.12190836  |

Raw Canonical Coefficients for the WITH Variables

|         |               | W1           | W2           |
|---------|---------------|--------------|--------------|
| math    | math score    | 0.0669826768 | -0.120142451 |
| science | science score | 0.0482406314 | 0.1208859811 |

Standardized Canonical Coefficients for the VAR Variables

|       |               | V1     | V2      |
|-------|---------------|--------|---------|
| read  | reading score | 0.6486 | 1.0642  |
| write | writing score | 0.4668 | -1.1555 |

Standardized Canonical Coefficients for the WITH Variables

|         |               | W1     | W2      |
|---------|---------------|--------|---------|
| math    | math score    | 0.6275 | -1.1255 |
| science | science score | 0.4776 | 1.1969  |



|       |               | V1     | V2      |
|-------|---------------|--------|---------|
| read  | reading score | 0.9272 | 0.3746  |
| write | writing score | 0.8539 | -0.5205 |

Correlations Between the WITH Variables and Their Canonical Variables

|         |               | W1     | W2      |
|---------|---------------|--------|---------|
| math    | math score    | 0.9288 | -0.3706 |
| science | science score | 0.8734 | 0.4870  |

Correlations Between the VAR Variables and the Canonical Variables of the WITH Variables

|       |               | W1     | W2      |
|-------|---------------|--------|---------|
| read  | reading score | 0.7166 | 0.0088  |
| write | writing score | 0.6599 | -0.0122 |

Correlations Between the WITH Variables and the Canonical Variables of the VAR Variables

|         |               | V1     | V2      |
|---------|---------------|--------|---------|
| math    | math score    | 0.7178 | -0.0087 |
| science | science score | 0.6750 | 0.0114  |

| Canonical Variables                                     |            |                       | Canonical Variables |            |                       |
|---|------------|-----------------------|---------------------|------------|-----------------------|
| Canonical Variable Number                               | Proportion | Cumulative Proportion | Canonical R-Square  | Proportion | Cumulative Proportion |
| 1   | 0.7995     | 0.7995                | 0.5973              | 0.4775     | 0.4775                |
| 2   | 0.2005     | 1.0000                | 0.0006              | 0.0001     | 0.4777                |
| Raw Variance of the WITH Variables Explained by         |            |                       |                     |            |                       |
| Their Own   |            |                       | The Opposite        |            |                       |
| Canonical Variables                                     |            |                       | Canonical Variables |            |                       |
| Canonical Variable Number                               | Proportion | Cumulative Proportion | Canonical R-Square  | Proportion | Cumulative Proportion |
| 1   | 0.8100     | 0.8100                | 0.5973              | 0.4838     | 0.4838                |
| 2   | 0.1900     | 1.0000                | 0.0006              | 0.0001     | 0.4839                |
| Standardized Variance of the VAR Variables Explained by |            |                       |                     |            |                       |
| Their Own   |            |                       | The Opposite        |            |                       |
| Canonical Variables                                     |            |                       | Canonical Variables |            |                       |
| Canonical Variable Number                               | Proportion | Cumulative Proportion | Canonical R-Square  | Proportion | Cumulative Proportion |
| 1   | 0.7944     | 0.7944                | 0.5973              | 0.4745     | 0.4745                |

| Their Own<br>Canonical Variables   |               |                          | The Opposite<br>Canonical Variables |            |                          |
|--|---------------|--------------------------|-------------------------------------|------------|--------------------------|
| Canonical<br>Variable<br>Number  | Proportion    | Cumulative<br>Proportion | Canonical<br>R-Square               | Proportion | Cumulative<br>Proportion |
| 1  | 0.8127        | 0.8127                   | 0.5973                              | 0.4854     | 0.4854                   |
| 2  | 0.1873        | 1.0000                   | 0.0006                              | 0.0001     | 0.4855                   |
| Squared Multiple Correlations Between the VAR Variables and<br>the First M Canonical Variables of the WITH Variables |               |                          |                                     |            |                          |
| M  |               | 1                        | 2                                   |            |                          |
| read   | reading score | 0.5135                   | 0.5136                              |            |                          |
| write  | writing score | 0.4355                   | 0.4356                              |            |                          |
| Squared Multiple Correlations Between the WITH Variables<br>and the First M Canonical Variables of the VAR Variables |               |                          |                                     |            |                          |
| M  |               | 1                        | 2                                   |            |                          |
| math   | math score    | 0.5152                   | 0.5153                              |            |                          |
| science  | science score | 0.4557                   | 0.4558                              |            |                          |

The output above shows the linear combinations corresponding to the first canonical correlation. At the bottom of the output are the two canonical correlations. These results indicate that the first canonical correlation is .772841. The F-test in this output

## Factor analysis

there are some common factors underlying the various test scores. We will use the principal components method of extraction, use a varimax rotation, extract two factors and obtain a scree plot of the eigenvalues. All of these options are listed on the **proc factor** statement.

```
proc factor data = "c:/mydata/hsb2" method=principal rotate=varimax nfactors=2 scree;  
    var read write math science socst;  
run;
```

Eigenvalues of the Correlation Matrix: Total = 5    Average = 1

|   | Eigenvalue | Difference | Proportion | Cumulative |
|---|------------|------------|------------|------------|
| 1 | 3.38081982 | 2.82344156 | 0.6762     | 0.6762     |
| 2 | 0.55737826 | 0.15058550 | 0.1115     | 0.7876     |
| 3 | 0.40679276 | 0.05062495 | 0.0814     | 0.8690     |
| 4 | 0.35616781 | 0.05732645 | 0.0712     | 0.9402     |
| 5 | 0.29884136 |            | 0.0598     | 1.0000     |

2 factors will be retained by the NFACTOR criterion.

The FACTOR Procedure

Initial Factor Method: Principal Components

|         |                      | Factor Pattern |          |
|---------|----------------------|----------------|----------|
|         |                      | Factor1        | Factor2  |
| READ    | reading score        | 0.85760        | -0.02037 |
| WRITE   | writing score        | 0.82445        | 0.15495  |
| MATH    | math score           | 0.84355        | -0.19478 |
| SCIENCE | science score        | 0.80091        | -0.45608 |
| SOCST   | social studies score | 0.78268        | 0.53573  |

Variance Explained by Each Factor

Final Communalities Estimates: Total = 3.938198

| READ       | WRITE      | MATH       | SCIENCE    | SOCST      |
|------------|------------|------------|------------|------------|
| 0.73589906 | 0.70373337 | 0.74951854 | 0.84945810 | 0.89958900 |

The FACTOR Procedure

Rotation Method: Varimax

Orthogonal Transformation Matrix

|   | 1        | 2       |
|---|----------|---------|
| 1 | 0.74236  | 0.67000 |
| 2 | -0.67000 | 0.74236 |

Rotated Factor Pattern

|         |                      | Factor1 | Factor2 |
|---------|----------------------|---------|---------|
| READ    | reading score        | 0.65029 | 0.55948 |
| WRITE   | writing score        | 0.50822 | 0.66742 |
| MATH    | math score           | 0.75672 | 0.42058 |
| SCIENCE | science score        | 0.90013 | 0.19804 |
| SOCST   | social studies score | 0.22209 | 0.92210 |

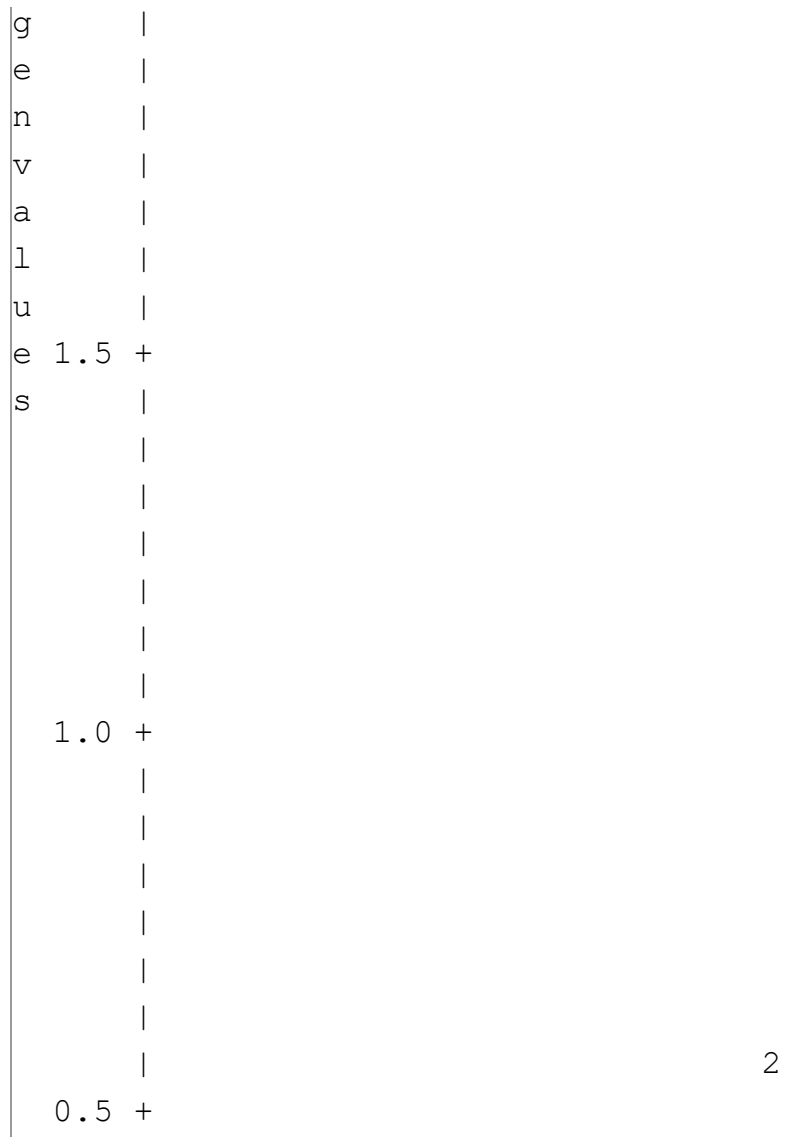
Variance Explained by Each Factor

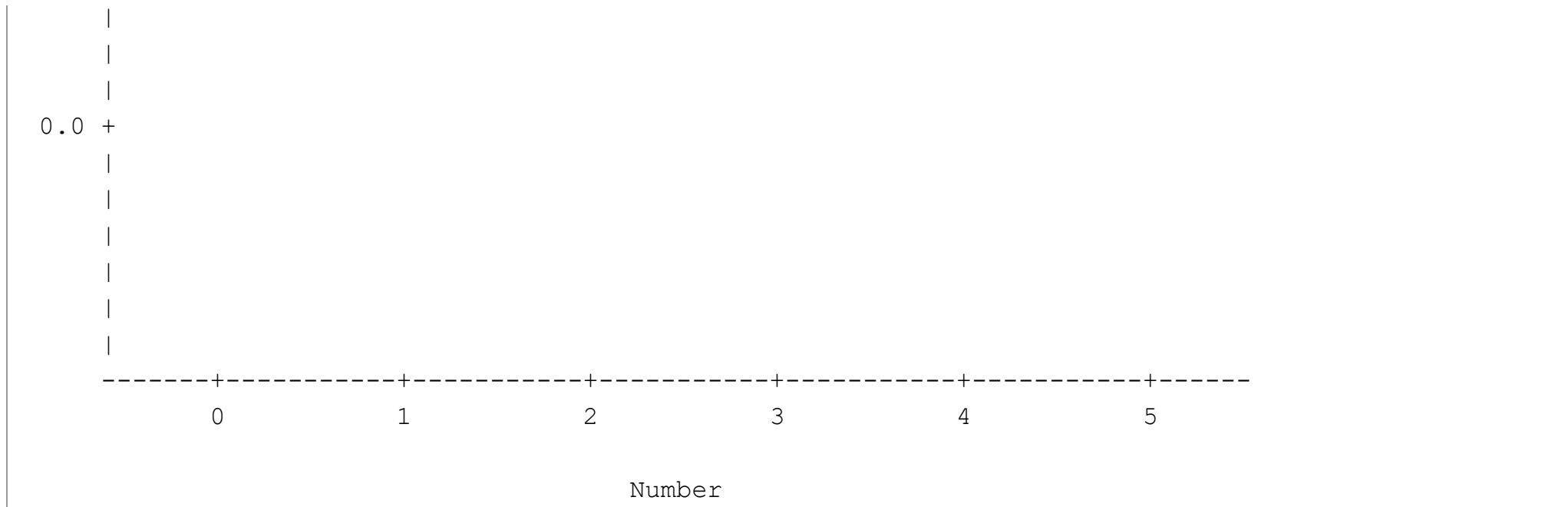
| READ       | WRITE      | MATH       | SCIENCE    | SOCST      |
|------------|------------|------------|------------|------------|
| 0.73589906 | 0.70373337 | 0.74951854 | 0.84945810 | 0.89958900 |

Communality (which is the opposite of uniqueness) is the proportion of variance of the variable (i.e., **read**) that is accounted for by all of the factors taken together, and a very low communality can indicate that a variable may not belong with any of the factors. From the factor pattern table, we can see that all five of the test scores load onto the first factor, while all five tend to load not so heavily on the second factor. The purpose of rotating the factors is to get the variables to load either very high or very low on each factor. In this example, because all of the variables loaded onto factor 1 and not on factor 2, the rotation did not aid in the interpretation. Instead, it made the results even more difficult to interpret. The scree plot may be useful in determining how many factors to retain.









## See also

- [SAS Library: Factor Analysis Using SAS PROC FACTOR \(/sas/library/sas-libraryfactor-analysis-using-sas-proc-factor/\)](/sas/library/sas-libraryfactor-analysis-using-sas-proc-factor/)

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