# 摘要

# 引言

# 理论背景

## The Solution of Ornstein-Uhlenbeck Process

In the stochastic volatility models, the volatility follows the Ornstein-Uhlenbeck Process, which shows the existence of mean reversion about the volatility(Heston,1993). Under the physical measure ,on the trade date , the stock price and volatility evolves as[[1]](#footnote-1):

Where and represent the drift rate and volatility of the stock return, respectively. is a constant but not .is the speed of mean reversion, and is the long-term mean of volatility, is the volatility of volatility..and are the wiener process of stock and volatility under the physical measure , respectively, and their correlation is .

The option price is , thus the partial differential equation of value is:

Where is the compensation of volatility risk(Huang and Darien, 2018), thus is the drift of volatility under the risk-neutral measure .is the risk free rate, which is set to a constant. To get the risk-neutral SDE of volatility and stocks, we use the Cameron-Martin-Girsanov lemma set the wiener process of volatility under Q , in which .Thus we get the SDE of volatility under Q measure:

To get the solution of volatility SDE under Q ,we substitute and into Equation (3) so that it takes the form Ornstein-Uhlenbeck process[]:

When the are much higher than (the volatility increase higher than ), the will be much more negative, thus will become negative with a greater probability (the volatility tends to move down even more); When the are much lower than (the volatility decrease lower than ), the will be much more positive, thus will become positive with a greater probability (the volatility tends to move up even more). We call this form of movement in volatility mean reverting. The solution of Equation (4) is[]:

Where and represent long-term mean and the speed of mean reversion under Q measure, respectively. We can get the risk-neutral volatility from the option price, which is called the implied volatility. The implied volatility follows the mean reversion process, and .From the Equation(5), we can get the math process of , include its possible values and the corresponding probability.

## Response of Volatility to Distance

According to the volatility SDE under Q Equation, the implied volatility fluctuates around long-term mean of volatility . For researching the response of volatility to distance between the and ,we let , thus:

Where . For the martingale properties of ITO integrals , ,thus and variance . Because the Ornstein-Uhlenbeck process belong to Markov process, .

We set the distance from implied volatility to long-term mean of volatility as and substitute it into Equation (6), then we get Equation (7):

Thus the partial derivative of with respect to is:

For , and , thus >0. This means there exists a positive relationship between and , and we call the as the intensity of mean reversion.

# 样本与变量

# 实证检验

# 稳健性检验

# 解释

# 结论

1. Heston(1993) set the SDE of volatility as , but we can change the with the to get the SDE in this paper easily. [↑](#footnote-ref-1)