Dual-Tone Radio Interferometric Positioning Systems Using a Single Mobile Anchor

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***Abstract*—In this paper, we propose a low-cost dual-tone radio interferometric positioning system using a single mobile anchor, named mDRIPS. There is no synchronization requirement between the mobile anchor and the target. In mDRIPS, the static target continuously transmits a dual-tone signal, and the mobile anchor receives the signal at different positions along its trajectory. The instability of the target clock is taken into account. An ESPRIT-type algorithm is developed to estimate frequencies of the dual-tone signal for the subsequent phase estimation. Furthermore, the time of arrival (TOA) is extracted from the phase estimate of the received dual-tone signal. After measuring several TOAs at different locations along the anchor's trajectory, the target can be located. The mDRIPS is robust to flat-fading channels. Since the frequency difference of the two tones of the dual-tone signal is designed to be smaller than the channel coherence bandwidth, the same fading effect on these two tones can be eliminated. Moreover, we investigate the integer ambiguity problem due to phase wrapping, and develop a localization algorithm to deal with a simplified ambiguity problem. Numerical results demonstrate the efficiency of the proposed mDRIPS.**

***Index Terms*—Ranging, localization, mobile anchor, radio interferometry, oscillator instability.**

I. Introduction

Location awareness has received lots of interest in many wireless systems such as cellular networks, wireless local area networks, and wireless sensor networks [1]. The positions of wireless terminals are indispensable for location-based services. A large set of wireless network applications [2] require device locations to meaningfully interpret the collected data. The high-cost of ultra-wideband (UWB) localization systems [3] discourage their popularity even with their high localization accuracy. Therefore, accurate and low-cost localization techniques are crucial for the location-based applications with cost-constrained terminals.

Recently, radio interferometric techniques are used for localization by measuring the relative phase offset of interfering radio signals. Maróti et al. propose a radio interferometric positioning system (RIPS) [4] to achieve both high accuracy and low cost. The RIPS uses a radio signal strength indicator (RSSI) to measure the low-frequency differential signal, which is generated by two interfering radios, and estimates its phase to extract range information. However, the RIPS only accommodates additive white Gaussian noise (AWGN) channels, retains approximation errors, and faces the integer ambiguity issue. The RIPS is further extended in [5] to track mobile nodes, where Doppler shifts are explored and velocity estimates of moving targets are achieved. Spinning anchors transmitting radio signals with fixed frequencies (SpinLoc) are employed in [6] to produce specified Doppler signals, and then angle of arrivals (AOAs) are estimated as localization metrics. An asynchronous RIPS (ARIPS) is proposed in [7], where time-difference-of-arrival (TDOA) based on radio interferometric technique is employed to localize asynchronous targets. Nevertheless, the ARIPS is still for AWGN channels and maintains the similar approximation errors as the original RIPS. Moreover, the localization techniques of the above papers [4]–[7] all require several anchors to be synchronized and cooperative with each other, which increase the cost and communication overheads.

In this paper, we propose a low-cost dual-tone radio interferometric positioning system using a single mobile anchor (mDRIPS) in the presence of noise, flat-fading channels and the target oscillator instability. Our work differs from [4]–[7] which require multi-anchor to coordinate. No communication and synchronization overheads are required in our system, since only a single anchor is used for locating the target. Hence, manipulating a single receiver is cost-efficient than operating several receivers simultaneously. In mDRIPS, the static target continuously transmits a dual-tone signal and the anchor receives the signal along its trajectory. The instability of the target clock is taken into account. An ESPRIT-type algorithm is developed to estimate frequencies of the dual-tone signal for subsequent phase estimation. Further, the time of arrival (TOA) can be extracted from the phase of the received dual-tone signal using the estimated frequencies. After measuring several TOAs at different positions along the anchor's trajectory, the target's location can be estimated by the mobile anchor. Since the frequencies of two tones transmitted by the target are only slightly different, and the difference is less than the channel coherence bandwidth. The same fading effect is experienced by the two tones, and it can be eliminated. Thus, the mDRIPS is robust to flat-fading channels. Furthermore, we investigate the integer ambiguity problem due to phase wrapping, and develop a localization algorithm to deal with the ambiguity problem.

In the rest of the paper, Section II presents the system model of the mDRIPS. The range estimation method is provided in Section III, and the localization algorithm is given in Section IV. Section V provides simulation results.

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II. SYSTEM MODEL

The localization scenario of the mDRIPS is shown in Fig. 1, the mobile anchor works in a periodic manner with a time-slot duration T. Each time slot is divided into receiving and moving mini slots. The target continuously transmits a dual-tone signal. The mobile anchor moves to a new position by every slot T , and receives the target signal at the receiving mini slot. The receiving mini-slot duration is T_d. The received signal at location L_0 will be used to estimate the received tone frequencies, which are used for subsequent TOA estimation. The receiving and moving steps repeat until several TOA measurements are collected. Finally, the localization algorithm is adopted to locate the target based on the estimated TOAs.

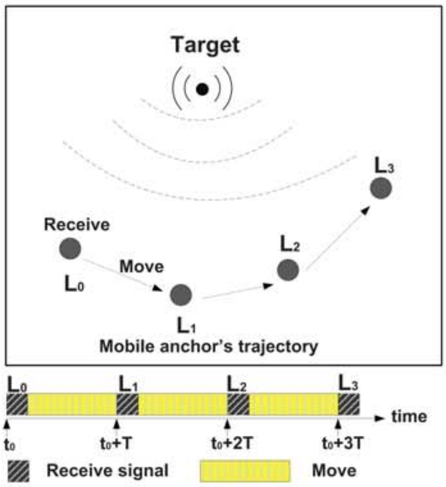

Fig. 1: Target localization by a mobile anchor.

The anchor clock is assumed to be calibrated and thus accurate. Taking the anchor clock as a reference, similar to [8], the target clock model is

$$c(t) = wt + \Delta, \tag{1}$$

where $w \triangleq 1 + \mu$ and is the target clock skew with respect to (w.r.t.) the anchor clock, Δ is the initial clock offset, and $c(t)$ is the target local time relative to the anchor's clock. Note that μ is typically very small, measured in parts per million (ppm). It is well known that the obvious time-varying clock skew phenomenon could be observed over day timescale for highly reliable CPU oscillators and seconds or minutes timescales for low-cost wireless sensors [8]. Hence, we assume that the target clock skew w is invariable during each localization procedure.

The dual-tone signal transmitted by the target is

$$s(t) = ae^{j2\pi(f_b+f_c)c(t)}(1 + e^{j2\pi g_b c(t)}), \tag{2}$$

where a is the real-valued amplitude of the dual-tone signal, f_c is the carrier frequency, f_b is the basic frequency and greater than zero, g_b is the small frequency difference between the two tones, and $c(t)$ is the local time of the target. Since g_b is smaller than the channel coherence bandwidth, the two tones will experience the same channel fading effect. As a result, a flat-fading channel model is applied to account for the fading effect. Although we only consider a single target localization problem, it is easy for us to expand to multi-target scenario by assigning different basic frequencies f_b to different targets.

In mDRIPS, a mobile anchor is adopted to receive the signal of the target along its trajectory. The mobile anchor estimates the corresponding TOAs of the target at N different positions $L_0, L_1, ..., L_{N-1}$. Taking the anchor clock as a reference, the unknown time instant of the target initiating the transmission w.r.t. the anchor's clock is t_0. When the anchor arrives at the lth position and starts to receive the signal, this instant can also be interpreted as

$$t_l = t_0 + lT, \quad 0 \le l \le N-1. \tag{3}$$

Without loss of generality, the received signal of the anchor at the lth position is down converted by f_c, and can be modeled as

$$r_l(t) = \beta_l s(t - t_l - \tau_l)e^{-j2\pi f_c t}, \tag{4}$$

where β_l is the complex channel coefficient for the flat-fading channel, and can be modeled as a zero-mean complex Gaussian random variable with variance σ_β^2 representing the average power of the flat-fading channel. Moreover, t_l is the transmission time instant of the target when the anchor arrives at the lth position, τ_l is the unknown propagation delay. It is related to the distance as $d_l = c\tau_l$, where c is the signal propagation speed, and d_l is the distance between the target and the lth position of the mobile anchor. Note that for simplicity noise is neglected in (4). However, the proposed algorithm is able to accommodate any noise. Plugging (2) into (4), we obtain

$$r_l(t) = \alpha\beta_l(e^{j2\pi f_1 t}e^{-j\theta_l} + e^{j2\pi f_2 t}e^{-j(\theta_l+\varphi_l)}). \tag{5}$$

where $f_1 = f_b w + f_c\mu$, $f_2 = f_b w + f_c\mu + g_b w$, $\theta_l = 2\pi w(f_b + f_c)(\tau_l + t_l)$, $\varphi_l = 2\pi g_b w(\tau_l + t_l - \Delta/w)$, and $\alpha = ae^{j2\pi(f_b+f_c)\Delta}$. Observe that the received tone frequencies f_1, f_2 are different from the transmission frequencies $f_b, f_b + g_b$ because of the clock skew. The accurate phase information φ_l cannot be obtained without the exact estimation of f_1, f_2. Hence, in Section III, the clock skew problem will be solved and the phase information will be estimated sequentially.

III. RANGE ESTIMATION IN THE PRESENCE OF OSCILLATOR INSTABILITY

The received dual-tone signal $r_l(t)$ is directly sampled with Nyquist rate f_s and M samples are collected into a vector $\mathbf{r}_l$ as follows

$$\mathbf{r}_l = \mathbf{H}_l\mathbf{y}_l, \tag{6}$$

where $\mathbf{y}_l = \alpha\beta_l[e^{-j\theta_l}, e^{-j(\theta_l+\varphi_l)}]^T$,
$\mathbf{H}_l = [\mathbf{\Phi}_l(f_1, f_s, t_l) \quad \mathbf{\Phi}_l(f_2, f_s, t_l)]$,
$\mathbf{\Phi}_l(f, f_s, t_l) = [e^{j2\pi f t_l}, \ldots, e^{j2\pi f(t_l+(M-1)/f_s)}]^T$.

We remark here the Nyquist rate $f_s > 2f_2$ is small according to the design of the tone frequencies, which is easy for handling even with cost-constraint devices. Moreover, instead of the squaring operation to generate a low-frequency differential signal as [4]–[6] , the direct sampling technique can get rid of the auto-correlation term of the noise and the cross-correlation term between the signal and the noise.

Note that in (6) the propagation delay (τ_l) is borne by the phase of the received dual-tone signal. The estimates of θ_l and φ_l depend on the accurate knowledge of the received dual-tone frequencies f_1, f_2. Therefore, we need to estimate frequencies at first, and then the estimated parameters will be used sequentially.

A. Frequency Estimation: ESPRIT

The ESPRIT algorithm is proposed in [9] to estimate direction-of-arrival (DOA) via rotational invariance techniques. In mDRIPS, an ESPRIT-type algorithm is designed to estimate f_1 and f_2 based on the received dual-tone signal at the first position $(l = 0)$ of the mobile anchor. Since the target clock skew is assumed to be invariant during the whole localization session, the estimated frequencies can be used in the rest ranging and localization steps.

As in [7], the sample vector $\mathbf{r}_0$ is rearranged into a matrix $\mathbf{R}$ of size $p \times q$ with $q = M - p + 1$ as

$$\mathbf{R} = \begin{bmatrix}[\mathbf{r}_0]_{1:p} & [\mathbf{r}_0]_{2:p+1} & [\mathbf{r}_0]_{3:p+2} & \cdots & [\mathbf{r}_0]_{q:M}\end{bmatrix}, \quad (7)$$

where $[\mathbf{a}]_{m:n}$ represents the vector composed of the mth to the nth elements of the vector $\mathbf{a}$. The matrix $\mathbf{R}$ can be factorized as

$$\mathbf{R} = [\mathbf{H}_0]_{1:p}\mathrm{diag}(\mathbf{y}_0)[\mathbf{H}_0]_{1:q}^T, \quad (8)$$

where $[\mathbf{H}_0]_{1:p}$ represents the first p rows of matrix $\mathbf{H}_0$, and the operator T is transposition. When $[\mathbf{H}_0]_{1:p}$ and $[\mathbf{H}_0]_{1:q}$ are tall matrices, the rank of $\mathbf{R}$ is no greater than 2 and $\mathbf{R}$ retains the shift-invariant property [9]. We take submatrices of $\mathbf{R}$ as

$$\mathbf{R}_1 = [\mathbf{I}_{p-1} \quad \mathbf{0}_{p-1}]\mathbf{R} = [\mathbf{H}_0]_{1:p-1}\mathrm{diag}(\mathbf{y}_0)[\mathbf{H}_0]_{1:q}^T, \quad (9)$$

$$\mathbf{R}_2 = [\mathbf{0}_{p-1} \quad \mathbf{I}_{p-1}]\mathbf{R} = [\mathbf{H}_0]_{1:p-1}\mathbf{\Sigma}\mathrm{diag}(\mathbf{y}_0)[\mathbf{H}_0]_{1:q}^T, \quad (10)$$

where $\mathbf{I}_{p-1}$ is the identity matrix with size $p-1$, $\mathbf{0}_{p-1}$ is a zero vector with size $p-1$ and $\mathbf{\Sigma} = \mathrm{diag}([e^{j2\pi f_1/f_s}, e^{j2\pi f_2/f_s}])$.

Therefore, by applying the ESPRIT algorithm based on the special relationship between $\mathbf{R}_1$ and $\mathbf{R}_2$, we achieve at

$$(\mathbf{R}_1)^\dagger\mathbf{R}_2 = \mathbf{T}^\dagger\mathbf{\Sigma}\mathbf{T}. \quad (11)$$

where $\mathbf{T} = \mathrm{diag}(\mathbf{y}_0)[\mathbf{H}_0]_{1:q}^T$, and the operator $\dagger$ is pseudo-inverse. The size of $(\mathbf{R}_1)^\dagger\mathbf{R}_2$ is $q \times q$, and its rank is 2. We apply eigenvalue decomposition and represent the eigenvalue estimates of $(\mathbf{R}_1)^\dagger\mathbf{R}_2$ as $\hat{\lambda}_0, \hat{\lambda}_1, \ldots, \hat{\lambda}_{q-1}, \hat{\lambda}_q$ in an ascending order by the amplitudes of the eigenvalues. Thus, the target dual-tone signal frequencies can be estimated based on the two largest eigenvalues as follows:

$$\hat{f}_1 = \frac{f_s}{2\pi}\arg(\hat{\lambda}_{q-1}) \quad (12)$$

$$\hat{f}_2 = \frac{f_s}{2\pi}\arg(\hat{\lambda}_q). \quad (13)$$

where $\arg(x)$ is the phase of variable x and $\hat{w} = \frac{(\hat{f}_1+f_c)}{f_b+f_c}$.

Moreover, $\mathbf{R}_1$ and $\mathbf{R}_2$ should be low-rank to employ the ESPRIT algorithm. Hence, $[\mathbf{H}_0]_{1:p-1}, [\mathbf{H}_0]_{1:q}$ should be strictly tall , $p > 3$ and $q > 2$ should be satisfied.

B. Ranging Estimation

With the estimates of f_1 and f_2, we construct $\hat{\mathbf{H}}_l = [\mathbf{\Phi}_l(\hat{f}_1, f_s, t_l) \quad \mathbf{\Phi}_l(\hat{f}_2, f_s, t_l)]$. Note $\mathbf{r}_l$ is the received signal vector of the mobile anchor's lth measurement and the estimation of $\mathbf{y}_l$ can be obtained through a least squares (LS) estimator

$$\hat{\mathbf{y}}_l = (\hat{\mathbf{H}}_l)^\dagger\mathbf{r}_l. \quad (14)$$

Based on (14), the phase of interest φ_l can be estimated as

$$\hat{\varphi}_l = \arg\{e^{-j2\pi g_b\hat{w}lT}[\hat{\mathbf{y}}_l]_1[\hat{\mathbf{y}}_l]_2^*\} + 2\pi m, \quad (15)$$

the operator $*$ is complex conjugate, and an unknown integer m is introduced due to the phase wrapping. We remark here that the effects of clock offset (Δ) and the flat-fading channel (β_l) are eliminated via (15). Hence, the mDRIPS is immune to the flat-fading channel. However, based on the phase estimation $\hat{\varphi}_l$ we can only obtain the biased TOA information, which is coupled with clock offset Δ and unknown time instant t_0, and faces the integer ambiguity issue.

C. Integer Ambiguity Issue

Since the integer ambiguity problem due to phase wrapping [10] exists in mDRIPS, the estimate of φ_l cannot be calculated from (15) because of the unknown integer m. Hence, we can only achieve $\hat{\tilde{\varphi}}_l$ instead of $\hat{\varphi}_l$, where $\hat{\tilde{\varphi}}_l$ is the estimate of $\tilde{\varphi}_l$ and $\tilde{\varphi}_l = \varphi_l - 2\pi\lfloor\frac{\varphi_l}{2\pi}\rfloor$. Recall that τ_l is coupled with t_0 and Δ as $\varphi_l = 2\pi g_b w(\tau_l + t_0 - \Delta/w)$. Let us define $\varepsilon_l = 2\pi g_b w\tau_l$ and $\tilde{\varepsilon}_l = \varepsilon_l - 2\pi\lfloor\frac{\varepsilon_l}{2\pi}\rfloor$, $\delta = 2\pi g_b w(t_0 - \Delta/w)$ and $\tilde{\delta} = \delta - 2\pi\lfloor\frac{\delta}{2\pi}\rfloor$, then we achieve at

$$\left\lfloor\frac{\varphi_l}{2\pi}\right\rfloor = \begin{cases} \left\lfloor\frac{\delta}{2\pi}\right\rfloor + \left\lfloor\frac{\varepsilon_l}{2\pi}\right\rfloor, & 0 \le \tilde{\delta} + \tilde{\varepsilon}_l < 2\pi \\ \left\lfloor\frac{\delta}{2\pi}\right\rfloor + \left\lfloor\frac{\varepsilon_l}{2\pi}\right\rfloor + 1, & 2\pi \le \tilde{\delta} + \tilde{\varepsilon}_l < 4\pi \end{cases}, \quad (16)$$

Based on (16) and $\tilde{\varphi}_l = \varphi_l - 2\pi\lfloor\frac{\varphi_l}{2\pi}\rfloor$, we simplify the integer ambiguity issue as

$$\tilde{\varphi}_l = \begin{cases} \tilde{\delta} + \tilde{\varepsilon}_l, & 0 \le \tilde{\delta} + \tilde{\varepsilon}_l < 2\pi \\ \tilde{\delta} + \tilde{\varepsilon}_l - 2\pi, & 2\pi \le \tilde{\delta} + \tilde{\varepsilon}_l < 4\pi \end{cases}. \quad (17)$$

From (17), the phase offset estimation exists two possible values, we could not confirm the exact value only based on the estimation of $\hat{\tilde{\varphi}}_l$ because of the integer ambiguity. In Section IV, the integer ambiguity problem can be solved through simplification and multiple measurements at different positions.

IV. Localization Algorithm in mDRIPS

Observe that $\tilde{\varepsilon}_l = \varepsilon_l - 2\pi\lfloor\varepsilon_l/2\pi\rfloor = \varepsilon_l - 2\pi k$, where k is an integer and related to ε_l. Note $\varepsilon_l = 2\pi g_b w\tau_l$. It is the phase offset due to the propagation delay and $\varepsilon_l = 2\pi g_b w d_l/c$. Given the maximum ranging distance between the target and the mobile anchor, the possible value of k can be determined. For example, if $0 < \varepsilon_l < 2\pi$ equal to $d_l < c/g_b w$, and

$k = 0$ is obtained. Usually $w \approx 1$ and g_b is designed to be small to benefit the dual-tone signal experiencing a flat-fading channel. Given $g_b = 50\,\text{kHz}$, the maximum ranging distance without ambiguity is $6\,\text{km}$. Moreover, if $0 < \varepsilon_l < 4\pi$, the possible value of k is 0 or 1 and the maximum ranging distance is $12\,\text{km}$ given $g_b = 50\,\text{kHz}$. In mDRIPS, the maximum value of k is bounded by $\Bbbk$ according to the ranging distance requirement. Hence, (17) can be transformed to

$$\frac{\tilde{\varphi}_l c}{2\pi g_b w} = \begin{cases} b_0 + d_l - \dfrac{kc}{g_b w}, & 0 \le \tilde{\delta} + \tilde{\varepsilon}_l < 2\pi \\ b_0 + d_l - \dfrac{(k+1)c}{g_b w}, & 2\pi \le \tilde{\delta} + \tilde{\varepsilon}_l < 4\pi \end{cases}, \tag{18}$$

where $b_0 = \tilde{\delta}c/2\pi g_b w$, $0 \le k \le \Bbbk$ and $0 \le l \le N-1$.

We collect all the phase estimates $\hat{\tilde{\varphi}}_l$ into a vector as $\mathbf{v} = c/2\pi g_b \hat{w}[\hat{\tilde{\varphi}}_0, \hat{\tilde{\varphi}}_1, \ldots, \hat{\tilde{\varphi}}_{N-1}]^T$. Consequently, the model of $\mathbf{v}$ can be given by

$$\mathbf{v} = \mathbf{d} + b_0 \mathbf{1}_N + \frac{c}{g_b \hat{w}}\mathbf{u}, \tag{19}$$

where $\mathbf{d} = [d_0, \ldots, d_{N-1}]^T$ with $d_l = ||\mathbf{s}_l - \mathbf{x}||$, $\mathbf{s}_l$ denotes the coordinate of the lth position of the mobile anchor, $\mathbf{x}$ is the coordinate of the target and $\mathbf{u} \in \Omega^N (\Omega = \{0, -1, \ldots, -(\Bbbk + 1)\})$.

It is difficult to solve (19) directly because the nonlinear relationship with unknown $\mathbf{x}$. Hence, we transfer (19) to a linear model at first. Let us assume the number of time slots for locating the target is small (e.g.$N = 16$ is enough to locate the target with high accuracy), and $\Bbbk \le 2$ for a large ranging area ($12\,\text{km}$ in mDRIPS). The value of $\mathbf{u}$ is in a limited set, and can be enumerated without high complexity. Hence, $\mathbf{u}$ is categorized as a known subset, while the unknown parameters $\mathbf{x}$ and b_0 as the other. Moving $c/g_b\hat{w}\mathbf{u}$ and $b_0\mathbf{1}_N$ to the left side of (19) and making element-wise multiplication, we obtain a linear model with the unknown parameters as

$$\boldsymbol{\Theta} - \tilde{\mathbf{v}} \odot \tilde{\mathbf{v}} = 2\mathbf{S}^T\mathbf{x} - 2b_0\tilde{\mathbf{v}} + (b_0 - ||\mathbf{x}||^2)\mathbf{1}_N, \tag{20}$$

where $\tilde{\mathbf{v}} = \mathbf{v} - c/g_b\hat{w}\mathbf{u}$, $\mathbf{S} = [\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_{N-1}]$, and $\boldsymbol{\Theta} = [||\mathbf{s}_0||^2, ||\mathbf{s}_1||^2, \ldots, ||\mathbf{s}_{N-1}||^2]^T$. We apply a brute-force search for all possible values of $\mathbf{u}$. The nth candidate of $\mathbf{u}$ is denoted by $\mathbf{u}^{(n)}$. With the candidate $\mathbf{u}^{(n)}$, the corresponding estimates of $\mathbf{x}^{(n)}$ and $b_0^{(n)}$ can be achieved based on (20) using an LS estimator. Substituting $\hat{\mathbf{x}}^{(n)}$ and $\hat{b}_0^{(n)}$ into (19), the estimation of $\hat{\mathbf{u}}^{(n)}$ can be caculated by a simple rounding operation as

$$\hat{\mathbf{u}}^{(n)} = \begin{cases} 0, & \text{if} \quad \hbar \ge 0 \\ -(\Bbbk+1), & \text{if} \quad \hbar \le -(\Bbbk+1) \\ \hbar, & \text{else} \end{cases}. \tag{21}$$

where $\hbar = \text{round}(g_b\hat{w}/c(\mathbf{v} - \hat{\mathbf{d}} - \hat{b}_0\mathbf{1}_N))$. The search process will be terminated if $\hat{\mathbf{u}}^{(n)} = \mathbf{u}^{(n)}$, and $\hat{\mathbf{x}}$ is the estimated coordinate of the target. Otherwise, we continue to search until all possible values of $\mathbf{u}$ are enumerated. The localization algorithm based on the brute-force search for $\mathbf{u}$ combined with the LS estimator is shown in Algorithm 1.

We remark here the observation matrix $\mathbf{A} = [\mathbf{S}^T, \tilde{\mathbf{v}}, \mathbf{1}_N]$ in (20) should be full rank to guarantee the identifiability of $\mathbf{x}$ by the LS estimator. Hence, the moving trajectory of the mobile anchor should not be linear.

Algorithm 1 Localization algorithm with brute-force search.

1: Initial step: enumerate all possible values of $\mathbf{u}^{(n)}$, set $n = 0$ and $\mathbf{u}^{(n)} = \mathbf{0}_N$;
2: Estimate $\hat{\mathbf{x}}^{(n)}$ and $\hat{b}_0^{(n)}$ given $\mathbf{u}^{(n)}$ based on the linear model (20) using an LS estimator;
3: Calculate $\hat{\mathbf{u}}^{(n)}$ given $\mathbf{x}^{(n)}$ and $\hat{b}_0^{(n)}$ using (21);
4: If $\hat{\mathbf{u}}^{(n)} = \mathbf{u}^{(n)}$, terminate and return $\mathbf{x}^{(n)}$;
5: $n = n + 1$, go back to step 2 unless reach the maximum iteration number.

V. Simulation and Results

In this section, the performance of the mDRIPS is evaluated from three aspects: frequency estimation by the ESPRIT-type algorithm, ranging and localization by the LS estimator. The median absolute error (MAE) is used as a performance metric in the simulations to mitigate the effects of the outliers due to deep fading. The amplitude of the dual-tone signal $a = 1$ and the carrier frequency $f_c = 434\,\text{MHz}$. The basic frequency of the dual-tone signal $f_b = 1\,\text{MHz}$ and the frequency difference $g_b = 50\,\text{kHz}$, which is smaller than the typical channel coherence bandwidth at the VHF band [11]. Moreover, the average channel power of the flat-fading channel is assumed to be 1, thus $\sigma_\beta^2 = 1$. In the simulations, we assume that the noise term is modeled as a zero mean complex Gaussian random process. The signal-to-noise ratio (SNR) is defined as $1/\sigma_n^2$, where σ_n^2 is the variance of the noise term. The clock skew w varies from each Monte Carlo run within a bound $[1 - 10\,\text{ppm}, 1 + 10\,\text{ppm}]\,(1\,\text{ppm} = 10^{-6})$. Note that this bound can cause a frequency offset up to $4.35\,\text{kHz}$ in the mDRIPS. The sampling duration T_d and time slot period T of the mobile anchor are $1\,\text{ms}$ and $10\,\text{s}$. For each evaluation, 1000 Monte-Carlo runs are carried out.

A. Estimation Accuracy of ESPRIT

Due to the target oscillator instability, the dual-tone signal frequencies f_1, f_2 are estimated by the ESPRIT-type algorithm before the ranging and localization steps. The true clock skew $w = 1 + 10\,\text{ppm}$, and the number of collected samples $M = 1024$. We let $p = 200$ and $q = M - p + 1$ to balance the estimation accuracy and computation complexity. The MAE of frequency estimates versus SNR is indicated in Fig. 2. They are in the range of $[10^{-1}\,\text{Hz}, 10^{-2}\,\text{Hz}]$ at high SNR. The MAE of $\hat{f}_1$ is slightly smaller than $\hat{f}_2$ because the true value of f_1 is less than f_2. As shown in Fig. 2, the MAE of clock skew estimation is in the range of $[10^{-8}, 10^{-12}]$.

B. Ranging Accuracy

In this subsection, we investigate the ranging accuracy of the mDRIPS using the estimated frequencies. We assume that there is no clock offset of the target ($\Delta = 0$) and $t_0 = 0$. The true clock skew is still $w = 1 + 10\,\text{ppm}$ in the ranging simulations. The true distance between the target and the anchor is set

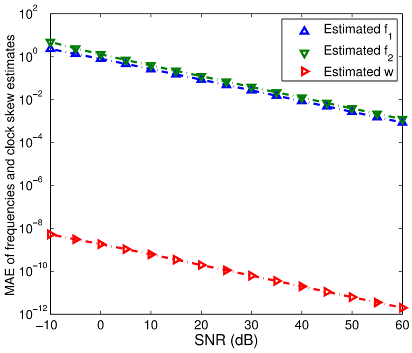

Fig. 2: MAE of frequencies, clock skew estimates vs. SNR

to be $1.5\,\text{km}$. To evaluate the ranging performance, the ranging accuracy using the exact frequencies f_1, f_2 and clock skew w is used as a benchmark. The comparisons of MAEs of distance estimates versus SNR using the estimated, the accurate and the nominated f_1, f_2, w are indicated in Fig. 3, respectively. Using the nominated f_1, f_2, w, the ranging always fails even at high SNR. On the other hand, using the estimated f_1, f_2, w, the ranging performance can achieve high accuracy at high SNR.

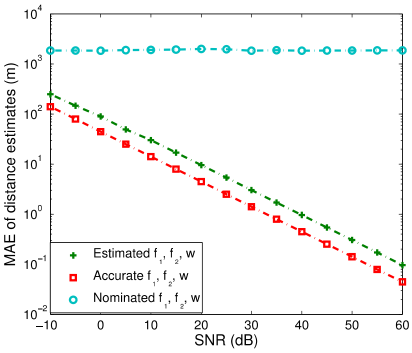

Fig. 3: MAE of distance estimates vs. SNR

C. Localization Accuracy

The localization performance is evaluated in a $6\,\text{km} \times 6\,\text{km}$ rectangle to simplify the integer ambiguity issue. Hence, $k = 0$ in (18). The target coordinate is fixed at $\mathbf{x} = [1.5, 2]\text{km}$ and the mobile anchor original position $\mathbf{L}_0 = [3, 3]\text{km}$. After ranging at previous position, the mobile anchor will move to the next position with a speed ($v = 30\,\text{m/s}$). The anchor chooses the moving directions randomly for simplicity in this simulation. In each Monte-Carlo run, the target clock offset Δ and unknown transmitting time t_0 of the target are uniformly distributed in the range of $[0, 1\,\text{s}]$, and the target clock skew w is uniformly distributed in the range of $[1 - 10\,\text{ppm}, 1 + 10\,\text{ppm}]$. The mobile anchor takes 8 periods ($N = 8$) to finish the localization steps. The comparisons of MAEs of position estimates versus SNR using the estimated, the accurate and the nominated f_1, f_2, w are indicated in Fig. 4, respectively. Without frequency estimation, the localization algorithm fails because of the huge ranging errors. However, localization based on the estimated frequencies and clock skew can approximately achieve the accuracy similarly as the ones based on accurate parameters, especially for high SNR.

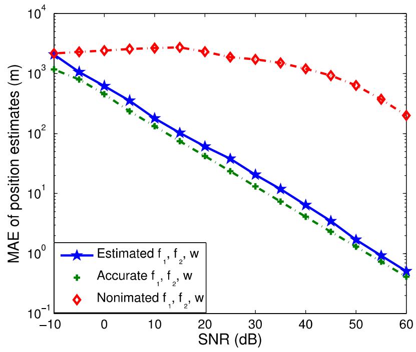

Fig. 4: MAE of position estimates vs. SNR

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