Poisson regression is for modeling count variables.

Please note: The purpose of this page is to show how to use various data analysis commands. It does not cover all aspects of the research process which researchers are expected to do. In particular, it does not cover data cleaning and checking, verification of assumptions, model diagnostics or potential follow-up analyses.

This example was done using SAS version 9.22.

Examples of Poisson regression

Example 1. The number of persons killed by mule or horse kicks in the Prussian army per year. von Bortkiewicz collected data from 20 volumes of *Preussischen Statistik*. These data were collected on 10 corps of the Prussian army in the late 1800s over the course of 20 years.

Example 2. A health-related researcher is studying the number of hospital visits in past 12 months by senior citizens in a community based on the characteristics of the individuals and the types of health plans under which each one is covered.

Example 3. A researcher in education is interested in the association between the number of awards earned by students at one high school and the students' performance in math and the type of program (e.g., vocational, general or academic) in which students were enrolled.

Description of the data

For the purpose of illustration, we have simulated a data set for Example 3 above: https://stats.idre.ucla.edu/wp-content/uploads/2016/02/poisson_sim.sas7bdat (https://stats.idre.ucla.edu/wp-content/uploads/2016/02/poisson_sim.sas7bdat). In this example, **num_awards** is the outcome variable and indicates the number of awards earned by students at a high school in a year, **math** is a continuous predictor variable and represents students' scores on their math final exam, and **prog** is a categorical predictor variable with three levels indicating the type of program in which the students were enrolled. It is coded as 1 = "General", 2 = "Academic" and 3 = "Vocational".

```
proc means data = poisson_sim n mean var min max;
  var num_awards math;
run;

The MEANS Procedure

Variable      Label           N           Mean        Variance         Minimum         Maximum
-----------------------------------------------------------------------------------------------
num_awards                  200      0.6300000       1.1086432               0       6.0000000
math          math score    200     52.6450000      87.7678141      33.0000000      75.0000000
-----------------------------------------------------------------------------------------------
```

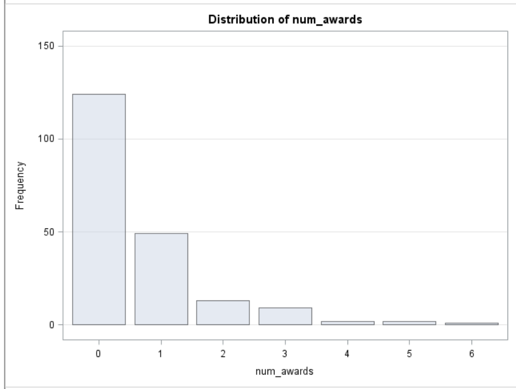
Each variable has 200 valid observations and their distributions seem quite reasonable. The *unconditional* mean and variance of our outcome variable are not extremely different. Our model assumes that these values, conditioned on the predictor variables, will be equal (or at least roughly so).

We can look at summary statistics by program type. The table below shows the mean and variance of numbers of awards by program type and seems to suggest that program type is a good candidate for predicting the number of awards, our outcome variable, because the mean value of the outcome appears to vary by **prog**. Additionally, the means and variances within each level of **prog**–the *conditional* means and variances–are similar. A frequency plot is also produced to display the distribution of the outcome variable.

```
run;
```

```
The MEANS Procedure

          Analysis Variable : num_awards

     type of      N
     program    Obs            Mean        Variance
---------------------------------------------------
           1     45       0.2000000       0.1636364

           2    105       1.0000000       1.6346154

           3     50       0.2400000       0.2677551
---------------------------------------------------
```

```
proc freq data=poisson_sim;
tables num_awards / plots=freqplot;
run;
```




```
proc freq data = poisson_sim;
  tables prog;
run;
```

```
The FREQ Procedure

                     type of program

                                    Cumulative    Cumulative
prog    Frequency     Percent     Frequency      Percent
-----------------------------------------------------------
   1          45       22.50            45        22.50
   2         105       52.50           150        75.00
   3          50       25.00           200       100.00
```

Analysis methods you might consider

Below is a list of some analysis methods you may have encountered. Some of the methods listed are quite reasonable, while others have either fallen out of favor or have limitations.

- Poisson regression – Poisson regression is often used for modeling count data. It has a number of extensions useful for count models.
- Negative binomial regression – Negative binomial regression can be used for over-dispersed count data, that is when the conditional variance exceeds the conditional mean. It can be considered as a generalization of Poisson regression since it has the same mean structure as Poisson

in the data, "true zeros" and "excess zeros". Zero-inflated models estimate two equations simultaneously, one for the count model and one for the excess zeros.

- OLS regression – Count outcome variables are sometimes log-transformed and analyzed using OLS regression. Many issues arise with this approach, including loss of data due to undefined values generated by taking the log of zero (which is undefined) and biased estimates.

Poisson regression analysis

At this point, we are ready to perform our Poisson model analysis. **Proc genmod** is usually used for Poisson regression analysis in SAS.

On the **class** statement we list the variable **prog**, since **prog** is a categorical variable. We use the global option **param = glm** so we can save the model using the **store** statement for future post estimations. The **type3** option in the model statement is used to get the multi-degree-of-freedom test of the categorical variables listed on the **class** statement, and the **dist = poisson** option is used to indicate that a Poisson distribution should be used. Statement "store" allows us to store the parameter estimates to a data set, which we call p1, so we can perform post estimation without rerunning the model.

```
  store p1;
run;
```

```
The GENMOD Procedure

          Model Information

Data Set              WORK.POISSON_SIM
Distribution                   Poisson
Link Function                      Log
Dependent Variable          num_awards


Number of Observations Read         200
Number of Observations Used         200


  Class Level Information

Class      Levels    Values

prog            3    1 2 3


             Criteria For Assessing Goodness Of Fit

Criterion                     DF           Value        Value/DF

Deviance                     196        189.4496          0.9666
Scaled Deviance              196        189.4496          0.9666
Pearson Chi-Square           196        212.1437          1.0824
Scaled Pearson X2            196        212.1437          1.0824
Log Likelihood                         -135.1052
Full Log Likelihood                    -182.7523
AIC (smaller is better)                 373.5045
AICC (smaller is better)                373.7096
BIC (smaller is better)                 386.6978


Algorithm converged.


                       Analysis Of Maximum Likelihood Parameter Estimates

                                     Standard     Wald 95% Confidence          Wald
Parameter         DF    Estimate        Error           Limits           Chi-Square    Pr > ChiSq

Intercept          1     -4.8773       0.6282     -6.1085     -3.6461         60.28        <.0001
prog        1      1     -0.3698       0.4411     -1.2343      0.4947          0.70        0.4018
prog        2      1      0.7140       0.3200      0.0868      1.3413          4.98        0.0257
prog        3      0      0.0000       0.0000      0.0000      0.0000           .             .
math               1      0.0702       0.0106      0.0494      0.0909         43.81        <.0001 Scale 0 1.0000 0.0000 1.0000 1.0000 NOTE: The scale parameter was held fixed. LR S

prog             2      14.57        0.0007
math             1      45.01        <.0001
```

- The output begins with the basic model information and then provides a list of goodness-of-fit statistics including the log likelihood, AIC, and BIC.
- Next you will find the Poisson regression coefficients for each of the variables along with standard errors, Wald Chi-Square statistics and intervals, and p-values for the coefficients. The coefficient for **math** is .07. This means that the expected increase in log count for a one-unit increase in **math** is .07. For our three-level categorical predictor **prog**, the model presents coefficients relating levels 1 and 2 to level 3. The indicator variable **prog(2)**

- To determine if **prog** itself, overall, is statistically significant, we can look at the Type 3 table in the outcome that includes the two degrees-of-freedom test of this variable. This is testing the null hypothesis that both **prog** estimates (level 1 vs. level 3 and level 2 vs. level 3) are equal to zero. We see there that **prog** is a statistically significant predictor.

To help assess the fit of the model, we can use the goodness-of-fit chi-squared test. This assumes the deviance follows a chi-square distribution with degrees of freedom equal to the model residual. From the first line of our Goodness of Fit output, we can see these values are 189.4495 and 196.

```
data pvalue;
  df = 196; chisq = 189.4495;
  pvalue = 1 - probchi(chisq, df);
run;
proc print data = pvalue noobs;
run;

 df     chisq      pvalue

196    189.450    0.61823
```

This is not a test of the model coefficients (which we saw in the header information), but a test of the model form: Does the poisson model form fit our data? We conclude that the model fits reasonably well because the goodness-of-fit chi-squared test is not statistically significant. If the test had been statistically significant, it would indicate that the data do not fit the model well. In that situation, we may try to determine if there are omitted predictor variables, if our linearity assumption holds and/or if there is an issue of over-dispersion.

Cameron and Trivedi (2009) recommend using robust standard errors for the parameter estimates to control for mild violation of the distribution assumption that the variance equals the mean. In SAS, we can do this by running **proc genmod** with the **repeated** statement in order to obtain robust standard errors for the Poisson regression coefficients.

```
  model num_awards = prog math /dist=poisson;
  repeated subject=id;
run;

             GEE Model Information

Correlation Structure              Independent
Subject Effect                 id (200 levels)
Number of Clusters                         200
Correlation Matrix Dimension                 1
Maximum Cluster Size                         1
Minimum Cluster Size                         1


Algorithm converged.

  GEE Fit Criteria

QIC          256.8581
QICu         257.6478


              Analysis Of GEE Parameter Estimates
               Empirical Standard Error Estimates

                     Standard   95% Confidence
Parameter   Estimate    Error       Limits            Z Pr > |Z|

Intercept    -4.8773   0.6297  -6.1116  -3.6430   -7.74   <.0001
prog      1  -0.3698   0.4004  -1.1546   0.4150   -0.92   0.3557
prog      2   0.7140   0.2986   0.1287   1.2994    2.39   0.0168
prog      3   0.0000   0.0000   0.0000   0.0000     .       .
math          0.0702   0.0104   0.0497   0.0906    6.72   <.0001
```

We can see that our estimates are unchanged, but our standard errors are slightly different.

We have the model stored in a data set called **p1**. Using **proc plm**, we can request many different post estimation tasks. For example, we might want to displayed the results as incident rate ratios (IRR). We can do so with a **data** step after using **proc plm** to create a dataset of our model estimates.

```
ods output ParameterEstimates = est;
proc plm source = p1;
  show parameters;
run;

data est_exp;
  set est;
  irr = exp(estimate);
  if parameter ^="Intercept";
run;
proc print data = est_exp;
run;
```

```
Obs    Parameter            prog    Estimate      StdErr      irr

 1     type of program 1     1       -0.3698      0.4411    0.69087
 2     type of program 2     2        0.7140      0.3200    2.04225
 3     type of program 3     3             0           .    1.00000
 4     math score            _       0.07015     0.01060    1.07267
```

The output above indicates that the incident rate for **prog**=2 is 2.04 times the incident rate for the reference group (**prog**=3). Likewise, the incident rate for **prog**=1 is 0.69 times the incident rate for the reference group holding the other variables constant. The percent change in the incident rate of **num_awards** is 100

```
log(num_awards) = Intercept + b₁(prog=1) + b₂(prog=2) + b₃math.
```

This implies:

```
num_awards = exp(Intercept + b₁(prog=1) + b₂(prog=2)+ b₃math) = exp(Intercept) * exp(b₁(prog=1)) *
exp(b₂(prog=2)) * exp(b₃math)
```

The coefficients have an *additive* effect in the log(y) scale and the IRR have a *multiplicative* effect in the y scale.

For additional information on the various metrics in which the results can be presented, and the interpretation of such, please see *Regression Models for Categorical Dependent Variables Using Stata, Second Edition* by J. Scott Long and Jeremy Freese (2006).

Below we use **lsmeans** statements in **proc plm** to calculate the predicted number of events at each level of **prog**, holding all other variables (in this example, **math**) in the model at their means.

We use the "**ilink**" option (for inverse link) to get the predicted means (predicted count) in addition to the linear predictions.

```
proc plm source = p1;
  lsmeans prog /ilink cl;
run;
```

```
                               prog Least Squares Means

type of               Standard
program    Estimate       Error    z Value    Pr > |z|     Alpha       Lower       Upper

1           -1.5540      0.3335      -4.66      <.0001      0.05     -2.2076     -0.9003
2           -0.4701      0.1381      -3.40      0.0007      0.05     -0.7407     -0.1995
3           -1.1841      0.2887      -4.10      <.0001      0.05     -1.7499     -0.6183

               prog Least Squares Means

                      Standard
type of               Error of       Lower       Upper
program        Mean        Mean        Mean        Mean

1            0.2114     0.07050      0.1100      0.4064
2            0.6249     0.08628      0.4768      0.8191
3            0.3060     0.08834      0.1738      0.5388
```

The first block of output above shows the predicted log count. The second block shows predicted number of events in the "mean" column.

In the output above, we see that the predicted number of events for level 1 of **prog** is about .21, holding **math** at its mean. The predicted number of events for level 2 of **prog** is higher at .62, and the predicted number of events for level 3 of **prog** is about .31. Note that the predicted count of level 1 of **prog** is (.2114/.3060) = 0.6908 times the predicted count for level 3 of **prog**. This matches what we saw in the IRR output table.

Below we will obtain the averaged predicted counts for values of **math** that range from 35 to 75 in increments of 10, using a data step and the **score** statement of **proc plm**.

```
    math = math_cat;
    output;
  end;
run;
proc plm source=p1;
  score data = toscore out=math /ilink;
run;
proc means data = math mean;
  class math_cat;
  var predicted;
run;
```

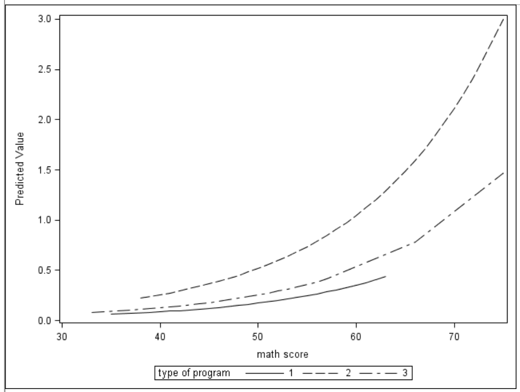
```
                  N
   math_cat     Obs            Mean
------------------------------------
         35     200       0.1311326

         45     200       0.2644714

         55     200       0.5333923

         65     200       1.0757584

         75     200       2.1696153
```

The table above shows that with **prog** at its observed values and **math** held at 35 for all observations, the average predicted count (or average number of awards) is about .13; when **math** = 75, the average predicted count is about 2.17.

If we compare the predicted counts at math = 35 and math = 45, we can see that the ratio is (.2644714/.1311326) = 2.017. This matches the IRR of 1.0727 for a 10 unit change: 1.0727^10 = 2.017.

You can graph the predicted number of events using **proc plm** and **proc sgplot** below.

```
  score data = poisson_sim out=pred /ilink;
run;
proc sort data = pred;
  by prog math;
run;
proc sgplot data = pred;
  series x = math y = predicted /group=prog;
run;
ods graphics off;
```



Things to consider

- When there seems to be an issue of dispersion, we should first check if our model is appropriately specified, such as omitted variables and functional forms. For example, if we omitted the predictor variable **prog** in the example above, our model would seem to have a problem with over-dispersion. In other words, a mis-specified model could present a symptom like an over-dispersion problem.
- Assuming that the model is correctly specified, you may want to check for overdispersion. There are several tests including the likelihood ratio test of over-dispersion parameter alpha by running the same regression model using negative binomial distribution.
- One common cause of over-dispersion is excess zeros, which in turn are generated by an additional data generating process. In this situation, a zero-inflated model should be considered.
- If the data-generating process does not allow for any 0s (such as the number of days spent in the hospital), then a zero-truncated model may be more appropriate.
- The outcome variable in a Poisson regression cannot have negative numbers.
- Poisson regression is estimated via maximum likelihood estimation. It usually requires a large sample size.

References

- Cameron, A. C. and Trivedi, P. K. 2009. *Microeconometrics Using Stata*. College Station, TX: Stata Press.
- Cameron, A. C. and Trivedi, P. K. 1998. *Regression Analysis of Count Data*. New York: Cambridge Press.
- Cameron, A. C. Advances in Count Data Regression Talk for the Applied Statistics Workshop, March 28, 2009. http://cameron.econ.ucdavis.edu/racd/count.html (http://cameron.econ.ucdavis.edu/racd/count.html) .
- Dupont, W. D. 2002. *Statistical Modeling for Biomedical Researchers: A Simple Introduction to the Analysis of Complex Data*. New York: Cambridge Press.
- Long, J. S. 1997. *Regression Models for Categorical and Limited Dependent Variables*. Thousand Oaks, CA: Sage Publications.
- Long, J. S. and Freese, J. 2006. *Regression Models for Categorical Dependent Variables Using Stata, Second Edition*. College Station, TX: Stata Press.

See also

/HTML/default/genmod_toc.htm)

- proc glimmix (http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#/documentation/cdl/en/statug/63033/HTML/default/glimmix_toc.htm)
- proc nlmixed (http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#/documentation/cdl/en/statug/63033/HTML/default/nlmixed_toc.htm)
- proc countreg (http://support.sas.com/documentation/cdl/en/etsug/60372/HTML/default/countreg_toc.htm)

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