Refactor Inference

January 5, 2022

Type variables	a, b		
Existential variables	$\widehat{\alpha}, \widehat{\beta}$		
Types	A, B, C	$::=$	$1 \mid a \mid \forall x.A \mid A \to B \mid \widehat{\alpha}$
Declarative Monotypes	τ	$::=$	$1 \mid a \mid \tau_1 \to \tau_2$
Algorithmic Monotypes	l, u	$::=$	$1 \mid a \mid \tau_1 \to \tau_2 \mid \widehat{\alpha}$
Context	Ψ	$::=$	$\cdot \mid \Psi, a$
Algorithmic worklist	Γ	$::=$	$\cdot \mid \Gamma, a \mid \Gamma \Vdash A \leq B$

$\boxed{\Gamma \longrightarrow \Gamma'}$ Γ reduces to Γ'.

$$
\begin{array}{rcl}
\Gamma, a & \longrightarrow & \Gamma \\
\Gamma, \widehat{\alpha} & \longrightarrow & \Gamma \\
\Gamma, \widehat{\alpha} & \longrightarrow & \Gamma \\
\Gamma \Vdash 1 \leq 1 & \longrightarrow & \Gamma \\
\Gamma \Vdash a \leq a & \longrightarrow & \Gamma \\
\Gamma \Vdash \widehat{\alpha} \leq \widehat{\alpha} & \longrightarrow & \Gamma \\
\Gamma \Vdash A_1 \to A_2 \leq B_1 \to B_2 & \longrightarrow & \Gamma \Vdash A_2 \leq B_2 \Vdash B_1 \leq A_1 \\
\Gamma \Vdash \forall a.\ A \leq B & \longrightarrow & \Gamma, \widehat{\alpha} \Vdash [\widehat{\alpha}/a]A \leq B \quad \text{when } B \neq \forall a.\ B' \\
\Gamma \Vdash A \leq \forall b.\ B & \longrightarrow & \Gamma, b \Vdash A \leq B \\
\Gamma \Vdash \widehat{\alpha} \leq \tau & \longrightarrow & \{\tau/\widehat{\alpha}\}\Gamma \\
\Gamma \Vdash \tau \leq \widehat{\alpha} & \longrightarrow & \{\tau/\widehat{\alpha}\}\Gamma \\
\Gamma \Vdash \widehat{\alpha} \leq A \to B & \longrightarrow & \{\widehat{\alpha}_1 \to \widehat{\alpha}_2/\widehat{\alpha}\}(\Gamma, \widehat{\alpha}_1, \widehat{\alpha}_2) \Vdash \widehat{\alpha}_1 \to \widehat{\alpha}_2 \leq A \to B \\
 & & \quad \text{when not monotype } (A \to B) \\
\Gamma \Vdash A \to B \leq \widehat{\alpha} & \longrightarrow & \{\widehat{\alpha}_1 \to \widehat{\alpha}_2/\widehat{\alpha}\}(\Gamma, \widehat{\alpha}_1, \widehat{\alpha}_2) \Vdash A \to B \leq \widehat{\alpha}_1 \to \widehat{\alpha}_2 \\
 & & \quad \text{when not monotype } (A \to B)
\end{array}
$$

$$\{A/\widehat{\alpha}\}\,\Gamma, \widehat{\alpha} \longrightarrow \Gamma$$

$\boxed{\{A/\widehat{\alpha}\}\,\Gamma \longrightarrow \Gamma'}$ SubstWL

$$\frac{}{\{A/\widehat{\alpha}\}\,\Gamma, \widehat{\alpha} \longrightarrow \Gamma}\ \texttt{WL\_}\widehat{\alpha} \qquad \frac{\{A/\widehat{\alpha}\}\,\Gamma \longrightarrow \Gamma' \qquad b \notin \mathrm{FV}(A)}{\{A/\widehat{\alpha}\}\,\Gamma, b \longrightarrow \Gamma', b}\ \texttt{WL\_b}$$

$$\frac{\{A/\widehat{\alpha}\}\,\Gamma \longrightarrow \Gamma' \qquad \widehat{\beta} \notin \mathrm{FV}(A)}{\{A/\widehat{\alpha}\}\,\Gamma, \widehat{\beta} \longrightarrow \Gamma', \widehat{\beta}}\ \texttt{WL\_}\widehat{\beta}\texttt{\_fresh} \qquad \frac{\{A/\widehat{\alpha}\}\,\Gamma[\widehat{\beta}, \widehat{\alpha}] \longrightarrow \Gamma' \qquad \widehat{\beta} \in \mathrm{FV}(A)}{\{A/\widehat{\alpha}\}\,\Gamma[\widehat{\alpha}], \widehat{\beta} \longrightarrow \Gamma'}\ \texttt{WL\_}\widehat{\beta}\texttt{\_inA}$$

$$\frac{\{A/\widehat{\alpha}\}\,\Gamma \longrightarrow \Gamma'}{\{A/\widehat{\alpha}\}\,\Gamma \Vdash B \leq C \longrightarrow \Gamma' \Vdash [A/\widehat{\alpha}]B \leq [A/\widehat{\alpha}]C}\ \texttt{WL\_sub}$$

$\boxed{\{A/\widehat{\alpha}\}\,\Gamma \mid \Gamma_o \longrightarrow \Gamma'}$ SubstWL with substution in the end

$$\frac{}{\{A/\widehat{\alpha}\}\,\Gamma, \widehat{\alpha} \mid \Gamma_o \longrightarrow \Gamma, [A/\widehat{\alpha}]\Gamma_o}\ \texttt{WL\_}\widehat{\alpha} \qquad \frac{\{A/\widehat{\alpha}\}\,\Gamma \mid b, \Gamma_o \longrightarrow \Gamma' \qquad b \notin \mathrm{FV}(A)}{\{A/\widehat{\alpha}\}\,\Gamma, b \mid \Gamma_o \longrightarrow \Gamma'}\ \texttt{WL\_b}$$

$$\frac{\{A/\widehat{\alpha}\}\,\Gamma \mid \widehat{\beta}, \Gamma_o \longrightarrow \Gamma' \qquad \widehat{\beta} \notin \mathrm{FV}(A)}{\{A/\widehat{\alpha}\}\,\Gamma, \widehat{\beta} \mid \Gamma_o \longrightarrow \Gamma'}\ \texttt{WL\_}\widehat{\beta}\texttt{\_fresh} \qquad \frac{\{A/\widehat{\alpha}\}\,\Gamma[\widehat{\beta}, \widehat{\alpha}] \mid \Gamma_o \longrightarrow \Gamma' \qquad \widehat{\beta} \in \mathrm{FV}(A)}{\{A/\widehat{\alpha}\}\,\Gamma[\widehat{\alpha}], \widehat{\beta} \mid \Gamma_o \longrightarrow \Gamma'}\ \texttt{WL\_}\widehat{\beta}\texttt{\_inA}$$

$$\frac{\{A/\widehat{\alpha}\}\,\Gamma \mid B \leq C, \Gamma_o \longrightarrow \Gamma'}{\{A/\widehat{\alpha}\}\,\Gamma \Vdash B \leq C \mid \Gamma_o \longrightarrow \Gamma'}\ \texttt{WL\_sub}$$