Shanghai Jiao Tong University
Department of Automation

EE353 Network Optimization

Fall 2014

Homework 2

Assigned on Fri. Oct. 31st, Due on Tue. Nov. 11th

Problem 1: What's the maximum flow of the directed graph (Fig.1)?

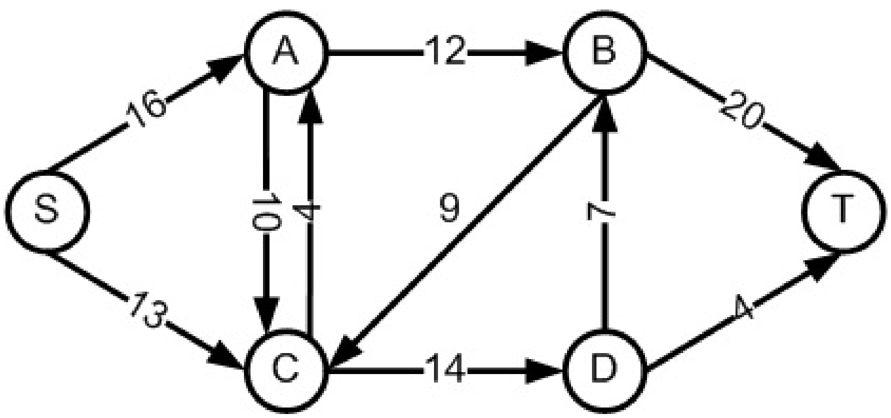

Figure 1: A directed graph with limited edge capacity

(a) Show the formulation of the maximum flow problem as a LP problem, and solve it by Matlab.

(b) Solve the problem by Ford Fulkerson algorithm. Indicate the augmenting path in each iteration and validate whether you achieve the same result as the previous one.

Problem 2: *SDP relaxations of two-way partitioning problem.* We consider the two-way partitioning problem,

$$\begin{array}{ll} \text{minimize} & x^T W x \\ \text{subject to} & x_i^2 = 1, \quad i = 1, \ldots, n, \end{array} \tag{1}$$

with variable $x \in \mathbf{R}^n$. The Lagrange dual of this (nonconvex) problem is given by the SDP

$$\begin{array}{ll} \text{minimize} & -\mathbf{1}^T \nu \\ \text{subject to} & W + \mathbf{diag}(\nu) \succeq 0 \end{array} \tag{2}$$

with variable $\nu \in \mathbf{R}^n$.The optimal value of this SDP gives a lower bound on the optimal value of the partitioning problem (1). In this exercise we derive another SDP that gives a lower bound on the optimal value of the two-way partitioning problem, and explore the connection between the two SDPs.

(a) *Two-way partitioning problem in matrix form.* Show that the two-way partitioning problem can be cast as

$$\begin{array}{ll} \text{minimize} & \mathbf{tr}(WX) \\ \text{subject to} & X \succeq 0, \quad \mathbf{rank}(X) = 1 \\ & X_{ii} = 1, \quad i = 1, \ldots, n, \end{array}$$

with variable $X \in \mathbf{S}^n$. *Hint.* Show that if X is feasible, then it has the form $X = xx^T$, where $x \in \mathbf{R}^n$ satisfies $x_i \in \{-1, 1\}$ (and vice versa).

(b) *SDP relaxation of two-way partitioning problem.* Using the formulation in part (a), we can form the relaxation

$$\begin{array}{ll} \text{minimize} & \mathbf{tr}(WX) \\ \text{subject to} & X \succeq 0 \\ & X_{ii} = 1, \quad i = 1, \ldots, n, \end{array} \tag{3}$$

with variable $X \in \mathbf{S}^n$. This problem is an SDP, and therefore can be solved efficiently. Explain why its optimal value gives a lower bound on the optimal value of the two-way partitioning problem (1). What can you say if an optimal point X^* for this SDP has rank one?

(c) We now have two SDPs that give a lower bound on the optimal value of the two-way partitioning problem (1): the SDP relaxation (3) found in part (b), and the Lagrange dual of the two-way partitioning problem, given in (2). What is the relation between the two SDPs? What can you say about the lower bounds found by them? *Hint* : Relate the two SDPs via duality.

Problem 3: *The pure Newton method.* Newton's method with fixed step size $t = 1$ can diverge if the initial point is not close to x^*. In this problem we consider two examples.

(a) $f(x) = \log(e^x + e^{-x})$ has a unique minimizer $x^* = 0$. Run Newton's method with fixed step size $t = 1$, starting at $x^{(0)} = 1$ and at $x^{(0)} = 1.1$.

(b) $f(x) = -\log x + x$ has a unique minimizer $x^* = 1$. Run Newton's method with fixed step size $t = 1$, starting at $x^{(0)} = 3$.

Plot f and f', and show the first few iterates.