# formulation

December 10, 2017

## 1 Random Walk on Hypergaph

### 1.1 Random walk defined by Dengyong Zhou

The random walk is done in two step: 1. Choose a hypteredge e over all hyperedges incodent with $u$ with the probability proportional to $w(e)$ 2. Choose a vertext $v \in e$ uniformly at random The transaction probably matrix $P$ is then

$$p(u,v) = \sum_{e \in E} \frac{h(u,e)}{d(u)} \frac{h(v,e)}{\delta(e)},$$

where for a vertex $v$

$$d(v) = \sum_{e \in E | v \in e} w(e)$$

or

$$d = HW$$

and $\delta(e)$ is the number of elements in the edge

$$\delta(e) = |e|$$

or

$$\delta = \mathbf{1}^T H$$

In matrix notation,

$$P = D_v^{-1} H W D_e^{-1} H^T$$

The stationary distrubtion $\pi$ of the random walk is

$$\pi(v) = \frac{d(v)}{\mathrm{vol} V}$$

This can be validated by

$$\sum_{u \in V} \pi(u) p(u,v) = \sum_{u \in V} \frac{d(v)}{\mathrm{vol} V} \sum_{e \in E} \frac{h(u,e)}{d(u)} \frac{h(v,e)}{\delta(e)} = \frac{1}{\mathrm{vol} V} \sum_{e \in E} w(e) \sum_{u \in E} h(u,e) \frac{h(v,e)}{\delta e} = \frac{1}{\mathrm{vol} V} \sum_{e \in E} w(e) h(v,e) = \frac{d(v)}{\mathrm{vol} V}$$

### 1.2 Random walk in this work

Degree of vertices:

$$d(v)$$

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$$p(u,v) = \sum_{e \in E} \frac{h(u,e)}{d(u)} \frac{h(v,e)}{\delta(e)},$$

where for a vertex $v$

$$d(v) = \sum_{e \in E | v \in e} w(e)$$

and $\delta(e)$ is the number of elements in the edge

$$\delta e = |e|$$

In matrix notation,

$$P = D_v^{-1} H W D_e^{-1} H^T$$

The stationary distrubtion $\pi$ of the random walk is

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