# 摘要

# 引言

波动率不会无限上涨或下跌，总是在经历一定的运动后反向回复。随机波动率模型使用Ornstein-Uhlenbeck过程来描述波动率的运动形式（heston,1993）。其中，波动率总是围绕一个长期均值上下变化，并且下一时刻的波动率变化又取决于当前的波动率水平（Yoon,2022）。Ornstein-Uhlenbeck过程从数学上证明，波动率变化的概率分布期望与波动率水平和长期均值的差负相关，表现均值回复特征。波动率衡量资产的风险和金融市场的不确定性，然而这种不确定性又具有清晰的确定性数学统计规律。

当前已经有文献研究了波动率的均值回复特征，它们研究重点聚焦于指数期权（Cao,2020；Chiu,2019）或大宗商品指数期权（Chi,2017）。相比于个股期权，指数期权会消除诸多金融市场细节特征（Bali,2014）。本文在时间横截面上，研究了个股期权的风险中性波动率的均值回复特征，包括均值回复强度、回复概率等内容。在随机波动率的框架下，本文证明了波动率均值回复的必然性以及波动率回复概率和所需时间的关系。个股期权与个股公司的财务特征具有紧密联系（Bing Han,2022），其中做空看涨期权的delta对冲收益率与股价、盈利能力负相关，而与现金流量、分析师预测偏差等负相关。但是它将研究目标仅仅聚集于期权本身的delta对冲收益率，而delta对冲收益率与波动率风险溢价负相关（Bakish,2003），与波动率的波动率风险溢价正相关（Darien,2018）。本文重点参考了这篇论文的研究方法，在类似的个股财务分析框架中将个股期权划分为不同的分位数组合，扩展研究了波动率的均值回复特征的内容。

本文研究发现，在个股期权的时间横截面上隐含波动率均表现出显著的均值回复特征，并且均值回复强度在各个财务框架组合中表现有所差异。本文使用现金流量方差描述个股的盈利风险程度（Haugena,1996），发现经营风险较高的行业具有较强的均值回复强度，即高风险个股期权的隐含波动率往往会在短期内实现较大的均值回复程度。同时本文将个股期权按照行业类别划分为不同的组合进行测试，研究发现以苹果和特斯拉等为代表的科技行业波动率具有更强的均值回复强度，而第一产业的行业波动率均值回复程度较低甚至无明显均值回复。总之，波动率的回复特征与个股的盈利能力、资产规模、现金流动能力负相关，而与负债水平、资产负债比率等负相关。

并且，本文在个股期权的横截面上研究了均值回复概率与所需时间之间的关系。本文使用Logit模型框架估计均值回复概率的期望值，研究发现：第一，波动率的单向时间变化越久，均值回复概率越低且所需时间越长；第二，波动率的单向变化程度越高，均值回复概率越大且所需时间越短；第三，波动率的单向变化越快，均值回复概率会随着时间的延长而降低。而在依据财务类别分组的框架下，盈利能力强、资产规模大、资金流动能力强的个股期权隐含波动率往往具有较低的均值回复概率，而经营风险高、负债水平高的行业往往具有较高的均值回复概率。

考虑到期权市场同样存在羊群效应（Bernales，2016），个股期权的隐含波动率的均值回复特征可能会受到波动率持续的影响。本文以历史波动率持续作为条件前提，研究发现：当波动率持续较长时，波动率的均值回复强度会变高，而均值回复概率会下降；当波动率持续较短时，一切相反。由于期权市场和个股市场的分歧（Ruslan，2019），本文认为波动率持续会改变投资者的心里预期，甚至长期的波动率持续会改变整个金融市场的短期固定参数，进而使得波动率均值回复不再出现。同时，考虑到波动率聚集现象以及由此带来的市场环境差别（Pearson,2020），本文分别研究了高波动市场和平稳市场中的均值回复现象，发现市场越平稳，波动率均值回复强度越弱甚至不存在。

最后，本文探讨了个股期权均值回复差异形成的原因。期权市场同样存在供需关系（Cheng,2019），投资者的对冲成本会随着波动率的上升而上升，进而其对冲需求会下降。本文研究了期权购买压力与均值回复强度之间的关系，发现当波动率上升时，均值回复强度会增加。并且，本文在4/2 heston模型框架下估计了波动率回归速度和波动率长期均值（Martino,2017），用其精确估值进一步验证了该结论。

未来波动率回复程度 = 过去波动率的变化程度 + 随机项

(产业类别、高低风险不同、个股特征详细于指数)

从理论推导上数学证明可行，且实证结果符合

测算未来回归概率

波动率持续）

需要推导出一个计量模型

# 理论背景

## The Solution of Ornstein-Uhlenbeck Process

In the stochastic volatility models, the volatility follows the Ornstein-Uhlenbeck Process, which shows the existence of mean reversion about the volatility(Heston,1993). Under the physical measure ,on the trade date , the stock price and volatility evolves as[[1]](#footnote-1):

Where and represent the drift rate and volatility of the stock return, respectively. is a constant but not .is the speed of mean reversion, and is the long-term mean of volatility, is the volatility of volatility..and are the wiener process of stock and volatility under the physical measure , respectively, and their correlation is .

The option price is , thus the partial differential equation of value is:

Where is the compensation of volatility risk(Huang and Darien, 2018), thus is the drift of volatility under the risk-neutral measure .is the risk free rate, which is set to a constant. To get the risk-neutral SDE of volatility and stocks, we use the Cameron-Martin-Girsanov lemma set the wiener process of volatility under Q , in which .Thus we get the SDE of volatility under Q measure:

To get the solution of volatility SDE under Q ,we substitute and into Equation (3) so that it takes the form Ornstein-Uhlenbeck process[]:

When the are much higher than (the volatility increase higher than ), the will be much more negative, thus will become negative with a greater probability (the volatility tends to move down even more); When the are much lower than (the volatility decrease lower than ), the will be much more positive, thus will become positive with a greater probability (the volatility tends to move up even more). We call this form of movement in volatility mean reverting. The solution of Equation (4) is[]:jfdjllingking suite by WRDS,

Where and represent long-term mean and the speed of mean reversion under Q measure, respectively. We can get the risk-neutral volatility from the option price, which is called the implied volatility. The implied volatility follows the mean reversion process, and .From the Equation(5), we can get the math process of , include its possible values and the corresponding probability.

## Response of Volatility to Distance

According to the volatility SDE under Q Equation, the implied volatility fluctuates around long-term mean of volatility . For researching the response of volatility to distance between the and ,we let , thus:

Where . For the martingale properties of ITO integrals , ,thus and variance . Because the Ornstein-Uhlenbeck process belong to Markov process, .

We set the distance from implied volatility to long-term mean of volatility as and substitute it into Equation (6), then we get Equation (7):

Thus the partial derivative of with respect to is:

For , and , thus >0. This means there exists a positive relationship between and , and we call the as the intensity of mean reversion.

# 样本与变量

# 实证检验

## Mean Reversion of Volatility

波动率回复强度与个股本身特征密切相关，包括个股的资产规模、盈利能力和成长能力等。我们在时间横截面上研究了个股的波动率回复特征，发现个股风险中性波动率在

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1 Regression Results of Mean Reversion | | | | | | | | | |
| **windows** | **10** | | | **30** | | | **90** | | |
|  | **coef** | **t** | **R** | **coef** | **t** | **R** | **coef** | **t** | **R** |
| **firm** |  |  |  |  |  |  |  |  |  |
| **BM** | 2.948 | 1.948 | 0.542 | 2.734 | 2.086 | 0.752 | 2.273 | 3.353 | 0.797 |
| **CFV** | 2.941 | 3.606 | 0.783 | 2.737 | 3.572 | 0.467 | 2.497 | 3.376 | 0.99 |
| **GK** | 3.251 | 2.116 | 0.626 | 2.903 | 3.183 | 0.22 | 2.593 | 3.006 | 0.672 |
| **ROA** | 3.49 | 2.621 | 0.686 | 3.208 | 2.561 | 0.692 | 2.847 | 3.473 | 0.489 |
| **TEF** | 2.989 | 3.587 | 0.901 | 2.764 | 2.318 | 0.093 | 2.43 | 2.339 | 0.396 |
| **ZS** | 3.311 | 3.274 | 0.874 | 3 | 3.11 | 0.786 | 2.65 | 2.801 | 0.712 |
| **assets** | 3.393 | 3.146 | 0.624 | 3.289 | 3.259 | 0.742 | 2.884 | 3.703 | 0.802 |
| **cash** | 3.074 | 4.455 | 0.686 | 2.851 | 2.565 | 0.514 | 2.459 | 3.709 | 0.641 |
| **debt** | 3.136 | 2.736 | 0.375 | 2.874 | 2.76 | 0.76 | 2.536 | 2.474 | 0.552 |
| **tf** | 2.778 | 2.931 | 0.752 | 2.479 | 2.776 | 0.651 | 2.278 | 3.089 | 0.674 |

此表展示了

## Probability of volatility

## Days used for Mean Reversion

# 稳健性检验

# 解释

# 结论

1. Heston(1993) set the SDE of volatility as , but we can change the with the to get the SDE in this paper easily. [↑](#footnote-ref-1)