Lecture 5: CNNs I - Architecture & Equivariance

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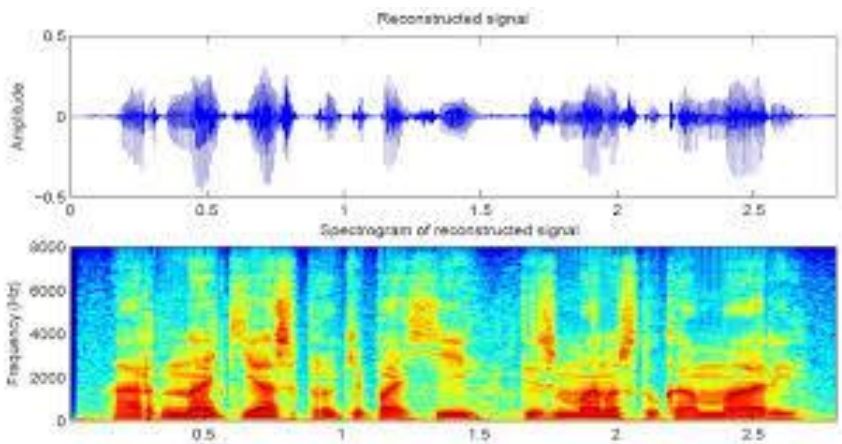
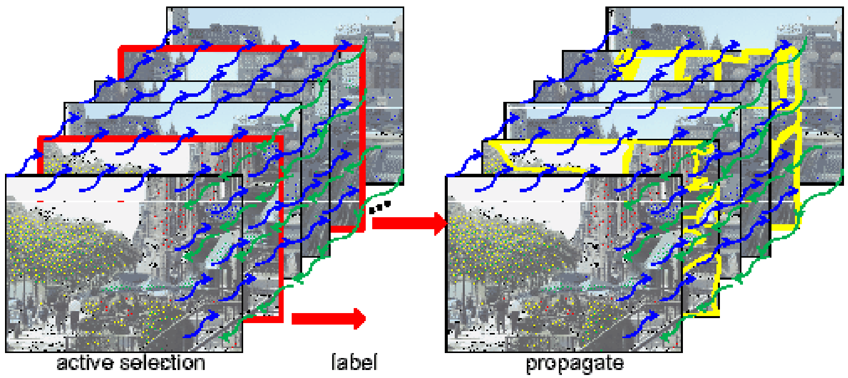
Logistics

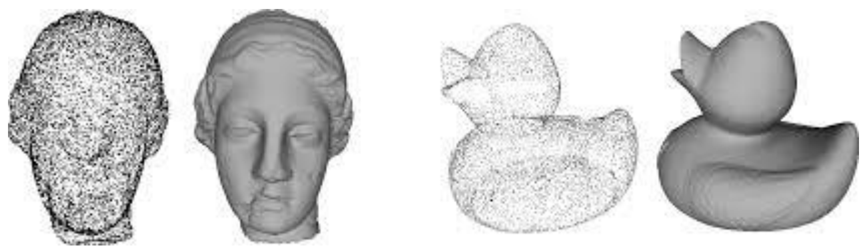
- Quiz 2 on Thursday
  - Every Tuesday afterwards
  - Scope: Lectures from the previous week
- A tentative course schedule on Piazza
- My office hour: 7pm-9pm Monday
- Tutorials: Friday evening

Overview

- In general, our goal is to learn a mapping from a signal to a ‘semantically meaningful’ representation.
  - Input signal has certain structures





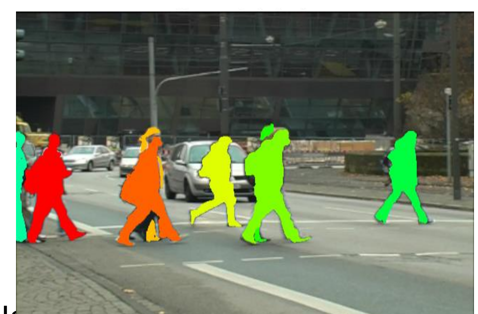


Overview

- In general, our goal is to learn a mapping from a signal to a 'semantically meaningful' representation.
  - Output can have many different forms:

red panda (*Ailurus fulgens*)

3	3	3	3	3	3	3	3	3	3	3	3	5	5	5	5	5	5
3	3	3	3	3	3	3	3	3	3	3	3	5	5	5	5	5	5
3	3	3	3	3	3	1	1	3	3	3	3	5	5	5	5	5	5
3	3	3	3	3	1	1	1	1	3	3	3	5	5	5	5	5	5
3	3	3	3	3	3	1	1	3	3	3	5	5	5	5	5	5	5
5	5	3	3	3	3	1	1	3	3	5	5	5	5	5	5	5	5
4	4	3	4	1	1	1	1	1	1	4	4	4	5	5	5	5	5
4	4	3	4	1	1	1	1	1	1	4	4	4	4	4	5	5	5
4	4	4	1	1	1	1	1	1	1	1	4	4	4	4	4	4	4
3	3	3	1	1	1	1	1	1	1	1	4	4	4	4	4	4	4
3	3	3	1	2	2	1	1	1	1	1	4	4	4	4	4	4	4
3	3	3	1	2	2	1	1	1	1	1	4	4	4	4	4	4	4



Overview

- In general, our goal is to learn a mapping from a signal to a 'semantically meaningful' representation.
  - Input signal has certain structures
  - Output can have many different forms
- Questions:
  - What kind of representations should we use?
    - Network structure design
  - How do we learn such representations from data?
    - Loss functions; Gradient descent algorithms
  - How good are those learned models?
    - Generalization; Robustness; Interpretability

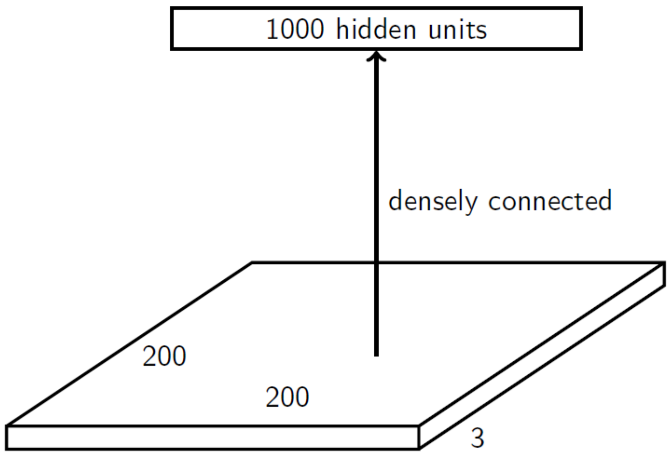
Outline

- Why Convolutional Neural Network (CNN)?
  - Motivation and overview
- What is the CNN?
  - Convolution and feature extraction
  - Convolution layers & model complexity
  - Closer look at activation functions
  - Pooling layers & model complexity
- Examples of CNNs

Acknowledgement: Roger Grosse@UofT & Feifei Li's cs231n notes

Motivation

- Visual recognition
  - Suppose we aim to train a network that takes a 200x200 RGB image as input



  - What is the problem with have full connections in the first layer?
    - Too many parameters! 200x200x3x1000 = 120 million
    - What happens if the object in the image shifts a little?

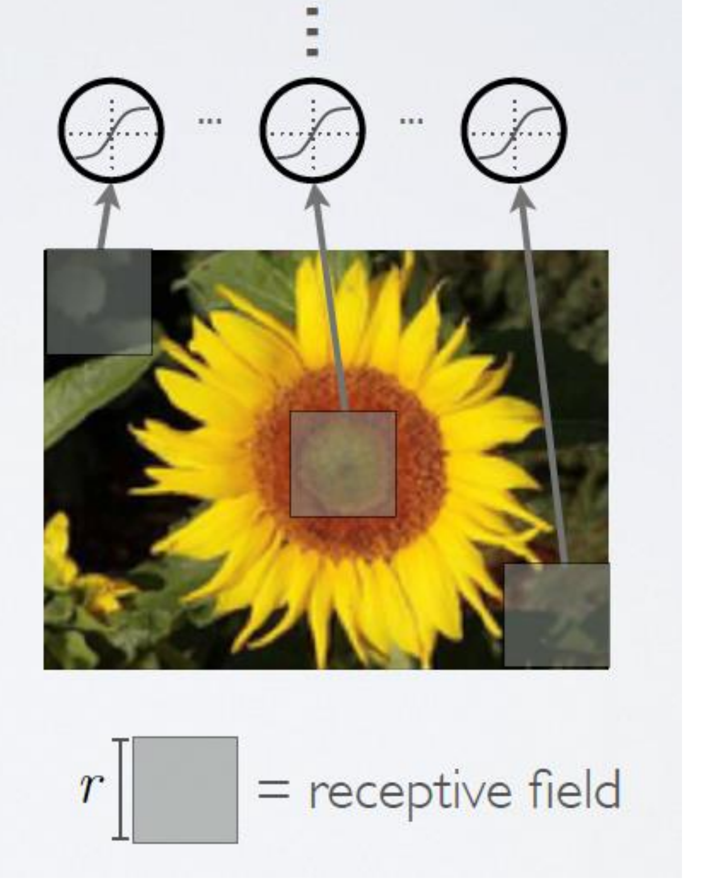
Our goal

- Visual Recognition: Design a neural network that
  - Much deal with very high-dimensional inputs
  - Can exploit the 2D topology of pixels in images
  - Can build in invariance/equivariance to certain variations we can expect
    - Translation, small deformations, illumination, etc.

- Convolution networks leverage these ideas
  - Local connectivity
  - Parameter sharing
  - Pooling/subsampling hidden units

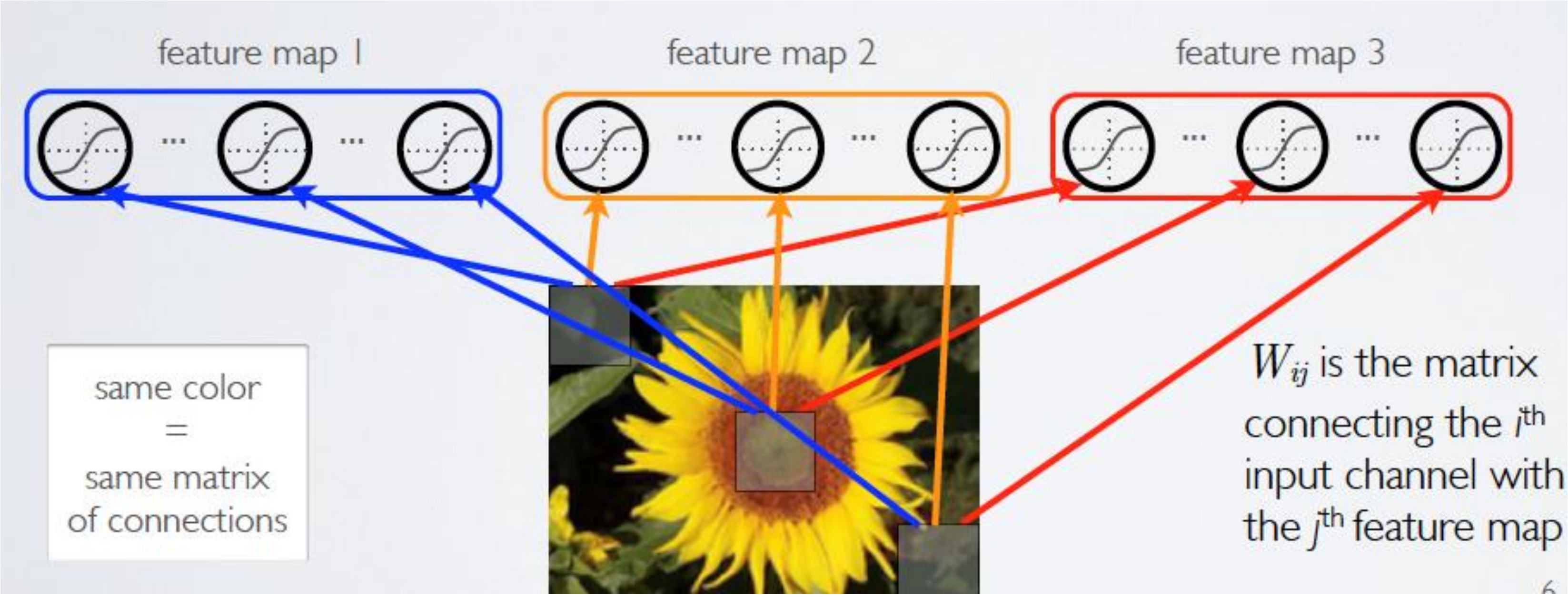
Overview of CNNs

- First idea: Use a local connectivity of hidden units
  - Each hidden unit is connected only to a subregion (patch) of the input image
  - Usually it is connected to all channels
  - Each neuron has a local receptive field



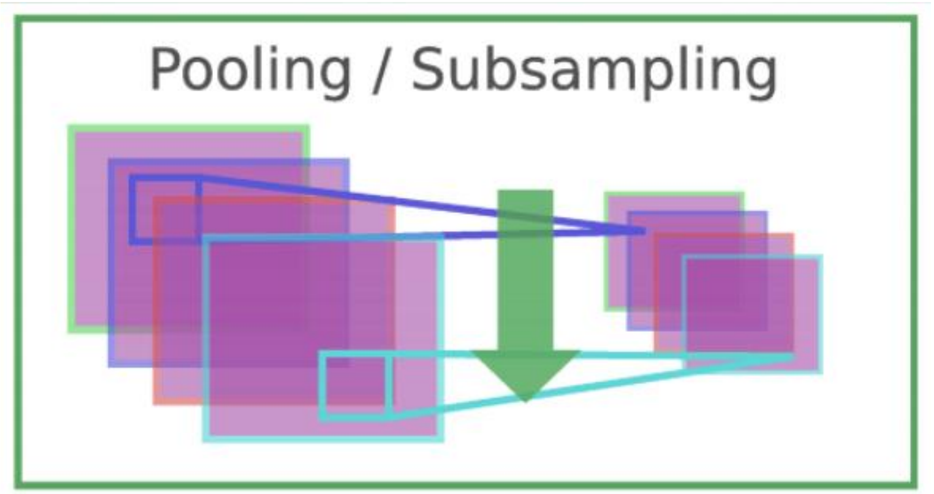
Overview of CNNs

- Second idea: share weights across certain units
  - Units organized into the same “feature map” share weight parameters
  - Hidden units within a feature map cover different positions in the image



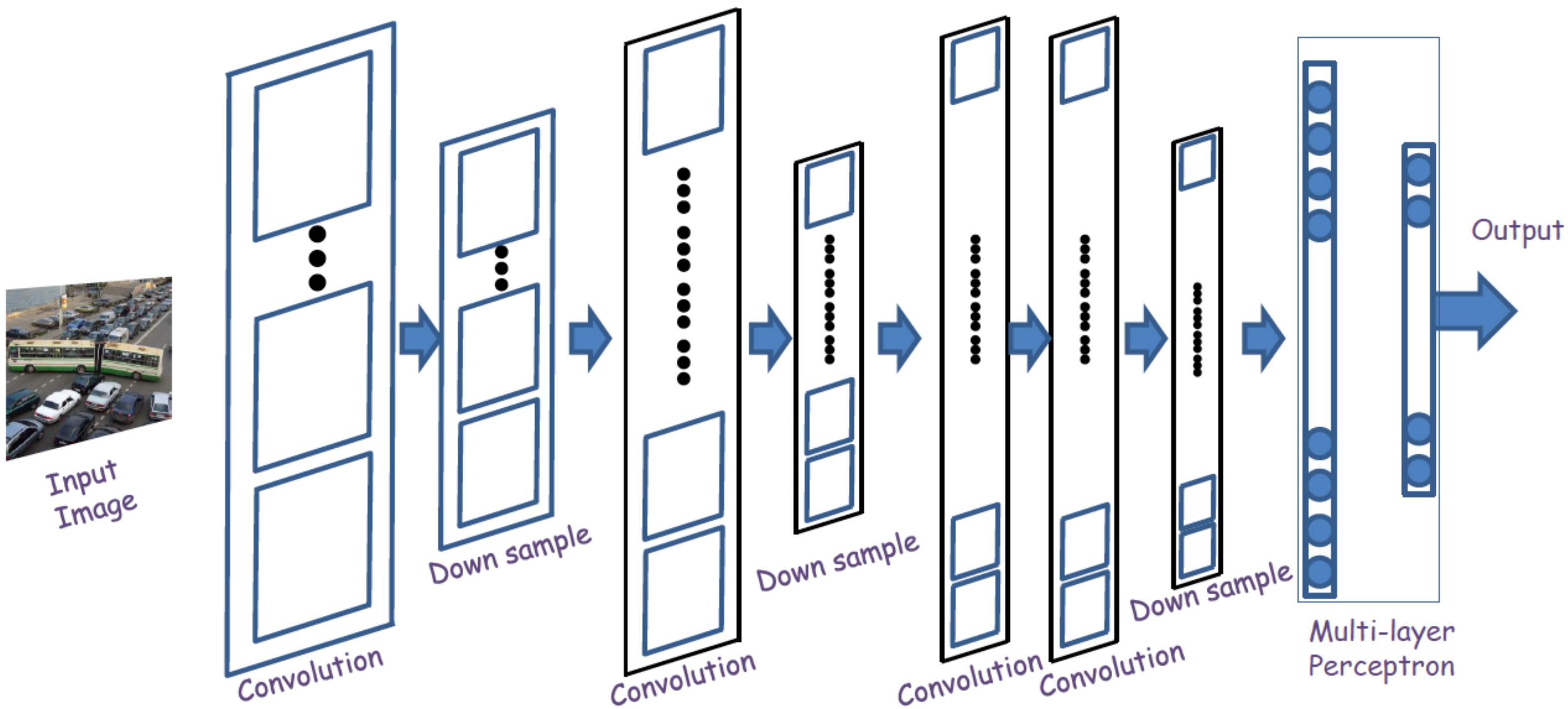
Overview of CNNs

- Third idea: pool hidden units in the same neighborhood
  - Averaging or Discarding location information in a small region
  - Robust toward small deformations in object shapes by ignoring details.



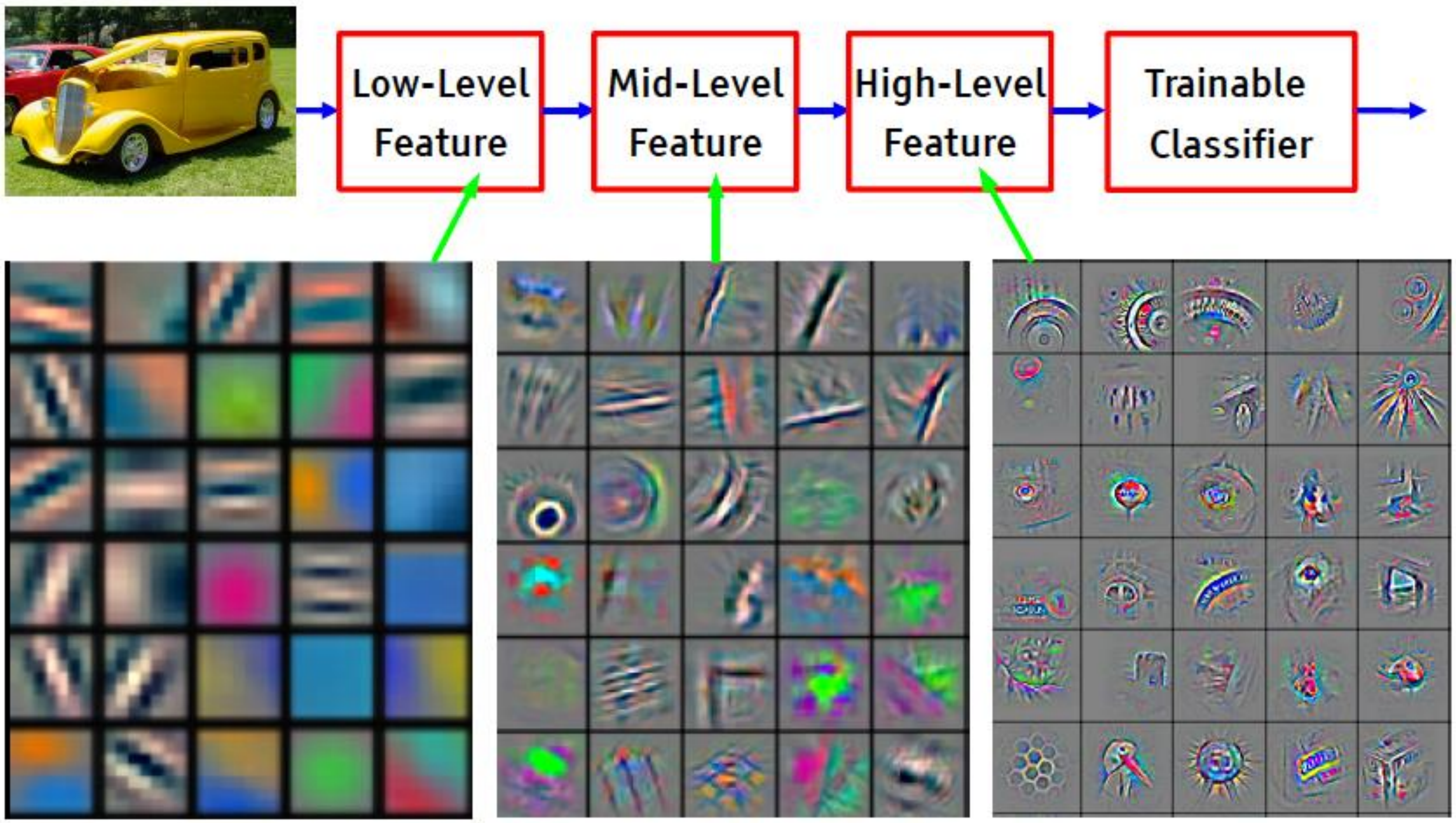
Overview of CNNs

- Fourth idea: Interleaving feature extraction and pooling operations
  - Extracting abstract, compositional features for representing semantic object classes



Overview of CNNs

- Artificial visual pathway: from images to semantic concepts (Representation learning)



Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

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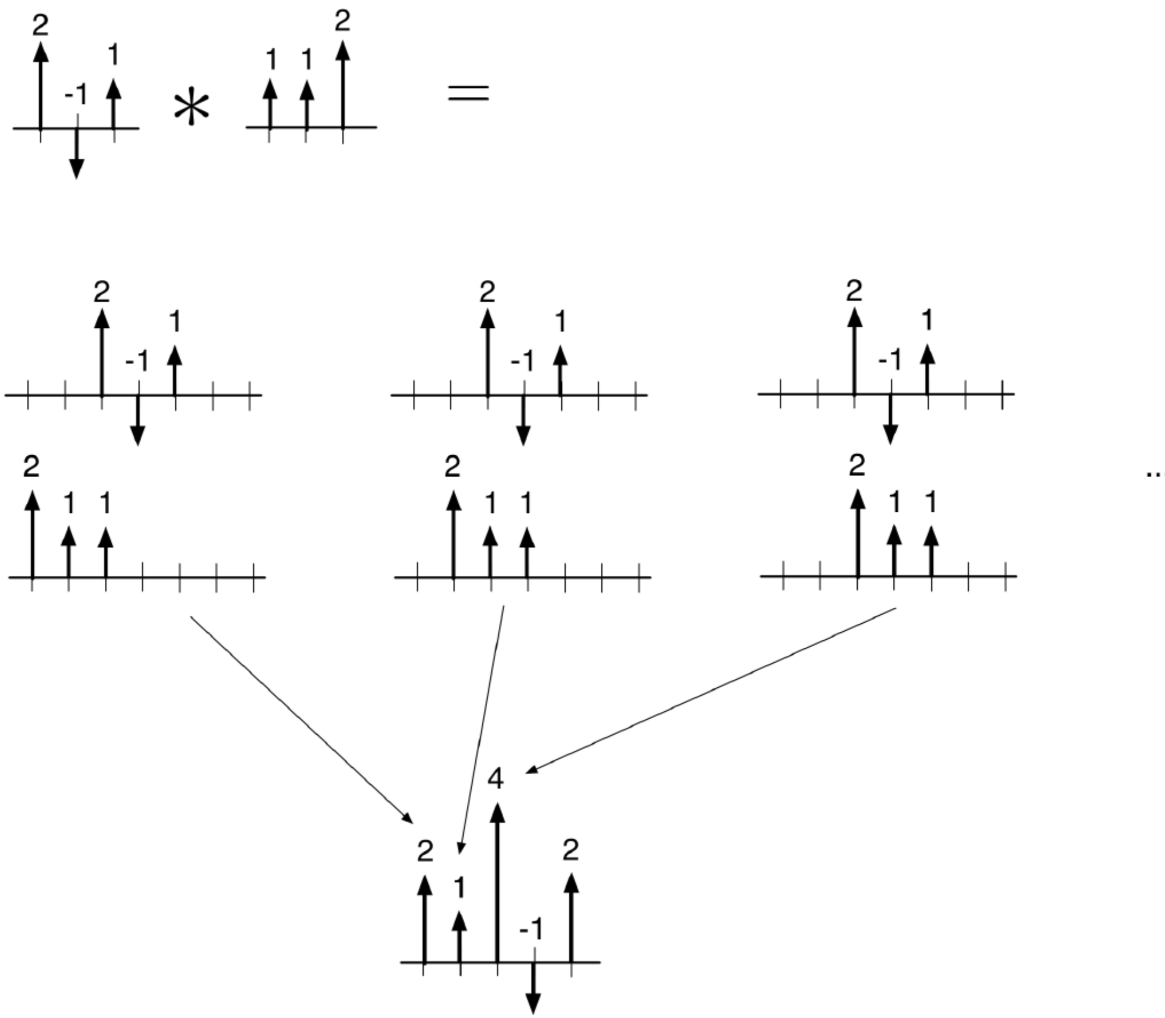
Convolution

- 1-D case:

$$\mathbf{c} = \mathbf{a} * \mathbf{b}$$

$$\mathbf{c}_t = \sum_{\tau} \mathbf{a}_{\tau} \mathbf{b}_{t-\tau}$$



Convolution

- Convolution can also be viewed as matrix multiplication

$$\mathbf{c} = \mathbf{a} * \mathbf{b} \qquad\qquad \mathbf{c}_t = \sum_\tau \mathbf{a}_\tau \mathbf{b}_{t-\tau}$$

$$(2,-1,1)*(1,1,2) = \begin{pmatrix} 1 & & \\ 1 & 1 & \\ 2 & 1 & 1 \\ & 2 & 1 \\ & & 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

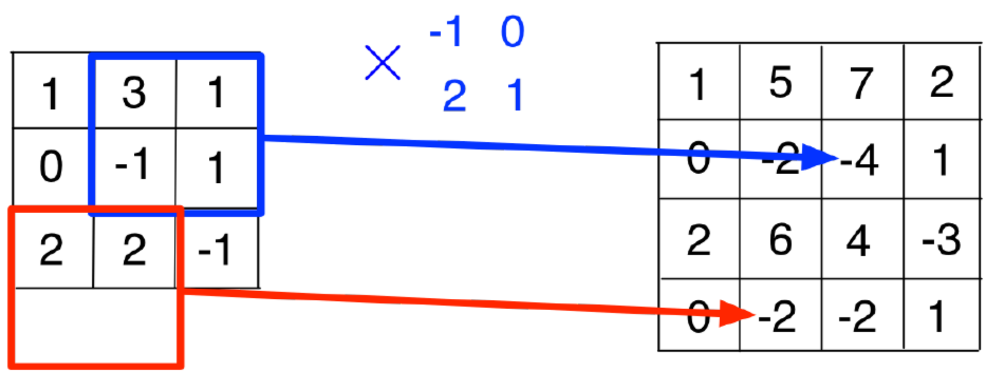
- Commutative $\quad \mathbf{a} * \mathbf{b} = \mathbf{b} * \mathbf{a}$
- Linear $\quad \mathbf{a} * (\lambda_1 \mathbf{x} + \lambda_2 \mathbf{y}) = \lambda_1 \mathbf{a} * \mathbf{x} + \lambda_2 \mathbf{a} * \mathbf{y}$
- Efficient implementation
  - FFT on CPU
  - More efficient on GPU

2D Convolution

If A and B are two 2-D arrays, then:

$$(A * B)_{ij} = \sum_s \sum_t A_{st} B_{i-s,j-t}.$$

$$\begin{array}{|c|c|c|}\hline 1 & 3 & 1 \\ \hline 0 & -1 & 1 \\ \hline 2 & 2 & -1 \\ \hline \end{array} \ast \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 0 & -1 \\ \hline \end{array}$$



2D Convolution

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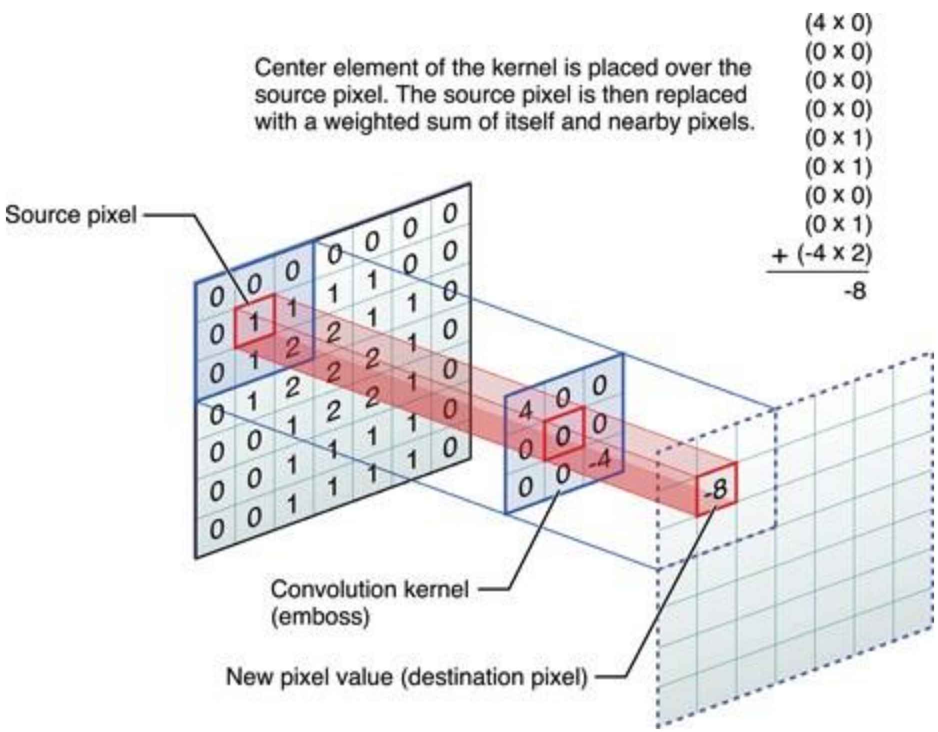

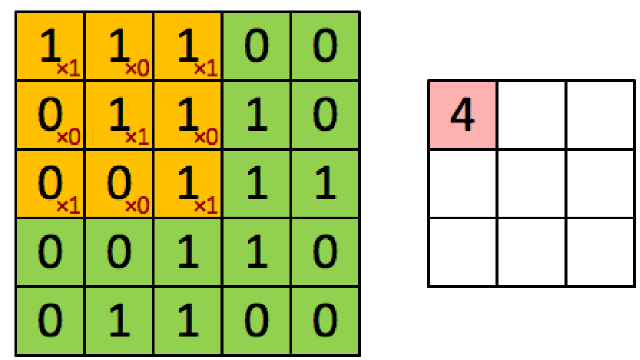

Image

Convolved Feature

Picture Courtesy: developer.apple.com

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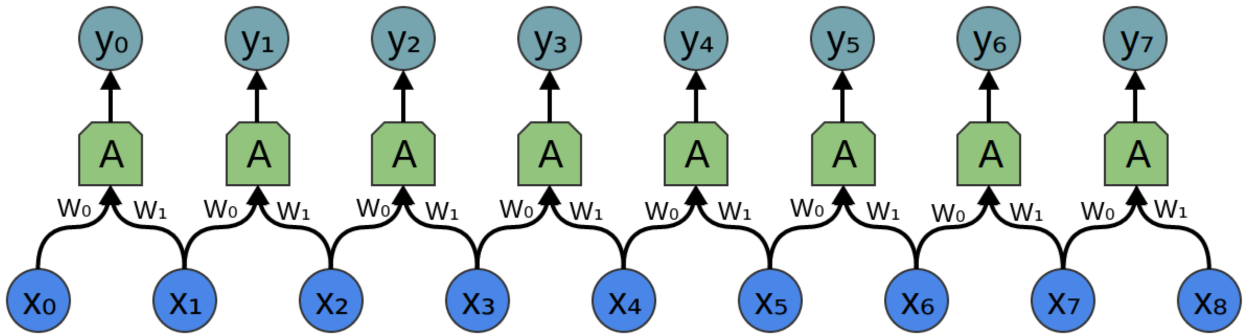
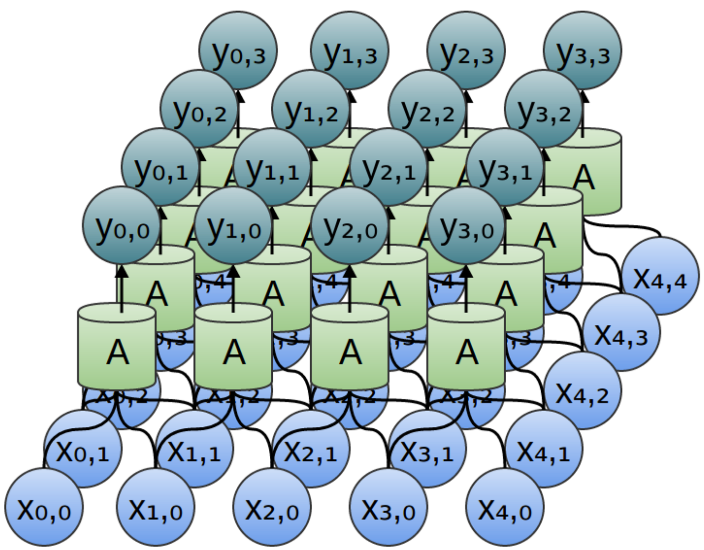
Convolution Layers

- 1D example

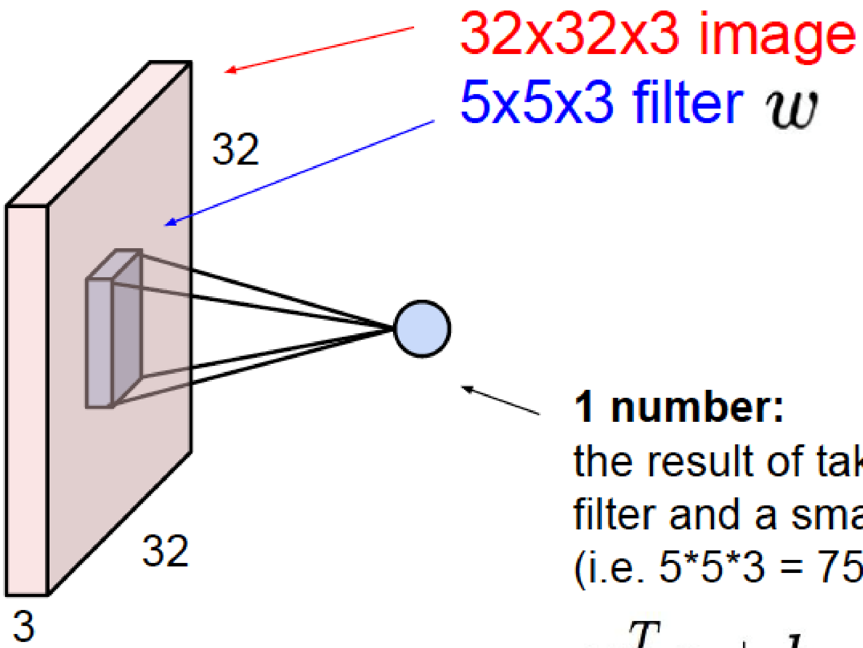


- 2D example



Convolution Layers

- Formal definition



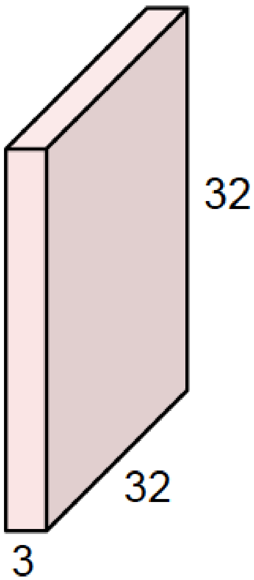
1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

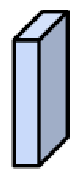
Convolution Layers

- Define a neuron corresponding to a 5x5 filter

32x32x3 image



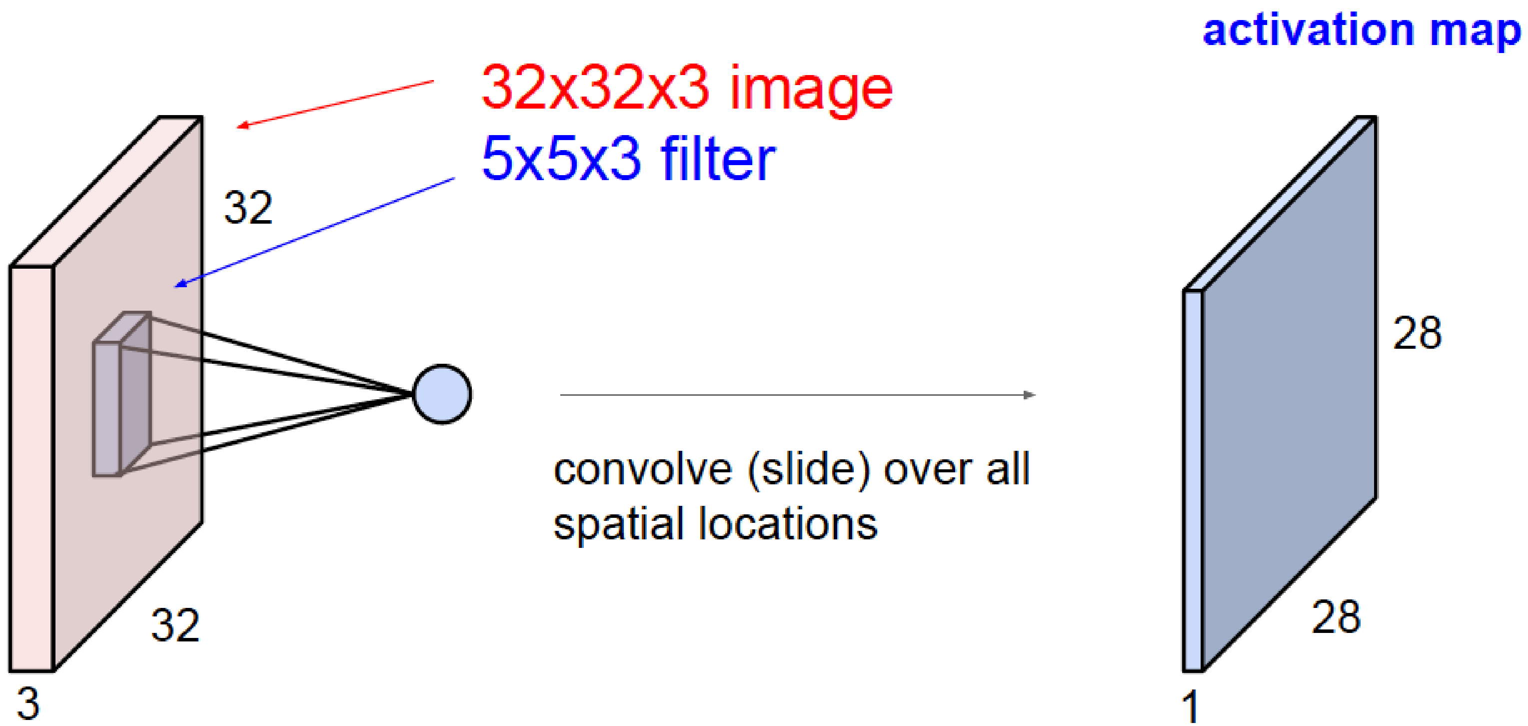
5x5x3 filter



Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

Convolution Layers

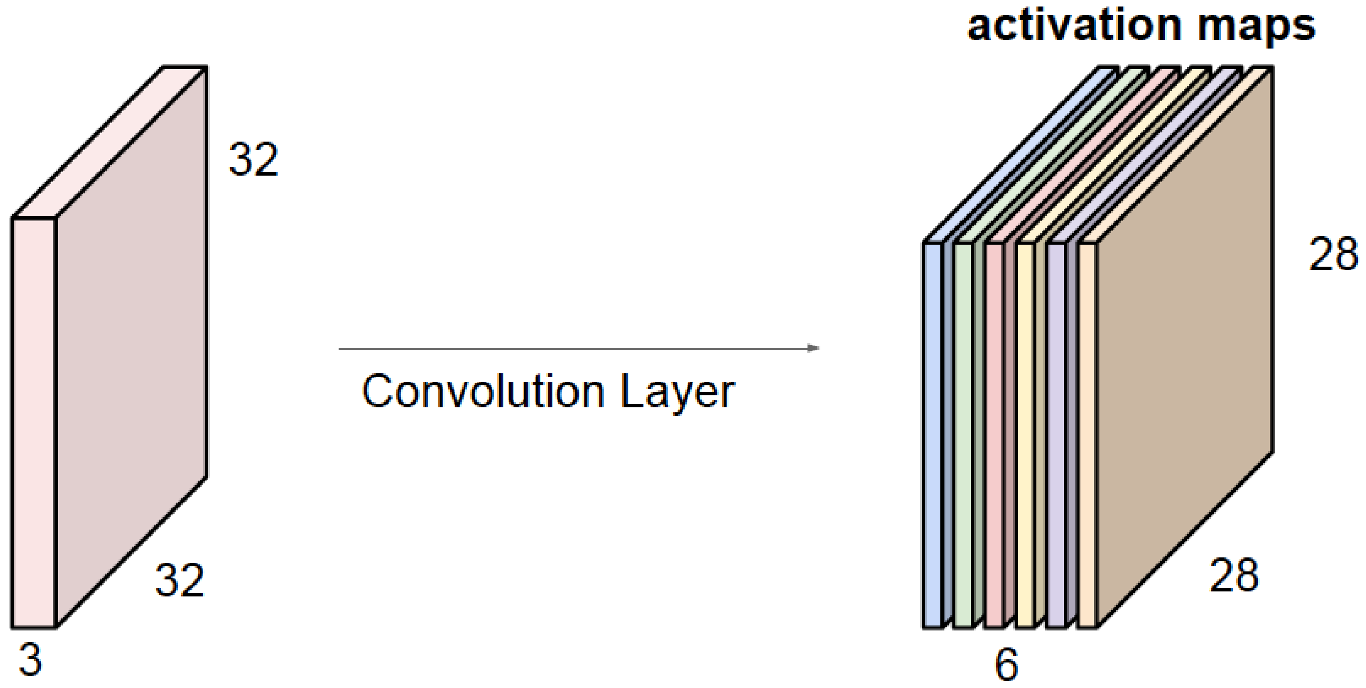
- Convolution operation
  - Parameter sharing
  - Spatial information



Convolution Layers

- Multiple kernels/filters

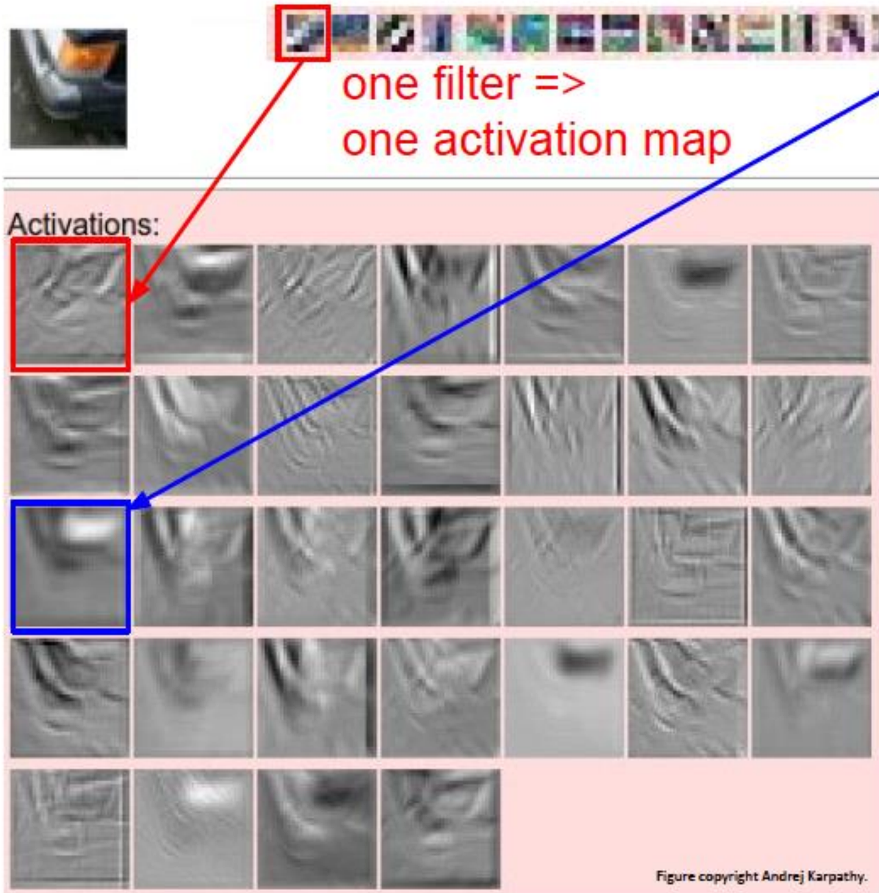
For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

Convolution Layers

- Visualizing the filters and their outputs



example 5x5 filters
(32 total)

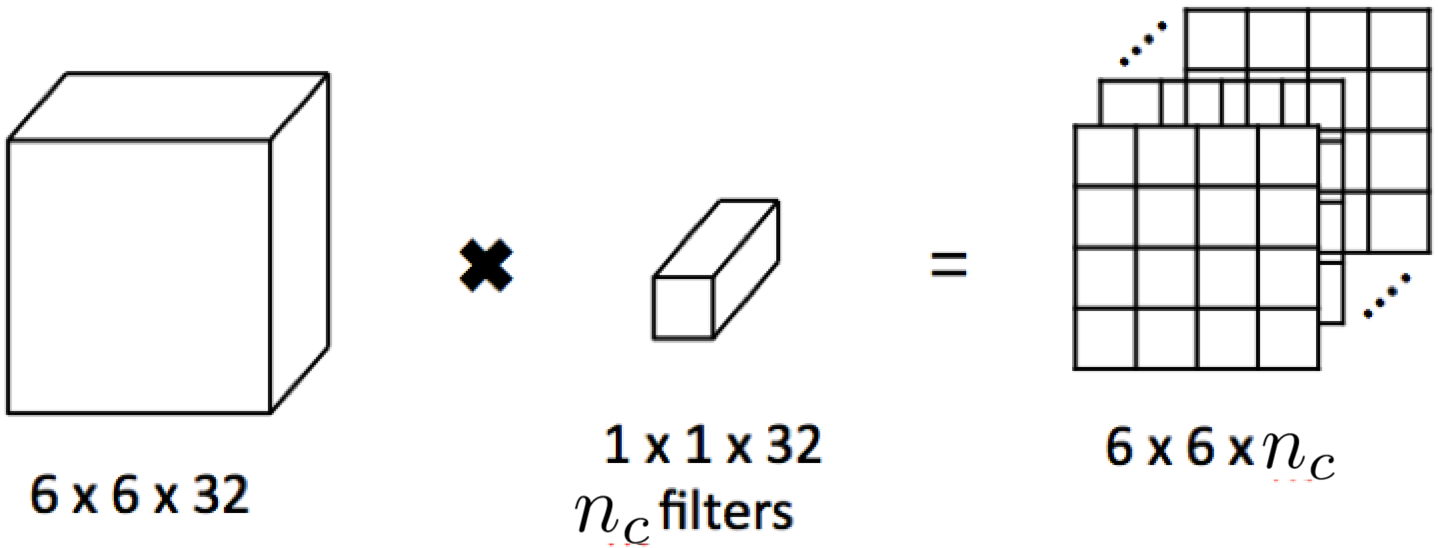
We call the layer convolutional because it is related to convolution of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

elementwise multiplication and sum of a filter and the signal (image)

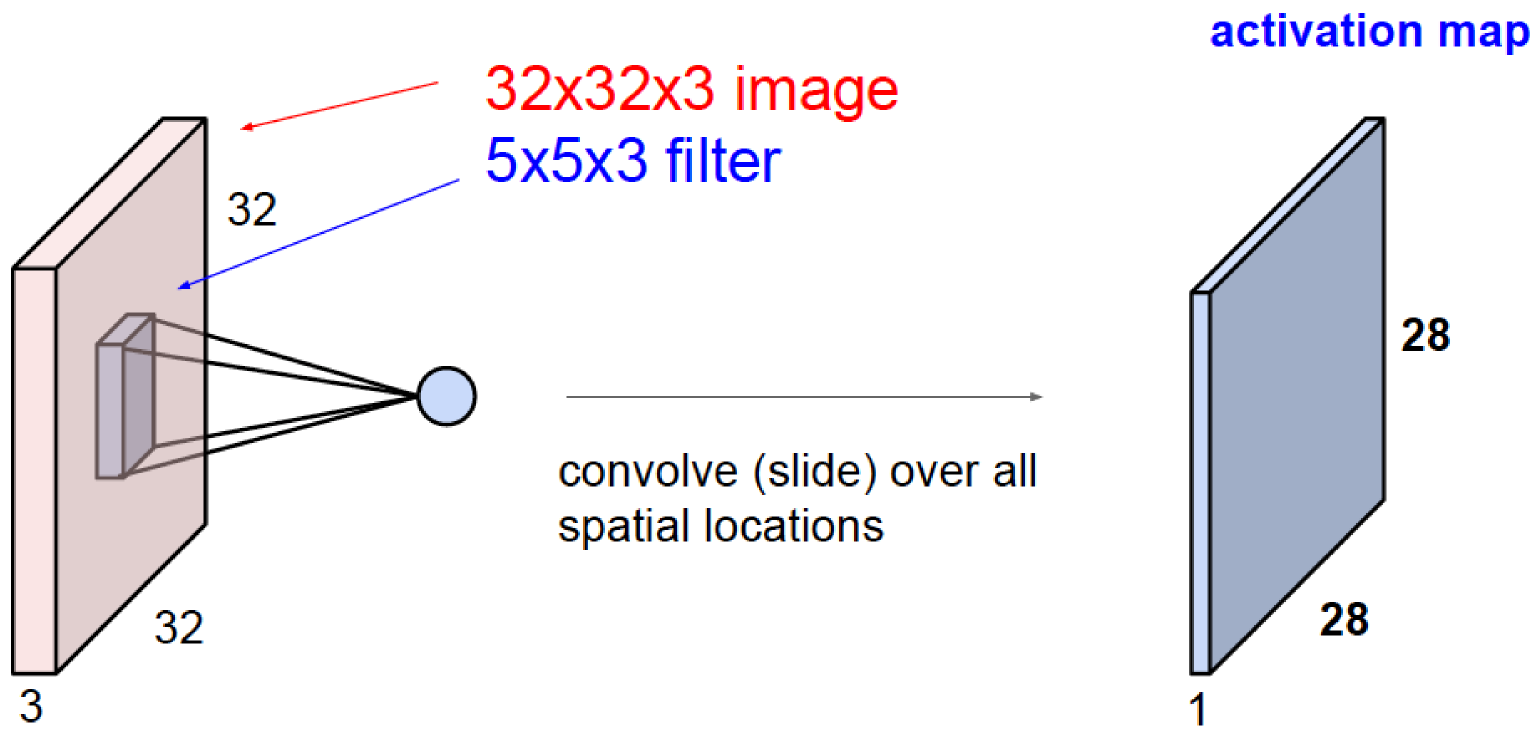
Special Convolutions

- 1x1 convolutions
  - Used in Network-in-network, GoogleNet
  - Reduce or increase dimensionality
  - Can be considered as 'feature pooling"



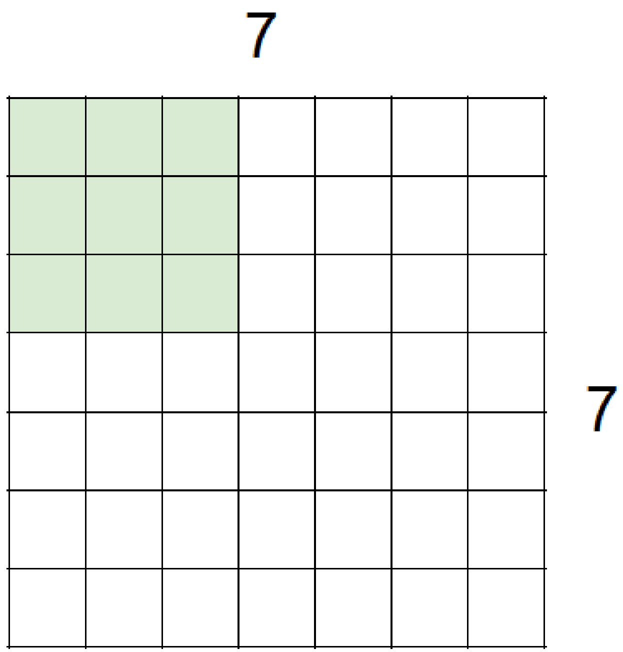
Complexity of Convolution Layers

- Sizes of activation maps and number of parameters



Complexity of Convolution Layers

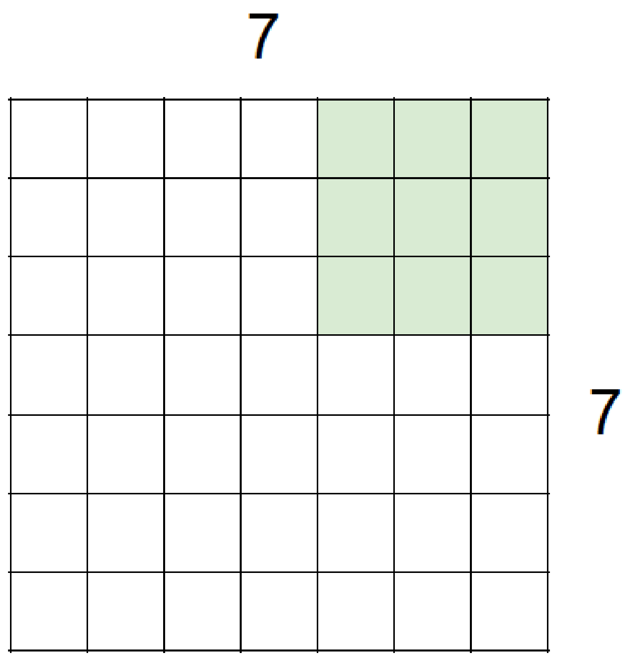
- Size of activation maps



7x7 input (spatially)
assume 3x3 filter

Complexity of Convolution Layers

- Size of activation maps

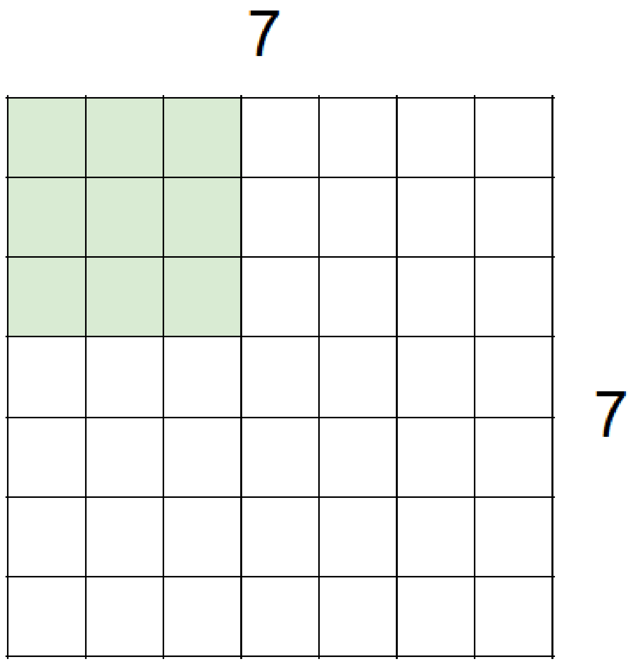


7x7 input (spatially)
assume 3x3 filter

=> 5x5 output

Complexity of Convolution Layers

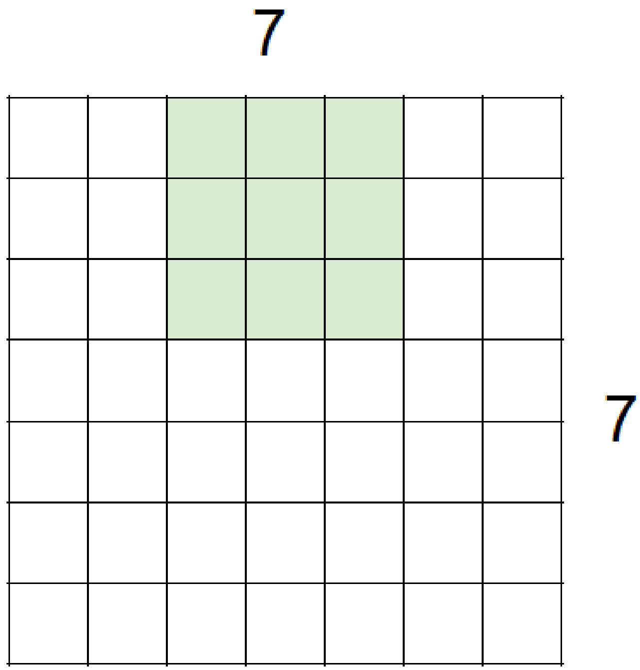
- Case: Stride > 1



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

Complexity of Convolution Layers

- Case: Stride > 1



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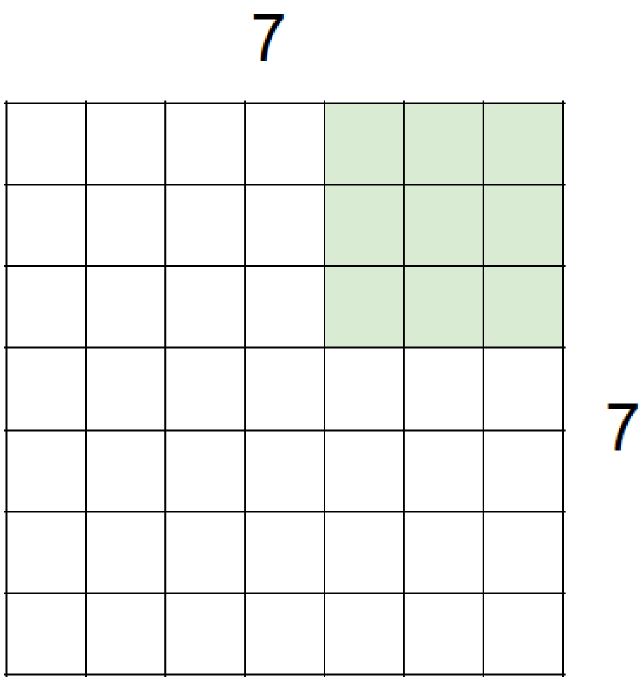
Complexity of Convolution Layers

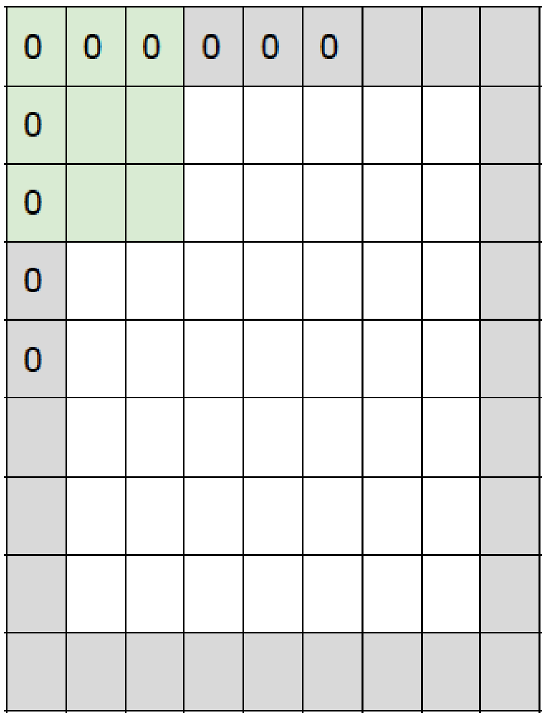
- Case: Stride > 1



7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

Complexity of Convolution Layers

- Zero padding to handle non-integer cases or control the output sizes



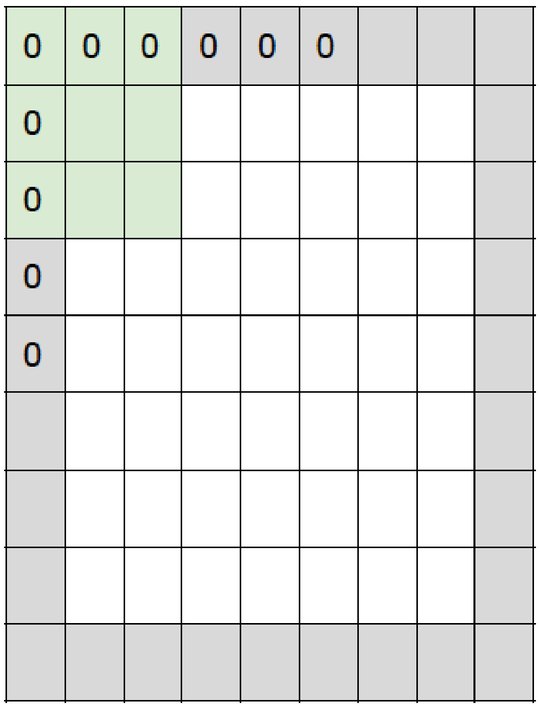
e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!

(recall:)
(N - F) / stride + 1

Complexity of Convolution Layers

- Zero padding to handle non-integer cases or control the output sizes

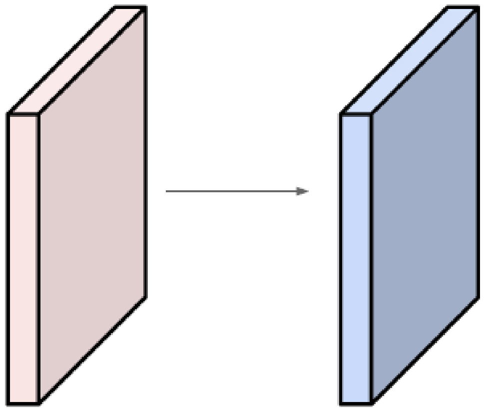


e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

Complexity of Convolution Layers

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

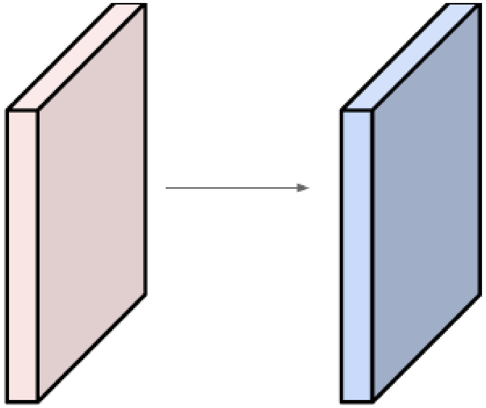
Output volume size:

(32+2*2-5)/1+1 = 32 spatially, so

32x32x10

Complexity of Convolution Layers

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

each filter has 5*5*3 + 1 = 76 params (+1 for bias)

=> 76*10 = **760**

Complexity of Convolution Layers

- Summary

- Accepts a volume of size $W_1 \times H_1 \times D_1$
- Requires four hyperparameters:
  - Number of filters K,
  - their spatial extent F,
  - the stride S,
  - the amount of zero padding P.
- Produces a volume of size $W_2 \times H_2 \times D_2$ where:
  - $W_2 = (W_1 - F + 2P)/S + 1$
  - $H_2 = (H_1 - F + 2P)/S + 1$ (i.e. width and height are computed equally by symmetry)
  - $D_2 = K$
- With parameter sharing, it introduces $F \cdot F \cdot D_1$ weights per filter, for a total of $(F \cdot F \cdot D_1) \cdot K$ weights and K biases.
- In the output volume, the d-th depth slice (of size $W_2 \times H_2$) is the result of performing a valid convolution of the d-th filter over the input volume with a stride of S, and then offset by d-th bias.

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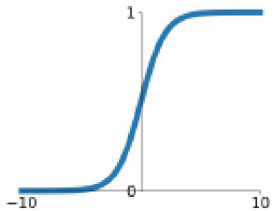
Review: Activation Function

- Zoo of Activation functions

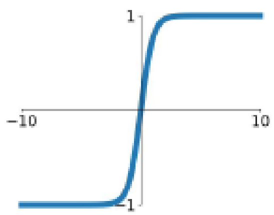
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



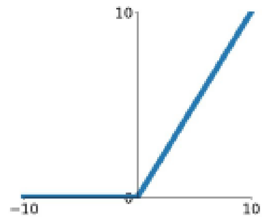
tanh

$\tanh(x)$



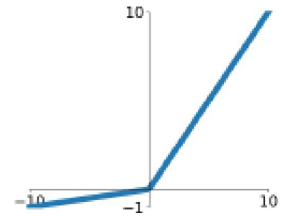
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

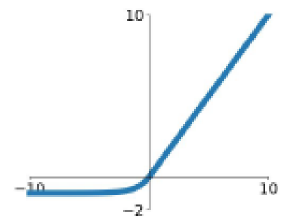


Maxout

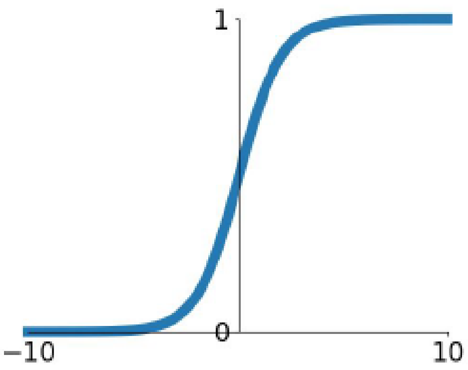
$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$



Sigmoid function



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

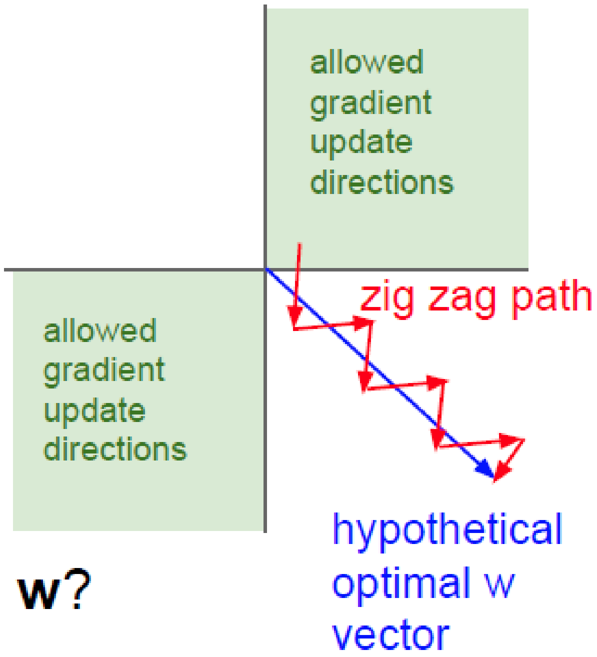
3 problems:

1. Saturated neurons "kill" the gradients
2. Sigmoid outputs are not zero-centered
3. exp() is a bit compute expensive

Sigmoid function

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

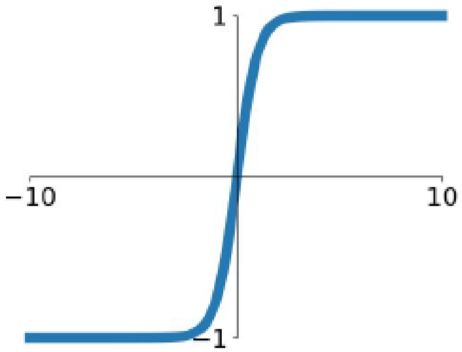


What can we say about the gradients on **w**?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

Tanh function



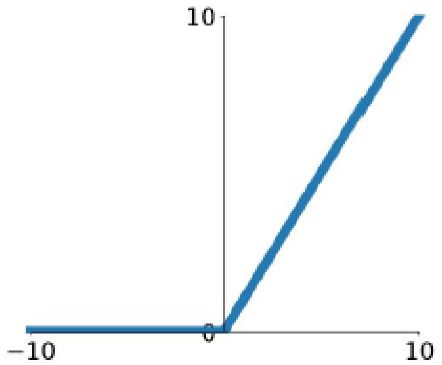
tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

- Recurrent neural networks: LSTM, GRU

Rectified Linear Unit



ReLU
(Rectified Linear Unit)

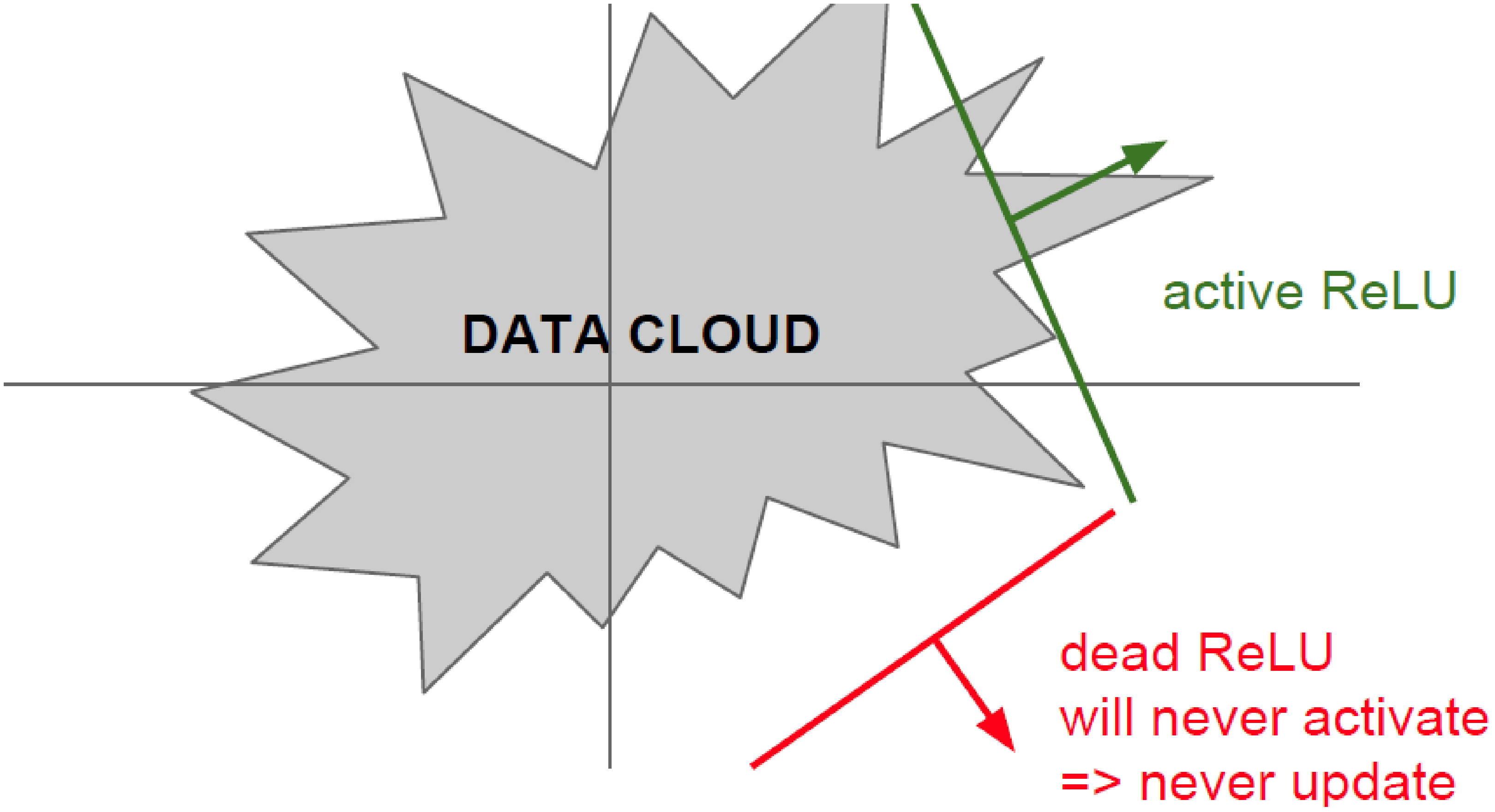
- Computes **f(x) = max(0,x)**

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Actually more biologically plausible than sigmoid

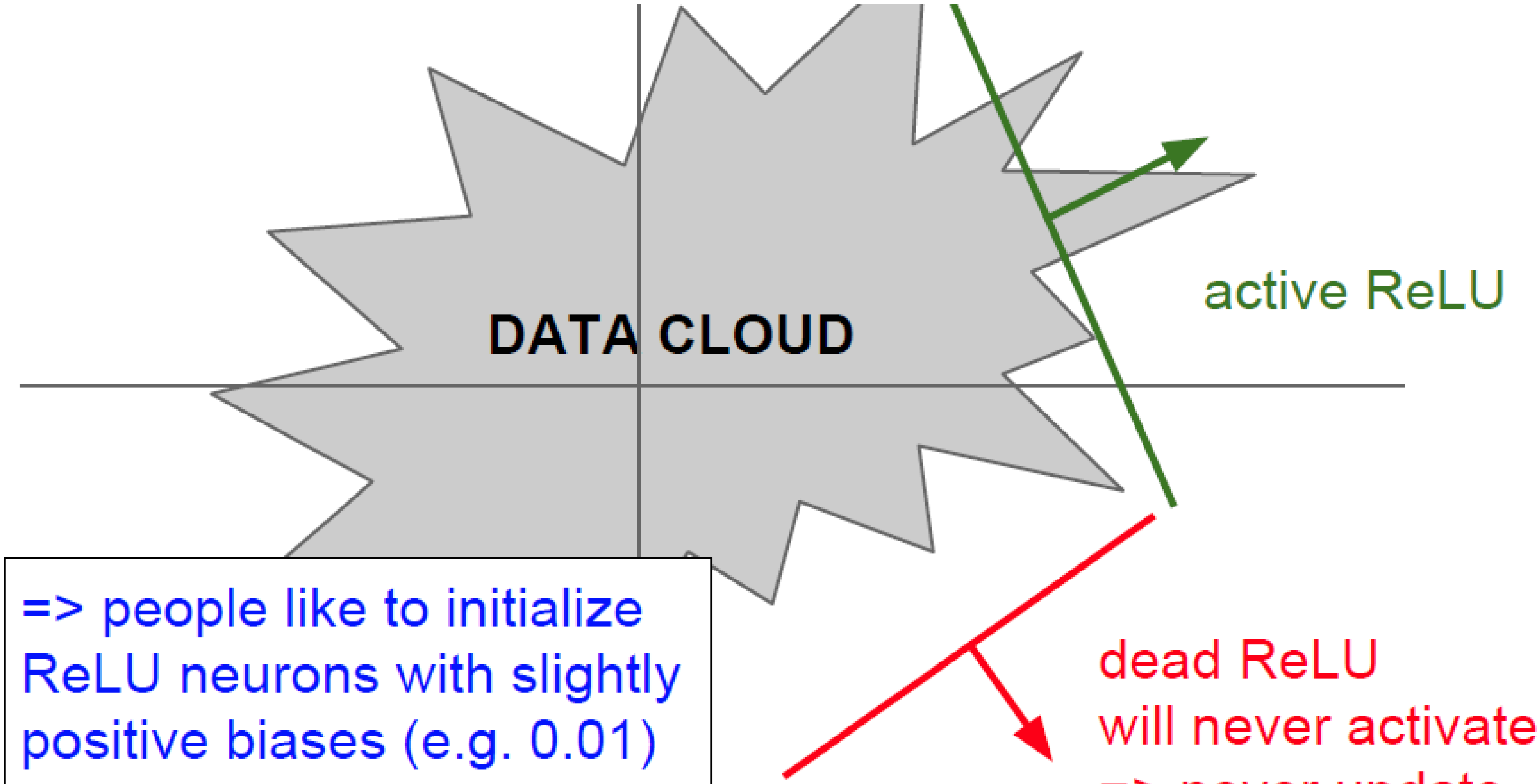
- Not zero-centered output
- An annoyance:

hint: what is the gradient when $x < 0$?

Rectified Linear Unit



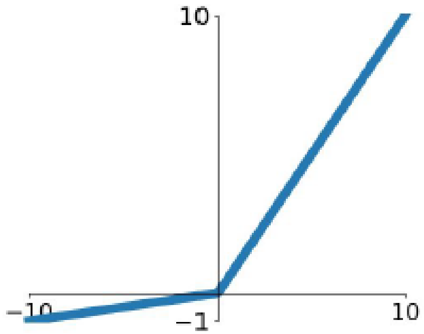
Rectified Linear Unit



=> people like to initialize ReLU neurons with slightly positive biases (e.g. 0.01)

Leaky ReLU

[Mass et al., 2013]
[He et al., 2015]



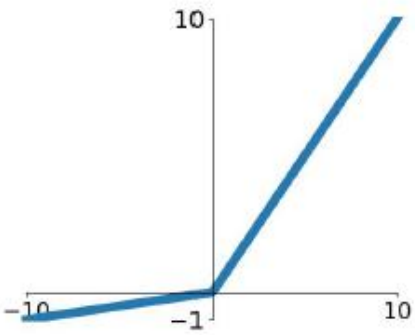
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Leaky ReLU

[Mass et al., 2013]
[He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Parametric Rectifier (PReLU)

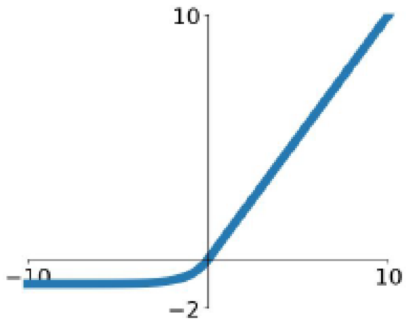
$$f(x) = \max(\alpha x, x)$$

backprop into \alpha
(parameter)

Exponential Linear Units (ELU)

[Clevert et al., 2015]

Exponential Linear Units (ELU)



- All benefits of ReLU
- Closer to zero mean outputs
- Negative saturation regime compared with Leaky ReLU adds some robustness to noise

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

- Computation requires exp()

Maxout function

Maxout "Neuron" [Goodfellow et al., 2013]

- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

Problem: doubles the number of parameters/neuron :(

Summary: Activation function

- For internal layers in CNNs
  - Use ReLU. Be careful with your learning rates
  - Try out Leaky ReLU / Maxout / ELU
  - Try out tanh but don't expect much
  - Don't use sigmoid
- For output layers
  - Task dependent
  - Related to your loss function

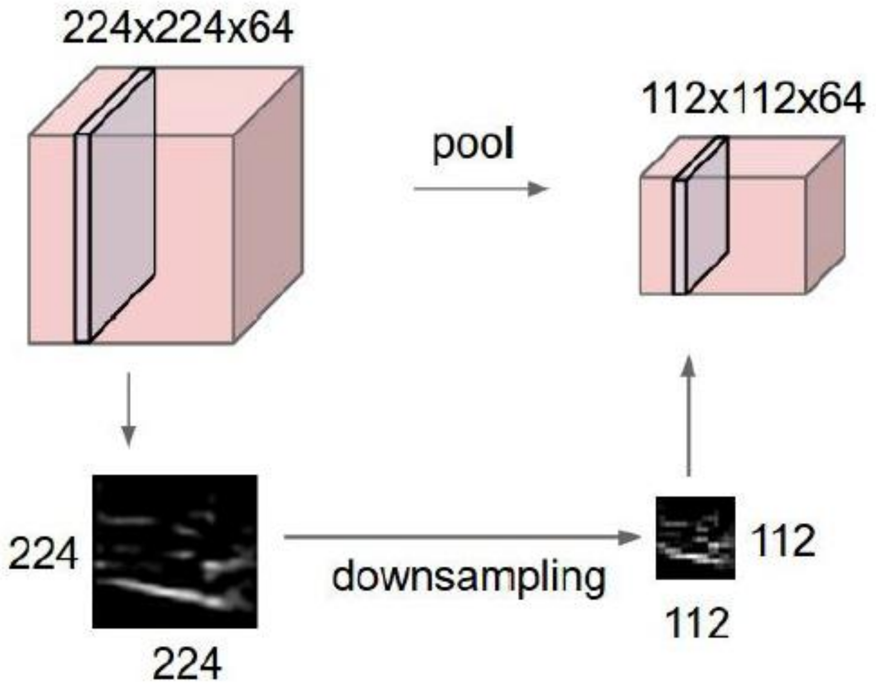
Outline

- Why Convolutional Neural Network (CNN)?
  - Motivation and overview
- What is the CNN?
  - Convolution and feature extraction
  - Convolution layers & model complexity
  - Closer look at activation functions
  - Pooling layers & model complexity
- Examples of CNNs

Acknowledgement: Roger Grosse@UofT & Feifei Li's cs231n notes

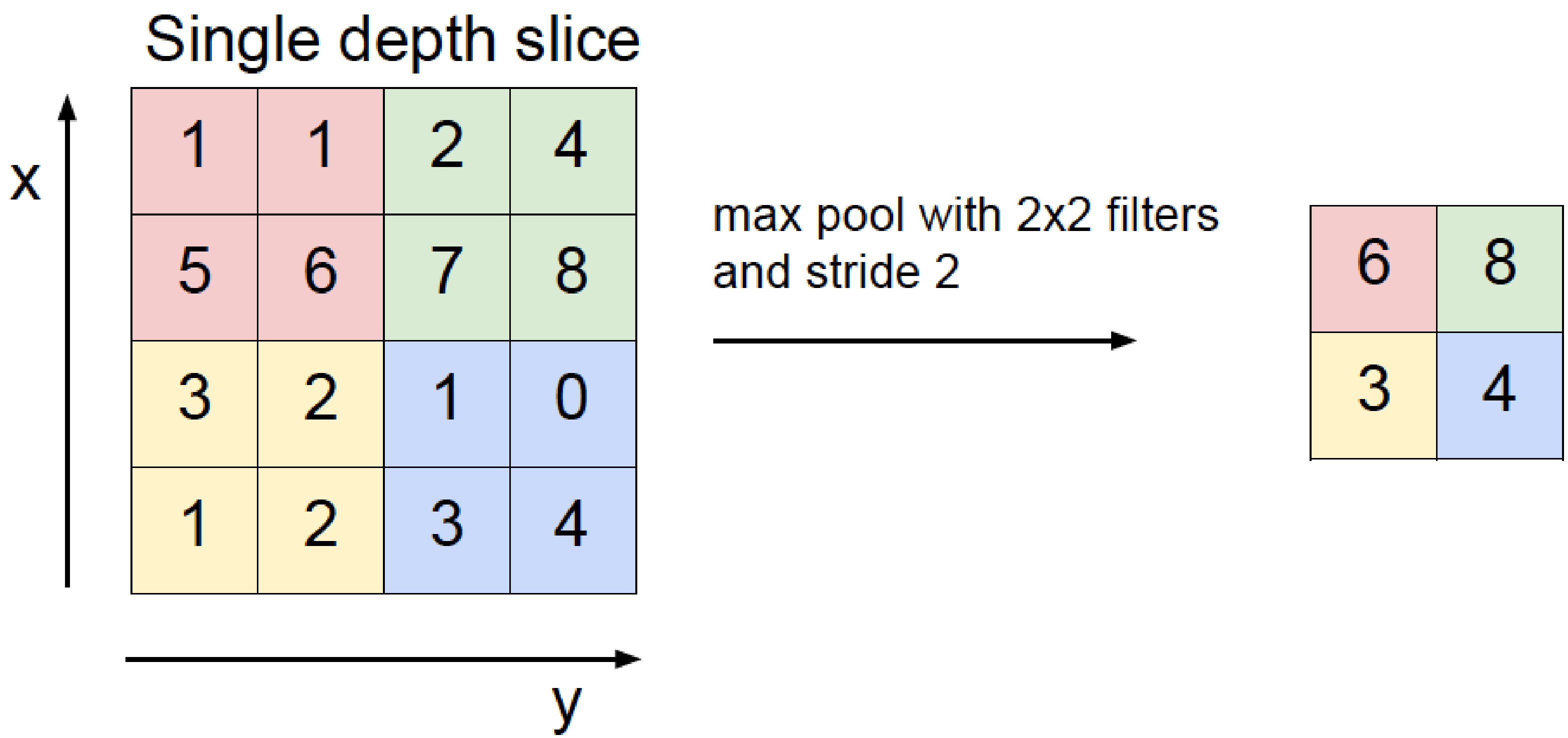
Pooling Layers

- Reducing the spatial size of the feature maps
  - Smaller representations
  - On each activation map independently
  - Low resolution means fewer details



Pooling Layers

- Example: max pooling



Complexity of Pooling Layers

- Summary

  - Accepts a volume of size $W_1 \times H_1 \times D_1$
  - Requires three hyperparameters:
    - their spatial extent F,
    - the stride S,
  - Produces a volume of size $W_2 \times H_2 \times D_2$ where:
    - $W_2 = (W_1 - F)/S + 1$
    - $H_2 = (H_1 - F)/S + 1$
    - $D_2 = D_1$
  - Introduces zero parameters since it computes a fixed function of the input
  - Note that it is not common to use zero-padding for Pooling layers

Math Properties of CNNs

- What representations a CNN can capture in general?
- Consider a representation ϕ as an abstract function

$$\phi : \mathbf{x} \to \phi(\mathbf{x}) \in \mathbb{R}^d$$

- We want to look at how the representation changes upon transformations of input image.
  - Transformations represent the potential variations in the natural images
  - Translation, scale change, rotation, local deformation etc.

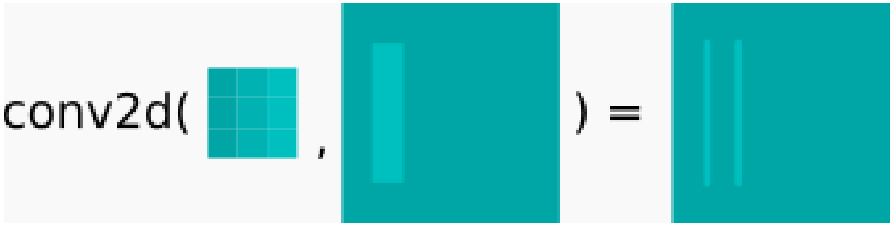
Math Properties of CNNs

- Two key properties of representations
  - Equivariance

A representation ϕ is equivariant with a transformation g if the transformation can be transferred to the representation output.

$$\exists \text{ a map } M_g : \mathbb{R}^d \to \mathbb{R}^d \text{ such that:}$$
$$\forall \mathbf{x} \in \mathcal{X} : \phi(g\mathbf{x}) \approx M_g \phi(\mathbf{x})$$

  - Example: convolution w.r.t. translation



Math Properties of CNNs

- Two key properties of representations
  - Invariance

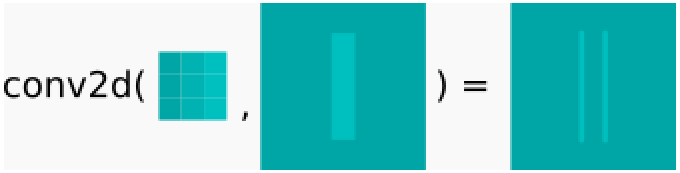
    A representation ϕ is invariant with a transformation g if the transformation has no effect on the representation output.

$$\forall \mathbf{x} \in \mathcal{X} : \phi(g\mathbf{x}) \approx \phi(\mathbf{x})$$

  - Example: convolution+pooling+FC w.r.t. translation

Math Properties of CNNs

- Recent results on convolution layers
  - Convolutions are equivariant to translation
  - Convolutions are not equivariant to other isometries of the sampling lattice, e.g., rotation



  - What if a CNN learns rotated copies of the same filter?
    - The stack of feature maps is equivariant to rotation.

Math Properties of CNNs

- Recent results on convolution layers
  - Ordinary CNNs can be generalized to Group Equivariant Networks (Cohen and Welling ICML'16, Kondor and Trivedi ICML'18)
    - Redefining the convolution and pooling operations
    - Equivariant to more general transformation from some group *G*
  - What if the underlying data is not on a grid-like topology?
    - Spherical CNNs (Cohen, Geiger, Kohler and Welling, ICLR'18)
    - Graph CNN (Niepert, Ahmed and Kutzkov, ICML'16)
    - Manifold-valued CNN (Monti, Boscaini, Masci, Rodola, Svoboda, Bronstein, CVPR'17)
    - We will cover some of them later in the course…

Summary of CNNs

- CNN properties [Bronstein et al., 2018]
  - Convolutional (Translation invariance)
  - Scale Separation (Compositionality)
  - Filters localized in space (Deformation Stability)
  - O(1) parameters per filter (independent of input image size n)
  - O(n) complexity per layer (filtering done in the spatial domain)
  - O(log n) layers in classification tasks

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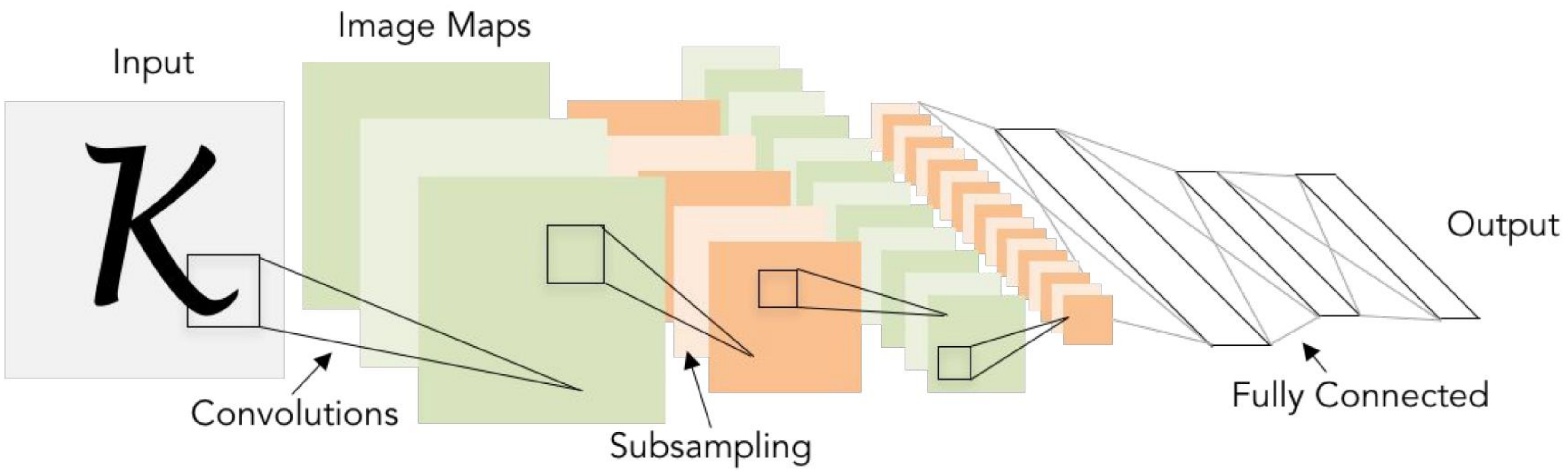
LeNet-5

- Handwritten digit recognition

[LeCun et al., 1998]

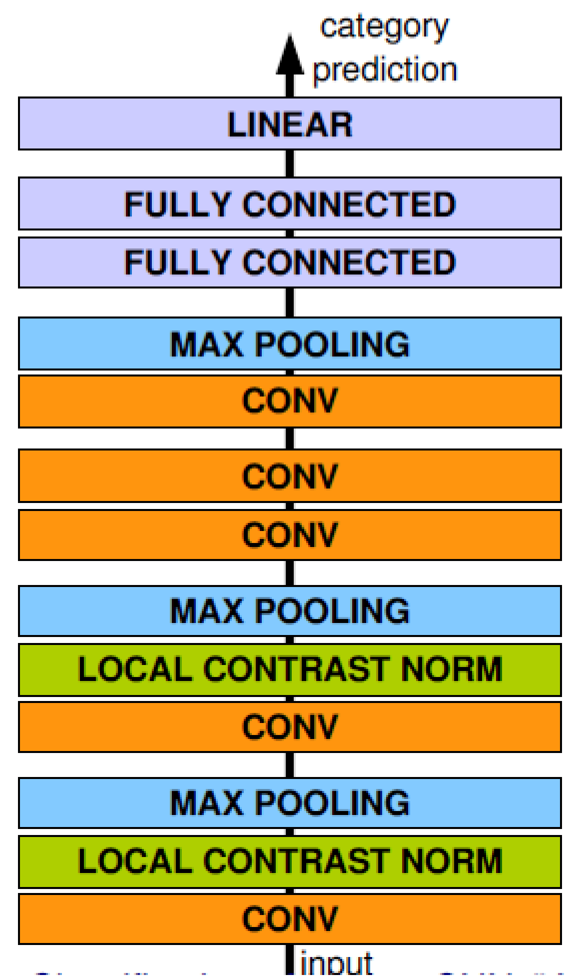


Conv filters were 5x5, applied at stride 1
Subsampling (Pooling) layers were 2x2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-FC-FC]

AlexNet

- Deeper network structure
  - More convolution layers
  - Local contrast normalization
  - ReLu instead of Tanh
  - Dropout as regularization

Architecture:
CONV1
MAX POOL1
NORM1
CONV2
MAX POOL2
NORM2
CONV3
CONV4
CONV5
Max POOL3
FC6
FC7
FC8



Summary

- Convolutional Neural Networks
  - Conv, Pooling, FC layers
- Next time …
  - Structure design of Modern CNNs