数学建模-作业4

1. SIR 模型可写作 $\frac{\mathrm{d}i}{\mathrm{d}t} = \mu i(\sigma s - 1)$,$\frac{\mathrm{d}s}{\mathrm{d}t} = -\lambda si$.由后一方程知 $\frac{\mathrm{d}s}{\mathrm{d}t} < 0, s(t)$ 单调减少.

   (a) 若 $s_0 > \frac{1}{\sigma}$,当 $\frac{1}{\sigma} < s < s_0$ 时 , $\frac{\mathrm{d}i}{\mathrm{d}t} > 0, i(t)$ 增加;当 $s = \frac{1}{\sigma}$ 时, $\frac{\mathrm{d}i}{\mathrm{d}t} = 0$ 达到最大值 i_m;当 $s < \frac{1}{\sigma}$ 时, $\frac{\mathrm{d}i}{\mathrm{d}t} < 0, i(t)$ 减少且 $i_m = 0$.

   (b) 若 $s_0 < 1\frac{1}{\sigma}, \frac{\mathrm{d}i}{\mathrm{d}t} < 0$,i(t) 单调递减至0.

2. 在图 12 坐标下铅球运动方程为

$$\ddot{x} = 0, \quad \ddot{y} = -g, \quad x(0) = 0, \quad y(0) = h$$

$$\dot{x}(0) = v \ \cos\alpha, \quad \dot{y}(0) = v \ \sin\alpha$$

   解出 $x(t), y(t)$ 后,可以求得铅球掷远为

$$R = \frac{v^2}{g}\sin\alpha\cos\alpha + \left(\frac{v^2}{g^2}\sin^2\alpha + \frac{2h}{g}\right)^{1/2} v\cos\alpha$$

   这个关系还可以表为 $R^2 g = 2v^2\cos^2\alpha(h + R\tan\alpha)$.
   由此计算 $\left.\frac{\mathrm{d}R}{\mathrm{d}\alpha}\right|_{\alpha^*} = 0$,得最佳出手角度 $\alpha^* = \sin^{-1}\frac{v}{\sqrt{2(v^2 + gh)}}$, 最佳成绩 $R^* = \frac{v}{g}\sqrt{v^2 + 2gh}$.设 $h = 1.5m, v = 10\ m/s$,则 $\alpha^* \approx 41.4°, R^* = 11.4\ m$.

3. 设 $f(p, v, s, \rho) = 0$ 量纲表达式: $[p] = L^2MT^{-3}, [v] = LT^{-1}, [s] = L^2, [\rho] = L^{-3}M$,解得 $F(\pi) = 0, \pi = p^{-1}v^3 s\rho$,故 $p = \lambda v^3 s\rho$.

4. 代码如下:

```
x = [464,788,229,13,127,13
     499,8605,1444,403,557,1223
     5,9,3,20,23,124
     62,527,128,163,67,146
     79,749,140,43,130,273
     146,1285,272,225,219,542];
x_all=[2918,16814,2875,1570,2341,5414];
a=x./x_all;
y=[1500;4200;3000;500;950;3000];
w=eye(6)-a;
q1=w\y
q2=w^-1
```