# Square-Root Variable Metric based elastic Full Waveform Inversion and Uncertainty Estimation

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# Motivation

- Full waveform inversion (FWI) has been widely studied.
- Its corresponding **Uncertainty Estimation** is still ignored.

# Challenges & Solution

## Challenges:

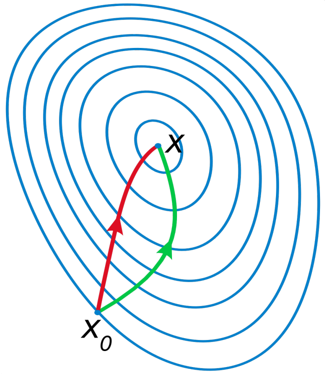
- Uncertainty Estimation is linked with (inverse) **Hessian** matrix
- Hessian matrix suffers from the "**Curse of Dimensionality**"

## Our solution:

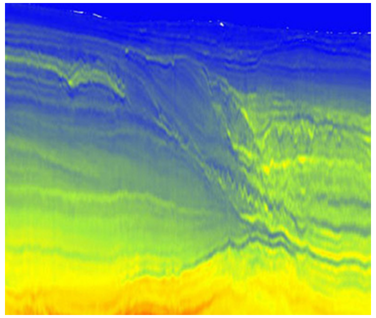
- Vector-Version Square-Root Variable Metric (**SRVM**) algorithm
- **Low-rank** approximation to the **inverse Hessian**

# Outline

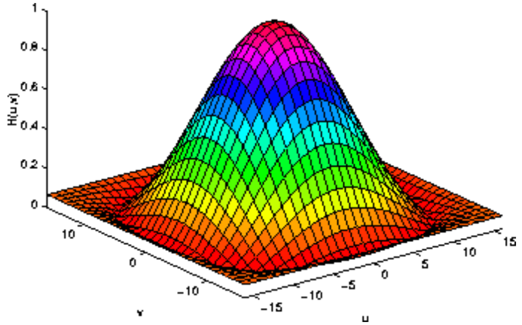
- **Theory on Square-Root Variable Metric (SRVM)**




- **SRVM into FWI**

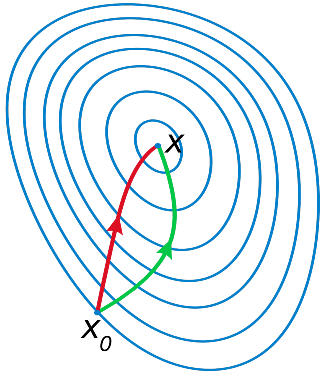


- **SRVM based Uncertainty Estimation**



# Outline

- **Theory on Square-Root Variable Metric (SRVM)**




- **SRVM into FWI**



- **SRVM based Uncertainty Estimation**

# What's ***SRVM*** ?

- BFGS: $$\mathbf{H}_{k+1} = \mathbf{H}_k - \frac{\mathbf{H}_k \Delta\mathbf{m}_k \Delta\mathbf{m}_k^T \mathbf{H}_k}{\Delta\mathbf{m}_k^T \mathbf{B}_k \Delta\mathbf{m}_k} + \frac{\Delta\mathbf{g}_k \Delta\mathbf{g}_k^T}{\Delta\mathbf{g}_k^T \Delta\mathbf{m}_k},$$

- DFP: $$\mathbf{B}_{k+1} = \mathbf{B}_k - \frac{\mathbf{B}_k \Delta\mathbf{g}_k \Delta\mathbf{g}_k^T \mathbf{B}_k}{\Delta\mathbf{g}_k^T \mathbf{B}_k \Delta\mathbf{g}_k} + \frac{\Delta\mathbf{m}_k \Delta\mathbf{m}_k^T}{\Delta\mathbf{g}_k^T \Delta\mathbf{m}_k},$$ *(Dual of BFGS)*

- SRVM: $$\mathbf{S}_{k+1}\mathbf{S}_{k+1}^T = \mathbf{S}_k \left(\mathbf{I} - (1/P_k)\mathbf{S}_k^T \mathbf{y}_k \mathbf{y}_k^T \mathbf{S}_k\right)\mathbf{S}_k^T,$$ *(Square-rooted DFP)*

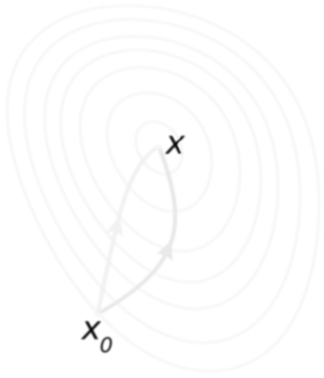
with $$\mathbf{B}_{k+1} = \mathbf{S}_{k+1}\mathbf{S}_{k+1}^T, \quad \mathbf{B}_k = \mathbf{S}_k \mathbf{S}_k^T, \quad \mathbf{y}_k = \mu_k \mathbf{g}_k + \Delta\mathbf{g}_k, \quad P_k = \mathbf{y}_k^T \mathbf{B}_k \Delta\mathbf{g}_k^T.$$

# Vector-version SRVM

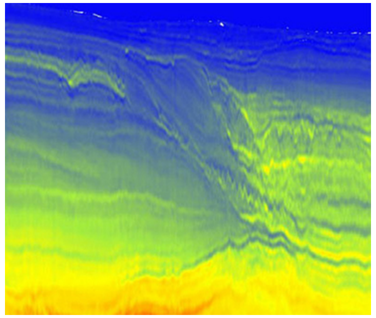
- **SRVM** algorithm in **vector version**
- The number of vectors = the number of iterations
- **Memory cost affordable** even for **large-scale** problems

# Outline

- Theory on Square-Root Variable Metric (SRVM)

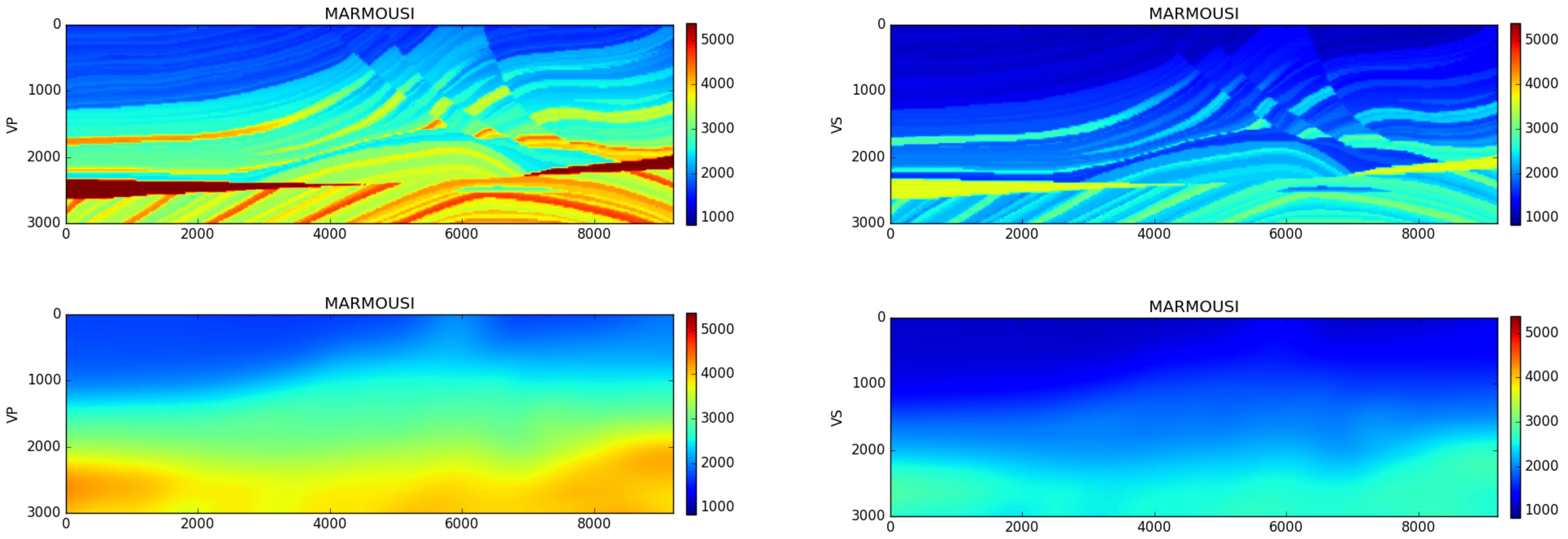



- **SRVM into FWI**

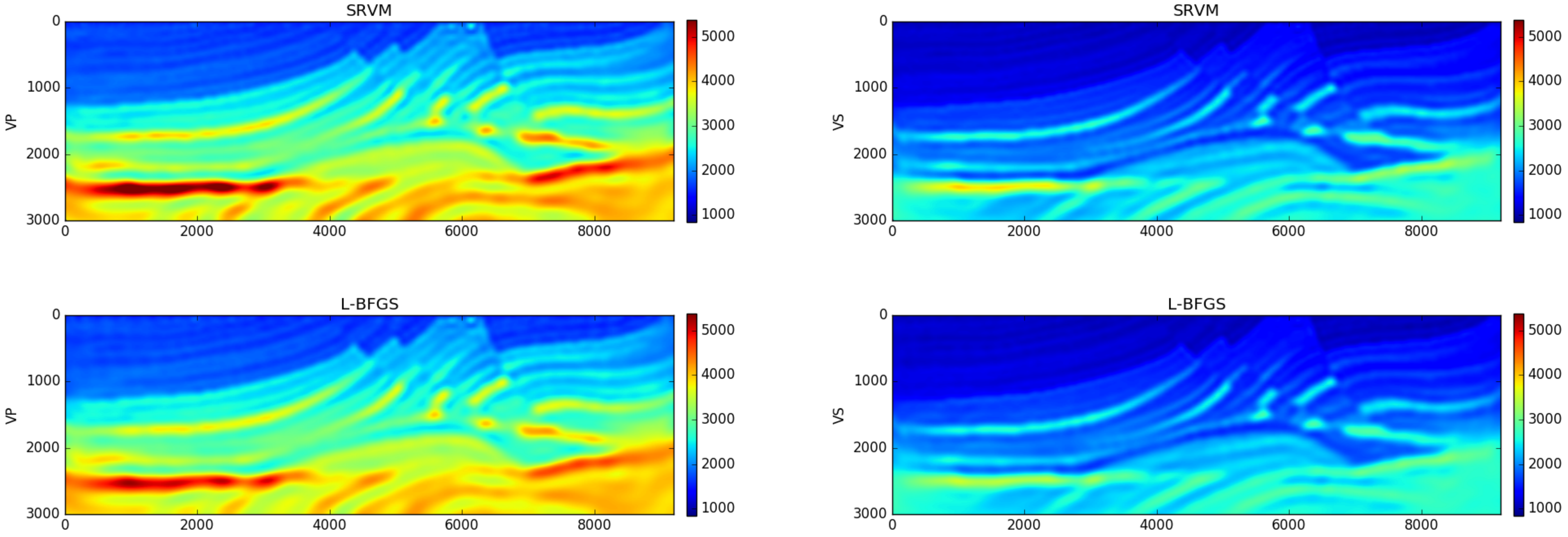


- SRVM based Uncertainty Estimation

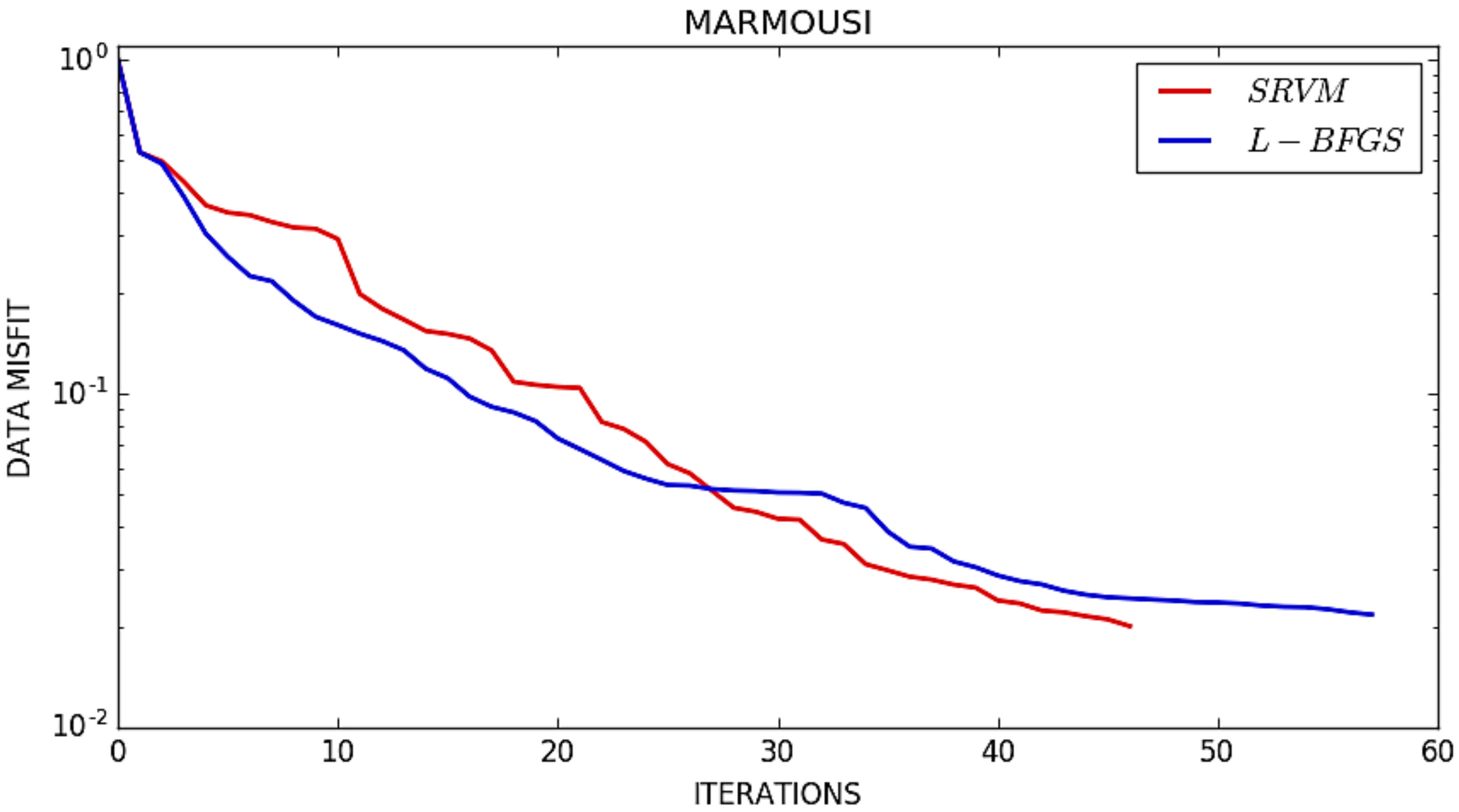
# Marmousi Benchmark *(**True** and **Init** models)*



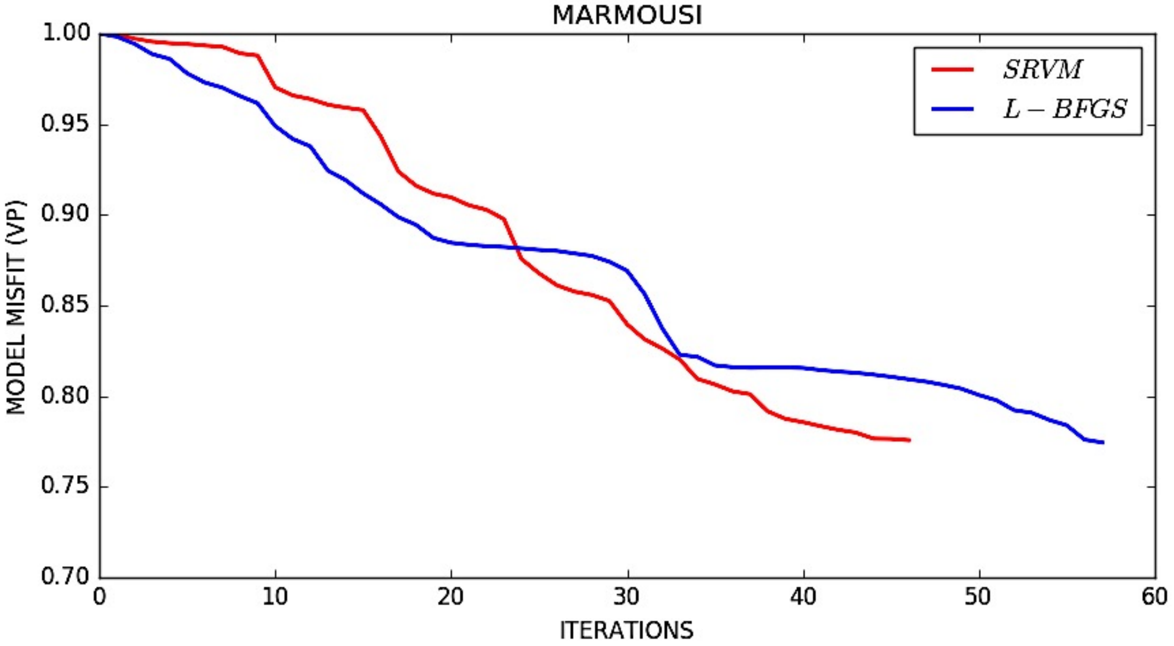
# SRVM *v.s.* L-BFGS in FWI

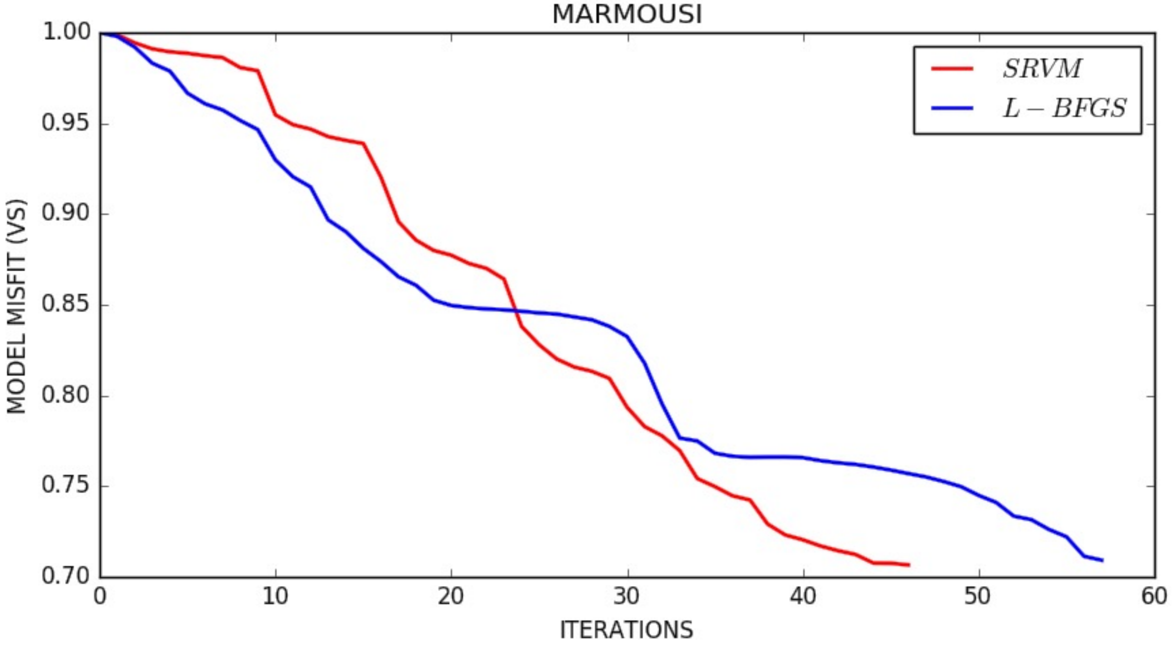


# SRVM *v.s.* L-BFGS: **data-misfit** curve

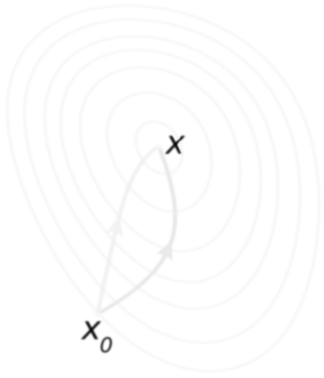


# SRVM *v.s.* L-BFGS: *Vp*, *Vs* **model-misfit** curves



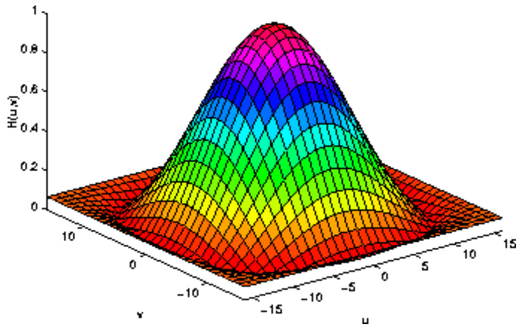
# Outline




- Theory on Square-Root Variable Metric (SRVM)

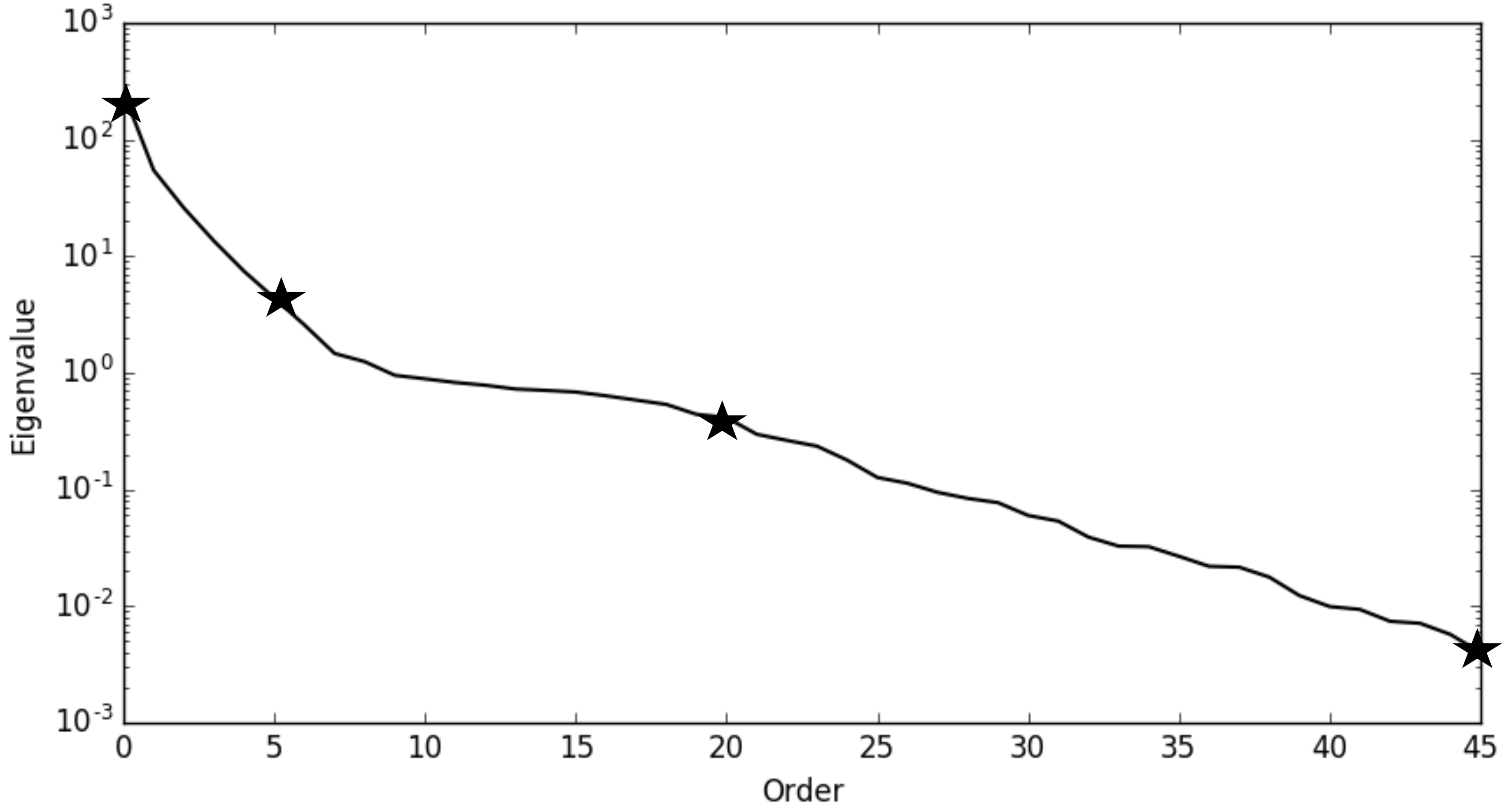


- SRVM into FWI

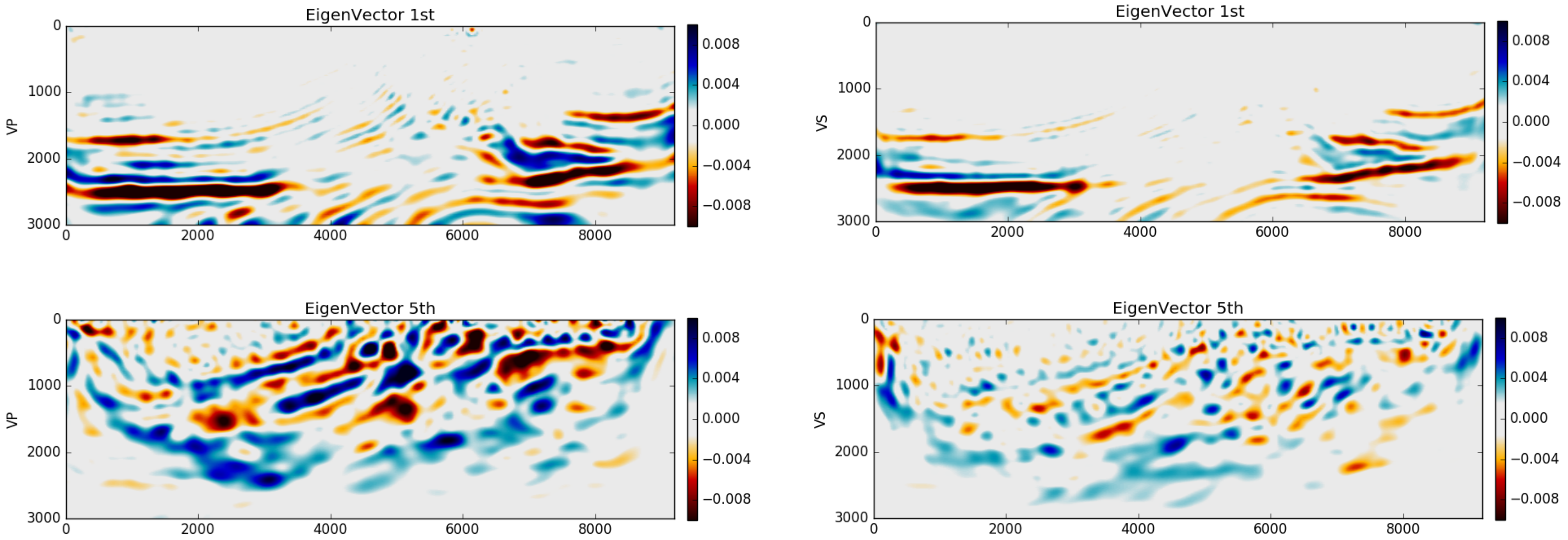


- **SRVM based Uncertainty Estimation**

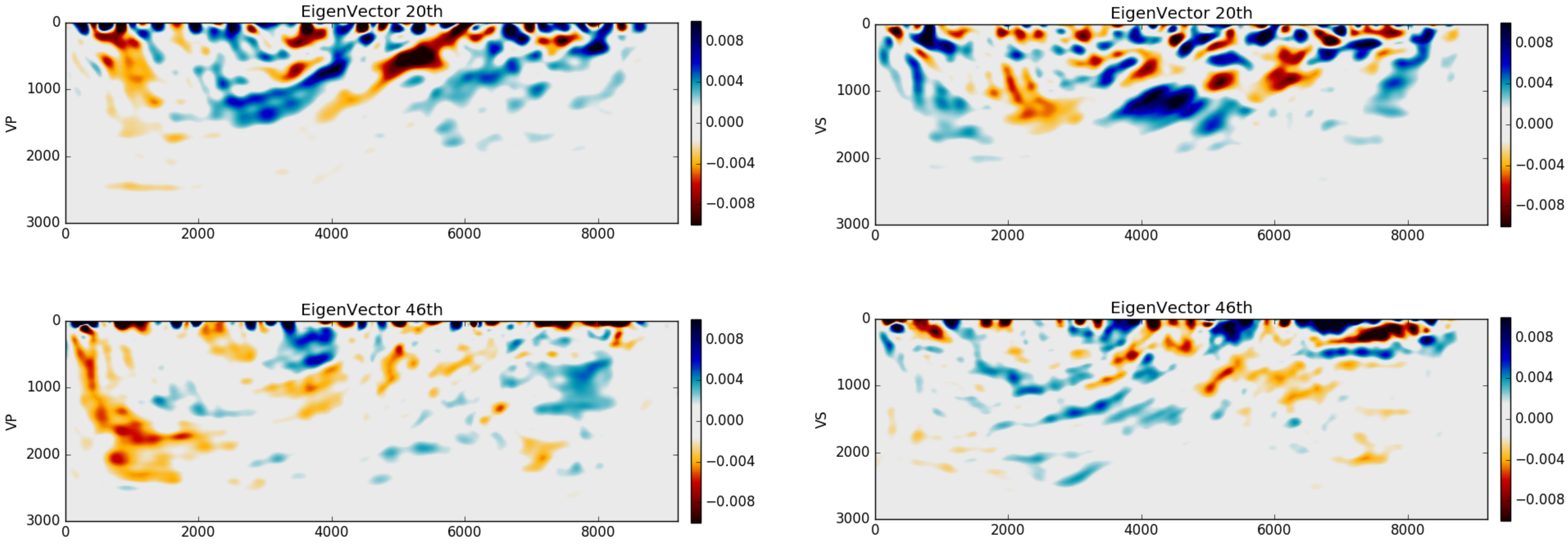
# SRVM with **Randomized SVD** on inverse Hessian



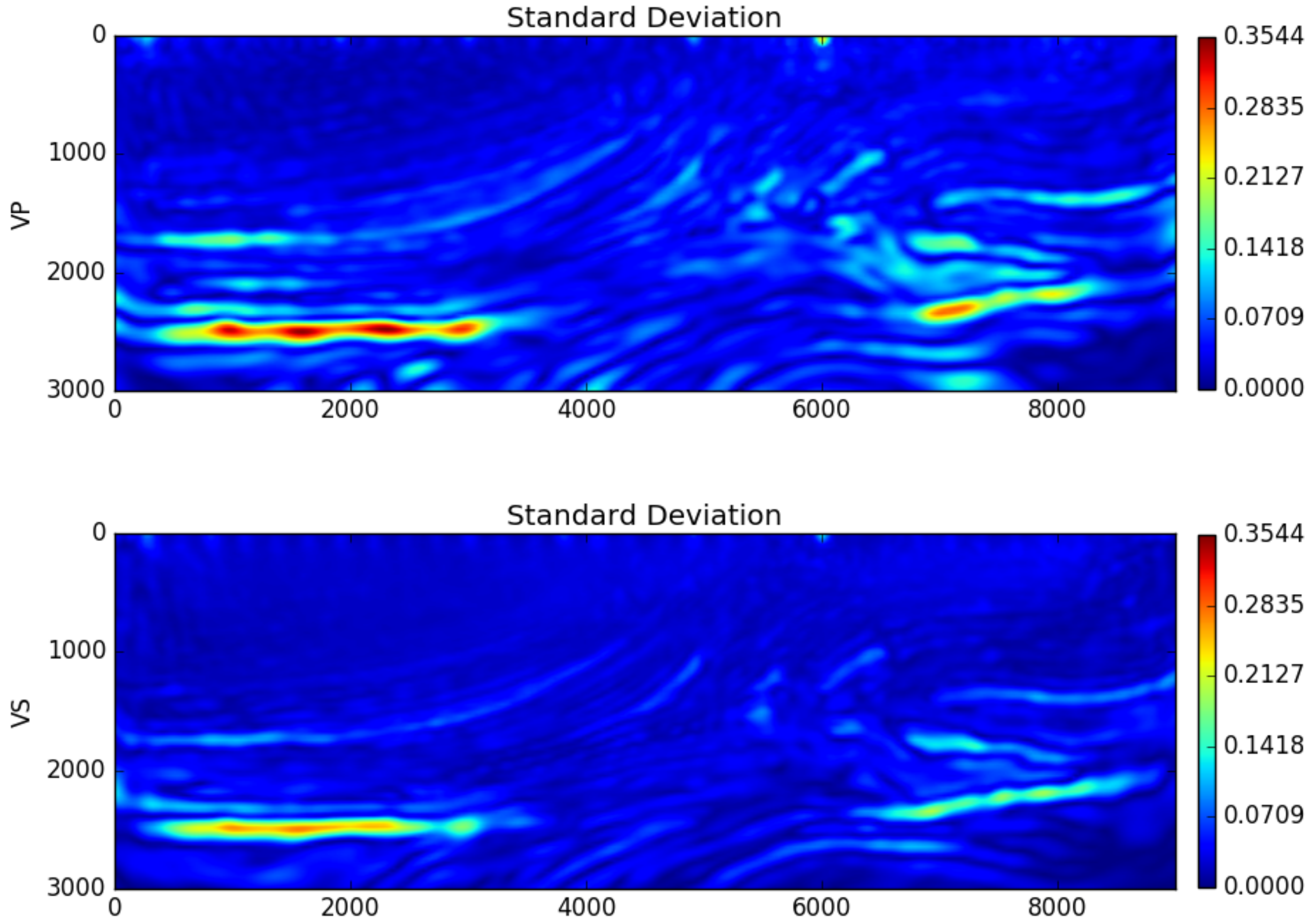
# **Eigenvectors** of inverse Hessian ($1^{st}$, $5^{th}$)



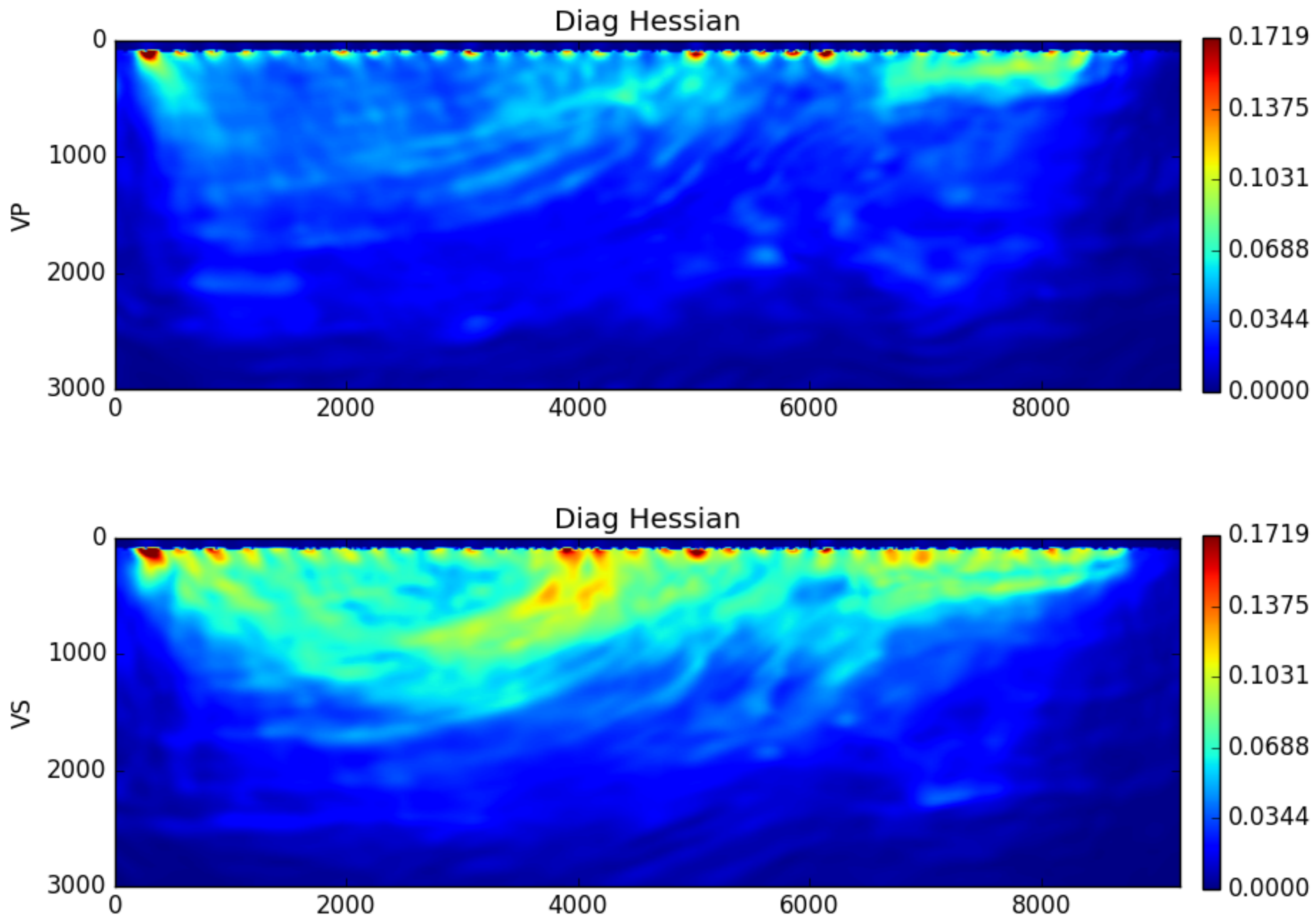
# **Eigenvectors** of inverse Hessian ($20^{th}$, $46^{th}$)



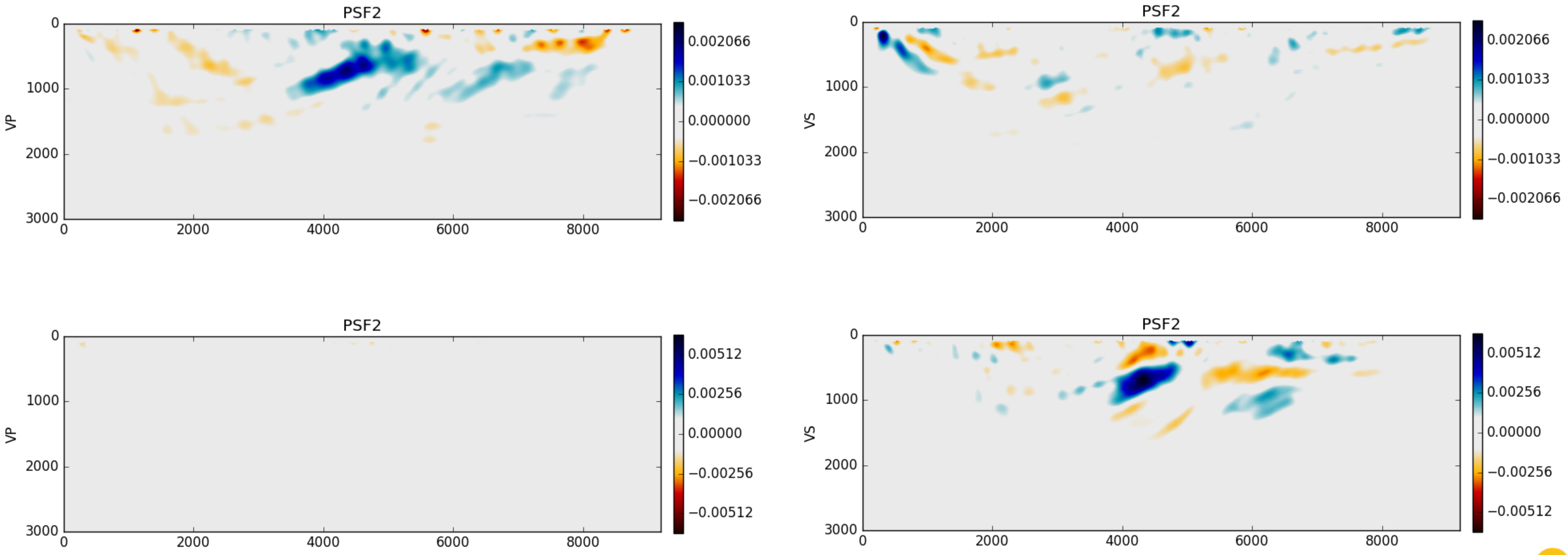
# Uncertainty map: **standard deviation** (*Vp*, *Vs*)



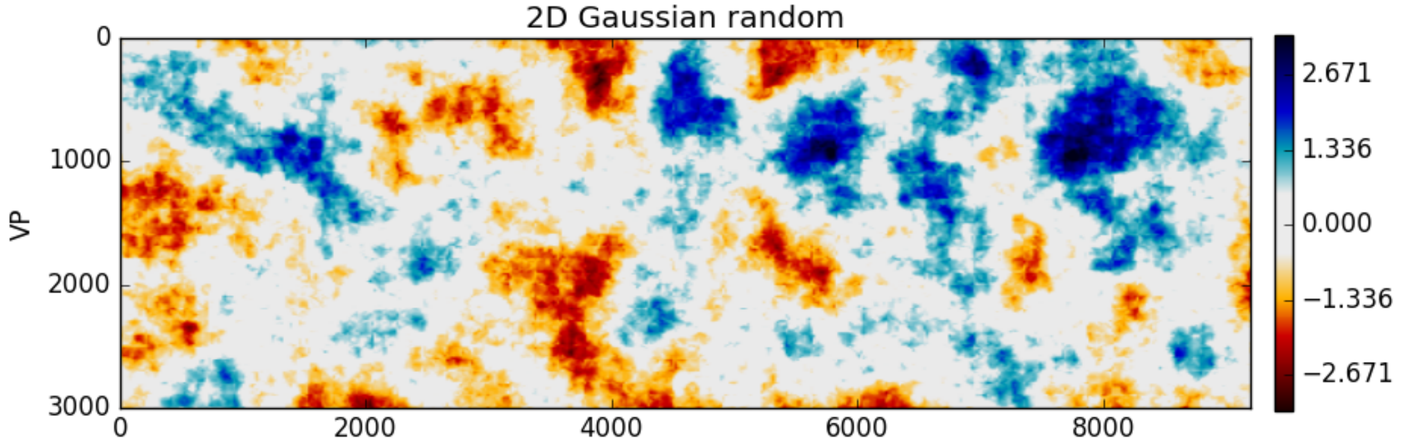
# Diagonal Hessian: *Vp*, *Vs*

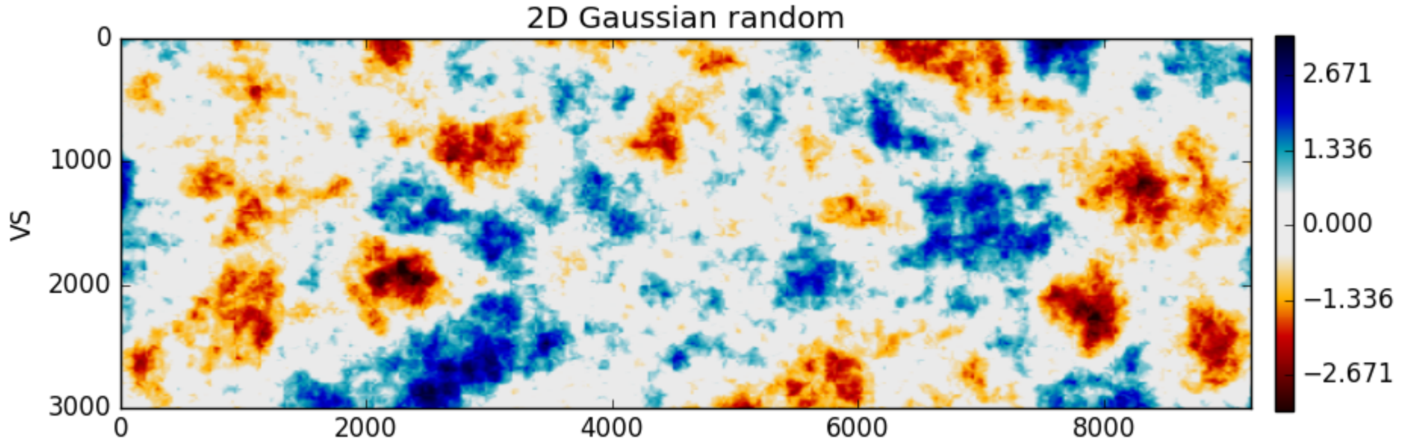


# Point-spread function (PSF): *Vp*, *Vs*

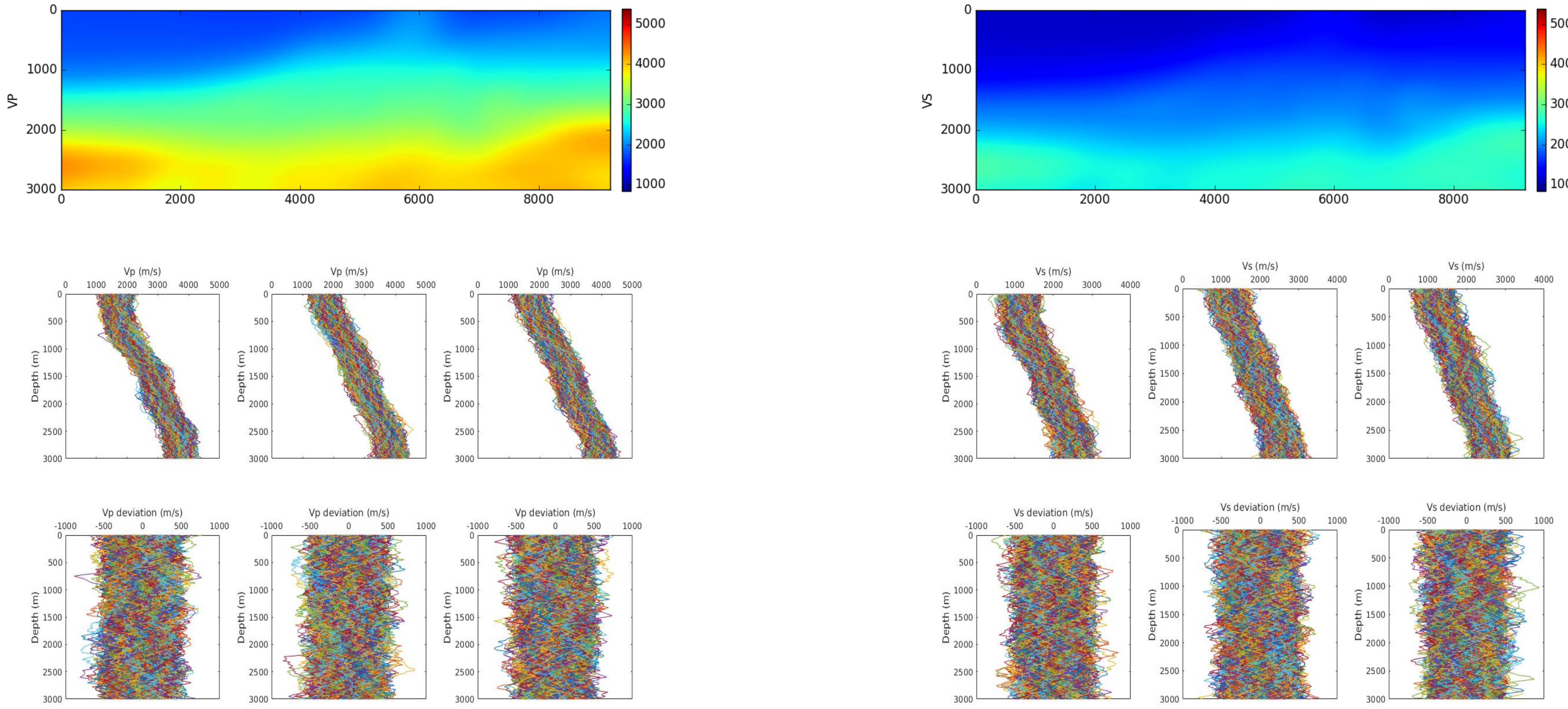


# **2D Gaussian randoms** for prior/posterior samplings

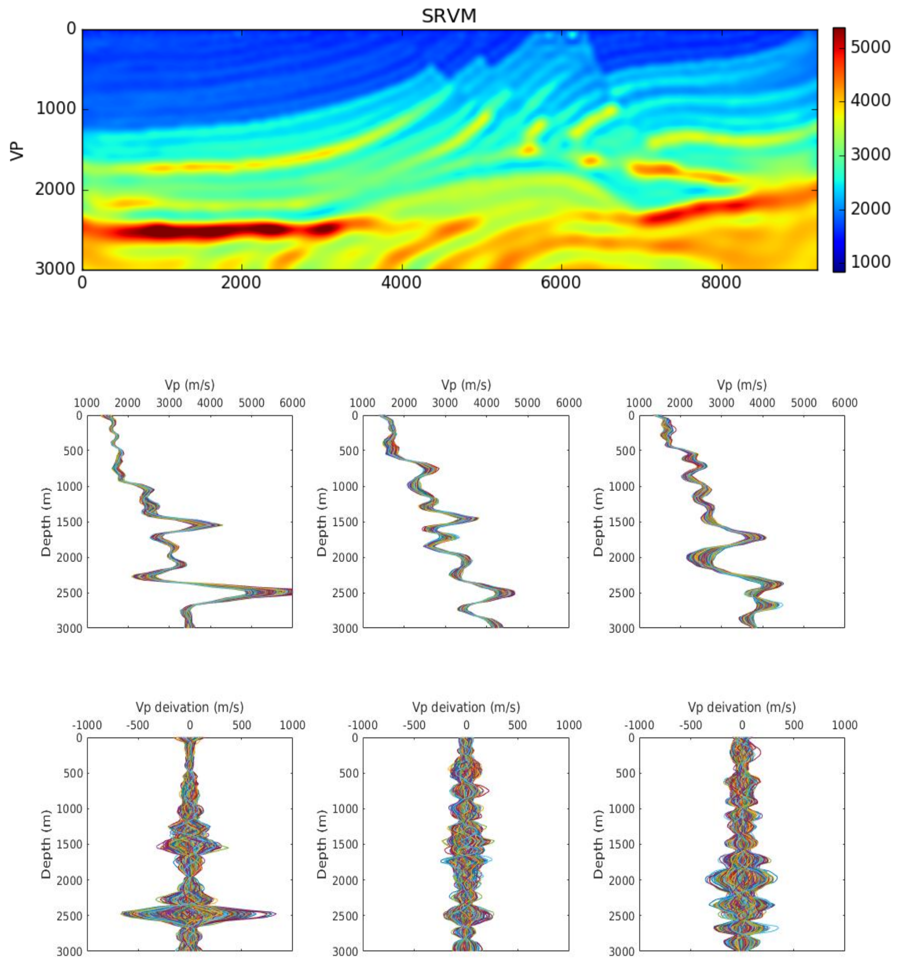


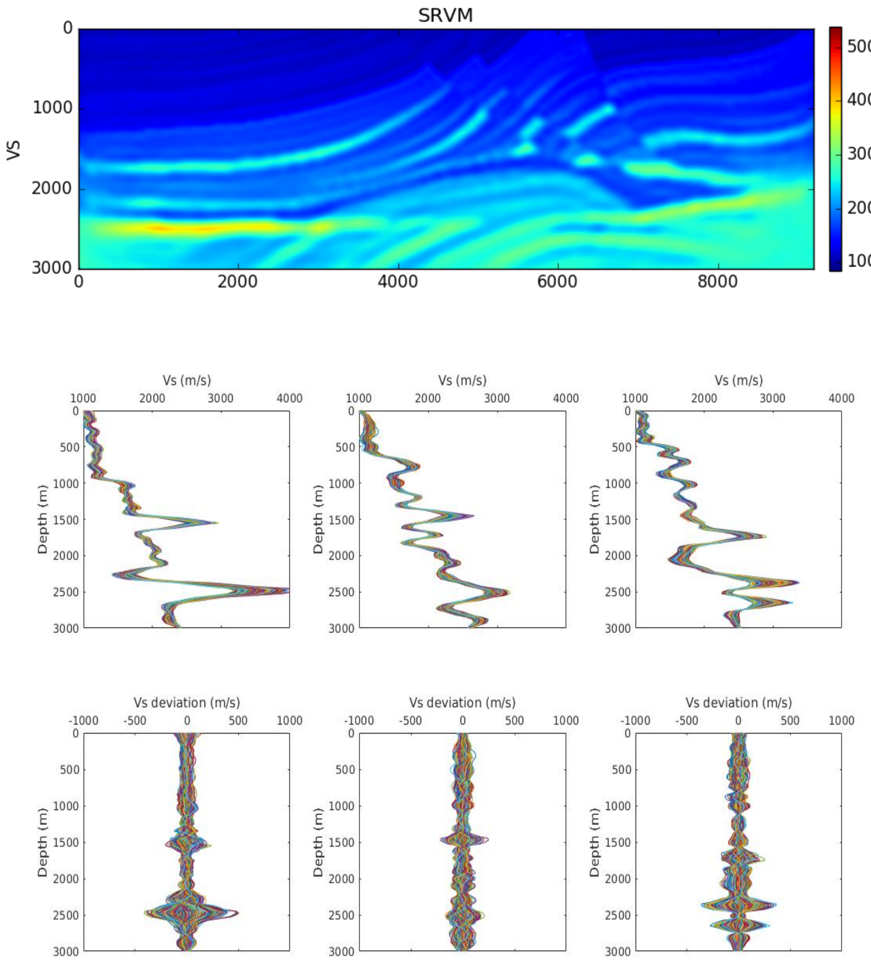



# Prior distribution: *Vp*, *Vs* *(boreholes at X=3100m, 4600m, 6900m)*

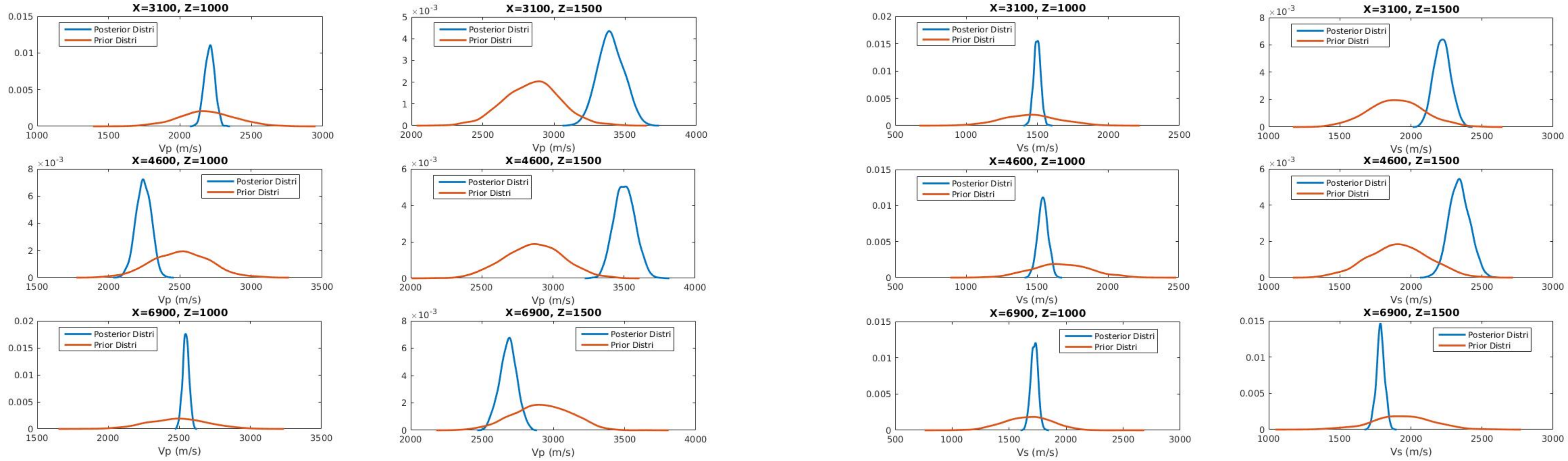


# Posterior distribution: *Vp*, *Vs* *(boreholes at X=3100m, 4600m, 6900m)*



# Prior *v.s.* Posterior Probability density functions: *Vp*, *Vs*



# Conclusions

- As a **quasi-Newton** method, **SRVM** performs as good as **L-BFGS** in our full-space Marmousi testing.

- After FWI is done, we can retrieve the information of inverse Hessian for **Uncertainty Estimation**.

- We employ randomized sampling to develop a **SRVM-RSVD** to facilitate the **Uncertainty Estimation** and **Posterior Sampling**.

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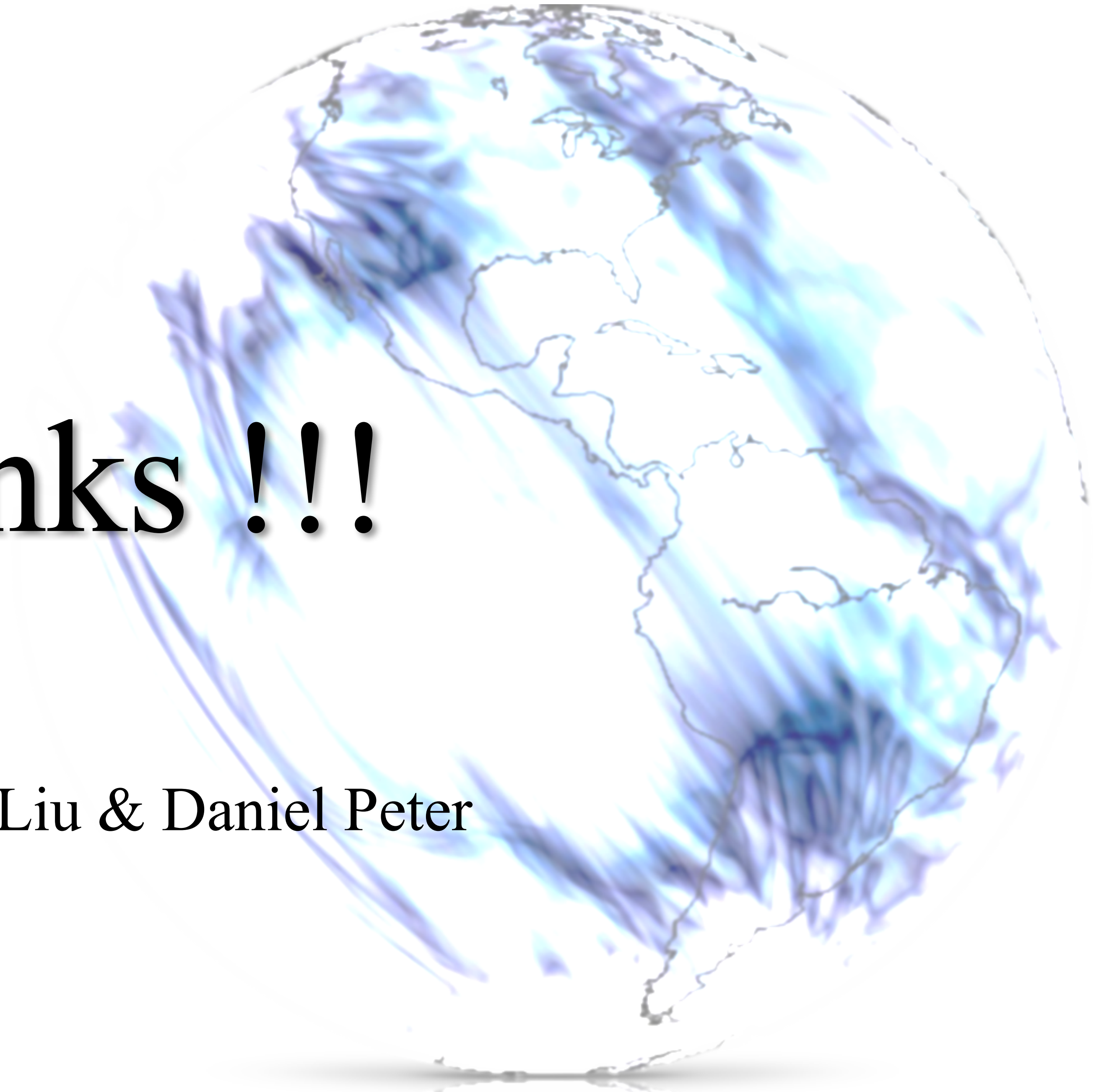

# Thanks !!!

*by* Qiancheng Liu & Daniel Peter