Chapter 13

Problem 13.1

Develop expressions for the mole fractions of reacting species as functions of the reaction coordinate for:

(a) A system containing 2 mol NH₃ and 5 mol O₂ and undergoing the reaction:

$$4 \text{ NH}_3(g) + 5 \text{ O}_2(g) \longrightarrow 4 \text{ NO}(g) + 6 \text{ H}_2\text{O}(g)$$

(b) A system initially containing 3 mol H₂S and 5 mol O₂ and undergoing the reaction:

$$2 H_2 S(g) + 3 O_2(g) \longrightarrow 2 H_2 O(g) + 2 SO_2(g)$$

(c) A system containing 3 mol NO₂, 4 mol NH₃, and 1 mol N₂ and undergoing the reaction:

$$6 \text{ NO}_2(g) + 8 \text{ NH}_3(g) \longrightarrow 7 \text{ N}_2(g) + 12 \text{ H}_2\text{O}(g)$$

Solution

The solution here is to substitute into Eq. (13.5)

$$y_i = \frac{n_{io} + v_i \varepsilon}{n_0 + v \varepsilon} \tag{13.5}$$

(a)

$$y_{\text{NH}_3} = \frac{2 - 4\varepsilon}{7 + \varepsilon}$$
$$y_{\text{O}_2} = \frac{5 - 5\varepsilon}{7 + \varepsilon}$$

(b)

$$y_{\text{H}_2\text{S}} = \frac{3 - 2\varepsilon}{8 - \varepsilon}$$
$$y_{\text{O}_2} = \frac{5 - 3\varepsilon}{8 - \varepsilon}$$

(c)

$$y_{\text{NO}_2} = \frac{3 - 6\varepsilon}{8 + 5\varepsilon}$$
$$y_{\text{NH}_3} = \frac{4 - 8\varepsilon}{8 + 5\varepsilon}$$
$$y_{\text{N}_2} = \frac{1 + 7\varepsilon}{8 + 5\varepsilon}$$

Problem 13.2

A system initially containing 2 mol C₂H₄ and 3 mol O₂ undergoes the reactions:

$$C_2H_4(g) + \frac{1}{2}O_2(g) \longrightarrow ((CH_2)_2)O(g)$$

$$C_2H_4(g) + 3O_2(g) \longrightarrow 2CO_2(g) + 2H_2O(g)$$

Develop expressions for the mole fractions of the reacting species as functions of the reaction coordinates for the two reactions.

Solution

The two chemical reactions need to be added and the same solution in Problem 13.1 is to be used.

$$2 C_2 H_4(g) + 3.5 O_2(g) \longrightarrow 2 CO_2(g) + 2 H_2 O(g) + ((CH_2)_2) O(g)$$

$$y_{C_2H_4} = \frac{2 - 2\varepsilon}{5}$$
$$y_{O_2} = \frac{3 - 3.5\varepsilon}{5}$$

Problem 13.3

A system formed initially of 2 mol CO₂, 5 mol H₂, and 1 mol CO undergoes the reactions:

$$CO_2(g) + 3 H_2(g) \longrightarrow CH_3OH(g) + H_2O(g)$$

 $CO_2(g) + H_2(g) \longrightarrow CO(g) + H_2O(g)$

Develop expressions for the mole fractions of the reacting species as functions of the reaction coordinates for the two reactions.

Solution

Similar to the previous problem.

$$2 \operatorname{CO}_2(g) + 4 \operatorname{H}_2(g) \longrightarrow \operatorname{CH}_3 \operatorname{OH}(g) + 2 \operatorname{H}_2 \operatorname{O}(g) + \operatorname{CO}(g)$$

$$y_{\text{CO}_2} = \frac{2 - 2\varepsilon}{8 - 2\varepsilon}$$
$$y_{\text{H}_2} = \frac{5 - 4\varepsilon}{8 - 2\varepsilon}$$
$$y_{\text{CO}} = \frac{1 + \varepsilon}{8 - 2\varepsilon}$$

Problem 13.4

Consider the water-gas-shift reaction:

$$H_2(g) + CO_2(g) \longrightarrow H_2O\left(g\right) + CO\left(g\right)$$

At high temperatures and low to moderate pressures the reacting species form an ideal-gas mixture. By Eq. (11.27):

$$G = \sum_{i} y_i G_i + RT \sum_{i} y_i \ln y_i$$

When the Gibbs energies of the elements in their standard states are set equal to zero, $G_i = \Delta G_{f_i}^{\circ}$, for each species, and then:

$$G = \sum_{i} y_i \Delta G_{f_i}^{\circ} + RT \sum_{i} y_i \ln y_i$$
 (A)

At the beginning of Sec. 13.2 we noted that Eq. (14.68) is a criterion of equilibrium. Applied to the water-gas-shift reaction with the understanding that T and P are constant, this equation becomes:

$$dG' = d(nG) = ndG + Gdn = 0 \qquad n\frac{dG}{d\varepsilon} + G\frac{dn}{d\varepsilon} = 0$$

Here, however, $dn/d\varepsilon = 0$. The equilibrium criterion therefore becomes:

$$\frac{dG}{d\varepsilon} = 0 \tag{B}$$

Once the y_i are eliminated in favor of ε , Eq. (A) relates G to ε . Data for $\Delta G_{f_i}^{\circ}$ for the compounds of interest are givent with Ex. 13.13. For a temperature of 1000 K (the reaction is unaffected by P) and for a feed of 1 mol H₂ and 1 mol CO₂:

- (a) Determine the equilibrium value of ε by application of Eq. (B).
- (b) Plot G vs. ε , indicating the location of the equilibrium value of ε determined in (a).

Solution

(a)

$$y_{\text{H}_2} = y_{\text{CO}_2} = \frac{1 - \varepsilon}{2}$$
$$y_{\text{H}_2\text{O}} = y_{\text{CO}} = \frac{\varepsilon}{2}$$

From Eq. (A)—the values of $\Delta G_{f_i}^{\circ}$ in J mol⁻¹ are given in Example 13.13:

$$G = \left(\frac{1-\varepsilon}{2}\right)(-395790) + \left(\frac{\varepsilon}{2}\right)(-200240 - 192420) + R(1000) \left[2\left(\frac{1-\varepsilon}{2}\right)\ln\left(\frac{1-\varepsilon}{2}\right) + 2\left(\frac{\varepsilon}{2}\right)\ln\left(\frac{\varepsilon}{2}\right)\right]$$

Differentiate with respect to ε and solve for the root as in Eq. (B):