Hw_4

Local Lattice Planner:

(1) Build an ego-graph of the linear modeled robot

(2) Select the best trajectory closest to the planning target

The modelling and how to select the best trajectory by using OBVP have been given here, please follow the annotation in the code, finish the homework step by step. Enjoy it~

1. Modelling
    a) Objective, minimize the integral of squared accelerate

$$J=\int_0^T g(x,u)dt=\int_0^T (1+u^T Ru)dt=\int_0^T (1+a_x^2+a_y^2+a_z^2)dt$$

    b) State, Input and System equation

$$x=\begin{pmatrix}p_x\\p_y\\p_z\\v_x\\v_y\\v_z\end{pmatrix}, u=\begin{pmatrix}a_x\\a_y\\a_z\end{pmatrix}, \dot{x}=f(x,u)=\begin{pmatrix}v_x\\v_y\\v_z\\a_x\\a_y\\a_z\end{pmatrix}$$

2. Solving
    a) Costate $\lambda=(\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6)^T$
    b) Define the Hamiltonian function

$$H(x,u,\lambda)=g(x,u)+\lambda^T f(x,u),$$
$$H(x,u,\lambda)=\left(1+a_x^2+a_y^2+a_z^2\right)+\lambda^T f(x,u)$$
$$\dot{\lambda}=-\nabla H(x^*,u^*,\lambda)=(0 \quad 0 \quad 0 \quad -\lambda_1 \quad -\lambda_2 \quad -\lambda_3)^T$$

    c) The costate is solved as

有微分方程λ_4' = -λ_1
P(x)=0,Q(x)=-λ_1
带入公式求解即得
λ_4=-λ_1t+C

$$\lambda=\begin{pmatrix}2\alpha_1\\2\alpha_2\\2\alpha_3\\-2\alpha_1 t-2\beta_1\\-2\alpha_2 t-2\beta_2\\-2\alpha_3 t-2\beta_3\end{pmatrix}$$

λ可简化为
[α
αt+β]
后续推导都使用单轴进行，然后拓展为3轴

    d) The optimal input is solved as

$$u^*=\arg\min_{a(t)} H(x^*(t),u(t),\lambda(t))=\begin{pmatrix}\alpha_1 t+\beta_1\\\alpha_2 t+\beta_2\\\alpha_3 t+\beta_3\end{pmatrix}$$

H=1+a^2+λ^T(v*,a)
其中v*最优，为常数
H'=2a+λ^_2
求导=0
则a=u*=-1/2 * λ^_2

    e) The optimal state trajectory is solved as

$$x^*=\begin{pmatrix}\frac{1}{6}\alpha_1 t^3+\frac{1}{2}\beta_1 t^2+v_{x0}t+p_{x0}\\\frac{1}{6}\alpha_2 t^3+\frac{1}{2}\beta_2 t^2+v_{y0}t+p_{y0}\\\frac{1}{6}\alpha_3 t^3+\frac{1}{2}\beta_3 t^2+v_{z0}t+p_{z0}\\\frac{1}{2}\alpha_1 t^2+\beta_1 t+v_{x0}\\\frac{1}{2}\alpha_2 t^2+\beta_2 t+v_{y0}\\\frac{1}{2}\alpha_3 t^2+\beta_3 t+v_{z0}\end{pmatrix}, initial\ state: x(0)=\begin{pmatrix}p_{x0}\\p_{y0}\\p_{z0}\\v_{x0}\\v_{y0}\\v_{z0}\end{pmatrix}$$

    f) α, β are solved as

$$\begin{pmatrix} \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 \\ 0 & 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 \\ \frac{1}{2}T^2 & 0 & 0 & T & 0 & 0 \\ 0 & \frac{1}{2}T^2 & 0 & 0 & T & 0 \\ 0 & 0 & \frac{1}{2}T^2 & 0 & 0 & T \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix}$$

$$\begin{pmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix} = \begin{pmatrix} p_{xf} - v_{x0}T - p_{x0} \\ p_{yf} - v_{y0}T - p_{y0} \\ p_{zf} - v_{x0}T - p_{z0} \\ v_{xf} - v_{x0} \\ v_{yf} - v_{y0} \\ v_{zf} - v_{z0} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0 & 0 \\ 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0 \\ 0 & 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} \\ \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0 & 0 \\ 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0 \\ 0 & 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} \end{pmatrix} \begin{pmatrix} \Delta p_x \\ \Delta p_y \\ \Delta p_z \\ \Delta v_x \\ \Delta v_y \\ \Delta v_z \end{pmatrix}$$

g) The cost

$$J = \int_0^T (1 + a_x^2 + a_y^2 + a_z^2)dt$$

$$J = T + \left(\frac{1}{3}{\alpha_1}^2T^3 + \alpha_1\beta_1T^2 + {\beta_1}^2T\right) + \left(\frac{1}{3}{\alpha_2}^2T^3 + \alpha_2\beta_2T^2 + {\beta_2}^2T\right) + \left(\frac{1}{3}{\alpha_3}^2T^3 + \alpha_3\beta_3T^2 + {\beta_3}^2T\right)$$

h) J only depends on T, and the boundary states (known), so we can even get an optimal T! 最小化T可以使用解析解和数值解

i. You can use the Mathematica obtain the optimal analytic expression of T

ii. You can use Numerical calculation method obtain the optimal approximate solution of T