Shanghai Jiao Tong University
Department of Automation

EE353 Network Optimization

Fall 2014

Homework 1

Assigned on Fri. Oct. 17th, Due on Fri. Oct. 31st

Problem 1: Which of the following sets are convex? Please explain the reason

(a) A *slab*, *i.e.*, a set of the form $\left\{x \in \mathbf{R}^n \mid \alpha \leq a^T x \leq \beta\right\}$.

(b) A *rectangle*, *i.e.*, a set of the form $\{x \in \mathbf{R}^n \mid \alpha_i \leq x_i \leq \beta_i, i = 1, \ldots, n\}$. A rectangle is sometimes called a *hyperrectangle* when $n > 2$.

(c) A *wedge*, *i.e.*, $\left\{x \in \mathbf{R}^n \mid a_1^T x \leq b_1, a_2^T x \leq b_2\right\}$.

(d) The set $\{x \mid x + S_2 \subseteq S_1\}$, where $S_1, S_2 \subseteq \mathbf{R}^n$ with S_1 convex.

Problem 2: *Running average of a convex function.* Suppose $f : \mathbf{R} \to \mathbf{R}$ is convex, with $\mathbf{R}_+ \subseteq \mathbf{dom} f$. Show that its *running average* F, defined as

$$F(x) = \frac{1}{x} \int_0^x f(t)\, dt, \quad \mathbf{dom}\, F = \mathbf{R}_{++}$$

is convex. You can assume f is differentiable.

Problem 3: *Dual of general LP.* Find the dual function of LP

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Gx \preceq h \\ & Ax = b. \end{array}$$

Give the dual problem, and make the implicit equality constraints explicit.

Problem 4:

(a) Show that the *log-sum-exp* function $f(x) = \frac{1}{\beta} \log\left(\sum_{i=1}^n e^{\beta x_i}\right)$ is the optimal value of the following optimization problem

$$\begin{array}{lll} \mathbf{P1:} & \max_{y \geq 0} & \displaystyle\sum_{i=1}^n y_i x_i - \frac{1}{\beta} \sum_{i=1}^n y_i \log y_i \\ & s.t. & \displaystyle\sum_{i=1}^n y_i = 1. \end{array}$$

(b) Solve the KKT conditions to obtain the optimal solution for Problem **P1**.
(Note: justify to yourself why the solution is optimal.)