Capstone Project:
Conventional vs FIRE Retirement Strategies

ST474

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Topic Preface

The topic we are interested in is the FIRE (Financial Independence, Retire Early) retirement strategy where the retiree would save a large nest egg early following Bill Bengen's 4% rule and draw from that portfolio annually (Kerr, 2022). We wanted to compare this strategy with the conventional saving strategy, which is saving until retirement at age 65. We want to simulate these two strategies with financial and mortality data of Canadians.

Some topics we want to investigate would be how successful the FIRE strategy would be: Would FIRE be more susceptible to volatility due to early retirement? How likely FIRE can last until a retiree's death compared to conventional strategy? We also wanted to challenge FIRE's 4% rule and conduct sensitivity analysis given changes to certain inputs.

The reason why we are interested in this topic is because most university students are not well educated about personal finance and retirement. By researching different retirement strategies, it can provide insights on how people can best apply these strategies to their own personal financial situations. FIRE strategy is deeply rooted in being frugal and minimalist, so we are passionate to see whether this strategy is feasible for Canadians in general. We believe this research topic would interest those who want to know different retirement strategies, those who seek an early retirement, and those who are interested in the impacts of building a large nest egg early.

Stochastic Models Introduced

The stochastic models we used in our investigation were for age of mortality and return of the market respectively.

Our stochastic model of mortality is based on an exponential model which demonstrates that as age increases, death rate increases exponentially as well. This model is rooted from the mortality data given from Stats Canada (See Appendix 5). When plotting each age group in a histogram, the data shows that the death rate grows exponentially as age increases. We can use this property to generate the age of death for each simulation. This age of death will allow the simulation to define the success of retirement if funds do not run out by death.

We used a stochastic model for return of the market that follows a Laplace distribution. We used a Laplace distribution for the model because returns in the stock market have fat tails, that is extreme events happen more frequently than expected, and a Laplace distribution fits the volatility in the stock market better than a normal distribution does. Using the generated returns, we were able to simulate the changes in the accumulated wealth a person could experience. The function used to generate returns was directly from course content.

Stochastic Model Explained

Stochastic Exponential Mortality Model

Given data from Statistics Canada, we constructed a histogram of mortality rate and fitted an exponential curve on the histogram to gain the pdf of the exponential distribution (See Appendix 5). The PDF is y=0.0148*EXP(0.5627x) where y represents the rate of mortality and x represents age at different stages, a stage being every 5 years. We used the PDF of mortality rate to compute the CDF, which is:

$$0.02630175937 * (EXP(0.5627 * x) - 1)$$

The inverse transform of the CDF is:

$$x = (1 / 0.5627) * \log(u / (0.0148 / 0.5627) + 1)$$

The x would range approximately from (0, 7), and by using IF and ELSE statements, the program would generate the death age (See Appendix 1). For example, given x = 2.5, which is between 2 and 3, it falls under the 70-74 age range. Age of death would equal to floor(70+(2.5-2)*5) = 72.

Stochastic Returns following a Laplace distribution

To generate random returns that follow a Laplace distribution, we knew that Z=X1-X2 follows a standard Laplace distribution where both X1 and X2 are independent EXP(1) random variables. We used the formula r=0.074+0.11*Z (Metzler, 2022, p. 05) to determine the annual market return. (See Appendix 2 for code)

Conventional Model

The key assumptions used for the conventional method simulation model were:

- Median annual income of $66,800 (Government Of Canada, 2022)
- Annual expense of $47008.58 (WOWA, 2022) (See Appendix 6)

- Annual saving rate (amount investing) = 0.0597 (ycharts, 2022)

In each generated realization, a person would accumulate wealth for 35 years, starting at age of 30 until 65. An average Canadian would save $3987.96 (66800*0.0697) annually, and the accumulated amount would be compounded by multiplying the market returns. By age 66, the model would change to subtracting $47008.58 annually and then compounding the remaining amount. This continues until death. The conventional model will run the simulation for 10000 times through a loop. In each simulation, if the amount of wealth after retirement is less than or equal to 0 at any point, that simulation would be considered a failure. Given 10000 simulations, we can determine the success rate of the conventional strategy. (See Appendix 3 for code)

Fire Model

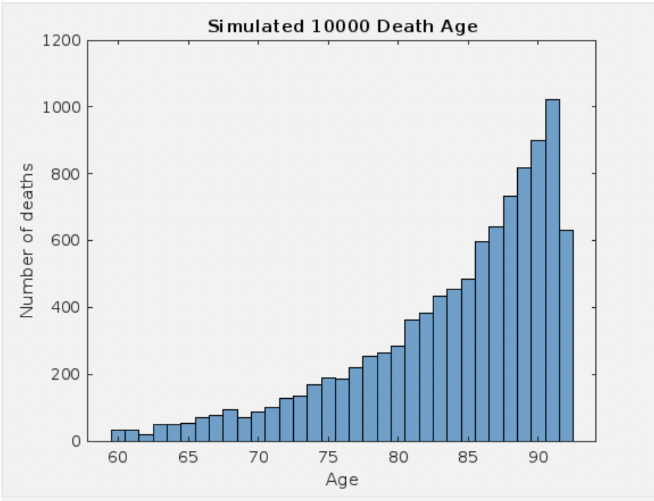
The key assumptions used to build the FIRE method simulation model were:

- Median annual income of $66,800 (Government Of Canada, 2022)
- Annual saving rate (amount investing) = 0.70 (Kerr, 2022)
- Bill Bengen 4% rule: ie. save until $501,000 = 66,800*(1-0.7)/0.04

In each generated realization, a person would accumulate wealth until they reached $501,000. The FIRE user would save $46,760 (66800*0.7) annually, and the accumulated amount would be compounded by multiplying the market returns. Once the user reaches $501,000, the model would change to subtracting $20,040 (66800*0.3) annually and then compounding the remaining amount. This continues until death. The FIRE model will run the simulation for 10000 times through a loop. In each simulation, if the amount of wealth after retirement is less than or equal to 0 at any point, that simulation would be considered a failure. Given 10000 simulations, we can determine the success rate of the FIRE strategy. (See Appendix 4 for code)

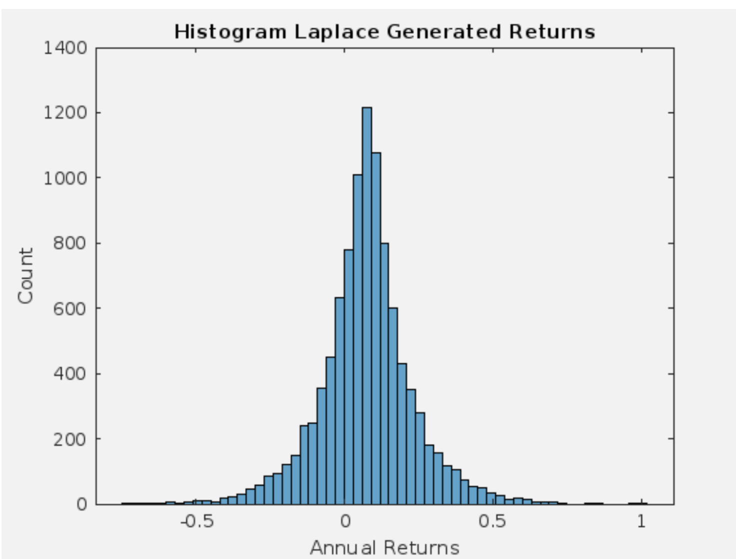
Simulation Outputs

Section 1 - 10000 Simulation Results from Mortality Model



Above image shows a histogram of the simulated results from the mortality model. As seen, it follows the exponential function we defined in the report. The average death age is 84 years old.

Section 2 - 10000 Simulation Results from Laplace Annual Return Model



Above image shows a histogram of the simulated annual returns following a laplace distribution.

Section 3 - General Key Metrics From Simulation

Figure 3.1 Key Metrics after 100 of 10000 Monte Carlo simulations

100 of 10000 Monte Carlo Simulations		
Strategy	Conventional	FIRE
Successful Retirement Rate (%)	66.08%	94.67%
Std of Success Rate	0.43%	0.24%

Table above shows the average success rate and its standard deviation for the Conventional and FIRE strategy after 100 simulations of 10000 Monte Carlo simulations.

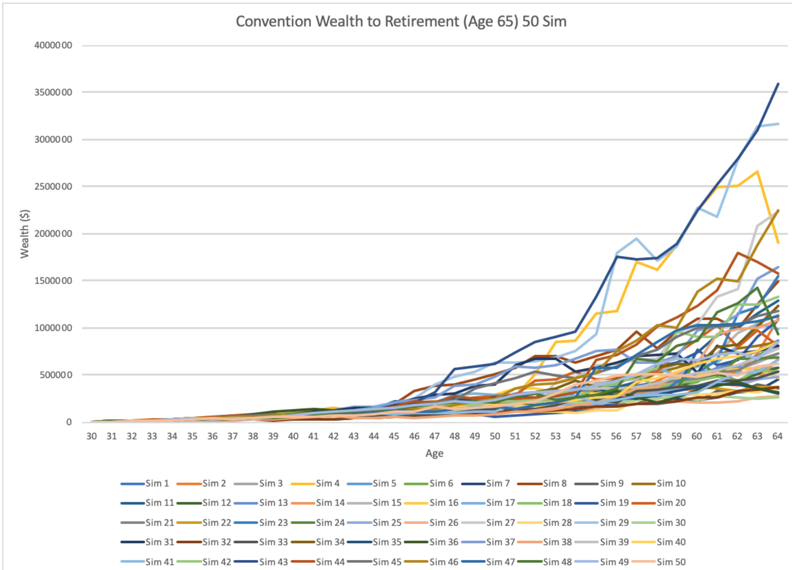
Figure 3.2 Key Metrics after 1 of 10000 Monte Carlo simulations

10000 Monte Carlo Simulations		
Strategy	Conventional	FIRE
Avg Age of retirement (#)	65.00	39.71
Avg Amount at retirement ($)	$ 848,940.06	$ 602,631.34
Std of amount at retirement ($)	$ 67,934.35	$ 167,114.73
Avg Amount at Death ($)	$ 2,324,232.05	$ 20,795,940.20
Std Amount at Death ($)	$ 6,962,503.07	$ 49,147,686.30
Given failure, avg years retirement (#)	11.6	29.6
Given failure, std years retirement (#)	5.3	8.8

Table above shows key metrics for the Conventional and FIRE strategy after 1 simulation of 10000 Monte Carlo simulations.

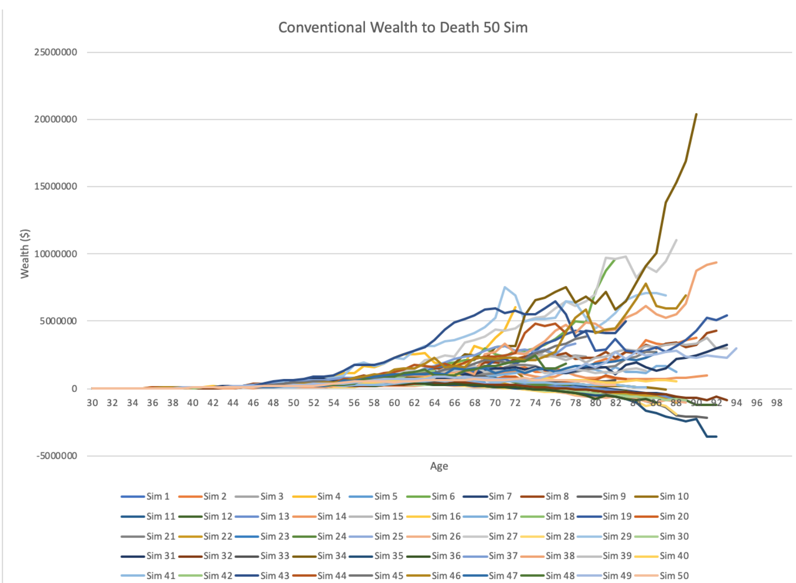
Section 4 - Plots of Simulated Retirement Paths (Conventional and FIRE)

Figure 4.1 Conventional Method Wealth to Retirement (Age 65) 50 Simulations



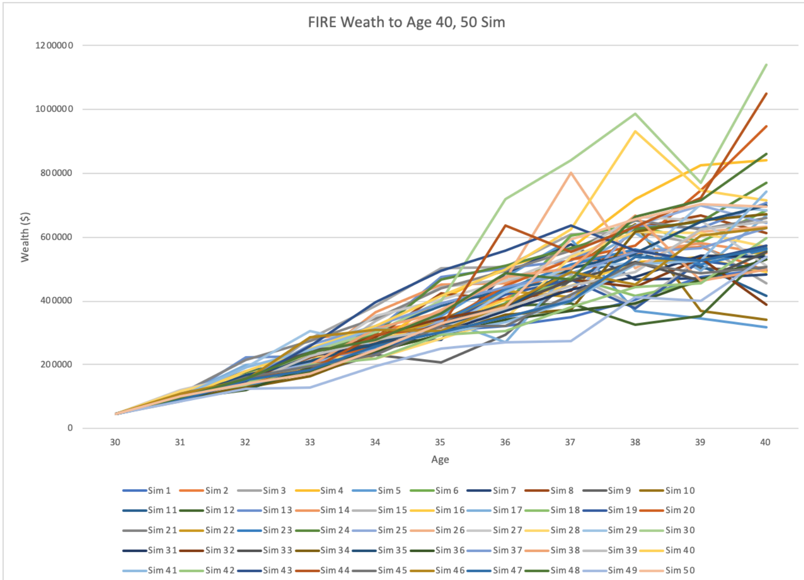
Above Image shows 50 independent simulation results of portfolio value from age 30 saved up to age 65 (retirement).

Figure 4.2 Conventional Method Wealth to Death 50 Simulations



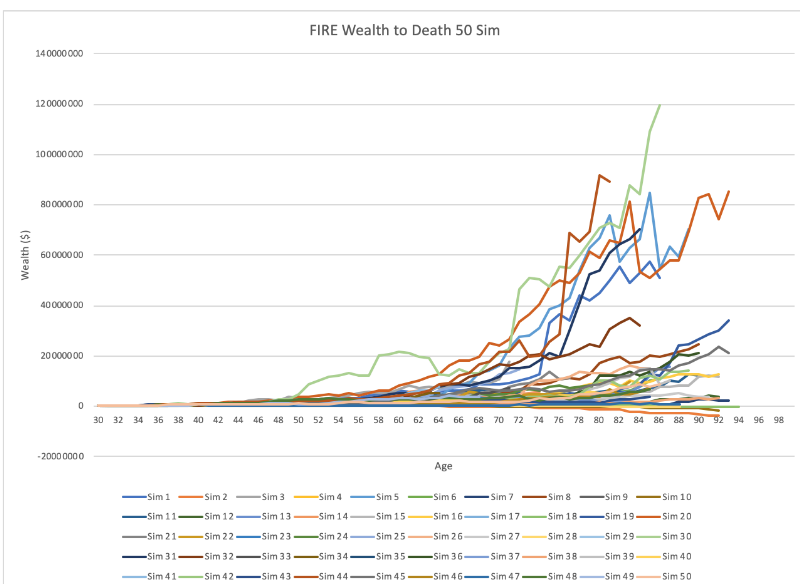
Above Image shows 50 independent simulation results of portfolio value from age 30 to death. Notice after age 65, portfolio can dip to negative since the person is withdrawing annually.

Figure 4.3 FIRE Method Wealth to Age 40, 50 Simulations



Above Image shows 50 independent simulation results of portfolio value from age 30 saved up to age 40. Age 40 was chosen since it is approximately the average retirement age for FIRE in our model.

Figure 4.4 FIRE Method Wealth to Death, 50 Simulations



Above Image shows 50 independent simulation results of portfolio value from age 30 to death. Notice that the portfolio can dip to negative since the person is withdrawing annually in retirement.

Section 5 - FIRE Method Sensitivity Analysis

Figure 5.1 Key Metrics when changing the saving rate (and spending rate)

Changing Saving Rate						Special Case
Saving Rate	0.9	0.8	0.7	0.6	0.5	0.141
Success Rate (100 of 10000 sims)	95.25%	94.68%	94.67%	94.93%	95.37%	98.67%
Success std	0.20%	0.23%	0.24%	0.27%	0.21%	0.12%
Age for retirement	34.0754	36.8767	39.7135	42.7608	46.0572	64.9758

Spending rate = (1-saving rate). Table above shows the changes in key metrics, like success rate and age for retirement, given changes in the saving and spending rate. Special case shows the saving rate required for the age of retirement of 65.

Figure 5.2 Key Metrics when changing the Spending Rate at Retirement

Changing Spending Rate at Retirement					
Spending Rate (1-Saving Rate+change)	0.2	0.25	0.3	0.35	0.4
Success Rate	99.09%	97.49%	94.67%	90.32%	84.34%
Success std	0.10%	0.16%	0.24%	0.30%	0.38%

Table above shows the changes of success rate when the annual expense increases or decreases during retirement.

Figure 5.3 Key Metrics when changing Bill Bengen's 4% rule

Changing 4% rule					
%	0.02	0.03	0.04	0.05	0.06
Amount need to retire	1002000	668000	501000	400800	334000
Success Rate	99.81%	98.50%	94.67%	87.38%	77.18%
Success std	0.04%	0.13%	0.24%	0.31%	0.47%
Age for retirement	44.7578	41.6358	39.7135	38.4329	37.4956

Table above shows the changes to key metrics to the retirement plan when the 4% rule is adjusted up or down.

Discussion and Conclusion

Looking at the figure 3.1, it was clear that the FIRE strategy has a much higher success rate than the conventional method. It is surprising how the average Canadian following the conventional method would only have 66.08% chance to have enough funds to last them until their death. While the FIRE method does promise a higher rate for success, it requires the person to be extremely frugal. We assume that they only spend $20,040 (66800*0.3) annually, which is significantly less than the average Canadian annual expense of $47,008.58.

Looking at figure 3.2, it was surprising to see that the average FIRE user can retire at the age of 39.71. This means that they can retire after 10 years of savings versus the 35 years of saving for the conventional method. The amount required for FIRE users to retire is $501,000, but it was interesting to see that the average amount FIRE users have by age 40 is $602,631.34. Given that FIRE users have a large nest egg early, we can see the effects of compounding through the average amount they have at death which is $20,795,940. This is much larger than the average amount at death for conventional methods ($2,324,232.05). The deviation at death for conventional and FIRE are a few times larger than the average. This is because there can be outliers with extreme returns, and also because the user can go into negatives when they draw money even though their portfolio is negative.

This negative amount prompted us to look into the average years of retirement given a failure; i.e. money did not last them until death. For the conventional method, it would last them 11.6 years in retirement, with a std of 5.3 years. This means that given failure, the average Canadian would run out of funds by 76.6 years old if the market returns poorly. On the other hand, FIRE users on average would have 29.6 years of retirement with a std of 8.8 years. This means that given failure, FIRE users would run out of funds by 69.6 years old. This could be a potential risk for FIRE users because if the market returns poorly, they would be out of funds earlier than the conventional method even though they retired early. However, we want to remind that the FIRE method has an approximately 95% success rate so this scenario is extremely rare.

The figures in section 4 shows a small sample of the Monte Carlo simulations for the conventional method and FIRE method. The charts really display the power of compounding. The FIRE method builds a large nest egg very early on, so as it compounds throughout time, it can grow significantly larger than the conventional method.

Finally, we felt that the frugality aspect of FIRE may not be realistic for every Canadian, so we did a sensitivity analysis by changing the inputs (results in section 5). In figure 5.1, it shows that the success rate increases as the saving rate deviates from 0.7. Moreover, as the savings rate decreases, the age for retirement increases. This increasing retirement age could be the reason why we see a higher success rate as the savings rate decreases. In the special case, if the FIRE user wants to retire at age 65, they would need a saving rate of 0.141, which is more than double the saving rate current Canadians have (0.0597). If they are able to have a saving rate of 0.141, they can achieve retirement success with a success rate of 98.67% and std of 0.12%. This shows that Canadians who follow the conventional method must significantly increase their annual savings rate and be consistent with their saving and spending habits in order to have a successful retirement.

In figure 5.2 and 4.3, we tested the flexibility of the FIRE method by changing the withdrawal amount after retirement and challenging Bill Bengen's 4% rule. As expected, increasing the spending rate during retirement decreases the success rate. We feel that a 90% success rate is still reasonable, so a FIRE user can actually spend up to 35% of their annual income during retirement. As we increase the 4% rule, the success rate decreases as well, but the age for retirement decreases. The success rate decreases significantly for every percent increase, so we feel that Bengen's 4% rule does hold as it keeps our success rate at 94.67%.

In conclusion, we feel that every Canadian can adjust the FIRE method to their own retirement strategy. Following the 0.7 savings rate is extreme, but Canadians can adjust that rate to suit their needs better. Everyone can achieve financial independence and retire early, as long as they increase their savings rate and be consistent with their savings and spending rate over a long period of time. For those who want to follow the conventional method, they would need to save and invest at a rate of 0.141.

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Appendix

Appendix 1 - Code for Mortality Model

```
u = rand;
finish = deathAge(u) - 30;

function age = deathAge(u)
    x = 1/0.5627*log(u/(0.0148/0.5627)+1);
        if x<2
            age = floor(65+(x-1)*5);
        elseif x<3
            age = floor(70+(x-2)*5);
        elseif x<4
            age = floor(75+(x-3)*5);
        elseif x<5
            age = floor(80+(x-4)*5);
        elseif x<6
            age = floor(85+(x-5)*5);
        elseif x<7
            age = floor(90+(x-6)*10);
        end
end
```

Appendix 2 - Code for Laplace

```
r = 0.074+0.11*my_laplace(1,finish);

function r = my_laplace(start, finish)
    r = ones(start, finish);
    for i = start:finish
        x1 = -log(1-rand);
        x2 = -log(1-rand);
        z = x1-x2;
        r(i) = z;
    end
end
```

Appendix 3 - Code for Conventional Model

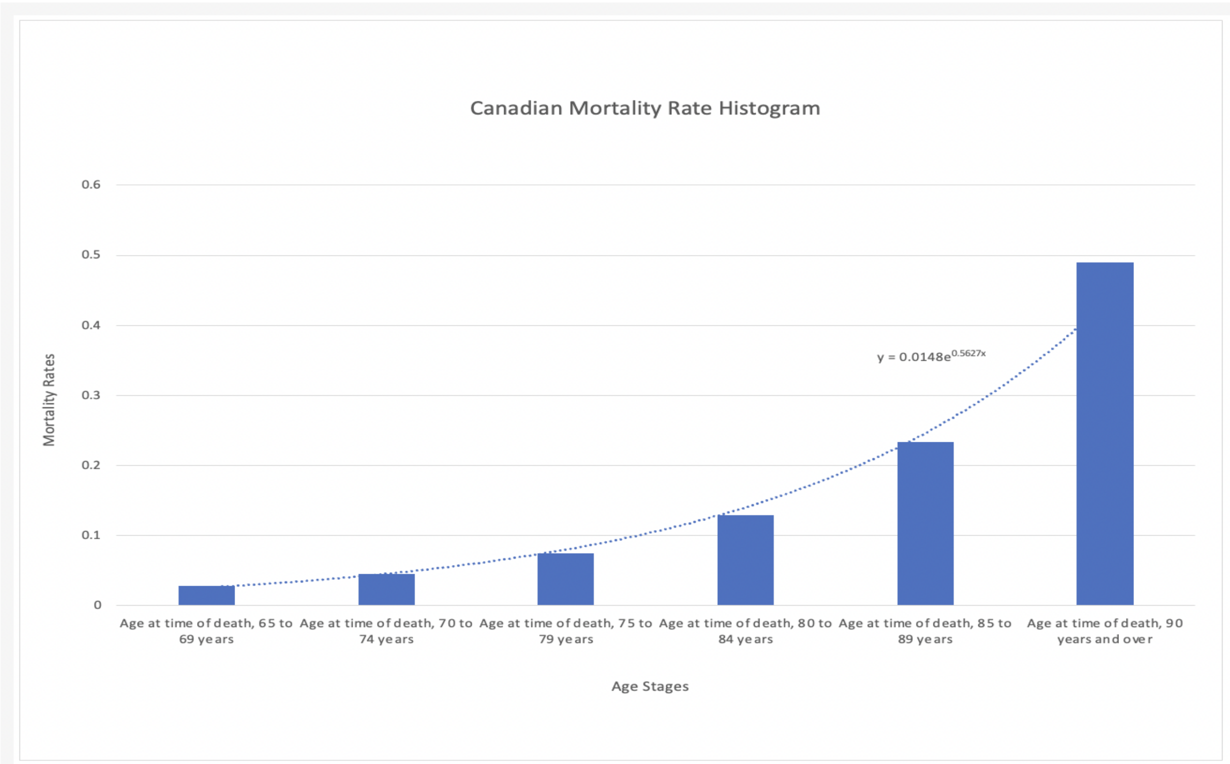
```
annualIncome = 66800;
savingRate = 0.0597;
annualExpense = 47008.58;

n = 10000;
n_sim = 1;
success = Inf*ones(1,n);
successAll = Inf*ones(1,n_sim);
wRetire = Inf*ones(1,n);
wDeath = Inf*ones(1,n);
failYear = [];

%siml = Inf*ones(n,70);

for k = 1: n_sim

    for j = 1:n
        success(j) = 1;

        u = rand;
        finish = deathAge(u) - 30;

        w = Inf*ones(1,finish);
        w(1) = annualIncome*savingRate;
        %siml(j,1) = w(1);
        r = 0.074+0.11*my_laplace(1,finish);

        for i = 2:finish
            if i <= 35
                w(i) = w(i-1)*(exp(r(i-1))) + w(1);
            else
                w(i) = (w(i-1)-annualExpense)*(exp(r(i-1)));

                if w(i) <= 0
                    if success(j) == 1
                        success(j) = 0;
                        failYear(end+1) = i+30;
                    end
                end
            end
            %siml(j,i) = w(i);
        end
        wDeath(j) = w(i);
        wRetire(j) = w(35);
    end
success;
numSuccess = sum(success);
successAll(k) = numSuccess;
end
```

Appendix 4 - Code for FIRE Model

```
annualIncome = 66800;
savingRate = 0.7;
percentRule = 0.04;

n = 10000;
n_sim = 100;
success = Inf*ones(1,n);
successAll = Inf*ones(1,n_sim);
wRetire = Inf*ones(1,n);
wDeath = Inf*ones(1,n);
failYear = [];
whenRetire = Inf*ones(1,n);

%siml = Inf*ones(n,70);

for k = 1:n_sim
    for j = 1:n
        success(j) = 1;

        u = rand;
        finish = deathAge(u)-30;
        w = Inf*ones(1,finish);
        w(1) = annualIncome*savingRate;
        %siml(j,1) = w(1);
        r = 0.074+0.11*my_laplace(1,finish);

        switchRetire = 0;
        for i = 2:finish
            if w(i-1)<(annualIncome*(1-savingRate)/percentRule) && switchRetire == 0
                    w(i) = w(i-1)*(exp(r(i-1))) + w(1);
            else
                switchRetire = 1;
                if whenRetire(j) == Inf
                    whenRetire(j) = i;
                end
                w(i) = (w(i-1)-annualIncome*(1-(savingRate)))*(exp(r(i-1)));
            end
            if switchRetire==1 && w(i) <= 0
                if success(j) == 1
                    success(j) = 0;
                    failYear(end+1) = i+30;
                end
            end
            %siml(j,i) = w(i);
        end
        wDeath(j) = w(i);
        wRetire(j) = w(10);
    end
success;
numSuccess = sum(success);
successAll(k) = numSuccess;
end
```

Appendix 5 - Mortality Data and Histogram from Statistics Canada

1	Age at time of death, 65 to 69 years	11.4	11.1	11.2	10.9	11.2	11.16	0.0286095
2	Age at time of death, 70 to 74 years	17.9	18	17.3	17.1	17.4	17.54	0.0449651
3	Age at time of death, 75 to 79 years	29.5	29.1	29.1	28	29	28.94	0.0741899
4	Age at time of death, 80 to 84 years	50.6	50.9	50.7	49.2	50.9	50.46	0.1293581
5	Age at time of death, 85 to 89 years	89.9	91.4	91.9	88.7	92.8	90.94	0.2331317
6	Age at time of death, 90 years and over	184.4	192.4	193.1	188.1	197.2	191.04	0.4897457



Appendix 6 - Weighted Average Canadian Expense

	Annual Average Expense	Population		Weighted Average of Expense
Ontario	55068	15007816		47008.5827
Newfounderland	34140	525972		
Nova scotia	44400	1002586		
Quebec	31500	8487628		Total Population
NewBrunck	35016	812061		38283550
Manitoba	39888	1390249		
Saskat	37092	1194803		
Alberta	49368	4543111		
BC	54684	5319324		

Given the annual average expense and population in each province in Canada, we were able to calculate the weighted average annual expense of an average Canadian.