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A Frequency Assignment Problem in Cellular Phone Networks

(Extended Abstract)

Preprint SC 97-27 (July 1997)

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July 10, 1997

Abstract

We present a mathematical formulation of a *frequency assignment problem* encountered in cellular phone networks: frequencies have to be assigned to stationary transceivers (carriers) such that as little interference as possible is induced while obeying several technical and legal restrictions.

The optimization problem is $\mathcal{NP}$-hard, and no good approximation can be guaranteed, unless $\mathcal{P} = \mathcal{NP}$. We sketch some starting and improvement heuristics, and report on their successful application for solving the frequency assignment problem under consideration. Computational results on real-world instances with up to 2877 carriers and 50 frequencies are presented.

Keywords: Frequency Assignment Problem, Cellular Phone Network, Heuristics, Graph Coloring

Mathematics Subject Classification (1991): 90B12 05C15 90C60 68Q25

1 Introduction

Radio spectrum is a scarce resource today. As a consequence, cellular phone network operators are granted only little bandwidth by the national regulators. The available frequency band is partitioned into intervals of equal size, called *frequencies* in the following. These frequencies are numbered consecutively for reference. Typically, the number of frequencies is less than a hundred. The available frequencies have to suffice to operate cellular phone networks with several thousand carriers. The *carriers* represent stationary transceivers operating as access points to the phone network for cellular phones. Every carrier uses one of the frequencies to

*This is a brief summary of a forthcoming paper on ongoing research carried out together with Ralf Borndörfer, Martin Grörtschel, and Alexander Martin at ZIB, Germany. The research is done in cooperation with `E-Plus` Mobilfunk GmbH, Germany. `E-Plus` operates a GSM1800 network. The full article on this work is going to be published soon. There, in total five heuristics are discussed and, using data from several real-world cellular phone networks, a more comprehensive assessment of the computational merits of these heuristics is given.

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run communication links with cellular phones near its location. If nearby carriers use the same frequency, so called *co-channel interference* is likely to occur. Interference between frequencies with adjacent numbers, called *adjacent-channel interference*, may also occur, but it is usually smaller than co-channel interference. Severe interference can result in communication link failure. This must be avoided. A *separation* parameter is sometimes specified for pairs of carriers. The separation parameter gives the minimum distance required for the frequencies (in terms of their numbers) used by the carriers. Typical separation values are 1, 2, and 3.

We give a mathematical model of the "frequency assignment problem" in cellular phone networks. Our objective is to find frequency assignments that cause little interference.

Several different problems have already been studied using the name "frequency assignment problem". Traditionally, a (generalized) graph coloring problem is derived from interference information, see [9], for example. The graph obtained is to be colored with as few frequencies as possible or with frequencies from an interval that is as narrow as possible. Additional restrictions may apply. In the case of cellular phone networks, however, the available frequencies are given. The main question is how to use them best. In recent publications, interference minimization in mobile systems networks with a fixed spectrum of available frequencies was considered [3, 6, 7, 11, 12]. We address this problem in the setting of cellular phone networks and present three heuristics that are fast enough for daily use in practice. Real-world networks are used to evaluate the heuristics' performance. Huge interference reductions are achieved in comparison to assignments currently used in practice.

2 A Frequency Assignment Problem

A frequency assignment for a cellular phone network fixes one frequency for every carrier. We use the sum over all co- and adjacent-channel interferences as our measure for the quality of an assignment.

Mathematical Formulation

Let (V, E) be an undirected graph. The nodes of the graph represent the carriers. The *spectrum* C is an interval of non-negative integers. The elements of the spectrum represent the frequencies and will be called *channels*. For every carrier v, a set $B_v \subseteq C$ of *blocked channels* is specified. The channels in $C \setminus B_v$ are called *available* at carrier v. B_v may be empty. Three functions, $d : E \to \mathbb{Z}_+$, $c^{co} : E \to [0,1]$, and $c^{ad} : E \to [0,1]$ with $c^{ad} \leq c^{co}$, are specified on the edge set. For an edge $vw \in E$, $d(vw)$ gives the *separation* necessary between channels assigned to v and w. $c^{co}(vw)$ and $c^{ad}(vw)$ denote the *co-channel* and *adjacent-channel interference*, respectively, which may occur between v and w. We will refer to the 7-tuple $N = (V, E, C, \{B_v\}_{v\in V}, d, c^{co}, c^{ad})$ as *carrier network*.

Definition 1 *An* assignment *for a carrier network* N *is a mapping* $y : V \to C$. *An assignment is* feasible *if every carrier* $v \in V$ *is assigned an available channel (from* $C \setminus B_v$*) and all separation requirements are met, i.e.,* $|y(v) - y(w)| \geq d(vw)$ *for all* $vw \in E$. *Given a carrier network* N*, we call the optimization problem*

$$\min_{y \text{ feasible}} \sum_{\substack{vw\in E:\\ y(v)=y(w)}} c^{co}(vw) \quad + \sum_{\substack{vw\in E:\\ |y(v)-y(w)|=1}} c^{ad}(vw) \tag{FAP}$$

frequency assignment problem.

It is easily observed that feasible assignments are a generalization of list colorings (see [13], for example) and are related to T-colorings of graphs [5].

Computational Complexity

Deciding whether there is a feasible assignment of cost no more than q is a decision problem underlying the FAP. We call this problem q–FAP. By reducing GRAPH K-COLORABILITY to q–FAP, the $\mathcal{NP}$-completeness of q–FAP can be shown. Finding good approximate solutions to FAP is also hard. Even on a particular subclass of FAP where a feasible solution is easily obtained, approximation of an optimal solution is at least as hard as approximation for MINIMUM GRAPH COLORING (see [1]).

3 Heuristics

We present several heuristics. One objective in their design was running time efficiency. Running time efficiency is a major requirement by practitioners who want to have algorithms that permit computing frequency assignments within minutes, even on large instances. We distinguish between starting and improvement heuristics. *Starting heuristics* compute an assignment from scratch, step-wise extending an initially empty assignment to an assignment on all carriers. *Improvement heuristics* take a (feasible) assignment as input and try to improve it.

DSATUR With Costs

This starting heuristic is a modification of `DSATUR` [2] incorporating ideas from [4]. The idea is to recursively deal with the carrier "hardest" to assign first. While there are unassigned carriers, the carrier with the least number of channels left available is picked. If there is more than one such carrier, an interference-based tie-breaking rule is applied. The carrier chosen is assigned a channel (locally) incurring the least interference.

We observed that the choice of the first carrier to assign has considerable impact on the quality of the assignment obtained. No generally good rule could be identified as to which carrier to start with. One may start with each carrier consecutively and pick the best assignment obtained. A running time reducing option is to choose some set of start carriers at random and then pick the best assignment computed this way.

Iterated 1-OPT

This improvement heuristic uses a neighborhood structure defined on the set of all assignments. Two assignments y and y' are considered *adjacent* if y' can be obtained from y by changing the channel of a single carrier. Given this neighborhood structure, an assignment y, and a carrier v, a *1-opt step* determines a least costly neighbor of y. If this neighbor is at most as costly as y, this neighbor becomes the new assignment. Otherwise, the assignment remains unchanged. An assignment y is considered less costly than an assignment y' if y implies fewer constraint violations, or, if both assignments violate equally many (or no) constraints, causes less interference than y'.

A sequence of 1-opt steps where every carrier is selected once is called a *pass*. Conceivably, several consecutive passes are capable of improving an assignment. Passes are performed until

Instance	$\lvert V\rvert$	$\lvert E\rvert$	density [%]	minimum degree	average degree	maximum degree	# components	component sizes	diameter of components	maximum clique in components
k	267	20164	57	2	151	238	1	267	3	69
f	2877	187753	5	0	130	453	58	1×2786 34×2 23×1	1×12 34×1 23×0	1×69 34×2 23×1

Table 1: Parameters of two instances supplied by `E-Plus`.

no more improvement is made. This variant is called *(multi-pass) Iterated 1-OPT heuristic*. The repeated application of this heuristic leads to an assignment that cannot be further improved by 1-opt steps. Of course, such an assignment need not be optimal. The algorithm might be trapped in a local minimum instead.

Min-Cost Flow

This improvement heuristics has a more global flavor than the Iterated 1-OPT, but imposes strong restrictions on the way the old and new assignments, y and y', respectively, differ. Roughly speaking, these restrictions can be summarized as follows: Whenever two carriers v and w are connected by an edge, the signs of $y(v) - y(w)$ and $y'(v) - y'(w)$ have to be the same. For example, if $y(v) > y(w)$ then $y'(v) > y'(w)$ has to hold, too.

A min-cost flow problem—giving the heuristic its name—is solved on a directed graph derived from the graph (V, E) with the arc directions implied by y. The functions d, c^{co}, and c^{ad} are used to compute cost coefficients and arc capacities, respectively. The dual variables of the solution are integral and give the assignment y'. The min-cost flow problem is solved using a Network Simplex Method implementation [10] which has worst-case running time exponential in the input size. Although there are strongly polynomial min-cost flow algorithms, we have chosen this implementation of the Network Simplex Algorithm since it turned out to be very fast in practice.

4 Computational Experiments

We performed tests on several instances stemming from real-world cellular phone networks. We chose two instances, named **k** and **f**, to give computational results here. Table 1 lists some parameters of the instances. Following the name of the instance, the remaining columns display properties of the underlying graph $G = (V, E)$. The graph underlying carrier network **k** is connected. That of the carrier network **f** has one major component, 34 isolated edges, and 23 singletons. The densities, the diameters (of the major component), and clique numbers all indicate that *both graphs are very far from being planar*. No edge $vw \in E$ satisfies $d(vw) = c^{co}(vw) = c^{ad}(vw) = 0$. For instance **k**, the maximum value d takes is four—which is one more than usual. A total of 1053 edges $vw \in E$ satisfy $d(vw) > 0$. Furthermore, $\sum_{vw\in E} c^{co}(vw) = 2857.44$ and $\sum_{vw\in E} c^{ad}(vw) = 28.87$. For instance **f**, these figures are as follows: The maximum value d takes is three. There is a total of 15210 edges $vw \in E$ satisfying $d(vw) > 0$. Finally, $\sum_{vw \in E} c^{co}(vw) = 29146.7$ and $\sum_{vw \in E} c^{ad}(vw) = 983.17$. In both instances almost all sets B_v of blocked channels are empty, and the size of the spectrum is 50.

The instances were supplied by `E-Plus` together with a partial frequency assignment. This assignment was generated using a commercial program with an extension that implements the algorithm described in [8]. In Tables 2 and 3 the quality of the supplied assignment is

Assignment	total interference	co-channel interference	adjacent-channel interference	separation violations	invalid assignments	unassigned carriers	time [secs]
Original	23.7416	23.2958	0.4458	8	6	21	—
+ (MCF 1-OPT)*	2.4341	2.2783	0.1557	0	0	0	5.13
RANDOM	54.1935	53.3958	0.7977	52	0	0	0.02
+ (MCF 1-OPT)*	2.6981	2.3785	0.3197	0	0	0	5.17
DSATUR 1%	1.0142	0.9665	0.0478	0	0	0	0.61
+ (MCF 1-OPT)*	0.9889	0.9387	0.0502	0	0	0	2.38
DSATUR 3%	1.1207	1.0254	0.0953	0	0	0	3.63
+ (MCF 1-OPT)*	1.0617	0.9658	0.0959	0	0	0	2.30
DSATUR 5%	0.9924	0.9655	0.0270	0	0	0	2.42
+ (MCF 1-OPT)*	0.9558	0.9288	0.0269	0	0	0	2.28
DSATUR 10%	0.9978	0.9649	0.0329	0	0	0	10.27
+ (MCF 1-OPT)*	0.9969	0.9640	0.0329	0	0	0	2.33

Table 2: Assignments computed for instance **k** with 50 channels.

shown for comparison. Each of the tables contains the following data. The first column lists the source of the assignment. In rows headed by a '+', the preceding assignment was used to improve on. In columns two, three, and four the incurred interference is listed. The column titled "separation violations" contains the number of violated minimal distance constraints. The next two columns show the number of invalidly assigned and unassigned carriers. A feasible assignment has to have zeros in all three columns mentioned last. The rightmost column lists the time consumed to run the heuristic. All computations were performed on a *SUN SPARCstation 20-501*.

"RANDOM" is a trivial starting heuristic that randomly assigns to every carrier an available channel. No attention is paid to possible separation constraint violations. "(MCF 1-OPT)*" stands for alternatingly applying MCF and Iterated 1-OPT until no more improvement is obtained during Iterated 1-OPT. The percentage listed following "DSATUR" tells how many of the carriers were checked out as a starting point for applying DSATUR With Costs. A summary of the performance of the heuristics is given below.

Assignment	total interference	co-channel interference	adjacent-channel interference	separation violations	invalid assignments	unassigned carriers	time [secs]
Original	52.1004	40.8344	11.2661	0	0	3	—
+ (MCF 1-OPT)*	20.3743	16.8944	3.4799	0	0	0	221.98
RANDOM	635.8770	599.3162	36.5608	812	0	0	0.12
+ (MCF 1-OPT)*	20.4380	16.2523	4.1857	0	0	0	133.84
DSATUR 1%	9.0211	7.9864	1.0346	0	0	0	68.95
+ (MCF 1-OPT)*	8.8203	7.8135	1.0068	0	0	0	120.50
DSATUR 3%	8.6832	7.4796	1.2037	0	0	0	273.01
+ (MCF 1-OPT)*	8.5304	7.3730	1.1574	0	0	0	113.30
DSATUR 5%	8.6698	7.5285	1.1414	0	0	0	483.60
+ (MCF 1-OPT)*	8.5262	7.4293	1.0969	0	0	0	78.20
DSATUR 10%	8.4091	7.2412	1.1680	0	0	0	929.84
+ (MCF 1-OPT)*	8.1669	7.0491	1.1178	0	0	0	79.67

Table 3: Assignments computed for instance **f** with 50 channels.

DSATUR With Costs

This starting heuristic produces assignments of comparatively excellent quality in little running time. Running this heuristic for some random starting points usually irons out the lack of a good deterministic choice for the carrier to start with. Selecting 3 to 10% of the carriers at random as starting point will usually give a good trade-off between quality of the assignment and running time. Quite often, the obtained assignments can be somewhat improved by MCF and Iterated 1-OPT.

Iterated 1-OPT

In several cases, Iterated 1-OPT does succeed in improving over results obtained by DSATUR With Costs. Depending on the quality of the initial assignment, the improvement ranges from minor to huge. The running time observed is slightly inferior to a single run of the DSATUR With Costs.

Min-Cost Flow

Considering the nature of changes MCF is able to perform on an assignment, it does not come as a surprise that improvements are typically small. The main purpose of MCF is to escape from local minima of the neighborhood structure underlying the Iterated 1-OPT heuristic. This goal is achieved often enough to recommend MCF in combination with Iterated 1-OPT.

5 Conclusions

We investigated three heuristics for solving a frequency assignment problem in cellular phone networks. Each of the heuristics is suitable for industrial application. We observed that DSATUR With Costs is a powerful starting heuristic when applied to a small percentage of randomly chosen starting points. The local search procedure Iterated 1-OPT is often able to improve the assignments produced by DSATUR With Costs in acceptable running time. MCF brings in a global optimization aspect, not elaborated here, which sometimes allows to escape from local minima of the neighborhood structure underlying the Iterated 1-OPT heuristic. We know from further computational experiments that the best values given in Tables 2 and 3 are not optimal. But achieving improvements is often quite time consuming.

All computational results were obtained on carrier networks stemming from `E-Plus`' cellular phone network. We were able to drastically improve over the original assignment: The assignments supplied by `E-Plus` for instances **k** and **f** were not feasible. By applying DSATUR 5% followed by MCF and Iterated 1-OPT alternatingly, we obtained feasible assignments and reduced the interference to 4.19% and 15.67% of the original value in cases of instances **k** and **f**, respectively.

`E-Plus` has integrated the heuristics presented here into their software system, thereby enhancing their network planning system with respect to frequency assignment considerably.

6 Acknowledgements

This work is done in cooperation with the German Cellular Phone Network operator `E-Plus` Mobilfunk GmbH. `E-Plus` most kindly supplied the data sets of carrier networks. We want to thank Axel Kaudewitz and Dr. Thomas Kürner for the excellent collaboration.

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