

# ChE Thermodynamics Problems

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## Chapter 10

### Example 10.1\*

Binary system acetonitrile(1)/nitromethane(2) conforms closely to Raoult's law. Vapor pressures for the pure species are given by the following Antoine equations:

$$\ln P_1^{\text{sat}}/\text{kPa} = 14.2724 - \frac{2945.47}{T - 49.15}$$
$$\ln P_2^{\text{sat}}/\text{kPa} = 14.2043 - \frac{2972.64}{T - 64.15}$$

- (a) Prepare a graph showing  $P$  vs.  $x_1$  and  $P$  vs.  $y_1$  for a temperature of 75 °C.
- (b) Prepare graph showing  $t$  vs.  $x_1$  and  $t$  vs.  $y_2$  for a pressure of 70 kPa.

**Solution:**

- (a)

$$P_\alpha = x_\alpha P_\alpha^*$$

Substitute this to the Antoine equation:

$$\ln \frac{P_\alpha/\text{kPa}}{x_\alpha} = A_\alpha - \frac{B_\alpha}{t/^\circ\text{C} + C_\alpha}$$

$$P = P_1^{\text{sat}} x_1 + P_2^{\text{sat}} (1 - x_1)$$

$$y_\alpha = \frac{P_\alpha}{P} = \frac{P_\alpha^{\text{sat}} x_\alpha}{P}$$

The plot is shown in Figure (1)

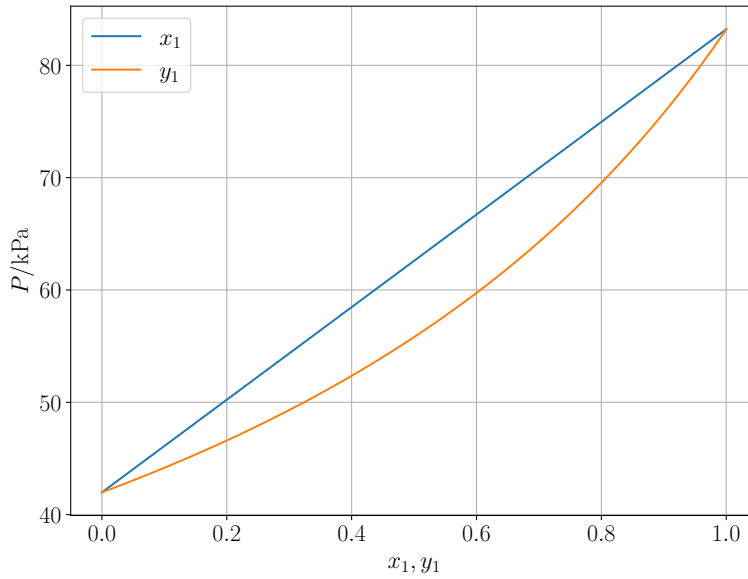


Figure 1: Plot of pressure versus liquid and vapor mole fraction of component 1 as described in Example 10.1a\*

(b) This is another form of the Antoine equation:

$$T_\alpha^{\text{sat}} = \frac{B_\alpha}{A_\alpha - \ln P} + C_\alpha$$

Solve for  $T_1^{\text{sat}}$  and  $T_2^{\text{sat}}$  as these comprise of the range of the graph. Solve for:

$$\ln P_\alpha^{\text{sat}} = A_\alpha - \frac{B_\alpha}{T + C_\alpha}$$

To solve for:

$$x_1 = \frac{P - P_2^{\text{sat}}}{P_1^{\text{sat}} - P_2^{\text{sat}}}$$

Which is derived from:

$$P = P_1^{\text{sat}} x_1 + P_2^{\text{sat}} (1 - x_1)$$

The plot is shown in Figure (2)

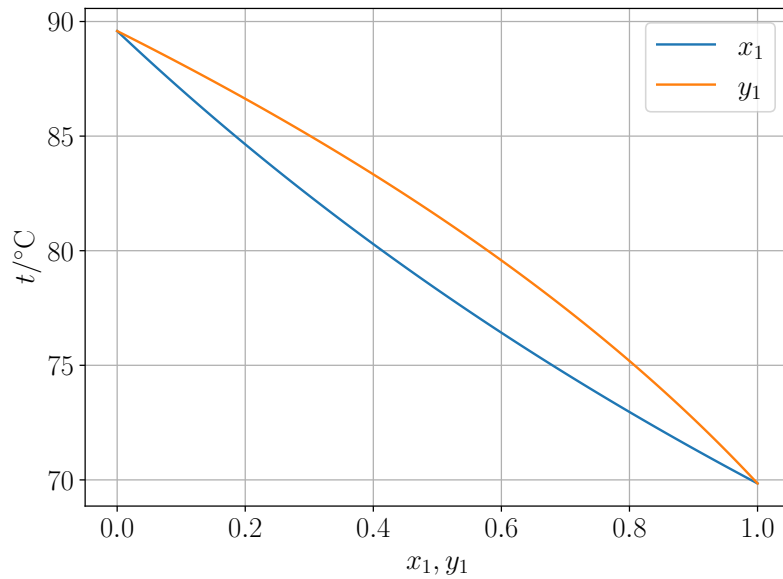


Figure 2: Plot of temperature (in °C) versus liquid and vapor mole fraction of component 1 as described in Example 10.1b\*

### Example 10.2\*

Assuming that carbonated water contains only  $\text{CO}_2(1)$  and  $\text{H}_2\text{O}(2)$ , determine the compositions of the vapor and liquid phases in a sealed can of "soda" and the pressure exerted on the can at 10 °C (283.15 K). Henry's constant for  $\text{CO}_2$  in water at 10 °C (283.15 K) is about 990 bar **Solution:**

Applying the phase rule:

$$F = 2 - \pi + N$$

$$F = 2$$

Only the temperature of the system is given. We must give another intensive variable. And since we are given the Henry's constant it is good that we limit the amount of  $C_{O_2}$  in the liquid,  $x_1 = 0.01$ .

$$\begin{aligned}y_1 P &= x_1 \mathcal{H}_1 & y_2 P &= x_2 P_2^{\text{sat}} \\ P &= x_1 \mathcal{H}_1 + x_2 P_2^{\text{sat}}\end{aligned}$$

The value of  $P_2^{\text{sat}}$  can be found at the steam table, at 10 °C, which is 0.01227 bar

$$\begin{aligned}P &= (0.01)(990) + (0.99)(0.01227) = \boxed{9.912 \text{ bar}} \\ y_2 &= \frac{x_2 P_2^{\text{sat}}}{P} = \frac{(0.99)(0.01227)}{9.912} = \boxed{0.0012} \\ y_1 &= \boxed{0.9988}\end{aligned}$$

### Example 10.3\*

For the system methanol(1)/methyl acetate(2), the following equations provide a reasonable correlation for the activity coefficients”

$$\begin{aligned}\ln y_1 &= Ax_2^2 & \ln y_2 &= Ax_1^2 \\ \text{where } A &= 2.771 - 0.00523T\end{aligned}$$

In addition, the following Antoine equations provide vapor pressures:

$$\begin{aligned}\ln P_1^{\text{sat}} &= 16.59158 - \frac{3643.31}{T - 33.424} \\ \ln P_2^{\text{sat}} &= 14.25326 - \frac{2665.54}{T - 53.424}\end{aligned}$$

where T is in kelvins and the vapor pressures are in kPa. Assuming the validity of Eq (), calculate

- $P$  and  $\{y_i\}$ , for  $t/T = 45$  °C/318.15 K and  $x_1 = 0.25$
- $P$  and  $\{x_i\}$ , for  $t/T = 45$  °C/318.15 K and  $y_1 = 0.60$
- $T$  and  $\{y_i\}$ , for  $P = 101.33$  kPa and  $x_1 = 0.85$
- $T$  and  $\{x_i\}$ , for  $P = 101.33$  kPa and  $y_1 = 0.40$
- The azeotropic pressure, and the azeotropic composition, for  $t/T = 45$  °C/318.15 K

$$y_i P = x_i \gamma_i P_i^{\text{sat}} \quad (i = 1, 2, \dots, N)$$

**Solution:**

(a) Solve for the activity coefficients:

$$A = 2.771 - (0.00523)(318.15) = 1.107$$

$$\gamma_1 = \exp(Ax_2^2) = 1.864$$

$$\gamma_2 = \exp(Ax_1^2) = 1.072$$

And then for pressure:

$$P = x_1\gamma_1P_1^{\text{sat}} + x_2\gamma_2P_2^{\text{sat}}$$

$$P = \boxed{73.50 \text{ kPa}}$$

And then for the vapor mole fractions:

$$y_i = x_i\gamma_iP_i^{\text{sat}}/P$$

$$y_1 = 0.281$$

$$y_2 = 0.719$$

(b) Figure (3) shows the diagram of the iteration solution to be used. The

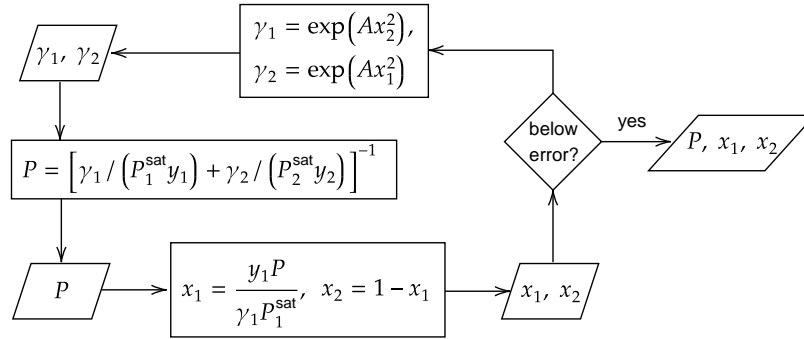


Figure 3: A schematic diagram for the iteration method to be used for Example 10.3b.

initial guesses are  $\gamma_1 = 1$  and  $\gamma_2 = 1$ . After the 12th iteration, the error is less than  $1 \times 10^{-6}$ .

$$P = 62.63 \text{ kPa}$$

$$x_1 = 0.82$$

$$x_2 = 0.18$$

(c)

$$T_i^{\text{sat}} = \frac{B_i}{A_i - \ln P} - C_i$$

$$T_1^{\text{sat}} = 337.71 \text{ K}$$

$$T_2^{\text{sat}} = 330.08 \text{ K}$$

A mole-fraction-weighted average of these values then provides an initial  $T$  for the iteration:

$$T = x_1 T_1 + x_2 T_2$$

$$T = 336.57 \text{ K}$$

Solve for either  $P_1^{\text{sat}}$  or  $P_2^{\text{sat}}$  and then solve for a new value of  $T$ . Using this  $T$ , solve for  $A$ ,  $\gamma_1$ , and  $\gamma_2$ . The iteration method is shown as a schematic diagram in Figure (4)

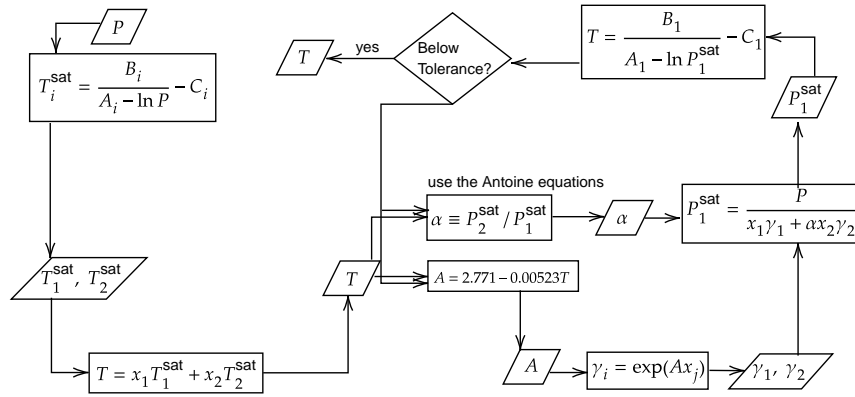


Figure 4: A schematic diagram showing the iteration method used in 10.3c.

$$T = 255.45 \text{ K}$$

$$x_1 = 0.4633$$

$$x_2 = 0.5367$$

(d) The iteration method to be done is shown in Figure (5)

### Example 10.4\*

### Example 10.5\*

The system acetone(1)/acetonitrile(2)/nitromethane(3) at 80 °C and 100 kPa has the overall composition,  $z_1 = 0.45$ ,  $z_2 = 0.35$ ,  $z_3 = 0.20$ . Assuming that

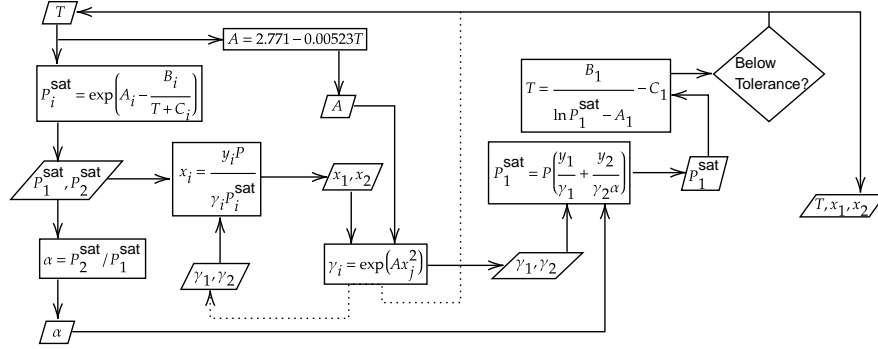


Figure 5: A schematic diagram showing the iteration method used in 10.3d.

Raoult's law is appropriate to this system, determine  $\mathcal{L}$ ,  $\mathcal{V}$ ,  $\{x_i\}$ ,  $\{y_i\}$ . The vapor pressures of the pure species at 80 °C are:

$$P_1^{\text{sat}} = 195.75 \quad P_2^{\text{sat}} = 97.84 \quad P_3^{\text{sat}} = 50.32 \text{ k}$$

**Solution:**

$$x_i = \frac{z_i}{1 + (V/F)(K_i - 1)}$$

$$1 = \frac{z_1}{1 + (V/F)(K_1 - 1)} + \frac{z_2}{1 + (V/F)(K_2 - 1)} + \frac{z_3}{1 + (V/F)(K_3 - 1)}$$

$$K_i = \frac{P_i^{\text{sat}}}{P}$$

### Problem 10.1

Assuming the validity of Raoult's law, do the following calculations for the benzene(1)/toluene(2) system:

- Given  $x_1 = 0.33$  and  $T = 100$  °C, find  $y_1$  and  $P$ .
- Given  $y_1 = 0.33$  and  $T = 100$  °C, find  $x_1$  and  $P$ .
- Given  $x_1 = 0.33$  and  $P = 120$  kPa, find  $y_1$  and  $T$ .
- Given  $y_1 = 0.33$  and  $P = 120$  kPa, find  $x_1$  and  $T$ .
- Given  $T = 105$  °C and  $P = 120$  kPa find  $x_1$  and  $y_1$ .

- (f) For part (e), if the *overall* mole fraction of benzene is  $z_1 = 0.33$ , what molar fraction of the two-phase system is vapor?
- (g) Why is Raoult's law likely to be an excellent VLE model for this system at the stated (or computed) conditions?

**Solution:**

$$\begin{aligned} A_1 &= 13.7819 & A_2 &= 13.9320 \\ B_1 &= 2726.81 & B_2 &= 3056.96 \\ C_1 &= 217.572 & C_2 &= 217.625 \end{aligned}$$

- (a)  $x_1 = 0.33$  and  $T = 100^\circ\text{C}$ , find  $P$  and  $y_1$

$$\ln P_i^{\text{sat}}/\text{kPa} = A_i - \frac{B_i}{t/^\circ\text{C} + C_i}$$

$$P_1^{\text{sat}} = 180.45 \text{ kPa}$$

$$P_2^{\text{sat}} = 74.26 \text{ kPa}$$

$$P = P_1^{\text{sat}}x_1 + P_2^{\text{sat}}(1 - x_1)$$

$$\boxed{P = 109.30 \text{ kPa}}$$

$$y_1 = \frac{x_1 P_1^{\text{sat}}}{P}$$

$$\boxed{y_1 = 0.5448}$$

- (b)  $y_1 = 0.33$  and  $T = 100^\circ\text{C}$ , find  $x_1$  and  $P$

$$x_1 = \frac{y_1 P}{P_1^{\text{sat}}}$$

$$P = y_1 P + P_2^{\text{sat}} \left( 1 - \frac{y_1 P}{P_1^{\text{sat}}} \right)$$

$$\boxed{P = 92.156 \text{ kPa}}$$

$$\boxed{x_1 = 0.169}$$

- (c)  $x_1 = 0.33$  and  $P = 120 \text{ kPa}$ , find  $y_1$  and  $T$

$$x_1 \cdot \exp \left( A_1 - \frac{B_1}{T + C_1} \right) + x_2 \cdot \exp \left( A_2 - \frac{B_2}{T + C_2} \right) = P$$

$$P_1^{\text{sat}} = A_1 - \frac{B_1}{T + C_1}$$

$$y_1 = \frac{x_1 P_1^{\text{sat}}}{P}$$

$$\boxed{T = 103.307^\circ\text{C}}$$

$$\boxed{y_1 = 0.542}$$



(d)  $y_1 = 0.33$  and  $P = 120$  kPa, find  $x_1$  and  $T$

$$P = x_1 P_1^{\text{sat}} + (1 - x_1) P_2^{\text{sat}}$$

$$x_1 = \frac{y_1 P}{P_1^{\text{sat}}} = y_1 P + \left(1 - \frac{y_1 P}{P_1^{\text{sat}}}\right) P_2^{\text{sat}}$$

$$P_i^{\text{sat}} = A_i - \frac{B_i}{T + C_i}$$

$T = 109.13 \text{ } ^\circ\text{C}$ $x_1 = 0.1726$
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(e)  $T = 105 \text{ } ^\circ\text{C}$ ,  $P = 120$  kPa, find  $x_1$ ,  $y_1$

$$P = y_1 P + \left(1 - \frac{y_1 P}{P_1^{\text{sat}}}\right) P_2^{\text{sat}}$$

$$P_i^{\text{sat}} = A_i - \frac{B_i}{T + C_i}$$

$$x_i = \frac{y_i P}{P_i^{\text{sat}}}$$

$y_1 = 0.4840$ $x_1 = 0.2818$
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(f)  $z_1 = 0.33$ , find  $\mathcal{V}$

$$z_1 = \frac{\mathcal{L}x_1 + \mathcal{V}y_1}{\mathcal{L} + \mathcal{V}}$$

Basis:  $\mathcal{L} + \mathcal{V} = 1$  mol

$$z_1 = \mathcal{L}x_1 + (1 - \mathcal{L})y_1$$

$$\mathcal{L} = 0.7616$$

$V = 0.2384$
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(g) Benzene and toluene are two very similar compounds on top of being non-polar, minimizing chemical interactions.

## Chapter 11