$$\frac{\partial f}{\partial t} + \xi_\beta \frac{\partial f}{\partial x_\beta} + \frac{F_\beta}{\rho}\frac{\partial f}{\partial \xi_\beta} = \Omega(f)$$

$$f_i\left(\boldsymbol{x} + \boldsymbol{c}_i \Delta t, t + \Delta t\right) = f_i(\boldsymbol{x}, t) - \frac{\Delta t}{\tau}\left(f_i(\boldsymbol{x}, t) - f_i^{\mathrm{eq}}(\boldsymbol{x}, t)\right)$$

$$\Omega_i(f) = -\frac{f_i - f_i^{\mathrm{eq}}}{\tau}\Delta t$$

1. The first part is collision (or relaxation),

$$f_i^\star(\boldsymbol{x}, t) = f_i(\boldsymbol{x}, t) - \frac{\Delta t}{\tau}\left(f_i(\boldsymbol{x}, t) - f_i^{\mathrm{eq}}(\boldsymbol{x}, t)\right)$$

$$f_i^\star(\boldsymbol{x}, t) = f_i(\boldsymbol{x}, t)\left(1 - \frac{\Delta t}{\tau}\right) + f_i^{\mathrm{eq}}(\boldsymbol{x}, t)\frac{\Delta t}{\tau}$$

2. The second part is streaming (or propagation),

$$f_i\left(\boldsymbol{x} + \boldsymbol{c}_i \Delta t, t + \Delta t\right) = f_i^\star(\boldsymbol{x}, t)$$

$$f_i^{\mathrm{eq}}(\boldsymbol{x}, t) = w_i \rho\left(1 + \frac{\boldsymbol{u}\cdot\boldsymbol{c}_i}{c_{\mathrm{s}}^2} + \frac{(\boldsymbol{u}\cdot\boldsymbol{c}_i)^2}{2c_{\mathrm{s}}^4} - \frac{\boldsymbol{u}\cdot\boldsymbol{u}}{2c_{\mathrm{s}}^2}\right)$$

$$v = c_{\mathrm{s}}^2\left(\tau - \frac{\Delta t}{2}\right)$$

$$\rho(\boldsymbol{x}, t) = \sum_i f_i(\boldsymbol{x}, t), \quad \rho\boldsymbol{u}(\boldsymbol{x}, t) = \sum_i \boldsymbol{c}_i f_i(\boldsymbol{x}, t)$$