```
x, y, z
type,\ t,\ A,\ B
                                    ::=
                                           int
                                           for all x.t
                                                                            \mathsf{bind}\;x\;\mathsf{in}\;t
                                           t_1 - > t_2
                                           \uparrow x
                                                                            S
                                           (t)
                                           [t_1/x]t_2
                                                                            Μ
var, v
                                    ::=
                                           \boldsymbol{x}
                                           \uparrow x
bounds, bs
                                    ::=
                                           bnil
                                           bs, t
work, w
                                    ::=
                                           \boldsymbol{x}
                                           bs <= \uparrow x <= bs'
                                           t_1 <: t_2
worklist, wl
                                    ::=
                                           wnil
                                           wl, w
formula
                                    ::=
                                           judgement
Monotype
                                    ::=
                                           \mathbf{mono}\ t
In Work list
                                    ::=
                                           v \ \mathbf{in} \ wl
InScope
                                    ::=
                                           wl[v_1][v_2]
Algorithmetic Subtyping
                                    ::=
                                           |-wl|
                                           wl|-w
                                           |wl| - wl'
judgement
                                    ::=
                                           Monotype
                                           In Work list
                                           InScope
```

Algorithmetic Subtyping

 $\mathbf{mono}\ t$

$$\begin{array}{ccc} \overline{\mathbf{mono}\,x} & \mathrm{LA_MONO_TVAR} \\ \hline \overline{\mathbf{mono}\,(\!\!\uparrow\!\!x)} & \mathrm{LA_MONO_TEXVAR} \\ \hline \overline{\mathbf{mono}\,\mathbf{int}} & \mathrm{LA_MONO_INT} \\ \hline \mathbf{mono}\,t_1 \\ \hline \mathbf{mono}\,t_2 \\ \hline \overline{\mathbf{mono}\,(t_1->t_2)} & \mathrm{LA_MONO_ARROW} \end{array}$$

 $v \mathbf{in} wl$

$$\overline{x \ in \ wl, x} \quad \text{IWL_HERE_VAR}$$

$$\overline{\uparrow x \ in \ wl, bs} <= \uparrow x <= bs' \quad \text{IWL_HERE_EXTVAR}$$

$$\overline{x \ in \ wl, bs} <= \uparrow y <= bs' \quad \text{IWL_THERE_VAR_VAR}$$

$$\overline{\frac{x \ in \ wl}{x \ in \ wl, y}} \quad \text{IWL_THERE_VAR_EXTVAR}$$

$$\frac{\uparrow x \ in \ wl}{\uparrow x \ in \ wl, y} \quad \text{IWL_THERE_EXTVAR_VAR}$$

$$\frac{\uparrow x \ in \ wl}{\uparrow x \ in \ wl, bs} <= \uparrow y <= bs' \quad \text{IWL_THERE_EXTVAR_EXTVAR}$$

$$\frac{x \ in \ wl}{x \ in \ wl, t_1 <: t_2} \quad \text{IWL_THERE_VAR_JUDGE}$$

$$\frac{\uparrow x \ in \ wl}{\uparrow x \ in \ wl, t_1 <: t_2} \quad \text{IWL_THERE_EXTVAR_JUDGE}$$

 $wl[v_1][v_2]$

$$\frac{v_1 \text{ in } wl}{wl, x[v_1][x]} \quad \text{INS_HERE_VAR}$$

$$\frac{v_1 \text{ in } wl}{wl, bs <= \uparrow x <= bs'[v_1][\uparrow x]} \quad \text{INS_HERE_EXTVAR}$$

$$\frac{wl[v_1][v_2]}{wl, w[v_1][v_2]} \quad \text{INS_THERE_W}$$

|-wl|

wl | - w

$$\frac{1}{wl - x}$$
 LA_W_WL_VAR

coq wl_worklist wl $\underline{\text{single_side_constraints } (bs')}$ LA_W_WL_EXTVAR_UNFOLD_LNIL

$$|wl| - \mathbf{bnil} \le \uparrow x \le bs'$$

 $|wl| - bs \le \uparrow x \le \mathbf{bnil}$

$$\frac{\text{coq wl_worklist wl double_side_constraint } (bs)}{wl|-bs <= \uparrow x <= bs'} \quad \text{LA_w_wl_extvar_unfold}$$

$$\frac{}{wl|-\mathbf{int}<:\mathbf{int}}\quad \text{LA_W_WL_SUB_INT}$$

$$\overline{wl|-x<:x}\quad \text{LA_W_WL_SUB_TVAR}$$

$$\frac{}{wl|-\uparrow x<:\uparrow x}\quad \text{LA_W_WL_SUB_EXTVAR}$$

$$\frac{wl, \mathbf{bnil} <= \uparrow y <= \mathbf{bnil} | - [\uparrow y/x]A <: B}{wl| - \mathbf{forall} \ x.A <: B} \quad \text{LA_W_WL_SUB_FORALLL}$$

$$\frac{wl,y|-[y/x]A<:B}{wl|-A<:\mathbf{forall}\,x.B}\quad \text{la_w_wl_sub_forallR}$$

$$wl | -A_2 <: B_2$$

$$\frac{wl, A_2 <: B_2 | -B_1 <: A_1}{wl| -A_1 -> A_2 <: B_1 -> B_2} \quad \text{LA_W_WL_SUB_ARROW}$$

|wl| - wl'

$$\frac{|-wl|}{wl|-\mathbf{wnil}} \quad \text{LA_WL_WL_NIL}$$

$$\frac{\text{coq }wl \text{ ++ }wl'}{wl|-wl',w'} \quad \text{LA_WL_WL_CONS}$$

Definition rules: 29 good Definition rule clauses: 51 good 0 bad