Negative binomial regression is for modeling count variables, usually for over-dispersed count outcome variables.

Please note: The purpose of this page is to show how to use various data analysis commands. It does not cover all aspects of the research process which researchers are expected to do. In particular, it does not cover data cleaning and checking, verification of assumptions, model diagnostics or potential follow-up analyses.

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Examples of negative binomial regression

Example 1. School administrators study the attendance behavior of high school juniors at two schools. Predictors of the number of days of absence include the type of program in which the student is enrolled and a standardized test in math.

Example 2. A health-related researcher is studying the number of hospital visits in past 12 months by senior citizens in a community based on the characteristics of the individuals and the types of health plans under which each one is covered.

Description of the data

Let's pursue Example 1 from above.

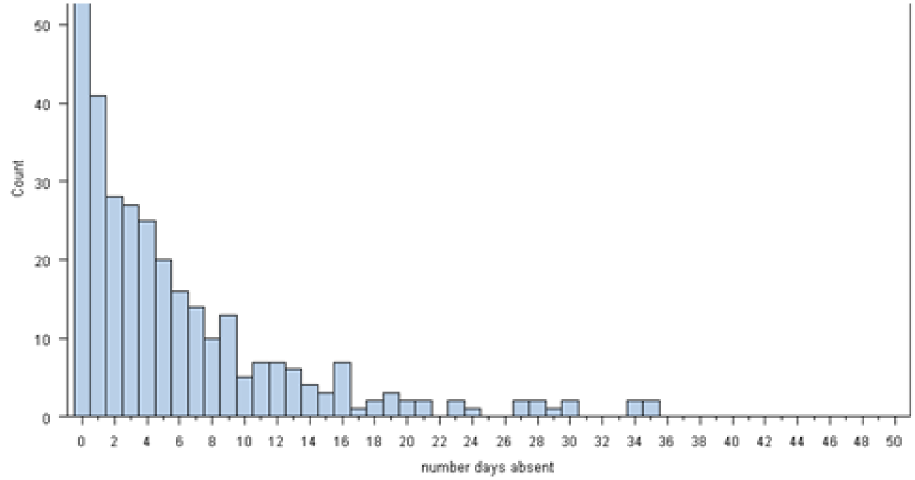
We have attendance data on 314 high school juniors from two urban high schools in the file https://stats.idre.ucla.edu/wp-content/uploads/2016/02/nb_data.sas7bdat (https://stats.idre.ucla.edu/wp-content/uploads/2016/02/nb_data.sas7bdat). The response variable of interest is days absent, **daysabs**. The variable **math** gives the standardized math score for each student. The variable **prog** is a three-level nominal variable indicating the type of instructional program in which the student is enrolled.

Let's look at the data. It is always a good idea to start with descriptive statistics and plots.

```
The MEANS Procedure

Variable    Label                  N          Mean       Std Dev       Minimum       Maximum
---------------------------------------------------------------------------------------------
DAYSABS     number days absent   314     5.9554140     7.0369576             0    35.0000000
MATH        ctbs math pct rank   314    48.2675159    25.3623913     1.0000000    99.0000000
---------------------------------------------------------------------------------------------
```

```
proc univariate data = nb_data noprint;
  histogram daysabs / midpoints = 0 to 50 by 1 vscale = count ;
run;
```



Each variable has 314 valid observations and their distributions seem quite reasonable. The mean of our outcome variable is much lower than its variance.

Let's continue with our description of the variables in this dataset. The table below shows the average numbers of days absent by program type and seems to suggest that program type is a good candidate for predicting the number of days absent, our outcome variable, because the mean value of the outcome appears to vary by **prog**. The variances within each level of **prog** are higher than the means within each level. These are the conditional means and variances. These differences suggest that over-dispersion is present and that a Negative Binomial model would be appropriate.

```
proc means mean var n data = nb_data;
  by prog;
  var daysabs;
run;
```

```
PROG=1

The MEANS Procedure

Analysis Variable : DAYSABS number days absent

        Mean        Variance      N
-----------------------------------
  10.6500000      67.2589744     40
-----------------------------------



PROG=2
```

```
-----------------------------------
   6.9341317      55.4474425    167
-----------------------------------



PROG=3

Analysis Variable : DAYSABS number days absent

        Mean        Variance      N
-----------------------------------
   2.6728972      13.9391642    107
-----------------------------------
```

Analysis methods you might consider

Below is a list of some analysis methods you may have encountered. Some of the methods listed are quite reasonable, while others have either fallen out of favor or have limitations.

- Negative binomial regression – Negative binomial regression can be used for over-dispersed count data, that is when the conditional variance exceeds the conditional mean. It can be considered as a generalization of Poisson regression since it has the

- Poisson regression – Poisson regression is often used for modeling count data. Poisson regression has a number of extensions useful for count models.
- Zero-inflated regression model – Zero-inflated models attempt to account for excess zeros. In other words, two kinds of zeros are thought to exist in the data, "true zeros" and "excess zeros". Zero-inflated models estimate two equations simultaneously, one for the count model and one for the excess zeros.
- OLS regression – Count outcome variables are sometimes log-transformed and analyzed using OLS regression. Many issues arise with this approach, including loss of data due to undefined values generated by taking the log of zero (which is undefined), as well as the lack of capacity to model the dispersion.

Negative binomial regression analysis

Negative binomial models can be estimated in SAS using **proc genmod**. On the **class** statement we list the variable **prog**. After **prog**, we use two options, which are given in parentheses. The **param**=**ref** option changes the coding of **prog** from effect coding, which is the default, to reference coding. The **ref**=**first** option changes the reference group to the first level of **prog**. We have used two options on the **model** statement. The **type3** option is used to get the multi-degree-of-freedom test of the categorical variables listed on the **class** statement, and the **dist** = **negbin** option is used to indicate that a negative binomial distribution should be used.

```
proc genmod data = nb_data;
  class prog (param=ref ref=first);
  model daysabs = math prog / type3 dist=negbin;
run;
```

Data Set	WORK.NB_DATA	
Distribution	Negative Binomial	
Link Function	Log	
Dependent Variable	DAYSABS	number days absent

Number of Observations Read	314
Number of Observations Used	314

Class Level Information

Class	Value	Design Variables	
PROG	1	0	0
	2	1	0
	3	0	1

Criterion	DF	Value	Value/DF
Deviance	310	358.5193	1.1565
Scaled Deviance	310	358.5193	1.1565
Pearson Chi-Square	310	339.8771	1.0964
Scaled Pearson X2	310	339.8771	1.0964
Log Likelihood		2151.5227	
Full Log Likelihood		-865.6289	
AIC (smaller is better)		1741.2578	
AICC (smaller is better)		1741.4526	
BIC (smaller is better)		1760.0048	

Algorithm converged.

Analysis Of Maximum Likelihood Parameter Estimates

Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits	Wald Chi-Square	Pr > ChiSq

```
PROG           2      1      -0.4408       0.1826      -0.7986      -0.0829            5.83         0.0158
PROG           3      1      -1.2787       0.2020      -1.6745      -0.8828           40.08         <.0001
Dispersion            1       0.9683       0.0995       0.7916       1.1844

NOTE: The negative binomial dispersion parameter was estimated by maximum likelihood.


     LR Statistics For Type 3 Analysis

                          Chi-
Source           DF     Square    Pr > ChiSq

MATH              1       5.61        0.0179
PROG              2      45.05        <.0001
```

- The output begins the Model Information table and the Criteria for Assessing Goodness of Fit table. The number of observations read and used is given. In this example, we have no missing data, so all 314 observations that are read in are used in the analysis. In the Criteria for Assessing Goodness of Fit table, we see the Pearson Chi-Square of 339.88. This is not a test of the model coefficients (which we saw in the header information), but a test of the model form: are the data overdispersed when modeled with a negative binomial distribution? A low p-value from this test suggests misspecification or other problems with the model. We can get the

```
  pval = 1 - probchi(339.8771, 310);
run;

proc print data = test; run;

Obs      pval
 1     0.11703
```

- The Analysis of Maximum Likelihood Parameter Estimates table is presented next, which gives the regression coefficients, standard errors, the Wald 95% confidence intervals for the coefficients, chi-square tests and p-values for each of the model variables. In this example, the variable **math** has a coefficient of -0.006, which is statistically significant. This means that for each one-unit increase in **math**, the expected log count of the days absent decreases by .0006. The indicator for **prog=2** is the expected difference in log count between group 2 and the reference group (**prog**=1). The expected log count for level 2 of **prog** is 0.44 lower than the expected log count for level 1. The indicator variable **prog=3** is the expected difference in log count between group 3 and the reference group. The expected log count for level 3 of **prog** is 1.28 lower than the expected log count for level 1. To determine if **prog** itself, overall, is statistically significant, we can look at the LR Statistics for Type 3 Analysis table that includes the two degrees-of-freedom test of this variable. The two degree-of-freedom chi-square test indicates that **prog** is a statistically significant predictor of **daysabs**. The chi-square value for this test is 45.05 with a p-value of .0001. This indicates that the variable **prog** is a statistically significant predictor of **daysabs**.

(variance greater than mean). An estimate less than zero suggests under-dispersion, which is very rare.

We can also see the results as incident rate ratios by using **estimate** statements with the **exp** option.

```
  estimate 'prog 2' prog 1 0 / exp;
  estimate 'prog 3' prog 0 1 / exp;
  estimate 'math'   math 1   / exp;
run;
```

< - some output omitted - >

Contrast Estimate Results

Label	Mean Estimate	Mean Confidence Limits		L'Beta Estimate	Standard Error	Alpha	L'Beta Confidence Limits		Chi-Square
prog 2	0.6435	0.4500	0.9204	-0.4408	0.1826	0.05	-0.7986	-0.0829	5.83
Exp(prog 2)				0.6435	0.1175	0.05	0.4500	0.9204	
prog 3	0.2784	0.1874	0.4136	-1.2787	0.2020	0.05	-1.6745	-0.8828	40.08
Exp(prog 3)				0.2784	0.0562	0.05	0.1874	0.4136	
math	0.9940	0.9892	0.9989	-0.0060	0.0025	0.05	-0.0109	-0.0011	5.71
Exp(math)				0.9940	0.0025	0.05	0.9892	0.9989	

The output above indicates that the incident rate for **prog=2** is 0.64 times the incident rate for the reference group (**prog=1**). Likewise, the incident rate for **prog=3** is 0.28 times the incident rate for the reference group holding the other variables constant. The percent change in the incident rate of **daysabs** is a 1% decrease (1 - .99) for every unit increase in **math**.

The form of the model equation for negative binomial regression is the same as that for Poisson regression. The log of the outcome is predicted with a linear combination of the predictors:

This implies:

```
daysabs = exp(Intercept + b1(prog=2) + b2(prog=3)+ b3math) = exp(Intercept) * exp(b1(prog=2)) *
exp(b2(prog=3)) * exp(b3math)
```

The coefficients have an *additive* effect in the log(y) scale and the IRR have a *multiplicative* effect in the y scale. The dispersion parameter in negative binomial regression does not effect the expected counts, but it does effect the estimated variance of the expected counts.

For additional information on the various metrics in which the results can be presented, and the interpretation of such, please see *Regression Models for Categorical Dependent Variables Using Stata, Second Edition* by J. Scott Long and Jeremy Freese (2006).

Below we use **estimate** statements to calculate the predicted number of events at each level of **prog**, holding all other variables (in this example, **math**) in the model at their means.

```
  estimate 'prog 1' intercept 1 prog 0 0 math 48.2675 / exp;
  estimate 'prog 2' intercept 1 prog 1 0 math 48.2675 / exp;
  estimate 'prog 3' intercept 1 prog 0 1 math 48.2675 / exp;
run;

< - some output omitted - >

                                    Contrast Estimate Results

                 Mean          Mean              L'Beta  Standard               L'Beta           Chi-
Label        Estimate   Confidence Limits      Estimate     Error   Alpha   Confidence Limits  Square

prog 1        10.2369    7.4291   14.1058        2.3260    0.1636    0.05    2.0054    2.6466  202.22
Exp(prog 1)                                     10.2369    1.6744    0.05    7.4291   14.1058
prog 2         6.5879    5.5916    7.7618        1.8852    0.0837    0.05    1.7213    2.0492  507.76
Exp(prog 2)                                      6.5879    0.5512    0.05    5.5916    7.7618
prog 3         2.8501    2.2720    3.5753        1.0473    0.1157    0.05    0.8207    1.2740   82.00
Exp(prog 3)                                      2.8501    0.3296    0.05    2.2720    3.5753
```

In the output above, we see that the predicted number of events for level 1 of **prog** is about 10.24, holding **math** at its mean. The predicted number of events for level 2 of **prog** is lower at 6.59, and the predicted number of events for level 3 of **prog** is about 2.85. Note that the predicted count of level 2 of **prog** is (6.5879/10.2369) = 0.64 times the predicted count for level 1 of **prog**. This matches what we saw in the after in the incident rate ratio output table.

```
  class prog (param=ref ref=first);
  model daysabs = math prog / type3 dist=negbin;
  estimate 'math 20' intercept 1 prog 0 0 math 20 / exp;
  estimate 'math 40' intercept 1 prog 0 0 math 40 / exp;
run;
```

Contrast Estimate Results

Label	Mean Estimate	Mean Confidence Limits		L'Beta Estimate	Standard Error	Alpha	L'Beta Confidence Limits		Chi-Square
math 20	12.1267	8.6305	17.0391	2.4954	0.1735	0.05	2.1553	2.8355	206.80
Exp(math 20)				12.1267	2.1043	0.05	8.6305	17.0391	
math 40	10.7569	7.8092	14.8172	2.3755	0.1634	0.05	2.0553	2.6958	211.38
Exp(math 40)				10.7569	1.7576	0.05	7.8092	14.8172	

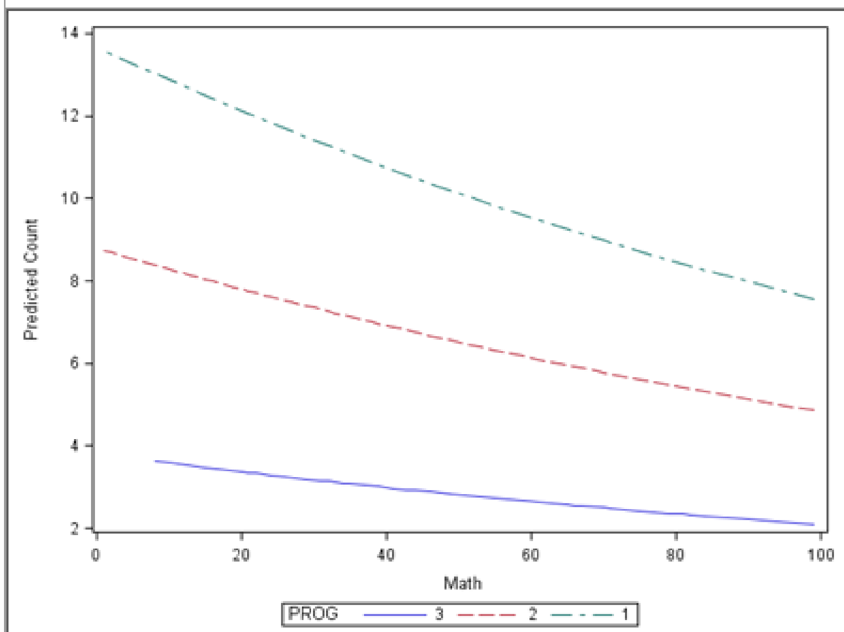
The table above shows that when **prog** held at its reference level and **math** at 20, the predicted count (or average number of days absent) is about 12.13; when **prog** held at its reference level and **math** at 40, the predicted count is about 10.76. If we compare the predicted counts at these two levels of **math**, we can see that the ratio is (10.7569/12.1267) = 0.887. This matches the IRR of 0.994 for a 20 unit change: 0.994^20 = 0.887.

You can graph the predicted number of events using the commands below. **Proc genmod** must be run with the **output** statement to obtain the predicted values in a dataset we called **pred1**. We then sorted our data by the predicted values and created a graph with **proc sgplot**.

```
  output out = nb_pred predicted = pred1;
run;

proc sort data = nb_pred;
  by pred1;
run;

proc sgplot data = nb_pred;
  series x=math y=pred1 / group = prog;
run;
```



possible to have more 0s than expected by the negative binomial model; in this case, a zero-inflated model (either zero-inflated Poisson or zero-inflated negative binomial) may be more appropriate.

- If the data generating process does not allow for any 0s (such as the number of days spent in the hospital), then a zero-truncated model may be more appropriate. Such models can be estimated with **proc countreg**.
- Count data often have an exposure variable, which indicates the number of times the event could have happened. This variable should be incorporated into your negative binomial model with the use of the **offset** option on the **model** statement.
- The outcome variable in a negative binomial regression cannot have negative numbers.

References

- Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables.* Thousand Oaks, CA: Sage Publications.
- Long, J. S. and Freese, J. 2006. *Regression Models for Categorical Dependent Variables Using Stata, Second Edition.* College Station, TX: Stata Press.
- Cameron, A. C. and Trivedi, P. K. 2009. *Microeconometrics Using Stata.* College Station, TX: Stata Press.
- Cameron, A. C. and Trivedi, P. K. 1998. *Regression Analysis of Count Data.* New York: Cambridge Press.
- Cameron, A. C. Advances in Count Data Regression Talk for the Applied Statistics Workshop, March 28, 2009. http://cameron.econ.ucdavis.edu/racd/count.html (http://cameron.econ.ucdavis.edu/racd/count.html) .
- Dupont, W. D. 2002. *Statistical Modeling for Biomedical Researchers: A Simple Introduction to the Analysis of Complex Data.* New York: Cambridge Press.

- proc genmod (http://support.sas.com/documentation/cdl/en/statug/63347/HTML/default/viewer.htm#genmod_toc.htm)

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