Class16 assignment

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3/5/2021

Problem 1

MA 677 Mar 4, 2021

Class 16 Assignment

1. machine → 4 feet / min, s.d = 5 inch = $\frac{5}{12}$ feet
independant. 1h : $P(Y \geq 250)$

Answer: Let X be ehe random variable that rope produces per minute.
Let Y be ehe total length that machine produces one hour.

So we have $X \sim N(4, (\frac{5}{12})^2)$ and

$$Y = 60X = \sum_{i=1}^{60} X_i \sim N(\sum_{i=1}^{60} \mu, \sum_{i=1}^{60} \sigma_i^2)$$

$Y \sim N(240, \frac{125}{12})$ we want $P(Y \geq 250)$

$$P(Y \geq 250) = 1 - P(Y < 250)$$

transform Y to a binomial distribu

$$Y \sim N(240, \frac{125}{12})$$

so $\frac{Y - 240}{\sqrt{\frac{125}{12}}} \sim N(0, 1)$

$$1 - P(Y < 250) \Rightarrow 1 - P\left(\frac{Y-240}{\sqrt{\frac{125}{12}}} < \frac{250-240}{\sqrt{\frac{125}{12}}}\right)$$

$$= 1 - \phi\left(\frac{10}{\sqrt{\frac{125}{12}}}\right)$$

$$= 1 - \phi\left(\frac{\sqrt{48}}{\sqrt{5}}\right)$$

$$= 1 - \phi(3.1)$$

$$= 0.001$$

Problem 2

Problem 2.

Let X be the number of defects on any bolt.

$X \sim Poi(\lambda = 5)$ approximate X using the normal distribution $X \sim N(\mu = 5, \sigma^2 = \frac{5}{125})$

$X \sim N(5, (\frac{1}{5})^2)$

So we want $P(X < 5.5) = P(\frac{X-5}{\frac{1}{5}} < \frac{5.5-5}{\frac{1}{5}})$

$= P(\frac{X-5}{\frac{1}{5}} < 2.5) = \Phi(2.5)$

≈ 0.99

Problem 3-4

Problem 3.

Let X be the number of defects in a given manufactured items, let n be the number of items.

$X \sim Bin(n=1, p=0.1, q=0.9)$

So $\mu = n \cdot p = 0.1$ $Var(X) = npq = 0.09$

So $\sigma^2 = 0.09$ $\sigma = 0.3$

We know $P(X < 0.13) = P\left(\frac{X-0.1}{\frac{0.3}{\sqrt{n}}} < \frac{0.13-0.1}{\frac{0.3}{\sqrt{n}}}\right) \geq 0.99$

$P\left(\frac{X-0.1}{0.3} \cdot \sqrt{n} < \frac{0.03}{0.3} \cdot \sqrt{n}\right) \geq 0.99$

$\phi(0.1 \cdot \sqrt{n}) \geq 0.99$ So $0.1\sqrt{n} \geq 2.33$

$n \geq 542.89$ n is an integer $\underline{n = 543}$

Problem 4.

Let X be the ~~digit~~ digits

$X \sim$ unif. from set $\{0, 1, \cdots, 9\}$

$\mu = (0 + 1 + \cdots + 9)/10 = 4.5$ $n = 16$

$\sigma^2 = Var(X) = \frac{n^2-1}{12} = \frac{33}{4} = 8.25$

$\sigma = 2.87$ $X \sim N(4.5, \frac{8.25}{16})$

So $P(4 \leq X \leq 6) = Pr\left(\frac{4-4.5}{\sqrt{\frac{33}{8}}} < \frac{X-4.5}{\sqrt{\frac{33}{64}}} < \frac{6-4.5}{\sqrt{\frac{33}{8}}}\right)$

S

$= \phi\left(\frac{4\sqrt{33}}{11}\right) - \phi\left(\frac{4\sqrt{33}}{33}\right)$

$= \phi(2.09) - \phi(-0.7)$

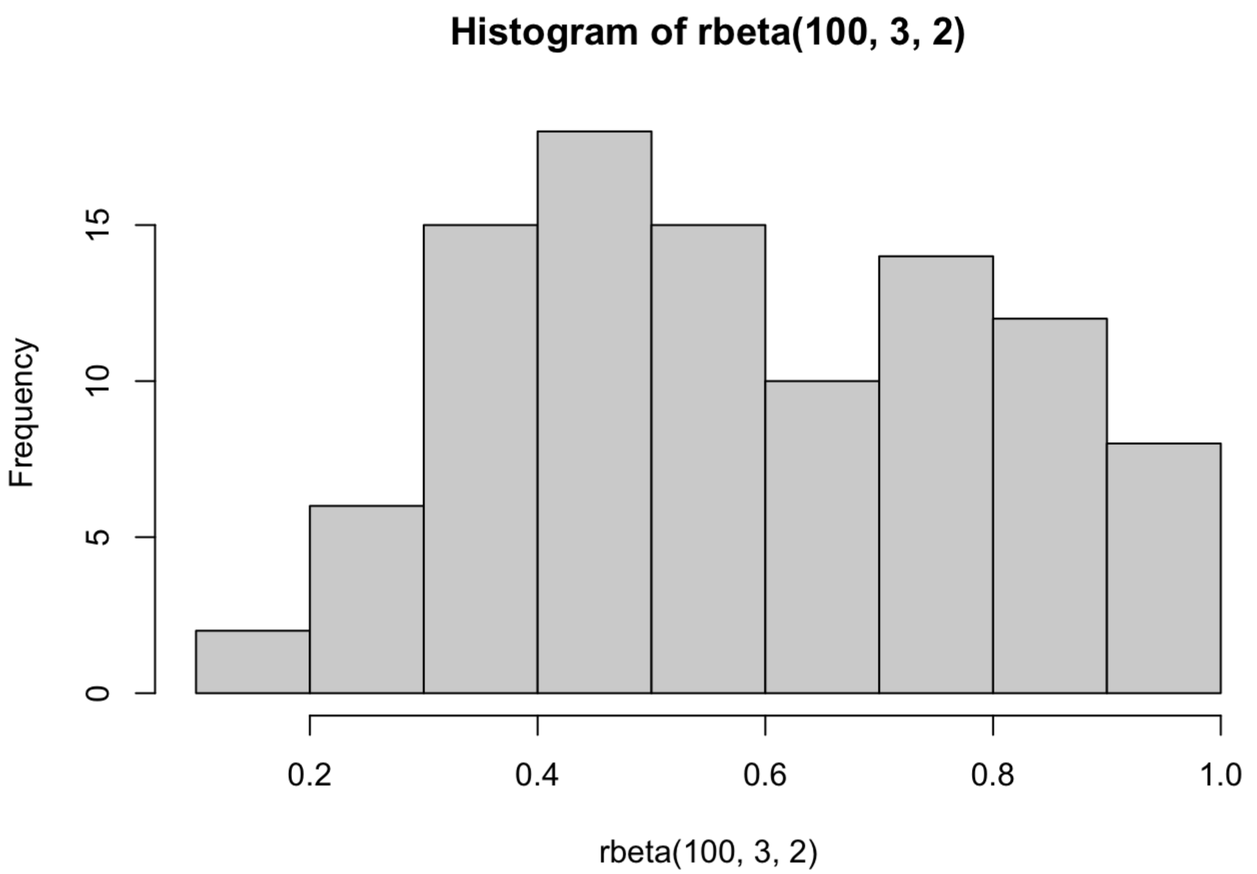
$= 0.982 - 0.243$

$= \boxed{0.739}$

Problem 5

I choose skewed Beta distribution Beta(alpha=3,beta=2) from which to sample. The mean of the skewed Beta distribution is E(X)=3/5 and the variance is Var(X)=1/25

```
#skewed Beta distribution
hist(rbeta(100,3,2))
```



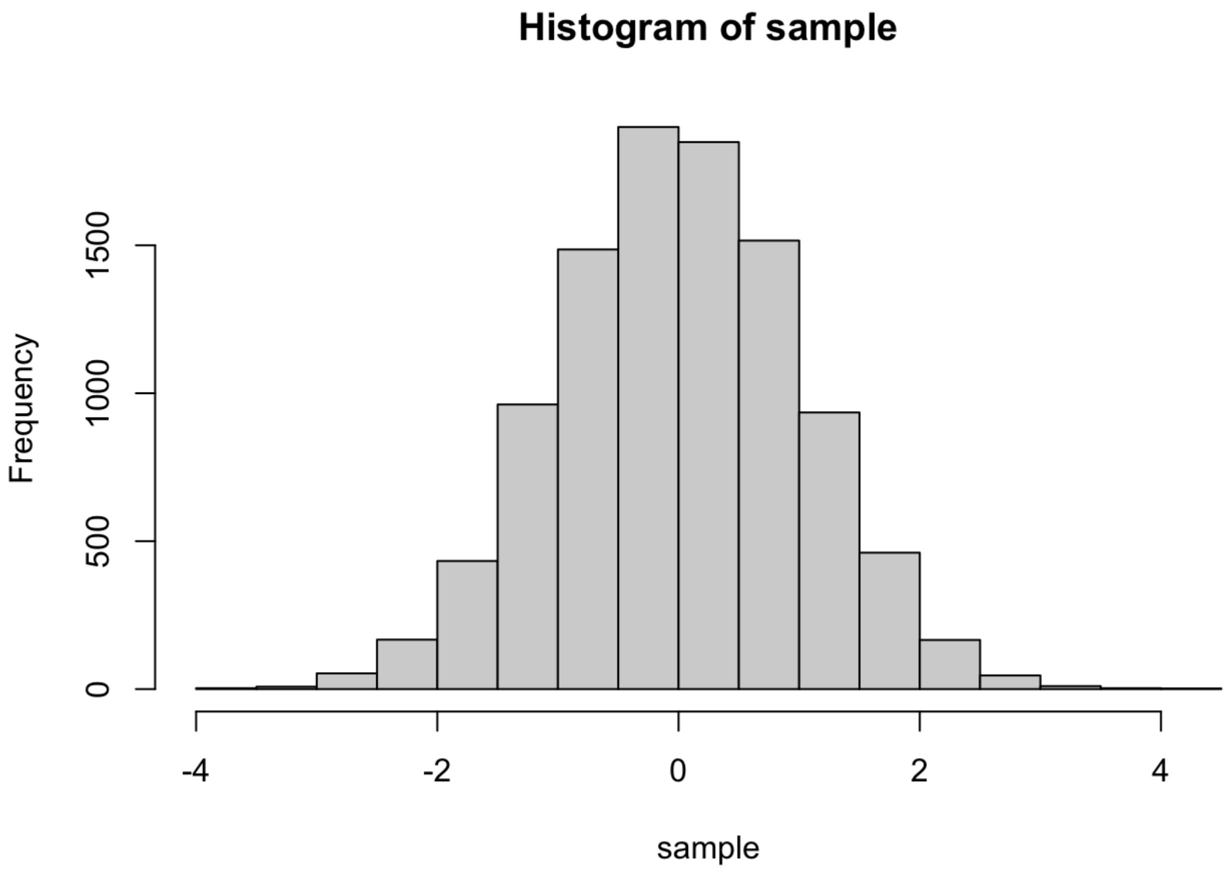

```
#Mean & Variance of Beta distribution
a=3
b=2
E=a/(a+b)
Var=(a*b)/((a+b)^2*(a+b+1))

#Make 10,1000 draws with 10,000 samples from Beta(3,2) each draw
sample=c()
k=10000
for(i in 1:10000){
  data2=rbeta(k,3,2)
  sample[i]=(sum(data2)-k*E)/(sqrt(k*Var))
}

#Mean & Variance of samples
mean(sample)
```

```
## [1] 0.002016343
```

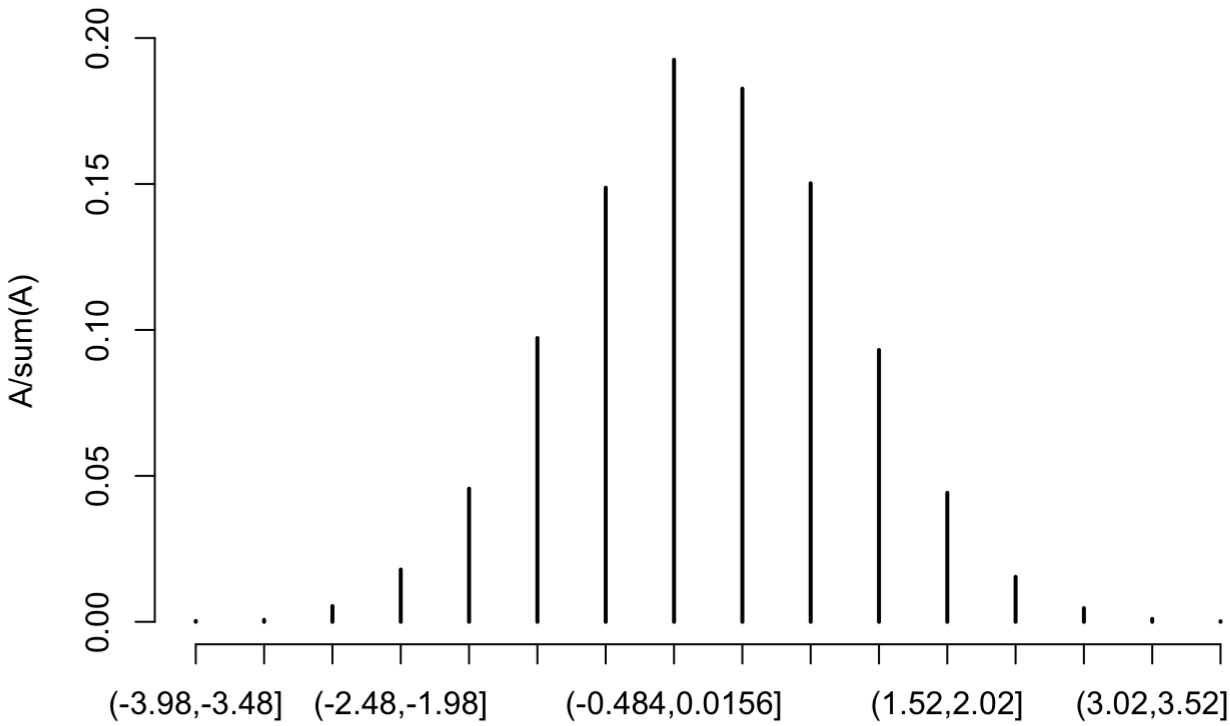
```
sd(sample)
```

```
## [1] 1.008105
```

```
#Graphically display of the sample distribution
hist(sample)
```




```
#Cut the sample into intervals and plot the probability histogram
s=seq(min(sample),max(sample),0.5)
A=table(cut(sample,br=s))
plot(A/sum(A))
```




```
#Use Pearson Chi test to validate whether the sample is standard Normal distributed.
q=pnorm(s,0,1);
n=length(q)
p=numeric(n-1)
p[1]=q[2]
p[n-1]=1-q[n-1]
for(i in 2:(n-2)){
  p[i]=q[i+1]-q[i]
}
chisq.test(A,p)
```

```
##
##  Pearson's Chi-squared test
##
## data:  A and p
## X-squared = 240, df = 225, p-value = 0.2348
```

```
#p value is larger than 0.05, we cannot reject the null hypothesis that the sample is
 a standard Normal distribution.
```