第八章 多采样率数字信号处理

Multirate Digital Signal Processing

8.1 信号的整数倍抽取

8.2 信号的整数倍内插

8.3 信号的任意有理数倍采样频变换及应用

第八章 多采样率数字信号处理

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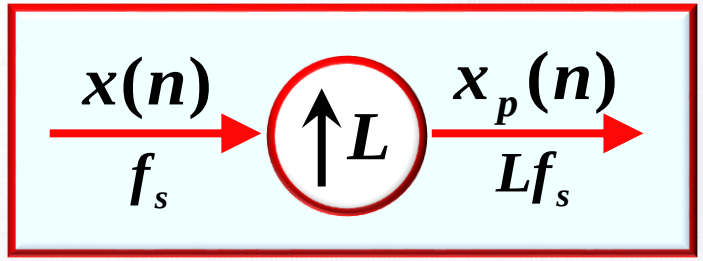
8.2 信号的整数倍内插

华东理工大学信息科学与工程学院 万永菁

一、信号的整数倍内插过程



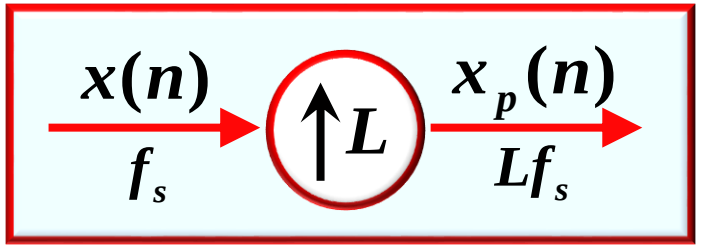
$$x_p(n)=\sum_{k=-\infty}^{\infty}x(k)\delta(n-KL)=\begin{cases}x(n/L) & n=0,\pm L,\pm 2L.....\\ 0 & \text{其他}n\end{cases}$$

$$X_p(e^{j\omega})=\sum_{n=-\infty}^{\infty}x_p(n)e^{-j\omega n}=\sum_{n=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty}x_p(k)\delta(n-kL)\right]e^{-j\omega n}$$

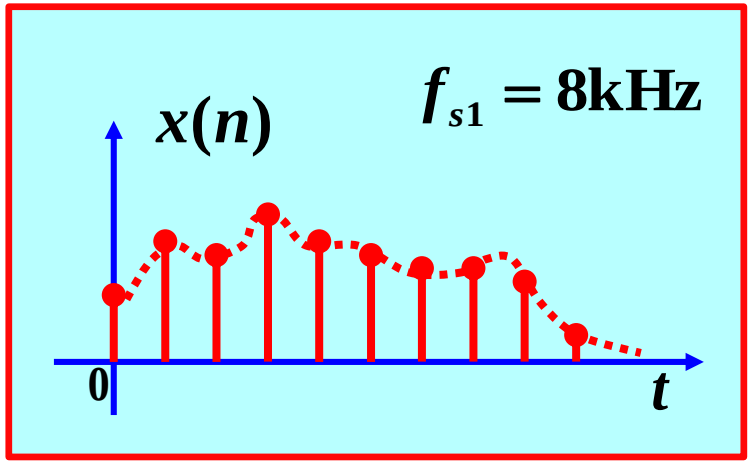
$$=\sum_{k=-\infty}^{\infty}x_p(k)\sum_{n=-\infty}^{\infty}\delta(n-kL)e^{-j\omega n}=\sum_{k=-\infty}^{\infty}x_p(k)e^{-j\omega kL}=X(e^{j\omega L})$$

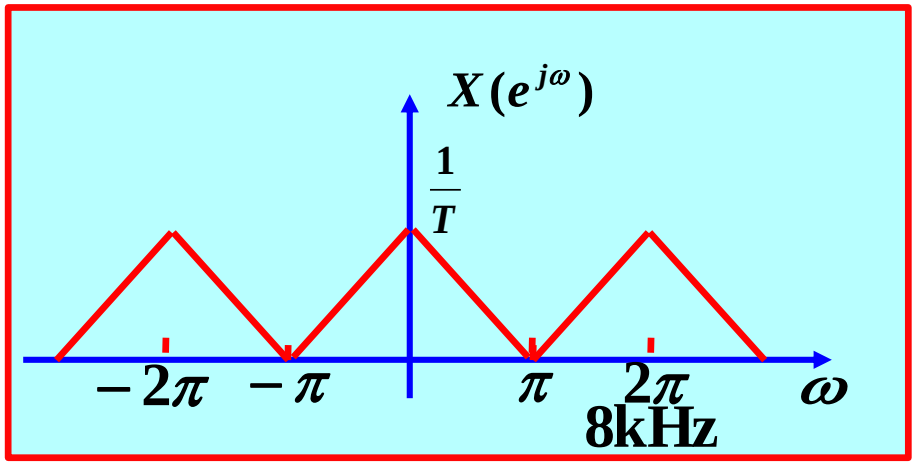
$n=kL$

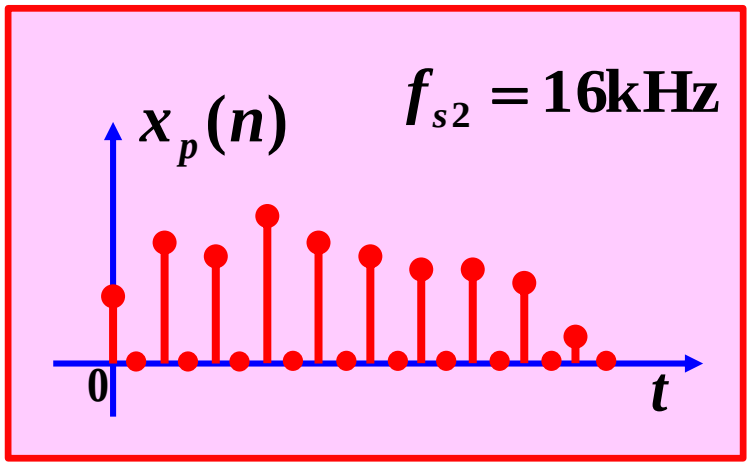
➢ 内插过程的理解(L=2)：

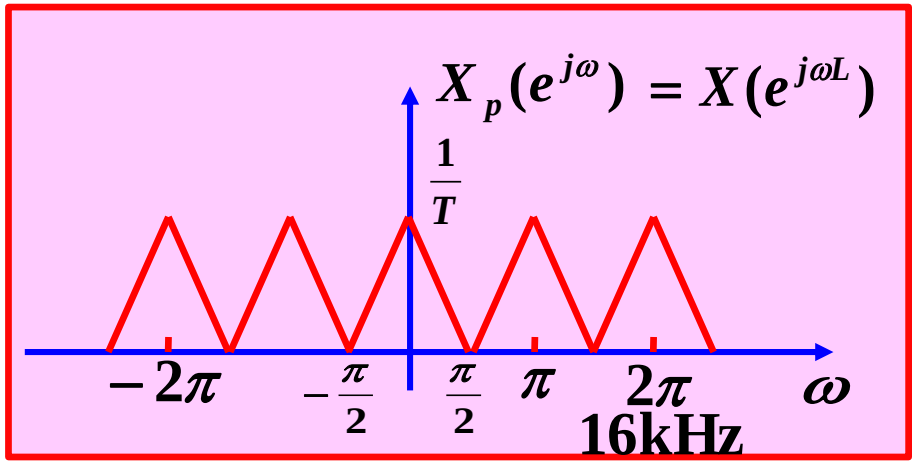


$X(e^{j\omega})$沿频率轴压缩L倍后得到$X_p(e^{j\omega})$，即周期2π缩为$2\pi/L$，出现了额外的L-1个“影像频谱”

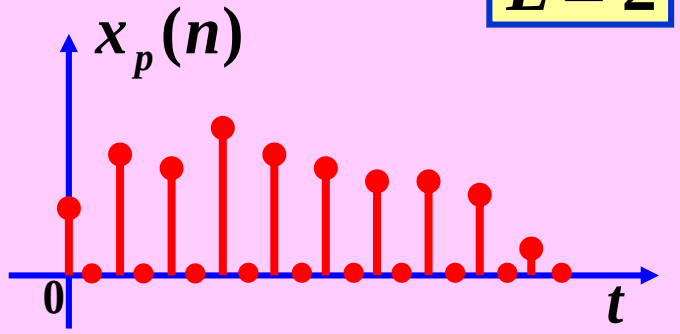




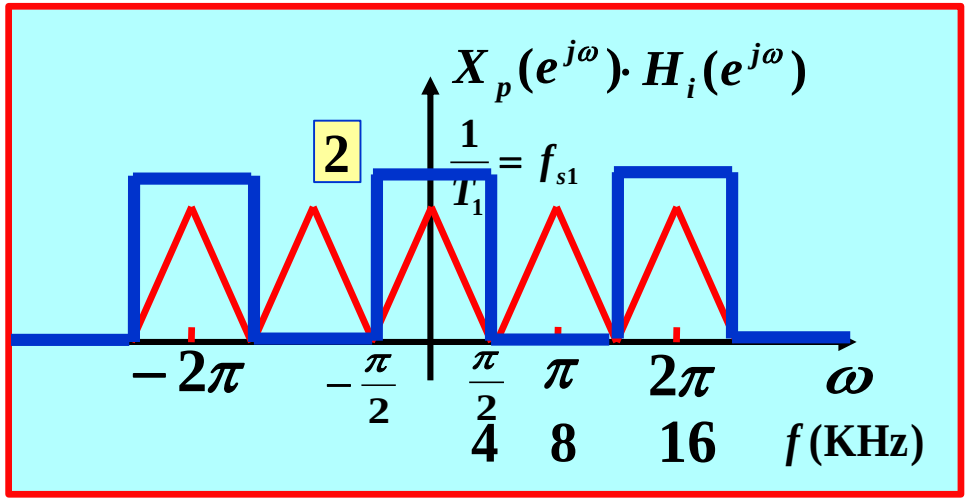




$L=2$

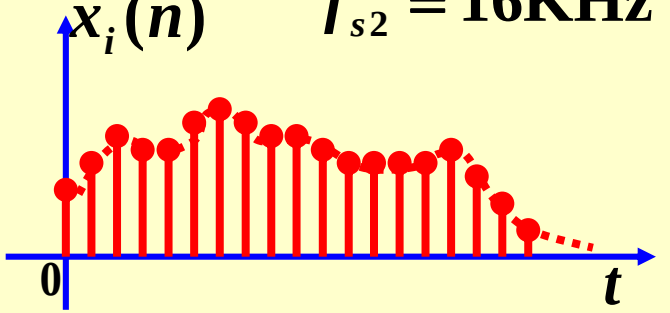


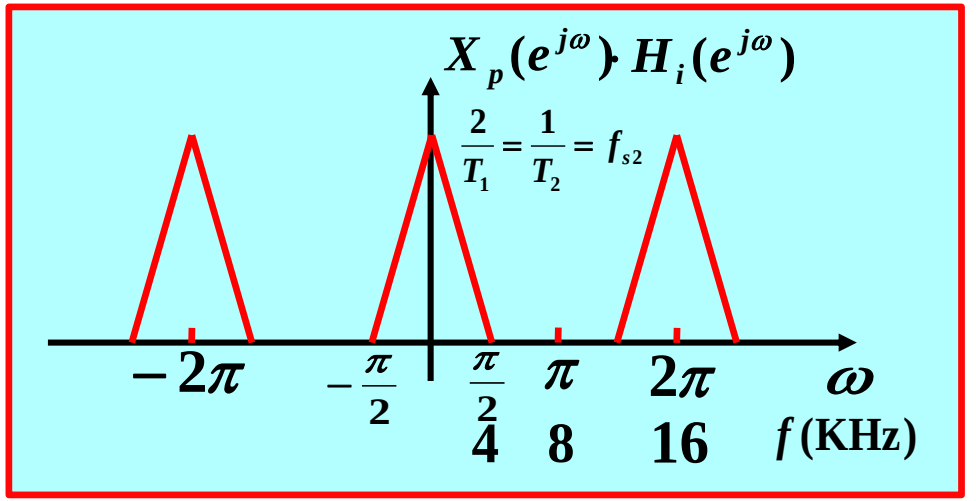
$f_{s1}=8\text{KHz}$ $f_{s2}=16\text{KHz}$



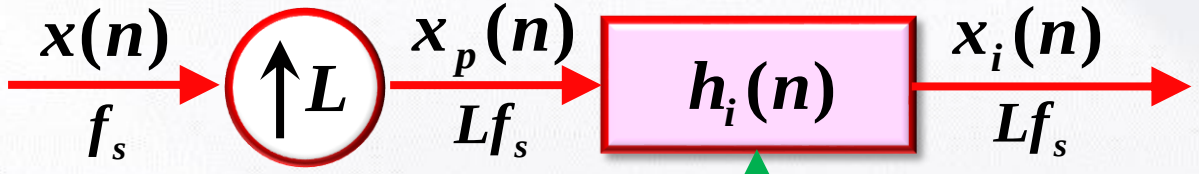
$$h_{\text{抗影像}}(n)=2\cdot\frac{\sin(0.5\pi n)}{\pi n}$$

$$H_{\text{抗影像}}(e^{j\omega})=\begin{cases}2, & |\omega|\le 0.5\pi\\ 0, & 0.5\pi<|\omega|\le\pi\end{cases}$$





二、内插器与内插滤波器(抗影像滤波器)级联



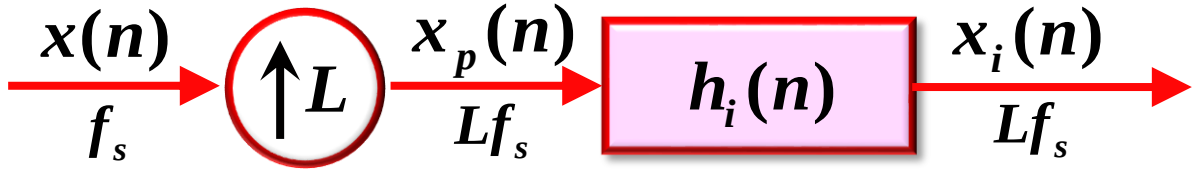
内插滤波器(抗影像滤波器)，起到消除影像频谱的作用

$$H_i(e^{j\omega})=\begin{cases}L & 0\le|\omega|<\pi/L\\ 0 & \pi/L\le|\omega|\le\pi\end{cases}$$

采样率增加L倍
频谱幅度增加L倍

IDTFT

$$h_i(n)=L\cdot\frac{\sin(\pi n/L)}{\pi n}=\frac{\sin(\pi n/L)}{\pi n/L}$$



$$x_i(n)=\sum_{l=-\infty}^{\infty}x_p(l)h_i(n-l)$$

$$h_i(n)=\frac{\sin(\pi n/L)}{\pi n/L}$$

$$=\sum_{l=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty}x(k)\delta(l-kL)\right]h_i(n-l)$$

$$=\sum_{k=-\infty}^{\infty}x(k)\sum_{l=-\infty}^{\infty}\delta(l-kL)h_i(n-l)$$

由$x(n)$求$x_i(n)$的内插公式

$$=\sum_{k=-\infty}^{\infty}x(k)h_i(n-kL)=\sum_{k=-\infty}^{\infty}x(k)\frac{\sin[\pi(n-kL)/L]}{\pi(n-kL)/L}$$

```
x=[1 0 2 0 3 0 4 0 4 0 3 0 2 0 1];
nx=0:length(x)-1;

nh=-10:10;
h = sinc(0.5*n);

subplot(211);stem(nh,h);grid on;

subplot(212)
stem(nx,x,'r','linewidth',2);
axis([-2 16 -1 5]);hold on;

y=conv(x,h);
ny=[nx(1)+nh(1):nx(end)+nh(end)];

stem(ny,y)
axis([-2 16 -1 5]);grid on;
```

$x(n)=\{\underline{1},0,2,0,3,0,4,0,4,0,3,0,2,0,1\}$

$h_{抗影像}(n)=2\cdot\dfrac{\sin(0.5\pi n)}{\pi n}$

MATLAB

$\mathrm{sinc}(t)=\dfrac{\sin(\pi t)}{\pi t}$

画$h_{抗影像}(n)$

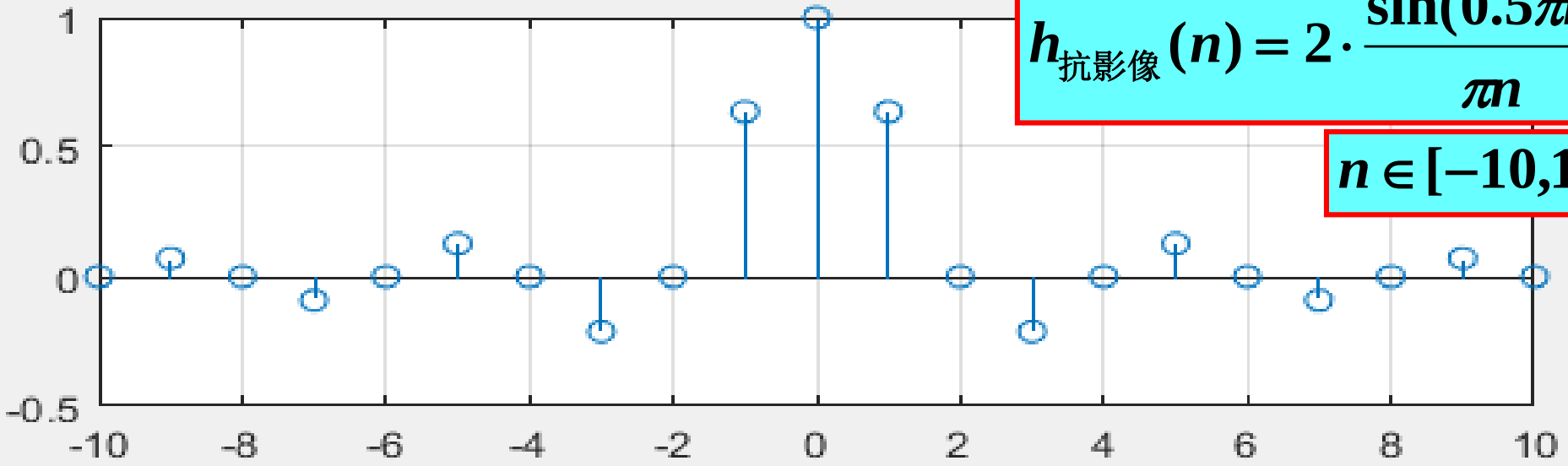
画 $x(n)$

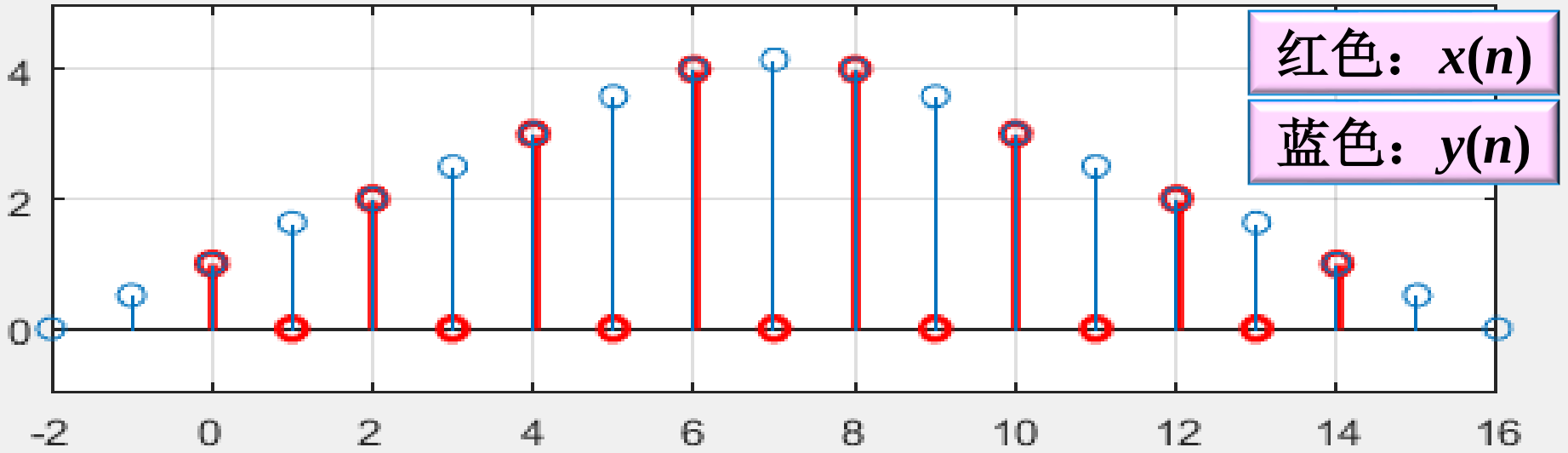
求 $y(n)=x(n)*h_{抗影像}(n)$

画 $y(n)$

$$h_{\text{抗影像}}(n) = 2 \cdot \frac{\sin(0.5\pi n)}{\pi n}$$

$n \in [-10,10]$





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