
The TEX book
TEX

., ., .$n \geqslant 5$,., . ,. .

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Contents

Chapter 1

1.1

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. , $\mathbb{N}$., $\mathbb{Z}$. $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Q}$. A a A , $a \in A$, a S . , b A , $b \notin A$.
A B , A B ,

$$a \in A \Rightarrow a \in B.$$

A B , $A \subseteq B$ $B \supseteq A$. $A \subseteq B$ $B \subseteq A$,

$$a \in A \Leftrightarrow a \in B,$$

A B , A B , $A = B$. A B B , A B , $A \subset B$ $B \supset A$., $\varnothing$..
. A () $a_1, \cdots, a_n (n \in \mathbb{N})$,

$$A = \{a_1, \cdots, a_n\}.$$

, . n n , $|A| = n$., $\boldsymbol{S}$ P

$$\{x \in \boldsymbol{S} \mid x \quad P\}.$$

: $\{0, \pm 2, \pm 4, \cdots\}$

$$\{n \in \mathbf{Z} \mid n \equiv 0 (\text{mod } 2)\}.$$

. . A B , A B , $A \cap B$,

$$A \cap B = \{x \mid x \in A \quad x \in B\}.$$

, n $A_1, \cdots, A_n$

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n = \{x \mid x \in A_i, 1 \leqslant i \leqslant n\}.$$

, $\{A_i \mid i \in I\}$ (I „ $i \in I$, A_i),

$$\bigcap_{i \in I} A_i = \{x \mid x \in A_i, \ i \in I\}.$$

, A B $A \cup B$,

$$A \cup B = \{x \mid x \in A \quad x \in B\}.$$

:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n = \{x \mid x \in A_i, \ i \in I\}.$$

A B , $B - A = \{x \mid x \in B, x \notin A\}$ A (B). Ω , $\Omega - A$ A , $\bar{A}$.
A B ,

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

A B . $A \times B$, $(a, b) = (a', b')$ $a = a'$ $b = b'$.

$$A_1 \times \cdots \times A_n = \prod_{i=1}^{n} A_i = \{(a_1, \cdots, a_n) \mid a_i \in A_i, 1 \leqslant i \leqslant n\}.$$

$$\prod_{i\in I} A_i = \{(a_i)_{i\in I} \mid a_i \in A_i,\ i \in I\}$$

,,. f A B , $a \in A$ B , a f , $f(a)$. f a $f(a)$ $a \to f(a)$. A B f f $f : A \to B$ $A \xrightarrow{f} B$. $f : A \to B$ $g : B \to C$., A C

$$g \circ f : A \to C, \quad (g \circ f)(a) = g(f(a)).$$

$g \circ f$ f g .

f g A B , f g ($f = g$), $a \in A$, $f(a) = g(a)$.

1 *()* $f : A \to B,\ g : B \to C,\ h : C \to D$,

$$h \circ (g \circ f) = (h \circ g) \circ f.$$

$a \in A$, $f(a) = b, g(b) = c, h(c) = d$.

$$(g \circ f)(a) = c,\ (h \circ g)(b) = d.$$

$(h \circ (g \circ f))(a) = h(c) = d, ((h \circ g) \circ f)(a) = (h \circ g)(b) = d.$ □

$f : A \to B$. A A' ,

$$f(A') = \{f(x) \mid x \in A'\},$$

B , A f ., B B' , $f^{-1}(B') = \{x \in A \mid f(x) \in B'\}$, A , B' . $f(A) = B$, B A (f), f . , A f B ,: $a,\ a' \in A,\ a \neq a' \Rightarrow f(a) \neq f(a')$, f ., $f : A \to B$, f .: A

$$1_A : A \to A,\ 1_A(a) = a.$$

A A . 1_A A . .

2 $f : A \to B$ $g : B \to A$, $f \circ g = 1_B, g \circ f = 1_A$.

f , $b \in B$, $a \in A$ $f^{-1}(b) = a$ (f , f)

$$g : B \to A,\ g(b) = f^{-1}(b).$$

$g \circ f = 1_A$ $f \circ g = 1_B$;.

, f , $b \in B$, $f^{-1}(b) = \varnothing$. $g : B \to A$, $(f \circ g)(b) = f(g(b) \neq b$. $f \circ g \neq 1_B$. f , $a,\ a' \in A,\ a \neq a'$, $f(a) = f(a') = b$. $g : B \to A, (g \circ f)(a) = g(b) = (g \circ f)(a'),\ g \circ f \neq 1_A$. $g : B \to A$ $f \circ g = 1_B$ $g \circ f = 1_A$. f . □

$f : A \to B$, $f \circ g = 1_B$ $g \circ f = 1_A$ $g : B \to A$.: $g : B \to A$ $f \circ g' = 1_B, g' \circ f = 1_A$, $g' = g' \circ 1_B = g' \circ (f \circ g) = (g' \circ f) \circ g = 1_A \circ g = g$. g f , f^{-1} .

A , $A \times A$ R A . $(a,b) \in R$, a b R , aRb . $\boldsymbol{R} \times \boldsymbol{R}$

$$R = \{(a,b) \in \boldsymbol{R} \times \boldsymbol{R} \mid a \ \ b\}$$

a b R a b ,. $a > b$. $\boldsymbol{R}$ $\geqslant$ ( ), $<$ (), $\leqslant$ (), $=$ (). A $\sim$,:

(1): $a \sim a$ ($a \in A$)

(2): $a \sim b, b \sim a$.

(3): $a \sim b,\ b \sim c,\ a \sim c$.

$\sim$ A . $a \sim b$, $b \sim a$. a b . $a \in A$, $[a]$ A a ,

$$[a] = \{b \in A \mid b \sim a\}.$$

$a \in [a]$, $[a]$ a , ($b, c \in [a]$, $b \sim a, a \sim c$, $b \sim c$).(?) S $\{[a_i] | i \in I\}$,. $[a_i]$ b ;($b_i \in [a_i]$), $R = \{b_i | i \in I\}$: A b , b . R S $\sim$.

$$A = \bigcup_{a\in R} [a]\,(). \tag{$*$}$$

, A $\{A_i | i \in I\}$, A ; , $\{A_i | i \in I\}$ S ., S A (**??**)., $\{A_i | i \in I\}$ A , A : $a, b \in A$,

$$a \sim b \Leftrightarrow a \ \ b \ \ A_i \ ,$$

. E A , P A , $f : E \to P$ $g : P \to E$ $f \circ g = 1_P,\ g \circ f = 1_E$, f **??**.,AA.

, F . F : $A, B \in F$,

$$A \sim B \Leftrightarrow A \ B .$$

F (: $1_A : A \to A$, $A \sim A$. : $f : A \to B$, $f^{-1} : B \to A$, $A \sim B \Rightarrow B \sim A$.**??**.) ,.,( ), $|A| = |B|$. N ,. R . E (), $\boldsymbol{N} \to E$, $n \to 2n$., A A !

A . $A \times A$ A

$$f : A \times A \to A$$

A().:

$$f : \boldsymbol{C} \times \boldsymbol{C} \to \boldsymbol{C},\ f(\alpha, \beta) = \alpha + \beta.$$

A $\cdot$, $a, b \in A$, $f(a, b)$ $a \cdot b (\in A)$ ab.

$\cdot$,

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c (\ a, b, c \in A).$$

$\cdot$,

$$a \cdot b = b \cdot a (\ a, b \in A).$$

(),. .

1. $B, A_i (i \in)$:
   (a) B $\cap (\bigcup_{i \in I} A_i) = \bigcup_{i \in I} (B \cap A_i)$,
   (b) B $\cup (\bigcap_{i \in I} A_i) = \bigcap_{i \in I} (B \cup A_i)$,
   (c) $\overline{\bigcup_{i \in I} A_i} = \bigcap_{i \in I} \overline{A_i}$, $\overline{\bigcap_{i \in I} A_i} = \bigcup_{i \in I} \overline{A_i}$.
2. $f : A \to B$, A .:
   (a) f $\Leftrightarrow$ $g : B \to A$, $g \circ f = 1_A$.
   (b) f $\Leftrightarrow$ $h : B \to A$. $f \circ h = 1_B$.
3. $f : A \to B$, $g : B \to C$, $g \circ f : A \to C$, $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
4. A , $P(A)$ A (), $|P(A)| = 2^{|A|}$., n 2^n .
5. $f : A \to B$. A : $a, a' \in A$, $a \sim a'$ $f(a) = f(a')$.,.
6. ,,.
7. A, B .
   (a) A B ?
   (b) A ?

1.2

.

S S $\cdot$. $(S, \cdot)$ S, $x \cdot y$ xy .

, $(S, \cdot)$. $x^2 = x \cdot x$, $x^{n+1} = x^n \cdot x (= x \cdot x^n$, $n \geqslant 1)$

S , $e \in S$ S , $x \in S$, $xe = ex = x$.

S e ,, e' , $e' = e'e = e$. S () 1_S 1 .

S . $y \in S$ $x \in S$, $xy = yx = 1$.

x ,. y' x , $xy' = y'x = 1$.

$$y = y \cdot 1 = y(xy') = (yx)y' = 1 \cdot y' = y'.$$

x , x^{-1} , $xx^{-1} = x^{-1}x = 1$.

G ,, G .,, G **(Abel)**.

.

1 M , $(M,+)$, 0 ,, 0 M .

$\boldsymbol{Z}, \boldsymbol{Q}, \boldsymbol{R}, \boldsymbol{C}$,,.
$(\boldsymbol{N}, \cdot)$ (), 1 .. $\boldsymbol{Q}*$, $(\boldsymbol{Q}*, \cdot)$, 1 , a a^{-1} . , $(\boldsymbol{R}*, \cdot)$ $(\boldsymbol{C}*, \cdot)$,.

2 $M_{m,n}(\boldsymbol{C})$ m n ,., $A = (a_{ij})$ $-A = (-a_{ij})$. $M_n(\boldsymbol{C})$ n , , I_n., n A $\det A \neq 0$. $M_n(\boldsymbol{C})$, $n \geqslant 2$, $M_n(\boldsymbol{C})$. $M_{m,n}(\boldsymbol{R})$, $M_n(\boldsymbol{Q})$.

3 A , $\sum(A)$ A A . $\sum(A)$. A 1_A .*1.1* **??**, $\sum(A)$ f f . $|A| > 1$, $\sum(A)$, $\sum(A)$.

4 $\boldsymbol{R}^2$. $\boldsymbol{R}^2$,,,, ,.

5 n , $\boldsymbol{Z}$ *:* a b ,

$$a \sim b \Leftrightarrow n \mid a - b(\ a \equiv b (\mathrm{mod}\ n))$$

, $\boldsymbol{Z}$ n *:* $\bar{0}, \bar{1}, \cdots, \overline{n-1}$, $\bar{i}$ i , $\bar{i} = \{m \in \boldsymbol{Z} \mid m \equiv i (\mathrm{mod}\ n)\}$. $\{0, 1, 2, \cdots, n-1\}$ Z m .
Z_n n . Z_n *:*

$$\bar{a} + \bar{b} = \overline{a+b}$$

,(n) , Z_n , $\bar{0}$, n .
Z_n

$$\bar{a}\bar{b} = \overline{ab}$$

Z_n , $\bar{1}$. $\bar{a}\bar{b} = \bar{1}$ $ab \equiv 1 (\mathrm{mod}\ n)$., a , b $ab \equiv 1 (\mathrm{mod}\ n)$ $(a, n) = 1$. a $(a, n) = 1$.

$(M, \cdot)$, $U(M)$ $M*$ M .
$(M, \cdot)$, $(U(M), \cdot)$.
$1_M^{-1} = 1_M$ $1 = 1_M \in U(M)$. $a, b \in U(M)$, a, b , $b^{-1}a^{-1}$ ab , $ab \in U(M)$. $\cdot$ $U(M)$. $U(M)$, $(U(M), \cdot)$. $U(M)$ a , $a^{-1} \in M$ (a a^{-1}), $a^{-1} \in U(M)$. $U(M)$ $U(M)$., $U(M)$. □
:

1. n , n , $GL(n, \boldsymbol{C})$. $GL(n, \boldsymbol{R})$,
   $GL(n, \boldsymbol{Q})$.
2. A , A , A , $S(A)$,(A A) A .
3. n , $\bar{a}$ a n .

$$Z_n^* = \{\bar{a} \mid (a, n) = 1\}$$

. $\varphi(n)$, $\varphi(n)$ 1 n n ($\varphi(n)$).
G . G , G ,. G n , G n n , $n = |G|$ G .
,,,.:
$(G, \cdot)$ $(G', \circ)$. $f : G \to G'$ G G' , $a, b \in G$
$f(a \cdot b) = f(a) \circ f(b)$($f(ab) = f(a)f(b)$). , f , f . f , f G G' ., G G' , $G = G'$ $f : G \xrightarrow{\sim} G'$.

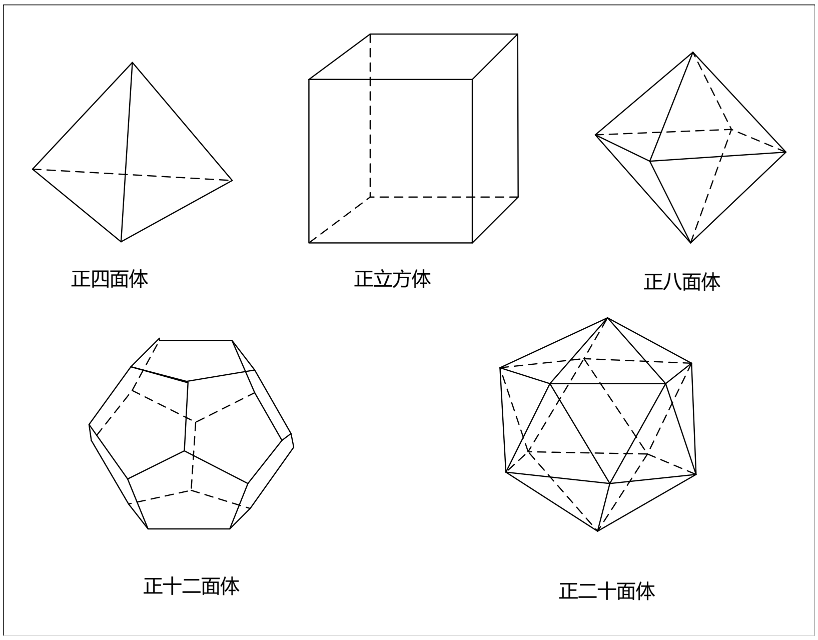

Figure 1.1:

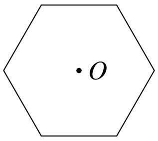

Figure 1.2: