Joint Time Synchronization and Localization for Underwater Acoustic Sensor Networks[*]

Youjie Xia, Yiyin Wang
Shanghai Jiao Tong University, China
{youjiexia,yiyinwang}@sjtu.edu.cn

Xiaoli Ma
Georgia Institute of Technology, USA
xiaoli@ece.gatech.edu

Cailian Chen, Xinping Guan
Shanghai Jiao Tong University, China
{cailianchen,xpguan}@sjtu.edu.cn

ABSTRACT

Time synchronization and localization for underwater acoustic sensor networks (UASNs) face new challenges in several aspects. Since an acoustic signal propagates slowly underwater, the propagation delay cannot be neglected. Hence, time synchronization is tightly coupled with localization. Furthermore, underwater sensor nodes have stringent energy constraints, and they are expected to have a long lifetime. To overcome these challenges, in this paper we propose a joint time synchronization and localization scheme for an asynchronous UASN, where static nodes are synchronized and localized with the help of a mobile node. The mobile node broadcasts packets for synchronization and localization periodically, and the period is carefully designed to avoid collisions. To save energy, the static nodes work in a (q, p) duty-cycle schedule, which impacts the packet receiving for the static node. Consequently, the probability for the static nodes to be successfully synchronized and localized is analyzed. It sheds lights on the design of system parameters to achieve a high probability. Moreover, simulation results indicate the efficiency of the proposed scheme.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design

General Terms

Algorithms, Design, Performance

Keywords

Time Synchronization, Localization, Probability, Underwater Acoustic Sensor Networks

1. INTRODUCTION

Due to a wide range of promising applications, underwater acoustic sensor networks (UASNs) are under intensive investigation [1], [2]. Sensor nodes in UASNs are capable of sensing and collecting data, which should be tagged with time and location for practical use. However, sensor nodes are asynchronous due to differences of individual clock devices, and are not aware of their positions. Therefore, time synchronization and localization are fundamental requirements in UASNs.

Since underwater acoustic medium is quite different from terrestrial radio one, new challenges are introduced for time synchronization and localization in UASNs. As the speed of an underwater acoustic signal is only approximately 1500 m/s, the propagation delay related to the senor position cannot be neglected in time synchronization. On the other hand, time-based localization methods are popular for their high accuracy. Time synchronization is essential for this kind of localization methods. Hence, time synchronization and localization are tightly coupled with each other. However, traditional methods normally treat them individually with the assumption that one of them is perfectly solved. Furthermore, it is not possible to recharge sensors' batteries frequently under water. Due to the limited energy, it is important to develop energy-efficient schemes for synchronization and localization.

Some recent works for time synchronization in UASNs take the propagation delay into account, such as TSHL [3], MU-Sync [4]. TSHL and MU-Sync utilize two-way message delivery and treat half of the round trip time as the propagation delay, which will cause synchronization errors. On the other hand, several underwater localization algorithms (e.g. ODAL [5]) uses differences of message arrival time to eliminate clock offsets. They can achieve an acceptable localization accuracy without time synchronization. Only a few research work deal with synchronization and localization jointly, such as JSL [6]. Most approaches mainly depend on the two-way message delivery and multiple rounds of message exchanges. Thus, they have a relatively high communication cost.

In this paper, we propose a joint time synchronization and localization scheme using one-way message delivery. Static

sensor nodes receive packets for synchronization and localization from a mobile node, which has a reference clock and knows its own position. The mobile node periodically broadcasts packets, and the period is carefully designed to avoid collisions at static sensor receivers. Furthermore, due to the strict energy constraint, the static node is designed to work in a (q, p) duty-cycle schedule to prolong its lifetime. The (q, p) duty-cycle schedule influences the packet receiving for the static node, which has to receive enough packets to estimate parameters for synchronization and localization. Thus, the probability for the static node to be successfully synchronized and localized at the same time is analyzed. It brings insight into the system parameter design to obtain a high probability.

The rest of paper is organized as follows. Section 2 gives the system model. The joint time synchronization and localization scheme is proposed in Section 3. The collision avoidance scheme is developed in Section 4. The probability for the static node to be successfully synchronized and localized at the same time is derived in Section 5. In Section 6, simulation results are shown.

2. SYSTEM MODEL

Consider a fully asynchronous UASN with a mobile node and uniformly distributed static nodes in a three-dimensional space. The mobile node could be an autonomous underwater vehicle (AUV) and serves as a clock reference. Its position information is well known by itself. It broadcasts packets with a length of T_p for synchronization and localization repeatedly with a period of T_b, as it moves along a pre-designed route with a speed of v.

All the static nodes are with different unknown clock skews and offsets, and not aware of their position information. They need to be synchronized and localized with the help of the mobile node. The communication ranges of the mobile and static nodes are the same and defined as D. Without loss of generality, we consider synchronization and localization for an arbitrary static node. The proposed method can be simply extended to multiple nodes. The coordinates of the static node is denoted as $\mathbf{x} \in \mathbb{R}^{3 \times 1}$. Normally in the 3D underwater space, the depth of the static node could be measured by a pressure sensor. Thus, $\mathbf{x}$ only has two unknown parameters. Note that the position of the static node may change due to various causes, such as ocean current. We assume that its position does not change during the synchronization and localization procedure.

The mobile node has a standard clock with reference time t, while the static node has an asynchronous clock with its local time $C(t)$. Their relationship can be given by a first-order linear model

$$C(t) = \alpha t + \beta \tag{1}$$

where α and β represent the unknown **clock skew** and the unknown **clock offset**, respectively. As the static node has a stringent energy constraint, it works in a (q, p) duty-cycle schedule to ensure a long lifetime. Furthermore, in the proposed one-way scheme, the static node keeps passively receiving from the mobile node during its active time.

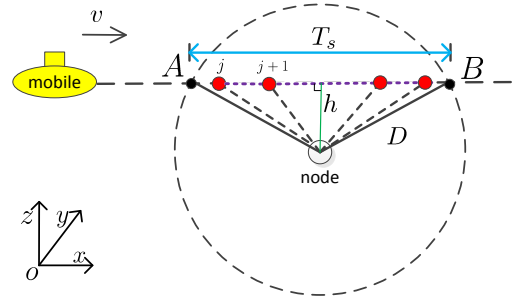## 3. JOINT TIME SYNCHRONIZATION AND LOCALIZATION

Figure 1: The illustration of a projected 2D case.

In this paper, we propose a novel scheme using the one-way message transmission for joint time synchronization and localization in UASNs. Furthermore, we propose an approach to estimate the unknown parameters for synchronization and localization.

The mobile node patrols along a preset route in a specific underwater region, and periodically broadcasts packets as shown in Fig. 1. The packet is used not only for synchronization, but also for localization. It includes both time and position stamps. For example, the jth packet is broadcast at the reference time t_j, and the position $\mathbf{a}_j$, where $\mathbf{a}_j \in \mathbb{R}^{3 \times 1}$. The jth packet contains information about t_j and $\mathbf{a}_j$. As a result, the jth packet arrives at the static node after a time lag d_j/c. The static node keeps receiving for T_p until the end of the packet arrives, and it records the corresponding time stamp r_j as

$$r_j = t_j + d_j/c + T_p \tag{2}$$

where $d_j = \| \mathbf{a}_j - \mathbf{x} \|$ the distance between the mobile node at the jth position and the static node, and c is the speed of the underwater acoustic signal. The notation $\| \cdot \|$ denotes the Euclidean norm throughout the paper. Note that T_p is normally several hundreds of milliseconds up to seconds and d_j/c is in the order of seconds up to tens of seconds. Thus, they cannot be ignored in underwater networks. The time instant r_j is measured by the static node's own clock as its local time $\tilde{r}_j$ based on (1) as

$$\tilde{r}_j = C(r_j) + w_j = \alpha(t_j + d_j/c + T_p) + \beta + w_j \tag{3}$$

where w_j is measurement noise and assumed to be independent and identically distributed (*i.i.d.*) Gaussian with zero mean and variance σ^2.

Accordingly, four parameters have to be estimated, including the static node's position $\mathbf{x}$ with two unknown parameters, the clock skew α and the clock offset β. If the static node receives four or more packets and measures the receiving time, it could estimate its four unknown parameters to achieve synchronization and localization. Let us assume the static node receives N packet in total. As a result, the unknown parameters can be found based on maximum likelihood estimation (MLE) as

$$(\hat{\mathbf{x}}, \hat{\alpha}, \hat{\beta}) = \arg \min_{(\mathbf{x}, \alpha, \beta)} \left\{ \frac{1}{2\sigma^2} \sum_{j=1}^{N} \left[\tilde{r}_j - \alpha(t_j + \frac{d_j}{c} + T_p) - \beta \right]^2 \right\} \tag{4}$$

which can be solved by various methods, for example gradient descent algorithms.

4. COLLISION AVOIDANCE SCHEME

This section develops the collision avoidance scheme. If different packets arrive at the same receiver simultaneously, a collision will happen. Since the mobile node moves and periodically broadcasts packets, the packets transmitted at different positions may cause collisions at the static node. A collision-free scheme [7] have been proposed for packet scheduling to minimize the localization time for UASN. With a different purpose, we design the broadcast period T_b for the mobile node to prevent the collision.

The broadcast period T_b is closely related to the mobile node's speed v and the packet duration T_p. Let us first consider the range of the time interval Δ_j. It is the time differences when the static node starts to receive packets broadcasted by the mobile node at two adjacent positions (e.g., the jth and the $(j+1)^{th}$ position). It can be modeled as

$$\Delta_j = T_b + (d_{j+1} - d_j)/c \quad (5)$$

According to the triangle inequality, we have $|d_j - d_{j+1}| \leq vT_b$. Hence, we have $\Delta_j \in [T_b - vT_b/c, T_b + vT_b/c]$. Recall that the packet duration is T_p. Thus, the general collision-free condition can be given as $|\Delta_j| > T_p, \ \forall j$. However, the speed of the mobile node v, normally 1 m/s up to 5 m/s, is much less than the speed of underwater acoustic signal c. Therefore, the collision-free condition must be $\Delta_j > T_p, \forall j$. We can obtain $T_b - vT_b/c \geq T_p$. As a result, the minimum T_b to prevent the collision for the static node can be obtained as

$$\underline{T_b} = cT_p/(c-v) \quad (6)$$

Finally, for any fixed $T_b \geq \underline{T_b}$ to avoid collisions, the time interval Δ_j has a upper bound, which is monotonically increasing in T_b. With a specific T_b considered, we define the upper bound of Δ_j as $\bar{\Delta}(T_b)$, given by

$$\bar{\Delta}(T_b) = (c+v)T_b/c \quad (7)$$

in which the static node may successfully receive at least one packet and the next packet arrives without collisions.

Recall that static node could finish receiving a single packet in T_p once it arrives. We can conclude that for the static node, time consumed by successfully receiving n consecutive packets without collisions, can be obtained as

$$R_n = (n-1)\bar{\Delta}(T_b) + T_p \quad (8)$$

where n is a positive integer.

5. PROBABILITY FOR JOINT TIME SYNCHRONIZATION AND LOCALIZATION

This section discusses the probability for the static node to be successfully synchronized and localized. Only a few existing work consider this kind of probability, especially in a (q, p) duty-cycle schedule. [8] has considered about the capture of stochastic events modeled by several distributions. However, different from stochastic events being captured, each packet has a duration and could not be captured instantaneously under water by an acoustic modem [1]. Moreover, the process of incoming packets could not always be treated independent from others. In the proposed scheme, if the static node successfully receives 4 or more packets, it could be synchronized and localized. However, due to the mobility of the mobile node, the limited communication range of the nodes, and the (q, p) duty-cycle scheme, the probability of successfully receiving at least 4 packets is not always 1.

Two necessary conditions for the static node to receive packets are

- **Spatial condition:** the mobile node must be in the communication range D of the static node.

- **Temporal condition:** the static node is in the awake mode (q time) when receiving packets.

Considering the spatial condition first, the number of captured packets must be determined by the mobile node's staying time T_s in the communication range of the static node. Note that we assume the mobile node moves along a straight line with a constant speed v at a fixed depth, and static nodes are uniformly deployed. The illustration of the scenario is shown in Fig. 1, where A is the mobile node's entry point of the static node's communication region, B is the exit point, and d_{AB} denotes the distance between A and B. Hence, the time for the mobile node to move from A to B is the staying time of the the mobile node in the communication range of the static node. Thus, we calculate the staying time of the mobile node as

$$T_s = d_{AB}/v = 2\sqrt{D^2 - h^2}/v \quad (9)$$

where h is the shortest distance between the static node and the mobile node's route. If the static node needs at least N_p packets for synchronization and localization, we must have $T_s \geq N_p T_b$, which is equivalent to $\lfloor T_s/T_b \rfloor \geq N_p$. This relationship serves to prove Theorem 1.

The temporal condition would be discussed in the proof of Theorem 1. The following theorem concerns, under the collision-free condition discussed in Section 4, the probability for a static node to successfully receive at least 4 packets in two consecutive periods. Since the clock skews and offsets may vary with time, we choose two consecutive periods to finish the joint task.

THEOREM 1 (**Capture probability**). *The static node follows a preset duty-cycle schedule* (q_0, p_0). *As a result, the duty-cycle schedule* (q_0, p_0) *in its local time corresponds to* (q, p) *in the reference time, where* $q = q_0/\alpha$, $p = p_0/\alpha$. *If the mobile node is in the communication range of the static node, the probability for the static node to successfully receive at least 4 packets in* 2 *consecutive periods* $(2p)$, *can be given by:*

$$P_r(4,2) = \frac{\tilde{q}_4}{p} + \frac{1}{p}\int_0^{p-\tilde{q}_4} f\left(\left\lfloor\frac{t}{\bar{\Delta}(T_b)}\right\rfloor + 5 - m, t\right)dt \quad (10)$$

$$m = \left(\left\lfloor\frac{q - T_p - p + t}{\bar{\Delta}(T_b)}\right\rfloor + 1\right)u(q - T_p - p + t) \quad (11)$$

$$f(X,t) = P_r\left(\left\lfloor\frac{T_s}{T_b}\right\rfloor \geq X\right)u(q - T_p + t - \bar{\Delta}(T_b)X) \quad (12)$$

where $\tilde{q}_4 = g(q - R_4)$ *with* $g(x) = xu(x)$ *and* $u(x)$ *is the unit step function, and* $R_4 = 3\bar{\Delta}(T_b) + T_p$ *as denoted in (8).*

PROOF. In consideration of the temporal condition, let us define t_{in} as the arrival time of the first packet after the mobile node enters the communication range of the static node. Without loss of generality, we can assume that t_{in} is uniformly distributed in $[0, p]$. Consequently, we consider two cases (i) $t_{in} \in [0, q - R_4]$ and (ii) $t_{in} \in (q - R_4, p)$.
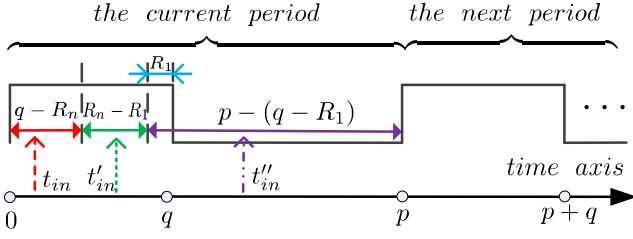
(i) $t_{in} \in [0, q - R_4]$

Figure 2: Illustration of the (q, p) duty-cycle schedule.

The probability for receiving at least 4 packets is given by

$$P_r^1(4,2) = \frac{q - R_4}{p} u(q - R_4) = \frac{g(q - R_4)}{p} \qquad (13)$$

The range of t_{in} in case (i) is shown by red dotted line in Fig. 2.

(ii) $t_{in} \in (q - R_4, p)$

If the first packet arrives in this subinterval, the static node could not finish receiving at least 4 packets in the current period. Only if the staying time T_s extends to the next awake period, it is possible to receive enough packets. Thus, the probability for the static node to successfully receive at least 4 packets is given as

$$P_r^2(4,2) = \frac{1}{p} \int_{q-R_4}^{p} P_r[T_s \geq (n_1 + 4 - n_2)T_b] d(t_{in}) \quad (14)$$

$$n_1 = \left\lfloor \frac{p - t_{in}}{\bar{\Delta}(T_b)} \right\rfloor + 1 \qquad (15)$$

$$n_2 = \left(\left\lfloor \frac{q - T_p - t_{in}}{\bar{\Delta}(T_b)} \right\rfloor + 1 \right) u(q - T_p - t_{in}) \qquad (16)$$

where n_1 describes the number of the arriving packets during the current period, while n_2 describes the number of successfully received packets within the subinterval $(q - R_4, q - T_p]$ of the current period. Thus, $4 - n_2$ represents the minimum number of the rest packets required in the next awake period. Hence, the staying time T_s should be no less than $(n_1 + 4 - n_2)T_b$. The range of t_{in} in case (ii) is given by two subintervals $(q - R_4, q - T_p]$ and $(q - T_p, p]$, corresponding to the green and purple dotted line in Fig. 2. Besides, the receiving time of the 4th packet should be in $[0, q]$ of the next period, i.e., $t_{in} + \bar{\Delta}(T_b)R_{(n_1+4-n_2)} \leq p + q$. Thus, we have a unit step function in (12). Referring to (15), (16) and the unit step function, we can rewrite (14) as (10). Recall the spatial condition, the probability could be given as:

$$P_r(4,2) = \begin{cases} P_r^1(4,2) + P_r^2(4,2) & , \quad d \leq D, \\ 0 & , \quad d > D, \end{cases} \qquad (17)$$

where d denotes the distance between the mobile node and the static node. It is obvious that the probability $P_r(4,2)$ is related to not only the duty-cycle schedule but also their positions. Hence, based on the relationship between the staying time T_s and the space positions h, D, we can rewrite the probability $P_r(4,2)$ in (17) in a spatial-temporal composite form, finally given as (10). T_s along with p and $\tilde{q}_4$ describes the probability from the spatial and temporal viewpoints. This completes the proof. $\square$

We remark here that in Theorem 1, $P_r(4,2)$ is a function of the following parameters: the broadcasting period T_b, the packet duration T_p, the periodic scheduling (q, p) and
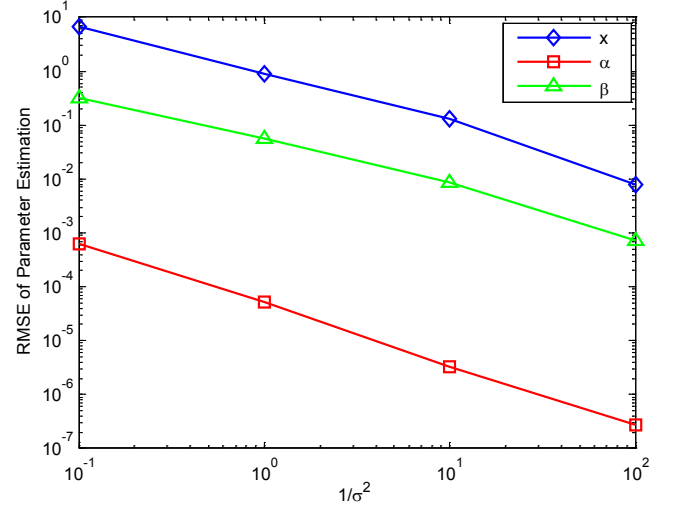

Figure 3: RMSE of the estimated position, clock skew and offset versus noise variance $1/\sigma^2$.

the staying time T_s. Since the probability density function (pdf) of T_s is determined by the static node distribution, D and v, these factors also affect $P_r(4,2)$. Among the above parameters, we may design T_b, T_p, (q, p), D and v to achieve a desired $P_r(4,2)$. Furthermore, we discuss the distribution of T_s with respect to (w.r.t.) the static node. Since the static node is uniformly distributed in the region, h is uniformly distributed in the range of $[0, D]$. As T_s is a function of h, we can obtain the closed-form pdf of T_s accordingly, as

$$f_{T_s}(t_s) = \frac{v^2 t_s}{2D\sqrt{(2D)^2 - v^2 t_s^2}}, \quad t_s \in [0, \frac{2D}{v}] \qquad (18)$$

6. SIMULATIONS AND DISCUSSIONS

For an arbitrary static node, the clock skew α and clock offset β are uniformly distributed in the range [1-100 ppm,1+100 ppm] and [-5 s,5 s], respectively. The communication range D of the static and mobile node is set as $500\,\mathrm{m}$. We assume that the speed of the underwater acoustic signal is constant as $c = 1500$ m/s. Furthermore, the mobile node moves in a straight line in the region with a speed of 2 m/s ($v = 2$ m/s) and broadcasts packets every 10 seconds ($T_b = 10$ s). The packet duration is 0.5 s ($T_p = 0.5\,\mathrm{s}$) for dealing about 200 bytes data [1], which is enough for message delivery in our scenario.

Based on (4) and N received packets, the static node can estimate unknown parameters. If $N \geq 4$, we can estimate the clock skew and offset without any ambiguity. However, we may obtain an estimate of the mirror position w.r.t. the true position. Since the mobile node moves in a straight line, this will cause an ambiguity for the position estimation. This ambiguity requires the follow-up correction. The unique position of the static node could be determined, if it receives at least one packet from the mobile node in a different patrol route. At last, we discard the mirror position.

A) Fig. 3 shows the root mean square error (RMSE) of the parameters **x**, α and β for joint estimation using 4 packets. Fig. 3 verifies the correctness of the proposed one-way message transmission scheme. Each point in the figures is the average results of 1000 rounds of Monte Carlo trial-
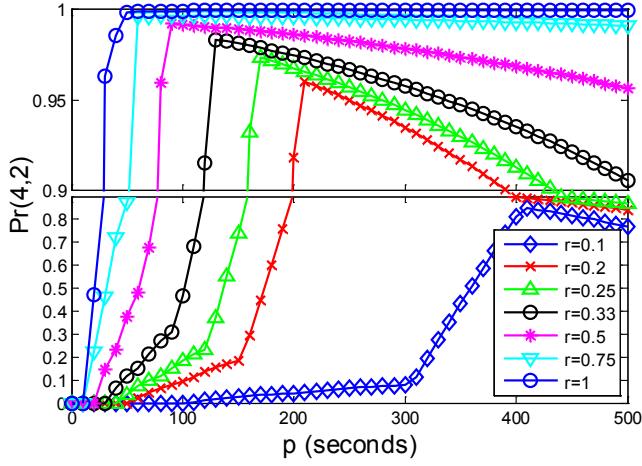
Figure 4: Plot of $P_r(4,2)$ versus p for different r.

Figure 5: Plot of $P_r(4,2)$ versus r for different T_b, using $q = R_5$.

s. With smaller noise variance σ^2, the estimation results of each parameter have a lower RMSE and become more accurate.

B) Fig. 4 plots the probability for the static node to successfully receive at least 4 packet in 2 consecutive periods under different (q, p) schedules. Let us define the duty ratio $r = q/p$. Notice that when r (or p) is fixed and p (or r) increases, it is more possible for the static node to be synchronized and localized in general. We can obtain that each curve has a transition point, where the rapid increment of $P_r(4,2)$ stops and $P_r(4,2) \approx 1$. Moreover, after the transition point, the increment of $P_r(4,2)$ is marginal with increasing r. Further, each curve has a peak and $P_r(4,2)$ goes down when p keeps increasing. The transition point overlaps with the peak point of the curve in some cases. These points are collected in Table 1. According to Fig. 4 and Table 1, when p varies from 0 s to the maximum $T_s = 500$ s with different r, the q corresponds to each transition point, denoted by q^*, is almost the same, approximately 40.6 s. And $P_r^*(4,2)$ denotes the probability when $q = q^*$. This consistent value of q^* happens to be $R_5 \approx 40.5533$ s. Hence, $q = R_5$ is useful to design the system parameters to achieve a high probability of synchronization and localization.

C) Fig. 5 plots $P_r(4,2)$ versus duty ratio r for different T_b, with $q = R_5$. We are interested at r^*, which makes $P_r(4,2) \geq 0.99$ with different T_b. Fig. 5 shows that when $q = R_5$, a decreasing T_b enable a smaller r^*, which satisfies $P_r(4,2) = 0.99$. These points are collected in Table 2. According to Table 2, the smallest r^* to make $P_r(4,2) \geq 0.99$ is linear w.r.t. T_b. The linear relationship could be written as $r^* = kT_b$, where $k = 0.04\,\mathrm{s}^{-1}$, if T_b is given and $q = R_5$. We can also obtain that when $T_b \geq 25$ s, it is not possible to achieve $P_r(4,2) \geq 0.99$, even with $r = 1$. The total packets broadcast by the mobile node in T_s decreases with a increasing T_b. It will cause a decrement in the probability.

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r	0.1	0.2	0.25	0.33	0.5	0.75
q^* (s)	40.6	40.6	40.56	40.557	40.56	40.575
$P_r^*(4,2)$	0.8522	0.9622	0.9746	0.9844	0.9926	0.9962

Table 1: q^* and $P_r^*(4,2)$ with different r.

T_b (s)	1	2	3	4	5	10	25	≥ 25
r^*	0.04	0.08	0.12	0.16	0.2	0.43	0.98	NaN

Table 2: r^* for $P_r(4,2) = 0.99$ with different T_b, using $q = R_5$.