

$\rightarrow N(0,1)$   
 $p(z)$  已知, 目的是学会  $p(x|z)$

$\downarrow$  极大似然

$$\max_{\theta} p(x) = \int_z p(x|z) p(z) dz$$

由于  $z$  是连续变量, 上式无法直接优化

$$x \rightarrow q_{\phi}(z|x) \rightarrow z \rightarrow p_{\theta}(x|z) \rightarrow \hat{x}$$

用  $q_{\phi}(z|x)$  逼近  $p(z|x)$

$$\begin{aligned} & \min_{\phi} D_{KL}(q_{\phi}(z|x) | p(z|x)) \\ &= \min_{\phi} \int_z q_{\phi}(z|x) \log \frac{q_{\phi}(z|x)}{p(z|x)} dz \\ &= \int_z q_{\phi}(z|x) \log \frac{q_{\phi}(z|x) p(x)}{p(x,z)} dz \\ &= \int_z q_{\phi}(z|x) \log \frac{q_{\phi}(z|x)}{p(x,z)} dz + \int_z q_{\phi}(z|x) \log p(x) dz \\ &= - \left( \int_z q_{\phi}(z|x) \log \frac{q_{\phi}(z|x)}{p(x,z)} dz \right) + \log p(x) \\ & \quad \downarrow \\ & \quad L(\phi, \theta) \\ &= -L(\phi, \theta) + \log p(x) \end{aligned}$$