ShanghaiTech University

CS240 Algorithm Design and Analysis

Spring 2019

Problem Set 4

Due: 23:59, Apr. 24, 2019

1. Submit your solutions to Gradescope (www.gradescope.com).
2. In "Account Settings" of Gradescope, set your FULL NAME to your Chinese name and enter your STUDENT ID correctly.
3. If you want to submit a handwritten version, scan it clearly. Camscanner is recommended.
4. When submitting your homework, match each of your solution to the corresponding problem number.

Note: When proving problem A is NP-complete, please clearly divide your answer into three steps:

(1) Prove that problem A is in NP.

(2) Choose an NP-complete problem B and for any B instance, construct an instance of problem A.

(3) Prove that the yes/no answers to the two instances are the same.

Problem 1:

Given a conjunctive normal form formula and an integer k, can this formula be satisfied by an assignment in which at most k variables are true? Prove this problem is NP-complete.

Problem 2:

You are given a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The Zero-Weight-Cycle Problem is to decide if there is a simple cycle in G so that the sum of the edge weights on this cycle is exactly 0. Prove that this problem is NP-complete. (Note: You must use the Directed-Hamiltonian-Cycle problem in your reduction.)

Problem 3:

Two graphs $G = (V, E)$ and $G' = (V', E')$ are said to be equivalent if there is a one-to-one mapping $f : V \to V'$ such that $(v, w) \in E$ if and only if $(f(v), f(w)) \in E'$. Also, we say that G' is a subgraph of G if $V' \subseteq V$, and $E' = \{(u, v) \in E | u, v \in V'\}$. Given two graphs G and G', show that the problem of determining whether G' is equivalent to a subgraph of G is NP-complete.

Problem 4:

Suppose you are going to schedule courses for the SIST and try to make the number of conflicts no more than K. You are given 3 sets of inputs: $C = \{...\}, S = \{...\}, R = \{\{...\}, \{...\}, ...\}$. C is the set of distinct courses. S is the set of available time slots for all the courses. R is the set of requests from students, consisting of a number of subsets, each of which specifies the courses a student

wants to take. A conflict occurs when two courses are scheduled at the same slot even though a student requests both of them. Prove this schedule problem is NP-complete.
Example:

$$K = 1; \quad C = \{a, b, c, d\}, \quad S = \{1, 2, 3\}, \quad R = \{\{a, b, c\}, \{a, c\}, \{b, c, d\}\}$$

An acceptable schedule is:

$$a \to 1; \quad b \to 2; \quad c, d \to 3;$$

Here only one confilct occurs. An unacceptalbe schedule is:

$$a \to 1; \quad b, c \to 2; \quad d \to 3;$$

Here two ($> K$) conflicts occur.

Problem 5:

SIST allows students to work as TAs but would like to avoid TA cycles. A TA cycle is a list of TAs $(A_1, A_2, \ldots, A_k)$ such that A_1 works as a TA for A_2 in some course, A_2 works as a TA for A_3 in some course, $\cdots$, and finally A_k works as a TA for A_1 in some course. We say a TA cycle is simple if it does not contain the same TA more than once. Given the TA arrangements of SIST, we want to find out whether there is a simple TA cycle containing at least K TAs. Prove this problem is NP-complete.

Problem 6:

Consider the Knapsack problem. We have n items, each with weight a_j and value c_j $(j = 1, ..., n)$. All a_j and c_j are positive integers. The question is to find a subset of the items with total weight at most b such that the corresponding profit is at least k (b and k are also integers). Show that Knapsack is NP-complete by a reduction from Subset Sum.