

A Combined SDC-SDF Architecture for Normal I/O Pipelined Radix-2 FFT

Zeke Wang, Xue Liu, Bingsheng He, and Feng Yu

Abstract—We present an efficient combined single-path delay commutator-feedback (SDC-SDF) radix-2 pipelined fast Fourier transform architecture, which includes $\log_2 N - 1$ SDC stages, and 1 SDF stage. The SDC processing engine is proposed to achieve 100% hardware resource utilization by sharing the common arithmetic resource in the time-multiplexed approach, including both adders and multipliers. Thus, the required number of complex multipliers is reduced to $\log_4 N - 0.5$, compared with $\log_2 N - 1$ for the other radix-2 SDC/SDF architectures. In addition, the proposed architecture requires roughly minimum number of complex adders $\log_2 N + 1$ and complex delay memory $2N + 1.5 \log_2 N - 1.5$.

Index Terms—Fast Fourier transform (FFT), pipelined architecture, single-path delay commutator processing engine (SDC PE).

I. INTRODUCTION

Fast Fourier transform (FFT) has played a significant role in digital signal processing field, especially in the advanced communication systems, such as orthogonal frequency division multiplexing (OFDM) [1] and asymmetric digital subscriber line [2]. All these systems require that the FFT computation must be high throughput and low latency. Therefore, designing a high-performance FFT circuit is an efficient solution to the abovementioned problems. In particular, the pipelined FFT architectures have mainly been adopted to address the difficulties due to their attractive properties, such as small chip area, high throughput, and low power consumption.

To the best of our knowledge, two types of pipelined FFT architectures can be found in this brief: delay feedback (DF) and delay commutator (DC). Further, according to the number of input data stream paths, they can be classified into multiple-path (M) or single-path (S) architectures. The two classifications form four kinds of pipelined FFT architectures [e.g., single-path DC (SDC)]. Multiple-path (M) architectures [3]–[9], are often adopted when the throughput requirement is beyond the theoretical limitation that the single-path architecture can offer at a given clock frequency. However, they require concurrent read (write) operations for the multipath input (output) data. Therefore, single-path (S) architectures could be appropriate in some cases when the system cannot ensure concurrent operations. However, the arithmetic utilization is relatively low, compared with 100% utilization of the existing MDF/MDC architectures [4]. In this brief, we focus on the SDC radix-2 pipelined FFT architecture, which can also achieve 100% multiplier utilization by reordering the inner data sequence.

For single-input data stream, the conventional radix-2 SDF FFT architecture [10] requires $2 \log_2 N$ complex adders and $\log_2 N - 1$

complex multipliers, where N is the FFT size. Both Chang [11] and Liu *et al.* [12] present the novel SDC architectures to reduce 50% complex adders by reordering inner data sequences. However, the utilization of the corresponding complex multipliers still remains 50% for the both architectures. We therefore study whether the complex multiplier unit can be modified to achieve the 100% utilization.

In the radix-2 FFT architectures, there is a common observation that one half data (sum part of butterfly operation) do not involve complex multiplication (W_N^0) at all, while the other half (difference part) indeed involves complex multiplication (W_N^k). Hence, it has the opportunity to achieve the objective that reduces the arithmetic resource of the conventional complex multipliers by a factor of 2, leading to 100% utilization. It is ideal for two consecutive complex input data to contain a complex number, which needs to execute complex multiplication. If so, we can minimize the reordering memory requirement while achieving the above objective that reduces 50% the arithmetic resource of complex multipliers.

Fortunately, the improved SDC architecture can produce the sum and the corresponding difference results of a butterfly operation in consecutive two cycles. The sum part is directly passed to the next stage, while the difference part needs to execute complex multiplication before passing to the next stage. Therefore, the SDC architecture is ideal for our efficient pipelined radix-2 FFT architecture. However, the SDF architecture does not meet the above constraint well since the sums of the all butterflies in the stage are produced first, followed by the corresponding differences.

In this brief, we present an efficient combined SDC-SDF radix-2 pipelined FFT architecture, which includes $\log_2 N - 1$ SDC stages, 1 SDF stage, and 1 bit reverser. The SDC processing engine (SDC PE) in each SDC stage achieves the 100% hardware resource utilizations of both adders and multipliers. We include the SDF stage to reorder the data sequence, and then the delay memory of the bit reverser is reduced to $N/2$. The proposed architecture can produce the same normal output order as [26].

II. REVIEW OF PIPELINED FFT ARCHITECTURE

A. FFT Review

The N -point DFT is defined by

$$X(k) = \sum_{n=0}^{N-1} x(n) \times W_N^{nk} \quad k = 0, 1, \dots, N-1$$

where $x(n)$ is the input data, W_N^{nk} is the coefficient ($W_N^{nk} = e^{-2\pi jnk/N}$), and N is any integer power of two.

It is well known that the radix-2 FFT can be deduced from DFT by factorizing the N -point DFT recursively into many 2-point DFTs. The data flow graph (DFG) of 16-point radix-2 FFT is shown in Fig. 1.

B. Review of Pipelined FFT Implementations

Assuming that the input data enters the FFT circuit serially in a continuous flow, the radix-2 MDC and SDF architectures can be directly deduced according to the DFG in Fig. 1. The radix-2 MDC architecture [9] is the most direct implementation approach of pipelined FFT, but its hardware utilization is only 50%. Compared with [9], the radix-2 SDF design [10] reduces the required memory size. However, the utilizations of adders and multipliers are still 50%.

Manuscript received May 8, 2013; revised September 13, 2013 and November 28, 2013; accepted April 16, 2014. Date of publication May 19, 2014; date of current version April 22, 2015. This work was supported by the National Science and Technology Support Program of China under Grant 2012BAK24B01.

Z. Wang is with Zhejiang University, Hangzhou 310027, China, and also with Nanyang Technological University, Singapore 639798 (e-mail: wangzeke@zju.edu.cn).

X. Liu is with the College of Information Science and Engineering, Institute of Cyber-Physical Systems Engineering, Northeastern University, Shenyang 110004, China (e-mail: liuxue0512@gmail.com).

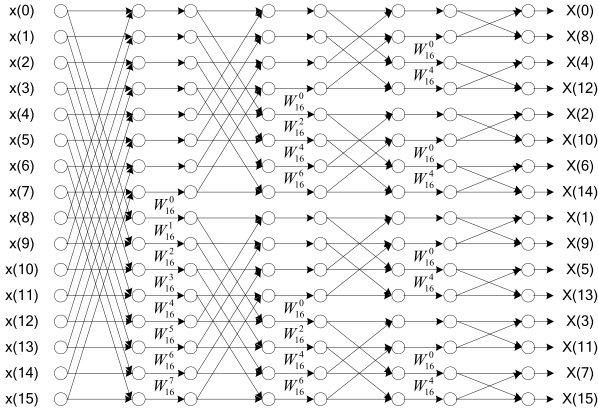
B. He is with the School of Computer Engineering, Nanyang Technological University, Singapore 639798 (e-mail: bshe@ntu.edu.sg).

F. Yu is with the Institute of Digital Technology and Instrument, Zhejiang University, Hangzhou 310027, China (e-mail: osfengyu@zju.edu.cn).

Digital Object Identifier 10.1109/TVLSI.2014.2319335

1063-8210 © 2014 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.

See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

Fig. 1. DFG of DIF radix-2 FFT ($N = 16$).

Besides the basic radix-2 architectures, various high-radix pipelined FFT architectures have also been proposed to address the arithmetic resource utilization problem. They are radix-4 MDC [4], [8], [23], radix-4 SDC [13], radix-4 SDF [18], radix-2² SDF [14], [21], radix-2³ SDF [15], radix-2⁴ SDF [16], radix-2⁵ SDF [17], radix- r^k SDC/SDF [19], and radix-2^k feedforward [20]. Compared with the radix-2 architectures, the high radix architecture can only process the FFT, whose size is a power of its high radix, not just 2.

In order to extend the application scope of the FFT architectures, the new dynamic data scaling architectures [22] for pipelined FFTs have been proposed to implement both 1-D and 2-D applications. The MDC-based FFT architecture [23] has been proposed for the MIMO-OFDM systems with variable length. Employing folding transformation and register minimization techniques, the novel parallel pipelined architecture [24] for complex and real valued FFT has been proposed to significantly reduce power consumption.

III. COMBINED SDC-SDF RADIX-2 PIPELINED FFT

For single-input data stream, we propose an efficient combined SDC-SDF radix-2 pipelined FFT architecture, and the proposed SDC PE structure can reduce 50% complex multipliers.

A. Proposed FFT Architecture

The proposed FFT architecture consists of 1 pre-stage, $\log_2 N - 1$ SDC stages, 1 post-stage, 1 SDF stage, and 1 bit reverser, shown in Fig. 2(a). The pre-stage shuffles the complex input data to a new sequence that consists of real part followed by the corresponding imaginary part, shown in Table I. The corresponding post-stage shuffles back the new sequence to the complex format. The SDC stage t ($t = 1, 2, \dots, \log_2 N - 1$) contains an SDC PE, which can achieve 100% arithmetic resource utilization of both complex adders and complex multipliers. The last stage, SDF stage, is identical to the radix-2 SDF, containing a complex adder and a complex subtracter. By using the modified addressing method [12], the data with an even index are written into memory in normal order, and they are then retrieved from memory in bit-reversed order while the ones with an odd index are written in bit-reversed order. Final, the even data are retrieved in normal order. Thus, the bit reverser requires only $N/2$ data buffer.

Table I illustrates the inner data sequence of 16-point FFT computation. The complex input data at cycle m are $(m-r, m-i)$, where $m-r$ and $m-i$ ($m = 0, 1, \dots, 15$) represent the real and imaginary parts, respectively. We only include the pre-stage, SDC stage 1, 2, 3, and post-stage, since the SDF stage has the same sequence as the post-stage except the 8-cycle delay, and the bit reverser, 8-cycle delay over the SDF stage [12], produces normal output sequence.

TABLE I
DATA OUTPUT ORDER OF THE PROPOSED PIPELINED
ARCHITECTURE FOR 16-POINT FFT

Cycle	pre-stage	stage 1	stage 2	stage 3	post-stage
1	0_r, 1_r	-	-	-	-
2	0_i, 1_i	-	-	-	-
3	2_r, 3_r	-	-	-	-
...
9	8_r, 9_r	-	-	-	-
10	8_i, 9_i	0_r, 8_r	-	-	-
11	10_r, 11_r	0_i, 8_i	-	-	-
12	10_i, 11_i	2_r, 10_r	-	-	-
13	12_r, 13_r	2_i, 10_i	-	-	-
14	12_i, 13_i	4_r, 12_r	-	-	-
15	14_r, 15_r	4_i, 12_i	0_r, 4_r	-	-
16	14_i, 15_i	6_r, 14_r	0_i, 4_i	-	-
17	-	6_i, 14_i	2_r, 6_r	-	-
18	-	1_r, 9_r	2_i, 6_i	0_r, 2_r	-
19	-	1_i, 9_i	8_r, 12_r	0_i, 2_i	0_r, 0_i
20	-	3_r, 11_r	8_i, 12_i	4_r, 6_r	2_r, 2_i
21	-	3_i, 11_i	10_r, 14_r	4_i, 6_i	4_r, 4_i
...
32	-	-	-	13_r, 15_r	11_r, 11_i
33	-	-	-	13_i, 15_i	13_r, 13_i
34	-	-	-	-	15_r, 15_i

The *pre-stage* shuffles the complex input data to a new sequence that consists of real part followed by the corresponding imaginary part, and *post-stage* undoes the data shuffling carried out by the *pre-stage*. The *SDC stage* t ($t = 1, 2, 3$), preserves the data sequence generated by its *data commutator*.

B. Single-Path DC Processing Engine

The SDC PE, shown in Fig. 2(b), consists of a data commutator, a real add/sub unit, and an optimum complex multiplier unit. In order to minimize the arithmetic resource of the SDC PE, the most significant factor is to maximize the arithmetic resource utilization via reordering the data sequences of the above three units.

In the stage t , the data commutator shuffles its input data (Node-A) to generate a new data sequence (Node-B), whose index difference is $N/2^t$, where t is the index of stage. The new data sequence (Node-B) is critical to the real add/sub unit, where one real adder and one real subtracter can both operate on two elements for each input data. The sum and difference results (Node-C) overlap the places of the two input elements. Therefore, it preserves the data sequence, requires only one real adder and one real subtracter.

For the optimum complex multiplier unit, its output data sequence (Node-E) should be the same as its input data sequence (Node-C). If so, its output sequence (Node-E), which is also the output sequence of the SDC stage t , can become the direct input data sequence (Node-A) of the SDC stage $t+1$. The implementation detail is described in Section III-C.

Table II illustrates the data sequence of SDC stage 1 of 16-point FFT computation, including the data sequences of the above three units.

C. Optimum Complex Multiplier Unit

As shown in Fig. 2(b), it contains 2 multiplexers ($M1$ and $M2$), 1.5-word memory ($G1$, $G2$, and $G3$), 2 Real Multipliers and 1 Real Adder. The signal s controls the behavior of two multiplexers ($M1$ and $M2$): through or swap. The signal s also controls the behavior of the Real Adder, which supports both addition and subtraction operations.

For the input couple $(0_r, 8_r)$ and $(0_i, 8_i)$ at the Node-C in Table II the sum part data 0_r and 0_i will directly pass to the delay memory $G1$ to generate 0_r^* and 0_i^* with one cycle delay in consecutive two cycles, while the difference part 8_r and 8_i will directly enter the Real Multipliers (Node-D) to generate $(c \times 8_r,$

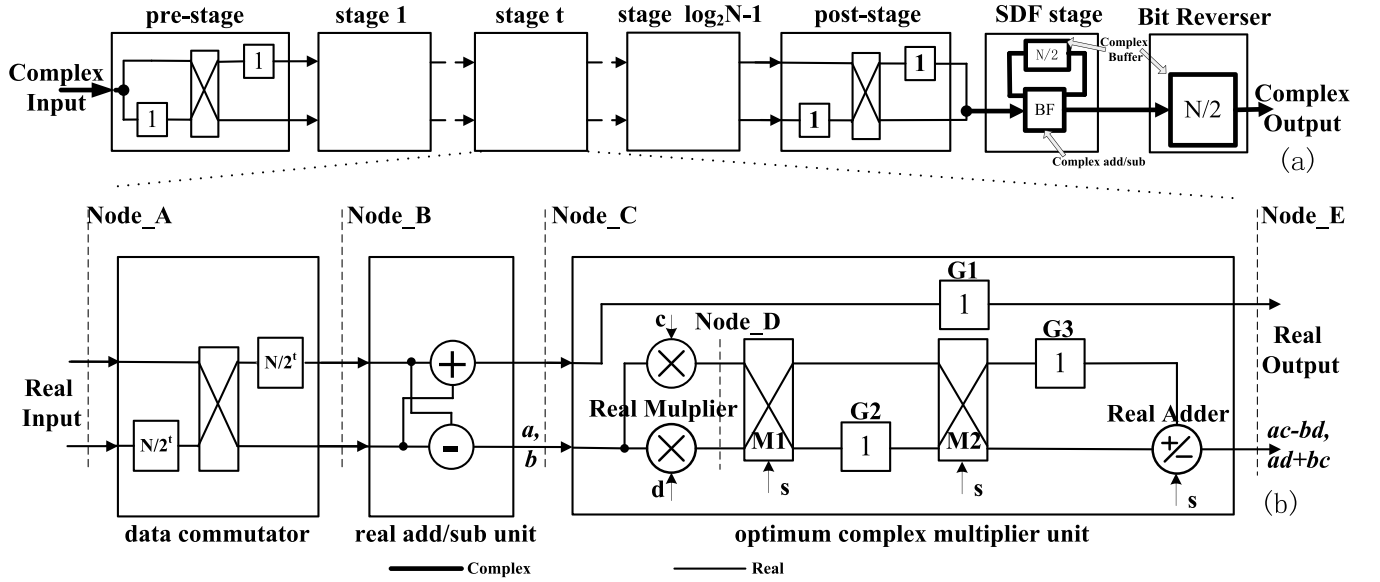


Fig. 2. (a) Block diagram of the proposed FFT architecture. (b) Block diagram of the SDC PE, including a data commutator, a real add/sub unit, and an optimum complex multiplier unit. a (b) means the real (imaginary) part of subtraction result, c (d) means the real (imaginary) part of the twiddle factor. $G1$, $G2$, and $G3$ mean three one-cycle delay elements. The signal s controls the behavior of two multiplexers ($M1$ and $M2$) and the Real Adder. When s is 1 (0), both multiplexers perform “through” (“swap”) and the Real Adder performs addition (subtraction) operation.

TABLE II
DATA SEQUENCE OF THE PROPOSED STAGE 1 OF 16-POINT FFT

Cycle	Node_A	Node_B/C	Node_D	G1	G2	G3	s	Real Adder	Node_E	New_Label
1	0 _r , 1 _r	-	-	-	-	-	-	-	-	-
2	0 _i , 1 _i	-	-	-	-	-	-	-	-	-
...
9	8 _r , 9 _r	0 _r , 8 _r	$c \times 8_r, d \times 8_r$	-	-	-	-	-	-	-
10	8 _i , 9 _i	0 _i , 8 _i	$c \times 8_i, d \times 8_i$	0 _r	$d \times 8_r$	$c \times 8_r$	0	sub	$0_r, c \times 8_r - d \times 8_i$	$0_{r^*}, 8_{r^*}$
11	10 _r , 11 _r	2 _r , 10 _r	$c \times 10_r, d \times 10_r$	0 _i	$c \times 8_i$	$d \times 8_r$	1	add	$0_i, c \times 8_i + d \times 8_r$	$0_{i^*}, 8_{i^*}$
12	10 _i , 11 _i	2 _i , 10 _i	$c \times 10_i, d \times 10_i$	2 _r	$d \times 10_r$	$c \times 10_r$	0	sub	$2_r, c \times 10_r - d \times 10_i$	$2_{r^*}, 10_{r^*}$
13	12 _r , 13 _r	4 _r , 12 _r	$c \times 12_r, d \times 12_r$	2 _i	$c \times 10_i$	$d \times 10_r$	1	add	$2_i, c \times 10_i + d \times 10_r$	$2_{i^*}, 10_{i^*}$
14	12 _i , 13 _i	4 _i , 12 _i	$c \times 12_i, d \times 12_i$	4 _r	$d \times 12_r$	$c \times 12_r$	0	sub	$4_r, c \times 12_r - d \times 12_i$	$4_{r^*}, 12_{r^*}$
15	14 _r , 15 _r	6 _r , 14 _r	$c \times 14_r, d \times 14_r$	4 _i	$c \times 12_i$	$d \times 12_r$	1	add	$4_i, c \times 12_i + d \times 12_r$	$4_{i^*}, 12_{i^*}$
16	14 _i , 15 _i	6 _i , 14 _i	$c \times 14_i, d \times 14_i$	6 _r	$d \times 14_r$	$c \times 14_r$	0	sub	$6_r, c \times 14_r - d \times 14_i$	$6_{r^*}, 14_{r^*}$
...
24	-	7 _i , 15 _i	$c \times 15_i, d \times 15_i$	7 _r	$d \times 15_r$	$c \times 15_r$	0	sub	$7_r, c \times 15_r - d \times 15_i$	$7_{r^*}, 15_{r^*}$
25	-	-	-	7 _i	$c \times 15_i$	$d \times 15_r$	1	add	$7_i, c \times 15_i + d \times 15_r$	$7_{i^*}, 15_{i^*}$

For the *stage 1*, *Node_A* denotes the sequence just after the *pre-stage*. *Node_B* denotes the new sequence after the *data commutator* shuffles the data, whose index difference is $N/2$. *Node_C* denotes the data sequence after the *real add/sub unit*. *Node_D* denotes the difference part of *real add/sub unit*, multiplied by the twiddle factor ($c+di$), where ‘ c ’ (‘ d ’) stands for the real (imaginary) part of twiddle factor for each data. ‘ $G1$ ’ is used to delay the sum part of *real add/sub unit*. ‘ $G2$ ’ and ‘ $G3$ ’ have been used to re-order the internal data sequence. The signal ‘ s ’ controls the behaviors of the *Real Adder*, “add” or “sub”, and the two multiplexers ($M1$ and $M2$), “through” or “swap”. *Node_E*, equal to “*New_Label*” (m_{r^*} and m_{i^*}), denotes complex butterfly computation result. “*New_Label*” is just the *Node_A* for the next stage (e.g., *stage 2*).

$d \times 8_{-r}$) and ($c \times 8_{-i}, d \times 8_{-i}$) before reordering. The reordering process is performed as follows.

- 1) In the first cycle, when 8_{-r} comes, the signal s ($s = 1$) selects “through”; that is, the up (down) input of the multiplexer ($M1$ or $M2$) connects to the up (down) output. Then, the $G2$ (or $G3$) would be $d \times 8_{-r}$ (or $c \times 8_{-r}$) in the second cycle.
- 2) In the second cycle, when 8_{-i} comes, the signal s ($s = 0$) selects “swap”; that is, the up (down) input of the multiplexer ($M1$ or $M2$) connects to the down (up) output. Then, the $G2$ (or $G3$) would be $c \times 8_{-i}$ (or $d \times 8_{-r}$) in the third cycle. The s will make the *Real Adder* perform subtraction operation and then $c \times 8_{-r} - d \times 8_{-i}$ (8_{-r^*}) would appear at the *Node_E*.

- 3) In the third cycle, the signal s ($s = 1$) selects “through” for $M1$ and $M2$, and chooses addition operation for *Real Adder*. Then, $d \times 8_{-r} + c \times 8_{-i}$ (8_{-i^*}) would appear at the *Node_E*.

Consequently, the complex result data couple ($0_{-r^*}, 8_{-r^*}$) and ($0_{-i^*}, 8_{-i^*}$) would come out at *New_Label* (*Node_E*) with one clock delay in consecutive two cycles.

The above mechanism can be iterated by applying to the other couples in the stage 1, e.g., ($2_{-r}, 10_{-r}$) and ($2_{-i}, 10_{-i}$), and so on. If we carry the above process toward the $\log_2 N - 1$ stages to completion, we can complete the majority part of the radix-2 FFT computation.

TABLE III
HARDWARE RESOURCE COMPARISON FOR THE VARIOUS PIPELINED FFT ARCHITECTURES

Architecture	Internal Memory	Overall Memory	Adder	General Multiplier (Utilization)	Constant Multiplier	Throughput	Latency	Critical Path Delay
R2SDF [10]	$N-1$	$2N-1$	$2\log_2 N$	$2\log_4 N-1$ (50%)	-	$1/N$	$2N-1$	$T_M + 2T_A + T_{MUX}$
R2SDC [19]	$2N-2$	$3N-2$	$2\log_2 N$	$2\log_4 N-1$ (50%)	-	$1/N$	N	$T_M + 2T_A + 2T_{MUX}$
Chang [11]	$1.5N$	$2N$	$\log_2 N+1$	$2\log_4 N-1$ (50%)	-	$1/N$	$2N$	$T_M + 2T_A + T_{MUX}$
Liu [12]	$1.5N+2 \times (\log_2 N-2)$	$2N+2 \times (\log_2 N-2)$	$\log_2 N+1$	$2\log_4 N-1$ (50%)	-	$1/N$	$2N+2 \times (\log_2 N-2)$	$T_M + 2T_A + 2T_{MUX}$
R2 ² SDF [14]	$N-1$	$2N-1$	$2\log_2 N$	$\log_4 N-1$ (75%)	-	$1/N$	$2N-1$	$T_M + 2T_A + T_{MUX}$
Yang [21]	$4(N-1)/3$	$(7N-4)/3$	$\log_2 N$	$\log_4 N-1$ (75%)	-	$1/N$	$(7N-4)/3$	$T_M + 2T_A + 4T_{MUX}$
R2 ³ SDF [16]	$N-1$	$2N-1$	$2\log_2 N$	$0.67\log_4 N-1$ (87.5%)	$0.67\log_4 N-1$	$1/N$	$2N-1$	$T_M + 2T_A + T_{MUX}$
R2 ⁴ SDF [16]	$N-1$	$2N-1$	$2\log_2 N$	$0.5\log_4 N-1$ (93.75%)	$0.5\log_4 N-1$	$1/N$	$2N-1$	$T_M + 2T_A + T_{MUX}$
Proposed	$1.5N+1.5 \times \log_2 N-1.5$	$2N+1.5 \times \log_2 N-1.5$	$\log_2 N+1$	$\log_4 N-0.5$ (100%)	-	$1/N$	$2N+\log_2 N-1$	$T_M + 2T_A + 3T_{MUX}$

TABLE IV
COMPARISONS OF TRANSISTOR REQUIREMENT AND LATENCY

Architecture	Components	Transistors	Latency	Transistors * Latency
Chang [11]	1024 16-bit SRAMs	230748 (135%)	512	118142976 (133%)
	32 16-bit Adders			
	28 16-bit Multipliers			
Liu [12]	1048 16-bit SRAMs	233052 (136%)	524	122119248 (138%)
	32 16-bit Adders			
	28 16-bit Multipliers			
R2 ² SDF [14]	1022 16-bit SRAMs	167138 (98%)	511	85407518 (96%)
	38 16-bit Adders			
	12 16-bit Multipliers			
R2SD ² F [21]	1192 16-bit SRAMs	175378 (103%)	591	103648398 (117%)
	22 16-bit Adders			
	12 16-bit Multipliers			
R2 ³ SDF [16]	1022 16-bit SRAMs	163614 (96%)	511	83606754 (94%)
	37.6 16-bit Adders			
	11.2 16-bit multipliers			
R2 ⁴ SDF [16]	1022 16-bit SRAMs	145992 (85%)	511	74601912 (84%)
	35.6 16-bit Adders			
	7.2 16-bit Multipliers			
Proposed	1045 16-bit SRAMs	171087 (100%)	519	88794153 (100%)
	25 16-bit Adders			
	14 16-bit Multipliers			

The R2³SDF, using mixed radix architecture, roughly needs 2.8 general multipliers while R2⁴SDF roughly needs 1.8 general multipliers [16].

In summary, the SDC PE can reduce 50% the arithmetic resource of complex multipliers in the time-multiplexing approach, at the expense of 1.5 complex delay memory overhead for each SDC PE.

IV. COMPARISON AND ANALYSIS

Table III presents the hardware requirement of our design and the other pipelined architectures. The internal memory denotes the complex internal memory and the overall memory shows the complex total memory when the bit reverser is included. The typical SDF design requires the minimum overall memory $2N$. The overall memory of the proposed design is $2N+1.5\log_2 N-1.5$. It includes: 1) $N-2$ for the data commutators in the SDC stages; 2) $1.5\log_2 N-1.5$ for the optimum complex multiplier units to reorder inner data sequences; 3) 2 for the pre-stage and post-stage; 4) $N/2$ for the SDF stage; and 5) $N/2$ for the bit reverser.

Table III also lists the required numbers of complex adders and complex multipliers. The proposed design requires roughly minimum number $\log_2 N + 1$ of complex adders, and requires only $\log_4 N - 0.5$ complex multipliers compared with $\log_2 N - 1$ for the

TABLE V
AREA AND PERFORMANCE OF THE PROPOSED FFT ARCHITECTURES FOR 16 BITS

FFT Size	Area				Freq (MHz)	Latency (ns)
	LUTs	FFs	DSPs	BRAMs		
16	672	522	4	0	322	140
64	1110	752	8	0	303	498
256	1733	1073	12	0	297	1834
1024	2804	1589	16	3	298	7028
4096	8391	2780	20	4	295	27975

other radix-2 designs. The multiplier requirement is approximately as same as radix-2² [14]–[21], and more than R2³SDF [14] and R2⁴SDF [16], since the high radix designs theoretically require fewer multipliers than the radix-2 designs. The proposed design preserves the radix-2 nature and achieves 100% multiplier utilization, while the other radix-2 designs only achieve 50% utilization $((2^1 - 1)/2^1)$. Furthermore, the high radix designs require more complex adders than the proposed design (except R2SD²F [21]), and can only process the FFT, whose size is a power of its high radix. For example, the 128-point FFT cannot be directly mapped to either one high radix design [14]–[16], but the radix-2 design can. Beyond the scope of this brief, the mixed radix design can implement the 128-point FFT with relatively complex control logic.

Since all of the FFT designs are single-path, their throughput is $1/N$. Since the latency is roughly proportional to the size of the overall memory, the latency of the proposed design is $2N+\log_2 N-1$. The critical path delay of the proposed design is $T_M + 2T_A + 3T_{MUX}$, where T_M , T_A , and T_{MUX} are computation time of a multiplier, adder, and multiplexer, respectively. Since the T_{MUX} is greatly less than T_M and T_A , the critical path delay of all designs is roughly same.

In the following, we consider a 256-point pipelined FFT with word length 16 bits. The multiplier, adder, and 16-bit SRAM are taken to be 4153, 505, and 96 transistors [25], respectively.

We compare the proposed design, in terms of transistor, with the other FFT designs, shown in Table IV. We observe that the proposed design requires fewer transistors than the other radix-2 architectures [11], [12] because of the reduction in complex multipliers. It is roughly the same as that of radix-2², and is more than the R2³SDF and R2⁴SDF. However, the high radix architecture can only process the FFT, whose size is a power of its high radix.

We also implement the proposed FFT architecture on the Xilinx Virtex-5 FPGA, XC5VSX240T-2 FF1738, and the corresponding place and route results for the different FFT sizes are shown in Table V. Our results show that the proposed FFT architecture can achieve small area and high frequency.

V. CONCLUSION

We propose a combined SDC-SDF pipelined FFT architecture which produces the output data in the normal order. The proposed SDC PE mainly reduces 50% complex multipliers, compared with the other radix-2 FFT designs. Therefore, the proposed FFT architecture is very attractive for the single-path pipelined radix-2 FFT processors with the input and output sequences in normal order.

REFERENCES

- [1] L. J. Cimini, "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 33, no. 7, pp. 665–675, Jul. 1985.
- [2] J. M. Cioffi, *The Communications Handbook*. Boca Raton, FL, USA: CRC Press, 1997.
- [3] Y. W. Lin, H. Y. Liu, and C. Y. Lee, "A 1-GS/s FFT/IFFT processor for UWB applications," *IEEE J. Solid-State Circuits*, vol. 40, no. 8, pp. 1726–1735, Aug. 2005.
- [4] C. Cheng and K. K. Parhi, "High throughput VLSI architecture for FFT computation," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 54, no. 10, pp. 339–344, Oct. 2007.
- [5] S. N. Tang, J. W. Tsai, and T. Y. Chang, "A 2.4-GS/s FFT processor for OFDM-based WPAN applications," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 57, no. 6, pp. 451–455, Jun. 2010.
- [6] Y. Jung, H. Yoon, and J. Kim, "New efficient FFT algorithm and pipeline implementation results for OFDM/DMT applications," *IEEE Trans. Consum. Electron.*, vol. 49, no. 1, pp. 14–20, Feb. 2003.
- [7] M. Shin and H. Lee, "A high-speed, four-parallel radix-2⁴ FFT processor for UWB applications," in *Proc. IEEE ISCAS*, May 2008, pp. 960–963.
- [8] J. H. McClellan and R. J. Purdy, "Applications of digital signal processing to radar," in *Applications of Digital Signal Processing*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1978, ch. 5.
- [9] L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1975, pp. 604–609.
- [10] E. H. Wold and A. M. Despain, "Pipeline and parallel-pipeline FFT processors for VLSI implementation," *IEEE Trans. Comput.*, vol. C-33, no. 5, pp. 414–426, May 1984.
- [11] Y. N. Chang, "An efficient VLSI architecture for normal I/O order pipeline FFT design," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 55, no. 12, pp. 1234–1238, Dec. 2008.
- [12] X. Liu, F. Yu, and Z. K. Wang, "A pipelined architecture for normal I/O order FFT," *J. Zhejiang Univ. Sci. C*, vol. 12, no. 1, pp. 76–82, Jan. 2011.
- [13] G. Bi and E. V. Jones, "A pipelined FFT processor for word-sequential data," *IEEE Trans. Acoust. Speech Signal Process.*, vol. 37, no. 12, pp. 1982–1985, Dec. 1989.
- [14] S. He and M. Torkelson, "Designing pipeline FFT processor for OFDM (de)modulation," in *Proc. URSI ISSSE*, vol. 29, Sep./Oct. 1998, pp. 257–262.
- [15] T. Sansaloni, A. Perez-Pascual, V. Torres, and J. Valls, "Efficient pipeline FFT processors for WLAN MIMO-OFDM systems," *Electron. Lett.*, vol. 41, no. 19, pp. 1043–1044, Sep. 2005.
- [16] J. Y. Oh and M. S. Lim, "Area and power efficient pipeline FFT algorithm," in *Proc. IEEE Workshop Signal Process. Syst. Design and Implementation*, Nov. 2005, pp. 520–525.
- [17] T. Cho, S. Tsai, and H. Lee, "A high-speed low-complexity modified radix-2⁵ FFT processor for high rate WPAN applications," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 21, no. 1, pp. 187–191, Jan. 2013.
- [18] A. M. Despain, "Fourier transform computer using CORDIC iterations," *IEEE Trans. Comput.*, vol. C-23, no. 10, pp. 993–1001, Oct. 1974.
- [19] A. Cortes, I. Velez, and J. F. Sevillano, "Radix r^k FFTs: Matricial representation and SDC/SDF pipeline implementation," *IEEE Trans. Signal Process.*, vol. 57, no. 7, pp. 2824–2839, Jul. 2009.
- [20] M. Garrido, J. Grajal, M. Sanchez, and O. Gustafsson, "Pipelined radix-2^k feedforward FFT architectures," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 21, no. 1, pp. 23–32, Jan. 2013.
- [21] L. Yang, K. Zhang, H. Liu, J. Huang, and S. Huang, "An efficient locally pipelined FFT processor," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 53, no. 7, pp. 585–589, Jul. 2006.
- [22] T. Lenart and V. Owall, "Architectures for dynamic data scaling in 2/4/8K pipeline FFT cores," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 14, no. 11, pp. 1286–1290, Nov. 2006.
- [23] K. Yang, S. Tsai, and G. Chuang, "MDC FFT/IFFT processor with variable length for MIMO-OFDM systems," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 21, no. 4, pp. 720–731, Apr. 2013.
- [24] M. Ayinala, M. Brown, and K. Parhi, "Pipelined parallel FFT architectures via folding transformation," *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 20, no. 6, pp. 1068–1081, Jun. 2012.
- [25] N. H. E. Weste and D. Harris, *CMOS VLSI Design: A Circuits and Systems Perspective*. Boston, MA, USA: Addison-Wesley, 2005.
- [26] B. Gold and C. M. Rader, *Digital Processing of Signal*. New York, NY, USA: McGraw-Hill, 1969, ch. 6.