Tracking an Asynchronous Sensor with Kalman Filters

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***Abstract*—In this paper, Kalman filters (KFs) without and with nonlinear constraints are proposed to track an asynchronous sensor node with the help of synchronous anchors. A linear model is derived from time-of-arrival (TOA) measurements, and used to design the KFs. Hence, the proposed KFs do not have the modeling errors compared to the extended Kalman filter (EKF), which employs the first-order Taylor approximation to linearize the nonlinear model. Furthermore, taking nonlinear constraints into account further improves the tracking performance of the KF. Simulation results indicate the superior performance of the proposed approach.**

***Index Terms*—Tracking, sensor network, time-of-arrival, Kalman filter**

I. Introduction

In wireless sensor networks (WSNs), the sensor nodes in general are not static. Therefore, tracking the location of a mobile sensor node is fundamental for the successful deployment of WSNs [1]–[3]. Due to its high accuracy and potentially low cost implementation, ultra-wideband (UWB) signaling allows for time-based ranging to obtain accurate time-of-arrival (TOA) or time-difference-of-arrival (TDOA) measurements [4]. In order to track a mobile node using these TOA or TDOA metrics, clock synchronization has to be considered [2], [3]. Furthermore, the conventional Kalman filter (KF) cannot be applied directly because of the nonlinear relations between these time-based ranging metrics and the coordinates of the mobile sensor node. To deal with nonlinear models, the extended Kalman filter (EKF) [5] is most widely used. However, the performance of the EKF is determined by the accuracy of the linear approximation and may suffer from a lack of convergence [6]. Moreover, the unscented Kalman filter (UKF) [7] and the particle filter (PF) [8] are developed to deal with nonlinear models and non-Gaussian noise for tracking. But both the UKF and the PF are computationally intensive. In [2], an EKF and a UKF are used to track a sensor node using TDOAs with the assistance of fixed anchors with known positions in asynchronous networks. In [3], a sequential Monte Carlo (SMC) method is employed for asynchronous WSNs to jointly estimate the clock offsets and the sensor trajectory, but this method has a high computational complexity.

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In this paper, we develop KFs without and with nonlinear constraints to track a sensor node with the help of anchors, where all the anchors are synchronized and the sensor clock runs freely. Since constraints provide extra information, the KF with constraints in general outperforms the one without [9], [10]. We consider a scenario where the sensor node broadcasts a ranging signal, and all the anchors record timestamps based on the TOAs of the ranging signal. Since we do not know the exact transmission instant of the ranging signal from the sensor node, it is modeled as an unknown parameter. Next, a linear measurement model is derived from these TOA metrics by element-wise multiplication, and nuisance parameters are eliminated by projection. Note that this linear model is quite different from the one obtained by the first order Taylor approximation of the EKF. Hence, the EKF would suffer from the modeling errors, while this is not an issue for the KF based on our linear model. Although similar linearization techniques are employed in [11], we consider a totally different scenario from [11]. So far the nonlinear relations among the unknown parameters are ignored, and thus information is lost. Therefore, we explore these nonlinear relations and propose a KF with nonlinear constraints to further improve the tracking performance. Note that such constraints are not investigated in [11] and cannot be easily made use of either.

II. Linearize the Measurement Model

Assuming M anchor nodes and one sensor node, we would like to track the mobile sensor node. All the nodes are distributed in an l-dimensional space, e.g., $l = 2$ (a plane (2-D)) or $l = 3$ (a space (3-D)). We define the known coordinates of the anchor nodes as $\mathbf{X}_a = [\mathbf{x}_1,\ \mathbf{x}_2,\ \ldots,\ \mathbf{x}_M]$, where the vector $\mathbf{x}_i = [x_{1,i},\ x_{2,i},\ \ldots,\ x_{l,i}]^T$ of length l is the ith anchor coordinates. The unknown coordinates of the sensor node at time k is denoted by a vector $\mathbf{x}(k)$ of length l. All the anchors are synchronized, while the sensor node is asynchronous to the anchors. When the sensor node transmits a ranging signal at its position $\mathbf{x}(k)$, all the anchors receive it and record a timestamp upon the arrival of the ranging signal independently. We define a vector $\mathbf{q}(k)$ of length M to collect all the distances corresponding to these timestamps, which is given by $\mathbf{q}(k) = [q_1(k),\ \ldots,\ q_M(k)]^T$. We employ $b(k)$ to denote the distance corresponding to the true sensor node transmission instant, which is unknown. Consequently, the distances related to the TOA measurements can be modeled as

$$\mathbf{q}(k) - b(k)\mathbf{1}_M \quad = \quad \mathbf{d}(k) + \mathbf{n}(k), \tag{1}$$

where $\mathbf{d}(k) = [d_1(k),\ d_2(k),\ \ldots,\ d_M(k)]^T$, with $d_i(k) = \|\mathbf{x}_i - \mathbf{x}(k)\|$ the true distance between the ith anchor node and the sensor node at time k. Moreover, $\mathbf{n}(k) = [n_1(k),\ \ldots,\ n_M(k)]^T$, where $n_i(k)$ denotes the distance error counting for the measurement error of the ith anchor at the kth time step. We can model it as a zero-mean random variable with variance $\sigma_i^2(k)$, and independent of each other $(E[n_i(k)n_j(k)] = 0, i \neq j)$.

Note that (1) is a nonlinear model with respect to (w.r.t.) $\mathbf{x}(k)$, the conventional KF cannot be applied directly for (1). Therefore, we employ similar linearization techniques as [11] to linearize (1) without any approximation. According to $\|\mathbf{x}_i - \mathbf{x}(k)\|^2 = \|\mathbf{x}_i\|^2 - 2\mathbf{x}_i^T\mathbf{x}(k) + \|\mathbf{x}(k)\|^2$, we can derive that $\mathbf{d}(k) \odot \mathbf{d}(k) = \boldsymbol{\psi}_a(k) - 2\mathbf{X}_a^T\mathbf{x}(k) + \|\mathbf{x}(k)\|^2\mathbf{1}_M$, where $\boldsymbol{\psi}_a = [\|\mathbf{x}_1\|^2,\ \ldots,\ \|\mathbf{x}_M\|^2]^T$. Element-wise multiplication at both sides of (1) is carried out, which leads to

$$\begin{aligned} & \mathbf{q}(k) \odot \mathbf{q}(k) - 2b(k)\mathbf{q}(k) + b^2(k)\mathbf{1}_M \\ = \quad & \boldsymbol{\psi}_a(k) - 2\mathbf{X}_a^T\mathbf{x}(k) + \|\mathbf{x}(k)\|^2\mathbf{1}_M - \mathbf{m}(k). \end{aligned} \tag{2}$$

where $\mathbf{m}(k) = -(2\mathbf{d}(k) \odot \mathbf{n}(k) + \mathbf{n}(k) \odot \mathbf{n}(k))$. Defining $\boldsymbol{\phi}(k) = \boldsymbol{\psi}_a - \mathbf{q}(k) \odot \mathbf{q}(k)$, $\mathbf{y}(k) = [\mathbf{x}^T(k), b(k), b^2(k) - \|\mathbf{x}(k)\|^2]^T$, and $\mathbf{G}(k) = [2\mathbf{X}_a^T,\ -2\mathbf{q}(k),\ \mathbf{1}_M]$, we can finally rewrite (2) as

$$\boldsymbol{\phi}(k) = \mathbf{G}(k)\mathbf{y}(k) + \mathbf{m}(k). \tag{3}$$

Alternatively, by canceling the terms $-2b(k)\mathbf{q}(k)$ and $(b^2(k) - \|\mathbf{x}(k)\|^2)\mathbf{1}_M$ in (2) via projection, we can also obtain the following equation only related to $\mathbf{x}(k)$:

$$\mathbf{b}(k) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{P}_d(k)\mathbf{P}\mathbf{m}(k) \tag{4}$$

where $\mathbf{P} = \mathbf{I}_M - \frac{1}{M}\mathbf{1}_M\mathbf{1}_M^T$,

$$\mathbf{P}_d(k) = \mathbf{I}_M - \frac{\mathbf{P}\mathbf{q}(k)\mathbf{q}^T(k)\mathbf{P}}{\mathbf{q}^T(k)\mathbf{P}\mathbf{q}(k)}, \tag{5}$$

$\mathbf{b}(k) = \mathbf{P}_d(k)\mathbf{P}\boldsymbol{\phi}(k)$, and $\mathbf{F}(k) = 2\mathbf{P}_d(k)\mathbf{P}\mathbf{X}_a^T$.

The statistical properties of the noise $\mathbf{m}(k)$ are given by

$$[\boldsymbol{\mu}(k)]_i = E[[\mathbf{m}(k)]_i] = -\sigma_i^2(k) \approx 0, \tag{6}$$

$$\begin{aligned} & [\boldsymbol{\Sigma}(k)]_{i,j} \\ = \quad & E[[\mathbf{m}(k)]_i[\mathbf{m}(k)]_j] - E[[\mathbf{m}(k)]_i]E[[\mathbf{m}(k)]_j] \\ = \quad & \begin{cases} 4d_i^2(k)\sigma_i^2(k) + 2\sigma_i^4(k) \approx 4d_i^2(k)\sigma_i^2(k), & i = j \\ 0, & i \neq j \end{cases}, \end{aligned} \tag{7}$$

where the higher order terms are ignored to obtain (7) and the noise mean $\boldsymbol{\mu}(k)$ is assumed to be zero under the condition of sufficiently small measurement errors. Note that the noise covariance matrix $\boldsymbol{\Sigma}(k)$ depends on the unknown $\mathbf{d}(k)$. Despite of the unknown $\mathbf{d}(k)$, we can plug in the predicted $\hat{\mathbf{d}}(k|k-1)$, which is calculated by the prediction $\hat{\mathbf{x}}(k|k-1)$ (we will define these notations later on). Furthermore, a larger $\mathbf{d}(k)$ indicates a larger $\boldsymbol{\Sigma}(k)$. The noise is clearly amplified due to the element-wise multiplication. The mean of $\mathbf{P}_d(k)\mathbf{P}\mathbf{m}(k)$ is $\boldsymbol{\zeta}(k) = \mathbf{P}_d(k)\mathbf{P}\boldsymbol{\mu}(k) \approx \mathbf{0}$ and its covariance matrix is given by $\boldsymbol{\Lambda}(k) = \mathbf{P}_d(k)\mathbf{P}\boldsymbol{\Sigma}(k)\mathbf{P}\mathbf{P}_d(k)$, which is rank-deficient due to the projections.

III. Kalman Filters without and with nonlinear constraints

We define the state at the kth time step as $\mathbf{s}(k) = [\mathbf{x}^T(k),\ \dot{\mathbf{x}}^T(k),\ \ddot{\mathbf{x}}^T(k)]^T$ with $\mathbf{x}(k)$ the coordinate vector, $\dot{\mathbf{x}}(k)$ the velocity vector and $\ddot{\mathbf{x}}(k)$ the acceleration vectors of the sensor node at the kth step. Furthermore, we assume a general linear state space model as [5]

$$\mathbf{s}(k+1) = \mathbf{A}(k)\mathbf{s}(k) + \mathbf{B}(k)\mathbf{u}(k) + \mathbf{w}(k), \tag{8}$$

where $\mathbf{A}(k)$, $\mathbf{B}(k)$ and $\mathbf{u}(k)$ are the state transition matrix, the input matrix, and the acceleration input vector, respectively. $\mathbf{w}(k)$ is a zero-mean driving noise vector with a covariance matrix $\mathbf{R}(k)$. Moreover, the measurement model (4) is rewritten using $\mathbf{s}(k)$ as

$$\mathbf{b}(k) = \mathbf{C}(k)\mathbf{s}(k) + \mathbf{P}_d(k)\mathbf{P}\mathbf{m}(k), \tag{9}$$

where $\mathbf{C}(k) = [\mathbf{F}(k),\ \mathbf{0}_{M\times 2l}]$.

Let us assume that $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ and $\mathbf{u}(k)$ are all known, and the driving and the measurement noise are independent. Based on (8) and (9), we can easily develop the corresponding KF tracker. The prediction equations are given by

$$\hat{\mathbf{s}}(k|k-1) = \mathbf{A}(k-1)\hat{\mathbf{s}}(k-1) + \mathbf{B}(k-1)\mathbf{u}(k-1), \tag{10}$$

$$\mathbf{P}_s(k|k-1) = \mathbf{A}(k-1)\mathbf{P}_s(k-1)\mathbf{A}(k-1)^T + \mathbf{R}(k-1), \tag{11}$$

whereas the update equations are given by

$$\begin{aligned} \mathbf{K}(k) = & \ \mathbf{P}_s(k|k-1)\mathbf{C}^T(k) \\ & \times(\mathbf{C}(k)\mathbf{P}_s(k|k-1)\mathbf{C}^T(k) + \boldsymbol{\Lambda}(k))^\dagger, \end{aligned} \tag{12}$$

$$\hat{\mathbf{s}}(k) = \hat{\mathbf{s}}(k|k-1) + \mathbf{K}(k)\left(\mathbf{b}(k) - \mathbf{C}(k)\hat{\mathbf{s}}(k|k-1)\right), \tag{13}$$

$$\mathbf{P}_s(k) = \left(\mathbf{I}_M - \mathbf{K}(k)\mathbf{C}(k)\right)\mathbf{P}_s(k|k-1), \tag{14}$$

where $\hat{\mathbf{s}}(k)$ (or $\hat{\mathbf{s}}(k|k-1)$) and $\hat{\mathbf{P}}_s(k)$ (or $\hat{\mathbf{P}}_s(k|k-1)$) are the estimates of $\mathbf{s}(k)$ and $\mathbf{P}_s(k)$ according to the measurements up to time k (or $k-1$), respectively. As $\mathbf{C}(k)\mathbf{P}_s(k|k-1)\mathbf{C}^T(k) + \boldsymbol{\Lambda}(k)$ may be rank-deficient, the pseudo-inverse denoted by $(\cdot)^\dagger$ is used here instead of the inverse.

From another point of view [9], [12], we can regard $\hat{\mathbf{s}}(k|k-1)$ as a noisy measurement of $\mathbf{s}(k)$. Therefore, it can be modeled as $\hat{\mathbf{s}}(k|k-1) = \mathbf{s}(k) + \mathbf{e}(k|k-1)$, where $\mathbf{e}(k|k-1)$ is the noise term with zero mean and covariance matrix $\mathbf{P}_s(k|k-1)$ according to (11). Collecting $\mathbf{b}(k)$ and $\hat{\mathbf{s}}(k|k-1)$ into a vector, we arrive at

$$\begin{bmatrix} \hat{\mathbf{s}}(k|k-1) \\ \mathbf{b}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{C}(k) \end{bmatrix} \mathbf{s}(k) + \begin{bmatrix} \mathbf{e}(k|k-1) \\ \mathbf{P}_d(k)\mathbf{P}\mathbf{m}(k) \end{bmatrix}. \tag{15}$$

The weighted LS (WLS) estimator based on (15) also produces the Kalman estimate and is given by

$$\hat{\mathbf{s}}(k)=\underset{\mathbf{s}(k)}{\operatorname{argmin}}\left(\|\hat{\mathbf{s}}(k|k-1)-\mathbf{s}(k)\|^2_{\mathbf{P}_s^{-1}(k|k-1)}\right.$$
$$\left.+\|\mathbf{b}(k)-\mathbf{C}(k)\mathbf{s}(k)\|^2_{\mathbf{\Lambda}^{-1}(k)}\right), \quad (16)$$

which can be simplified as (13).

Note that we eliminate nuisance parameters by projection to obtain the linear model (4) only related to $\mathbf{x}(k)$. On the other hand, the projection operation ignores the relations among parameters and thus loses information. In order to improve the tracking performance, we would like to take all the parameters into account, and further explore their nonlinear relations. Therefore, we return to (3), add $\hat{\mathbf{s}}(k|k-1)$ as a new measurement, and arrive at the linear model

$$\widetilde{\boldsymbol{\phi}}(k)=\mathbf{T}(k)\mathbf{z}(k)+\begin{bmatrix}\mathbf{e}(k|k-1)\\ \mathbf{m}(k)\end{bmatrix}, \quad (17)$$

where $\widetilde{\boldsymbol{\phi}}(k)=[\hat{\mathbf{s}}^T(k|k-1) \quad \boldsymbol{\phi}^T(k)]^T$, $\mathbf{z}(k)=[\mathbf{s}^T(k), b(k), b^2(k) - \|\mathbf{x}(k)\|^2]^T$ and

$$\mathbf{T}(k)=\begin{bmatrix} & \mathbf{I}_{3l\times 3l} & & \mathbf{0}_{3l} & \mathbf{0}_{3l}\\ 2\mathbf{X}_a^T & \mathbf{0}_{M\times l} & \mathbf{0}_{M\times l} & -2\mathbf{q}(k) & \mathbf{1}_M\end{bmatrix}. \quad (18)$$

We can then derive a constrained WLS (CWLS) estimator based on (17) as

$$\hat{\mathbf{z}}(k) = \underset{\mathbf{z}(k)}{\operatorname{argmin}}\, J_c(\mathbf{z}(k)), \quad (19)$$

with
$$J_c(\mathbf{z}(k))=\|\hat{\mathbf{s}}(k|k-1)-\mathbf{s}(k)\|^2_{\mathbf{P}_s^{-1}(k|k-1)}$$
$$+\|\boldsymbol{\phi}(k)-\mathbf{G}(k)\mathbf{y}(k)\|^2_{\mathbf{\Sigma}^{-1}(k)}$$
$$+\lambda\left(\mathbf{z}^T(k)\mathbf{J}\mathbf{z}(k)+\boldsymbol{\rho}^T\mathbf{z}(k)\right), \quad (20)$$

where λ is a Lagrangian multiplier, $\boldsymbol{\rho}=[\mathbf{0}^T_{3l+1},1]^T$ and

$$\mathbf{J} = \begin{bmatrix}\mathbf{I}_l & \mathbf{0}_{l\times 2l} & \mathbf{0}_l & \mathbf{0}_l\\ \mathbf{0}_{2l\times l} & \mathbf{0}_{2l\times 2l} & \mathbf{0}_{2l} & \mathbf{0}_{2l}\\ \mathbf{0}_l^T & \mathbf{0}_{2l}^T & -1 & 0\\ \mathbf{0}_l^T & \mathbf{0}_{2l}^T & 0 & 0\end{bmatrix}. \quad (21)$$

The nonlinear constraint $\mathbf{z}^T(k)\mathbf{J}\mathbf{z}(k)+\boldsymbol{\rho}^T\mathbf{z}(k)=0$, which explores the relations among $\mathbf{s}(k)$, $b(k)$ and $b^2(k)-\|\mathbf{x}(k)\|^2$, is taken into the cost function by the Lagrangian multiplier λ. A minimum point for (20) is given by

$$\hat{\mathbf{z}}(k) = (\mathbf{T}^T(k)\mathbf{W}(k)\mathbf{T}(k)+\lambda\mathbf{J})^{-1}$$
$$\times\mathbf{T}^T(k)\mathbf{W}(k)(\widetilde{\boldsymbol{\phi}}(k)-\frac{\lambda}{2}\boldsymbol{\rho}), \quad (22)$$

where $\mathbf{W}(k)$ is the weighting matrix given by

$$\mathbf{W}(k)=\begin{bmatrix}\mathbf{P}_s^{-1}(k|k-1) & \mathbf{0}_{3l\times(l+2)}\\ \mathbf{0}_{(l+2)\times 3l} & \mathbf{\Sigma}^{-1}(k)\end{bmatrix}. \quad (23)$$

Furthermore, λ is determined by plugging (22) into the following equation

$$\hat{\mathbf{z}}^T(k)\mathbf{J}\hat{\mathbf{z}}(k)+\boldsymbol{\rho}^T\hat{\mathbf{z}}(k) = 0. \quad (24)$$

We could find all the roots of (24) as in [13], or employ a bisection algorithm as in [14] to look for λ instead of finding all the roots. If we obtain roots as in [13], we discard the complex roots, and plug the real roots into (22). Finally, we choose the estimate $\hat{\mathbf{z}}(k)$, which fulfills (19).

In summary, the prediction equations for the KF with nonlinear constraints are the same as the KF without constraints. Meanwhile, the update equations for the KF with nonlinear constraints are given by (12), (14) and (19). We remark here that (12) and (14) are used to approximate the error covariance matrices of the KF with nonlinear constraints, which are overestimated in this case. Furthermore, the proposed method to derive the KF with nonlinear constraints is equivalent to the one proposed in [10] taking the optimal weighting matrix into account. In [10], the unconstrained KF estimates are first obtained and used as new measurements to derive a CLS estimator. If we follow the method in [10], but make use of the optimal weighting matrix given by $\mathbf{P}_s^{-1}(k|k-1)$ to arrive at a CWLS estimator instead of a CLS one , the same results will be achieved as for our proposed method.

IV. Simulation Results

We evaluate the performance of the proposed KFs without and with nonlinear constraints by Monte Carlo simulations. Five anchors with known positions are employed, where four anchors are located at the corners of a $100\,\mathrm{m}\times 100\,\mathrm{m}$ rectangular, and the fifth anchor is located at the center of the rectangular. Since the ranging signal is broadcast by the sensor node, we assume that $\sigma_i^2(k)$ is related to the distance based on the path loss model. Therefore, the average noise power is defined as $\bar{\sigma}^2=1/M\sum_{i=1}^M\sigma_i^2(k)$, where we choose the variances $\sigma_i^2(k)$ to satisfy the condition that all $\sigma_i^2(k)/d_i^2$ are equal. We employ a hidden Markov model (HMM) to mimic the source motion as in [15], where (8) is reduced to

$$\mathbf{s}(k+1)=\begin{bmatrix}\mathbf{I}_l & \mathbf{0}_{l\times 2l}\\ \mathbf{0}_{2l\times l} & \mathbf{0}_{2l\times 2l}\end{bmatrix}\mathbf{s}(k)+\begin{bmatrix}\mathbf{w}_{\mathbf{x}}(k)\\ \mathbf{0}_{2l}\end{bmatrix}, \quad (25)$$

where $\mathbf{w}_{\mathbf{x}}(k)$ is a zero mean white Gaussian process with covariance matrix $\sigma_w^2\mathbf{I}_l$. Note that we can employ other state space models as well. In each Monte Carlo run, the initial state estimate $\hat{\mathbf{s}}(-1|-1)$ is randomly generated according to $\mathcal{N}(\mathbf{s}(-1),\mathbf{P}_s(-1|-1))$, with $\mathbf{s}(-1)=[\mathbf{x}(-1)^T\ \mathbf{0}_{2l}^T]^T$ (the true initial state) and $\mathbf{P}_s(-1|-1)=100\mathrm{diag}([\mathbf{1}_l^T\ \mathbf{0}_{2l}^T]^T)$ (the covariance matrix of the initial state estimate). In each run, a trajectory of 50 points is generated according to the state model, and $b(k)$ is randomly generated in the range of $[1\mathrm{m}, 100\mathrm{m}]$. We use the root mean square error (RMSE) of $\hat{\mathbf{x}}(k)$ as the performance criterion, which is defined as $\sqrt{1/N_{exp}\sum_{j=1}^{N_{exp}}\|\hat{\mathbf{x}}^{(j)}(k)-\mathbf{x}(k)\|^2}$, where $\hat{\mathbf{x}}^{(j)}(k)$ is the estimate in the jth trial at the kth time step and $N_{exp}=500$. The rest of the parameters are assigned as $\mathbf{x}(-1)=[13\,\mathrm{m},\ 4\,\mathrm{m}]^T$, $\sigma_w^2=6$, which corresponds to a relatively large state model error, and $1/\bar{\sigma}^2=10\,\mathrm{dBm}$, which indicates that the measurement errors are several tens of centimeters and can be achieved by UWB signals.

The proposed KFs are compared to the EKF as well. Premultiplying the projection matrix $\mathbf{P}$ on both sides of (1) to eliminate the term $-b(k)\mathbf{1}_M$, we derive the EKF based on this new data model, which is only related to $\mathbf{x}(k)$

$$\mathbf{Pq}(k) = \mathbf{d}(k) - \bar{d}(k)\mathbf{1}_M + \mathbf{Pn}(k), \quad (26)$$

where $\bar{d}(k) = \frac{1}{M}\sum_{i=1}^{M} d_i(k)$. Defining the function $f(\mathbf{x}(k))$ as $f(\mathbf{x}(k)) = \mathbf{d}(k) - \bar{d}(k)\mathbf{1}_M$, and recalling that $d_i(k) = \|\mathbf{x}(k) - \mathbf{x}_i(k)\|$, we derive the Jacobian matrix $\mathbf{H}(k)$ of $f(\mathbf{x}(k))$ w.r.t. $\mathbf{x}(k)$ and evaluate its value at $\hat{\mathbf{x}}(k|k-1)$ as

$$[\mathbf{H}(k)]_{i,:} = \left.\frac{\partial[f(\mathbf{x}(k))]_i}{\partial \mathbf{x}(k)}\right|_{\mathbf{x}(k)=\hat{\mathbf{x}}(k|k-1)} = \left.\left(\frac{(\mathbf{x}(k) - \mathbf{x}_i)^T}{\|\mathbf{x}(k) - \mathbf{x}_i\|} - \frac{1}{M}\sum_{j=1}^{M}\frac{(\mathbf{x}(k) - \mathbf{x}_j)^T}{\|\mathbf{x}(k) - \mathbf{x}_j\|}\right)^T\right|_{\mathbf{x}(k)=\hat{\mathbf{x}}(k|k-1)}, \quad (27)$$

Note that the covariance matrix of the noise $\mathbf{Pn}(k)$ in (26) does not depend on $\mathbf{d}(k)$, in contrast to the covariance matrix of the noise in (3) and (4), which is an advantage of the EKF. Since we have developed all ingredients for the EKF, the standard prediction and update equations are omitted for brevity.

Fig. 1 shows a trajectory example, and the estimated trajectory by the KF with constraints called the CKF (the dashed line with "+" markers). The sensor moves inside the area of interest, which is a $100\,\mathrm{m} \times 100\,\mathrm{m}$ rectangular defined by four anchors at the corner. The estimated trajectory follows the true trajectory quite well. Fig. 2 illustrates the localization performance of the KF and EKF trackers, where the KF with nonlinear constraints (solid lines with "+" markers) outperforms all the other trackers. The performance gain by exploring constraint information is obvious. The performance of the EKF (the solid line with "o" markers) is the worst and varies quite a lot along the time index compared to the other trackers, whose performance is more stable. The estimated RMSEs of the trackers are calculated by $\sqrt{1/N_{exp}\sum_{j=1}^{N_{exp}}([\mathbf{P}^{(j)}(k)]_{1,1} + [\mathbf{P}^{(j)}(k)]_{2,2})}$ with $\mathbf{P}^{(j)}(k)$ the covariance estimate achieved in the jth trial at the kth time step. The estimated RMSEs of the KF with and without constraints (dashed lines with "+" and "×" markers, respectively) are the same, since these KF trackers employ the same error covariance prediction and update equations. The true RMSE of the KF without constraints (the solid line with "×" markers) closely follows the estimated one. The estimated RMSE of the EKF is too optimistic.

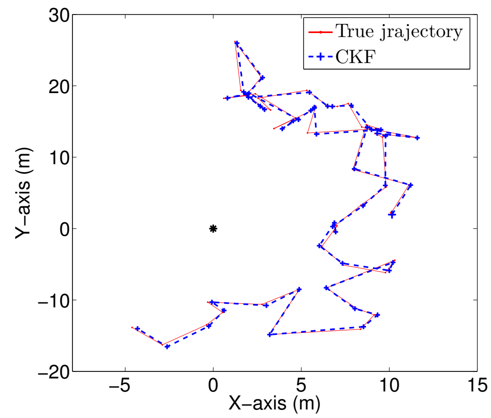

Fig. 1. An trajectory example

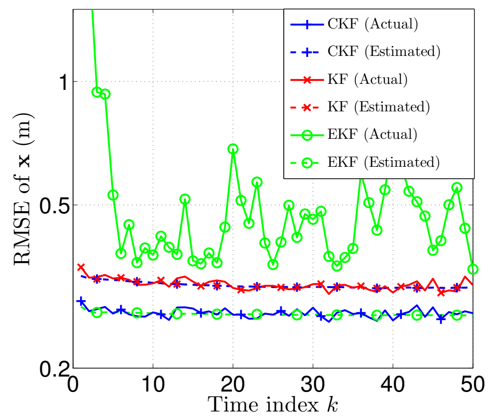

Fig. 2. Localization RMSE for the proposed KFs without and with nonlinear constraints as well as the EKF

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