Joint Synchronization and Doppler Scale Estimation Using Zadoff-Chu Sequences for Underwater Acoustic Communications

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***Abstract*—In this paper, a joint synchronization and Doppler scaling factor estimation algorithm has been proposed for underwater acoustic communications. The training sequence, which consists of two Zadoff-Chu (ZC) sequences being conjugate with each other, is utilized to synchronize and estimate Doppler scaling factor, time delay and carrier frequency offsets (CFOs). ZC sequences are well designed to show high robustness with the presence of CFOs. In the receiver, a two-dimensional cross-correlation and two auto-correlation operations can accomplish Doppler scaling factor estimation, propagation delay estimation and CFO estimation. Simulation results prove the proposed algorithm is effective for underwater acoustic communication systems.**

***Keywords*—ZC sequence, Doppler scaling factor, time delay, CFOs, underwater acoustic communications.**

I. Introduction

Recently, underwater acoustic communications have drawn more attention with the development of ocean explorations. However, the underwater acoustic channels have been the most complicated wireless channels with their unique characteristics, such as long propagation delay, multi-path propagation and significant Doppler scaling effect [1].

In particular, Doppler scaling effect, induced by the relative motion between the transmitter and the receiver, has a strong impact on the transmitted signals. If it is not compensated, the Doppler effect will severely affect the timing and frequency synchronization, and increase the error rate in decoding the message in the received signals. However, different from most terrestrial wireless systems, the Doppler scaling effect in underwater acoustic communication systems cannot be approximately handled as a fixed Doppler shift [2], because these systems are wideband and the Doppler effect causes different shifts with different frequencies. Thus, the Doppler scaling factor should be estimated separately and compensated properly in the receiver.

Several methods for the Doppler scaling factor estimation have been proposed in the recent studies. One typical approach is based on the Doppler-insensitive waveform, such as linear-frequency modulated (LFM) waveform and hyperbolic-frequency modulated (HFM) waveform [3][4]. These waveforms are used as the preamble and the postamble around the data and their times-of-arrival can be detected by the receiver with the help of the local templates. Thus, the Doppler-scaled time between the preamble and the postamble can be obtained, which in turn leads to the estimation of the Doppler scaling factor. Although this approach is practical, it requires that the packet should be received completely before the Doppler scaling factor can be estimated, which could be a waste of time and storage space.

Another approach is to take advantage of the null subcarrier and pilot for orthogonal frequency-division multiplexing (OFDM) communication systems [5][6]. The null subcarrier is used to facilitate residual Doppler compensation and the pilot subcarrier is useful for channel estimation and equalization.

Recently, Zadoff-Chu (ZC) sequences [7] have been proved to be an excellent candidate in time and carrier synchronization. ZC sequences have the following properties: firstly, it has great performance in autocorrelation and cross-correlation, which means it can be detected easily as a training sequence in the synchronization. Secondly, once the length of a ZC sequence is determined, the only parameter that one should consider is the root index. This property simplifies the design process of the ZC sequence. Moreover, with a proper selected root index, the ZC sequence shows robustness in the presence of the carrier frequency offsets (CFOs) [8][9]. However, [8][9] does not consider the Doppler scaling effect.

In this paper, we propose a joint algorithm to synchronize and estimate the Doppler scaling factor using the ZC sequence. Based upon the design principles presented by [8], we design the training sequence and propose the algorithm to estimate these parameters. The training sequence consists of two ZC sequences which are conjugate with each other and one of them uses the other as the template to calculate the correlation.

The rest of the paper is organized as follows. In Section II, we present the system model of the underwater acoustic channel and the structure of the training sequence. In Section III, we introduce the method of the parameter estimation using ZC

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sequences. Section IV shows the results of the simulations. Conclusions are drawn in Section V.

II. System Model

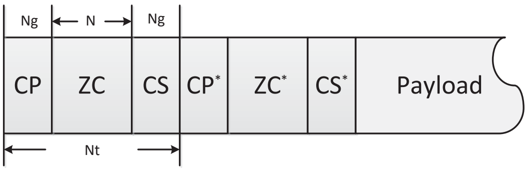

Fig. 1. The structure of the proposed training sequence

As shown in Fig.1, the transmitted training sequence consists of two ZC sequences which are conjugate with each other. Each sequence has a N_g sample-long cyclic prefix (CP) and a N_g sample-long cyclic suffix (CS). The expression is given by

$$s[n] = \begin{cases} e^{j\frac{\pi}{N}\mu(n-N_g)^2} & 0 \le n \le N_t - 1 \\ e^{-j\frac{\pi}{N}\mu(n-N_g-N_t)^2} & N_t \le n \le 2N_t - 1 \end{cases}, \quad (1)$$

where N is the length of the ZC sequence, μ is the root index which is relatively prime to N, and $N_t = N + 2N_g$ representing the total length of one ZC sequence with the CP and CS. The sampling period of the baseband signal is T_s. Thus, the baseband, continuous waveform of the transmitted signal can be written as

$$s(t) = \begin{cases} e^{j\frac{\pi\mu}{NT_s^2}(t-N_gT_s)^2} & 0 \le t < N_tT_s \\ e^{-j\frac{\pi\mu}{NT_s^2}(t-(N_g+N_t)T_s)^2} & N_tT_s \le t < 2N_tT_s \end{cases}. \quad (2)$$

The corresponding passband signal can be described as

$$\tilde{s}(t) = \sqrt{2}\mathrm{Re}\{e^{j2\pi f_{c,t}t}s(t)\}, \quad (3)$$

where $f_{c,t}$ is the carrier frequency at the transmitter.

We consider a time-varying underwater acoustic multipath channel. The channel impulse response (CIR) is characterized by

$$h(t,\tau) = \sum_{l=0}^{L-1} A_l(t)\delta(\tau - \tau_l(t)), \quad (4)$$

where L is the total number of the channel paths, $A_l(t)$ and $\tau_l(t)$ are the time-varying amplitude and propagation delay of the lth path, respectively. To simplify the calculation, we have two assumptions, as in [10][11]. One is that the amplitude remains constant in the transmission of the training signal. Another assumption is all paths share the same Doppler scaling factor, which means

$$\tau_l(t) = \tau_l - at. \quad (5)$$

Taking these assumptions into account, the simplified CIR is expressed as

$$h(t,\tau) = \sum_{l=0}^{L-1} A_l\delta(\tau - \tau_l + at). \quad (6)$$

Thus, the received passband signal can be written as

$$\begin{aligned} \tilde{y}(t) &= \tilde{s}(t) \otimes h(t,\tau) + \tilde{n}(t) \\ &= \sum_{l=0}^{L-1} A_l\tilde{s}((1+a)t - \tau_l) + \tilde{n}(t), \end{aligned} \quad (7)$$

where $\tilde{n}(t)$ represents the additive white Gaussian noise (AWGN). After sampling and down-conversion, the baseband signal is as follows,

$$y[n] = \begin{cases} e^{j2\pi f_\Delta nT_s}\sum_{l=0}^{L-1} A_l' e^{j\frac{\pi}{N}\mu((1+a)n-d_l-N_g)^2}, \\ \qquad 0 \le n \le N_t' - 1 \\ e^{j2\pi f_\Delta nT_s}\sum_{l=0}^{L-1} A_l' e^{-j\frac{\pi}{N}\mu((1+a)n-d_l-N_g-N_t)^2}, \\ \qquad N_t' \le n \le 2N_t' - 1 \end{cases} \quad (8)$$

where $f_\Delta = (1+a)f_{c,t} - f_{c,r}$ representing the total CFO caused by the Doppler scaling effects and the clock discrepancy between the transmitter and the receiver, $f_{c,r}$ is the carrier frequency at the receiver, $A_l' = A_l e^{-j2\pi f_{c,t}\tau_l}$ is the new amplitude, $d_l = \tau_l/T_s$ is the normalized delay of the lth path, $N_t' = \lfloor\frac{N_t}{1+a}\rfloor$ is the scaled length of the sequence.

III. The Method of Parameter Estimation

We propose a two-dimensional cross-correlation of the training sequence, which is given by,

$$r[k,m] = \sum_{n=0}^{m} y[n+k]y[n+k+m], \quad (9)$$

where k is the lag of the correlation and m is the scaled length of the ZC sequence with its CP and CS. The main purpose of the above cross-correlation is to find the location and the length of the first ZC sequence in the received training sequence. Both of the ZC sequences have gone through the same underwater acoustic channel and been equally affected by the Doppler scaling factor. Due to the fact that the two ZC sequences are conjugate, the cross-correlation between them can be regarded as the auto-correlation of the first ZC sequence with its conjugate template. Thus, the peak of the correlation should lie on the point where the length of the scaled sequence is matched. As an example, Fig. 2 shows the curve of correlation value versus the estimation of m. The original length $N_t = 2560$, the Doppler scaling factor $a = 0.001$, thus $N_t' = 2557$. Clearly, this is only a single sharp peak in the curve, which can be located easily. The derivation of (9) can be found in Appendix A.

A. Design of the ZC Sequence

As in [8], a well-designed ZC sequence is robust to the presence of CFOs and multi-path channel. The design of ZC sequence is actually the selection of the root index μ. The constraints are as follows:

1) According to the definition of the ZC sequence, the root index μ should be relatively prime to the length N.

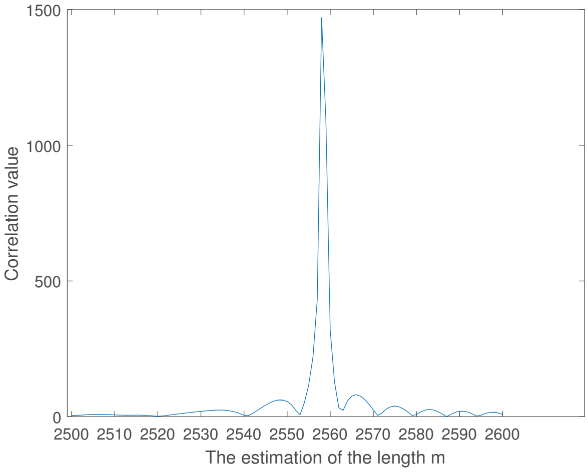

Fig. 2. Correlation value versus the estimation of the length m

2) To enable robust time synchronization in the presence of CFOs, [8] has proved the value of the root index μ should satisfy,

$$\mu(k_1 - N_g - d) = pN - 1, \tag{10}$$

where k_1 is the lag of the correlation corresponding to the minimum normalized CFO, $d = d_0$ representing the normalized propagation delay of the first arrival path and p is any integer. Specifically, [8] has pointed out that $\mu = 1$ is the simplest choice. The details of the derivation can be found in [8].

3) The length of the received ZC sequence is scaled and different from the original length.

Take all these constraints into account, we set the root index $\mu = 1$.

B. Finding the scaled length

Notice that (8) can be rewritten as

$$y[n] = \begin{cases} e^{j2\pi f_\Delta nT_s} \sum_{l=0}^{L-1} A_l' e^{j\frac{(1+a)\pi}{N'}\mu(n-d_l'-N_g')^2}, \\ \hfill 0 \le n \le N_t' - 1 \\ e^{j2\pi f_\Delta nT_s} \sum_{l=0}^{L-1} A_l' e^{-j\frac{(1+a)\pi}{N'}\mu(n-d_l'-N_g'-N_t')^2}, \\ \hfill N_t' \le n \le 2N_t' - 1 \end{cases} \tag{11}$$

where $x' = \frac{x}{1+a}$, $x \in \{N, d_l, N_g, N_t\}$. N', d_l', N_g', N_t' are the scaled length of the ZC sequence, the propagation delay, the guard time and the ZC sequence with its CP and CS, respectively.

To estimate N_t', the peak search of the 2D correlation is given by,

$$[\hat{k}, \hat{m}] = \arg\max_{k,m}\ r[k, m], \tag{12}$$

where $\hat{k}$ is an estimation of the propagation delay and $\hat{m}$ is an estimation of the scaled length N_t'.

C. Doppler Scaling Factor Estimation

Note that when the Doppler scaling factor $a > 0$, the signal through the channel is compressed, thus the scaled length of the received signal is less than the original length, which means $\hat{m} < N_t$. Similarly, when $a < 0$, the signal is extended, we have $N_t < \hat{m}$. Therefore, the Doppler scaling factor can be estimated as

$$a = \frac{N_t - \hat{m}}{N_t}, \tag{13}$$

where the sign of a represents whether the signal is compressed or extended in time.

D. CFO Estimation

With the estimated Doppler scaling factor, the received training signal can be resampled by the factor $\frac{1}{1+a}$ to restore the original signal. The restored signal is denoted by $y_r[n]$. Then we calculate the auto-correlation of the two ZC sequences with the local copies in the receiver, respectively. The expression is given by,

$$r_1[k] = \sum_{n=0}^{N-1} y_r[n+k] e^{-j\frac{\pi}{N}\mu n^2}, \tag{14}$$

$$r_2[k] = \sum_{n=0}^{N-1} y_r[n+k] e^{j\frac{\pi}{N}\mu n^2}. \tag{15}$$

The starting points of the two ZC sequences are respectively located at,

$$\hat{d}_i = \arg\max_k |r_i[k]|, \quad i = 1, 2. \tag{16}$$

Note that after resampling, the CFO caused by the Doppler scaling factor is eliminated. Thus, the residual CFO, denoted by f_Δ', is estimated as in [8],

$$\hat{f}_\Delta' = \frac{N_t + N_g - \hat{d}_2 + \hat{d}_1}{-2}. \tag{17}$$

IV. Simulation Results

In this section, we carry out Matlab simulations to illustrate the performance of the proposed algorithm. Table I shows the parameters of the training sequence used in the simulations. We design different lengths of training sequences to present how the length influences the effects of the estimation. The parameters of the channel are shown in Table II. The Doppler scaling factor $a = 0.001 > 0$ means the length of the scaled signal is shorter than that of the original signal. Besides, the carrier frequency f_c = 20 kHz, the sampling rate f_s = 96 kHz. The noise is additive white Gaussian noise.

Fig. 3 demonstrates the mean square error (MSE) of the Doppler scaling factor estimation changes with SNR changing. We can draw two conclusions from Fig. 3. One is that it proves the effectiveness of the proposed algorithm. The performance becomes better as SNR increases and when SNR exceeds 0 dB, the performance of the algorithm is rather reliable. The other is a longer training sequence can help increase the accuracy of the estimation. However, as the training sequence becomes longer, it takes more resource including time and storage

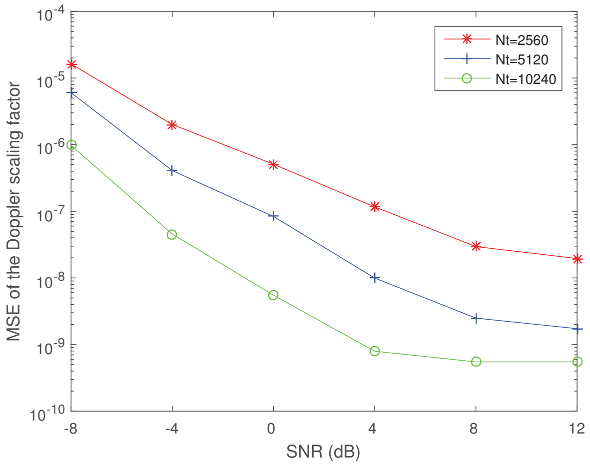

Fig. 3. MSE of the Doppler scaling factor estimation

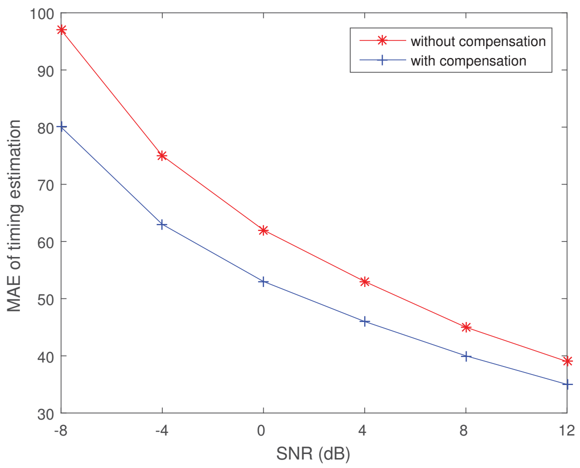

Fig. 4. MAE of the timing estimation

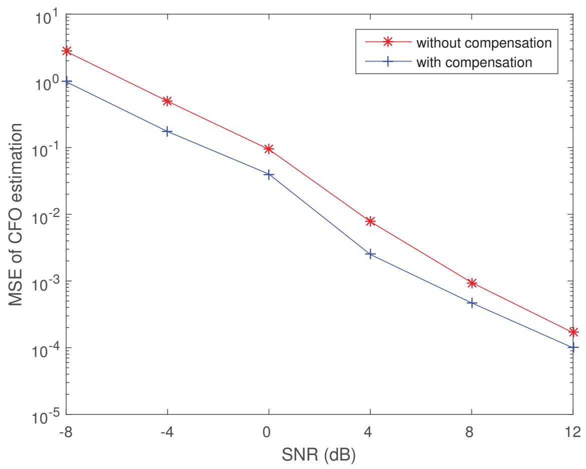

Fig. 5. MSE of the CFO estimation

TABLE I
TRAINING SEQUENCE PARAMETERS

Parameter	Notation	Values
The length of a ZC sequence	N	[2048 4096 8192]
The length of CP or CS	N_g	[256 512 1024]
The length of the complete ZC sequence (CP+ZC+CS)	N_t	[2560 5120 10240]
The root index	μ	1

TABLE II
UNDERWATER ACOUSTIC CHANNEL PARAMETERS

Parameter	Notation	Values
Channel gains	A_l	[1 0.6 0.3]
Channel delays (ms)	τ_l	[0 2 5]
Doppler scaling factor	a	0.001

space to accomplish the estimation. Thus, the selection of the length should make a balance between accuracy and resource consumption.

Fig. 4 depicts the mean absolute error (MAE) of the timing estimation versus SNR. Here, we compare the results of timing estimation with Doppler scaling compensation and without compensation. Fig. 4 shows the error decreases with SNR increasing and the Doppler scaling compensation can help further reduce the estimation error.

The MSE of the CFO estimaion is shown in Fig. 5. Although the CFO estimation algorithm without Doppler scaling compensation can accomplish the work, the estimation accuracy can always get better with the Doppler scaling compensation, no matter how SNR changes.

V. CONCLUSION

In this paper, we propose a joint synchronization and Doppler scaling factor estimation algorithm using ZC sequence. The training sequence is robust to the presence of CFOs and easy to design. Doppler scaling factor can be estimated by a 2D peak search of the correlation. Then the received signal is resampled and CFOs estimation is accomplished by the auto-correlation of the two ZC sequences. Simulations over multi-lag single scale underwater acoustic channels have shown the effectiveness and robustness of the proposed algorithm.

APPENDIX A

For the sake of simplification, we use the following signals $y_1[n]$ and $y_2[n]$ to represent the scaled signals in the receiver.

$$y_1[n] = e^{j\frac{\pi}{N}\mu(n-N_g)^2}, 0 \leq n \leq N_t - 1, \tag{18}$$

$$y_2[n] = e^{-j\frac{\pi}{N}\mu(n-N_g-N_t)^2}, N_t \leq n \leq 2N_t - 1. \tag{19}$$

The aim is to use a time window, the length of which is m, to estimate the scaled length N_t. Here, we only discuss the case where $m < N_t$. The case where $m > N_t$ can be discussed in the same way. Because the Doppler scaling factor is rather small, it is reasonable to set m close to N_t.

For any fixed m, there are basically three cases that should be discussed as the time window moves along the received signal.

Case 1:

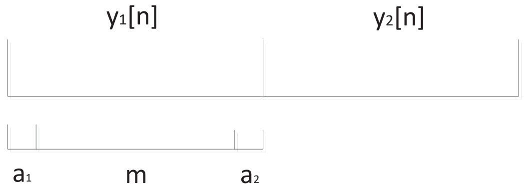

Fig. 6. The diagram for Case 1

As shown in Fig. 6, the length of the time window is m and a_1, a_2 represent the lengths of the parts where the ZC sequence exceeds the time window. Based on the fact that m is close to N_t, we can set a_1 and a_2 are smaller than N_g. Case 1 is a typical case and other cases can be derivate from it. Therefore, we start with the derivation for Case 1.

$$\begin{aligned}
|r[m]| &= |\sum_{n=a_1}^{a_1+a_2-1} e^{j\frac{\pi}{N}\mu(n-N_g)^2} e^{j\frac{\pi}{N}\mu(n+m-N_g)^2} + \\
&\quad \sum_{n=a_1+a_2}^{a_1+m-1} e^{j\frac{\pi}{N}\mu(n-N_g)^2} e^{-j\frac{\pi}{N}\mu(n-a_1-a_2-N_g)^2}| \\
&\le |\sum_{n=a_1}^{a_1+a_2-1} e^{j\frac{\pi}{N}\mu(n-N_g)^2} e^{j\frac{\pi}{N}\mu(n+m-N_g)^2}| + \\
&\quad |\sum_{n=a_1+a_2}^{a_1+m-1} e^{j\frac{\pi}{N}\mu(n-N_g)^2} e^{-j\frac{\pi}{N}\mu(n-a_1-a_2-N_g)^2}| \\
&\le a_2 + |e^{-j\frac{\pi}{N}\mu((a_1+a_2)^2+2(a_1+a_2)N_g)} \\
&\quad \sum_{n=a_1+a_2}^{a_1+m-1} e^{j\frac{2\pi}{N}\mu(a_1+a_2)n}|
\end{aligned} \tag{20}$$

$$\begin{aligned}
&= a_2 + |e^{-j\frac{\pi}{N}\mu((a_1+a_2)^2+2(a_1+a_2)N_g)} \\
&\quad (\sum_{n=0}^{N} e^{j\frac{2\pi}{N}\mu(a_1+a_2)n} - \sum_{n=0}^{a_1+a_2-1} e^{j\frac{2\pi}{N-1}\mu(a_1+a_2)n} + \\
&\quad \sum_{n=N}^{a_1+m-1} e^{j\frac{2\pi}{N}\mu(a_1+a_2)n})| \\
&\le a_2 + a_1 + a_2 + a_1 + a_2 = 2a_1 + 3a_2 \le 5N_g.
\end{aligned} \tag{21}$$

Using the parameters in the simulation, $N = 2048, N_g = 256, N_t = 2560$, we can get $5N_g < N_t$.

Case 2:

When the time window moves to the position where $a_1 = 0$ and $0 < a_2 < N_g$, we have Case 2. Case 2 is actually another version of Case 1. To save the page, we only put the conclusion here.

$$|r[m]| \le 3N_g. \tag{22}$$

Case 3: When the time window moves to the position where $0 < a_1 < N_g$ and $a_2 = 0$, we have Case 3. Similarly, we can get

$$|r[m]| \le 5N_g. \tag{23}$$

In conclusion, in all three cases where $m < N_t$, the maximum of the correlation is rather smaller than N_t. The same is true for cases where $m > N_t$. Only when $m = N_t$ and the time window perfectly matches $y_1[n]$, the correlation result is N_t. Therefore, we prove that the correlation of the received signal only has one sharp peak.

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