MASSEY UNIVERSITY

Geographic Data Visualization

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Abstract

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too.

Keywords: Keywords, for, your, thesis

Acknowledgements

The acknowledgments and the people to thank go here, don't forget to include your project advisor…

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List of Abbreviations

WSF	**W**hat (it) **S**tands **F**or
VR	**V**irtual **R**eality

List of Symbols

a	distance	m
P	power	W ($\mathrm{J\,s^{-1}}$)
ω	angular frequency	rad

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1 Chapter X

1.1 Ray Intersection

Detect collisions between ray and models is the key to allow user selecting objects in the VR would, which is one of the importent experience for user interaction.

A ray can be describe in a equation with known ray start position $\overrightarrow{R_0}$ and ray direction $\overrightarrow{R_d}$.

$$\overrightarrow{R(t)} = \overrightarrow{R_0} + \overrightarrow{R_d} \cdot t \tag{1.1}$$

1.1.1 Ray-Sphere

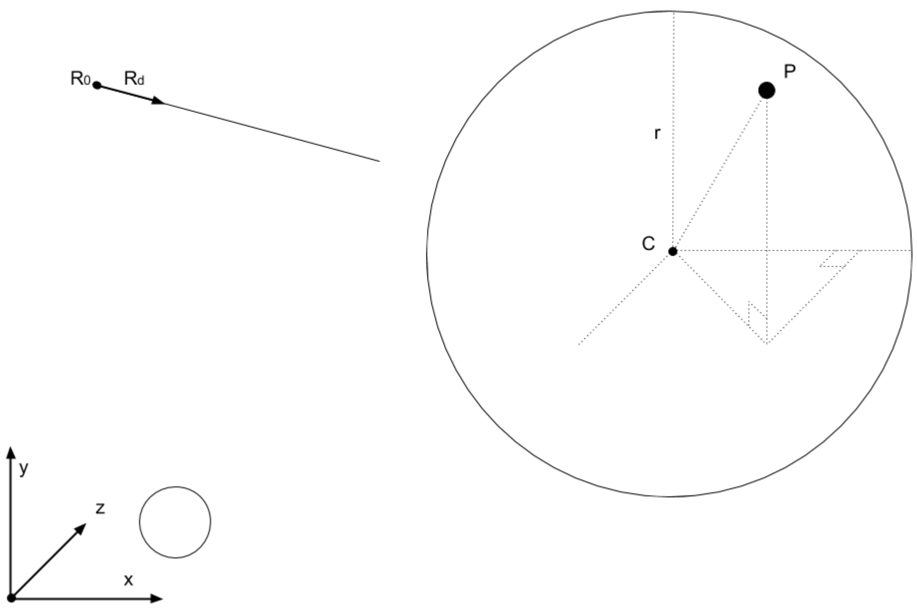

FIGURE 1.1: Ray-Sphere intersection

A point P on the surface of sphere should match the equation:

$$(x_p - x_c)^2 + (y_p - y_c)^2 + (z_p - z_c)^2 = r^2 \tag{1.2}$$

If the ray intersects with the sphere at any position$P$ must match the equation 1.1 and 1.2. Therefor the solution of t in the cointegrate equation implies whether or not the ray will intersect with the sphere:

$$
\begin{aligned}
(x_{R_0} + x_{R_d} \cdot t - x_c)^2 + (y_{R_0} + y_{R_d} \cdot t - y_c)^2 + (z_{R_0} + z_{R_d} \cdot t - z_c)^2 &= r^2 \\
&\vdots \\
x_{R_d}^2 \cdot t^2 + (2 \cdot x_{R_d} \cdot (x_{R_0} - x_c)) \cdot t + (x_{R_0}^2 - 2 \cdot x_{R_0} \cdot x_c + x_c^2) & \\
+ y_{R_d}^2 \cdot t^2 + (2 \cdot y_{R_d} \cdot (y_{R_0} - y_c)) \cdot t + (y_{R_0}^2 - 2 \cdot y_{R_0} \cdot y_c + y_c^2) & \\
+ z_{R_d}^2 \cdot t^2 + (2 \cdot z_{R_d} \cdot (z_{R_0} - z_c)) \cdot t + (z_{R_0}^2 - 2 \cdot z_{R_0} \cdot z_c + z_c^2) &= r^2
\end{aligned}
$$

It can be seen as a quadratic formula:

$$
a \cdot t^2 + b \cdot t + c = 0 \tag{1.3}
$$

At this point, we are able to solved the t:

$$
t = \begin{cases} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & \text{if } b^2 - 4ac > 0 \\ \frac{-b}{2a} & \text{if } b^2 - 4ac = 0 \\ \varnothing & \text{if } b^2 - 4ac < 0 \end{cases}
$$

Then, I take a further step to get rid of formula complexity.

$\because$ Equation 1.2, 1.3

$$
\begin{cases} a = x_{R_d}^2 + y_{R_d}^2 + z_{R_d}^2 \\ b = 2 \cdot (x_{R_d} \cdot (x_{R_0} - x_c) + y_{R_d} \cdot (y_{R_0} - y_c) + z_{R_d} \cdot (z_{R_0} - z_c)) \\ c = (x_{R_0} - x_c)^2 + (y_{R_0} - y_c)^2 + (z_{R_0} - z_c)^2 - r^2 \end{cases}
$$

&

$$
\begin{aligned}
|\overrightarrow{R_d}| &= \sqrt{x_{R_d}^2 + y_{R_d}^2 + z_{R_d}^2} = 1 \\
\overrightarrow{V_{c\_R_0}} &= \overrightarrow{R_0} - \overrightarrow{C} = \overrightarrow{(x_{R_0} - x_c,\ y_{R_0} - y_c,\ z_{R_0} - z_c)}
\end{aligned}
$$

$\therefore$

$$
\begin{cases} a = 1 \\ b = 2 \cdot \overrightarrow{R_d} \cdot \overrightarrow{V_{c\_R_0}} \\ c = \overrightarrow{V_{c\_R_0}} \cdot \overrightarrow{V_{c\_R_0}} \cdot r^2 \end{cases}
$$

$\because$ The formula for t can also be optimized

$$
\begin{cases} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -\alpha \pm \sqrt{\beta} \\ \alpha = 0.5b \\ \beta = \alpha^2 - c \end{cases}
$$

$\therefore$ The final solution for t

$$
t = \begin{cases} -\alpha \pm \sqrt{\beta} & \text{if } \beta > 0 \\ -\alpha & \text{if } \beta = 0 \\ \varnothing & \text{if } \beta < 0 \end{cases}
$$

And the collision position for each t is:

$$
\overrightarrow{P} = \overrightarrow{R_0} + \overrightarrow{R_d} \cdot t
$$

1.1.2 Ray-Plane

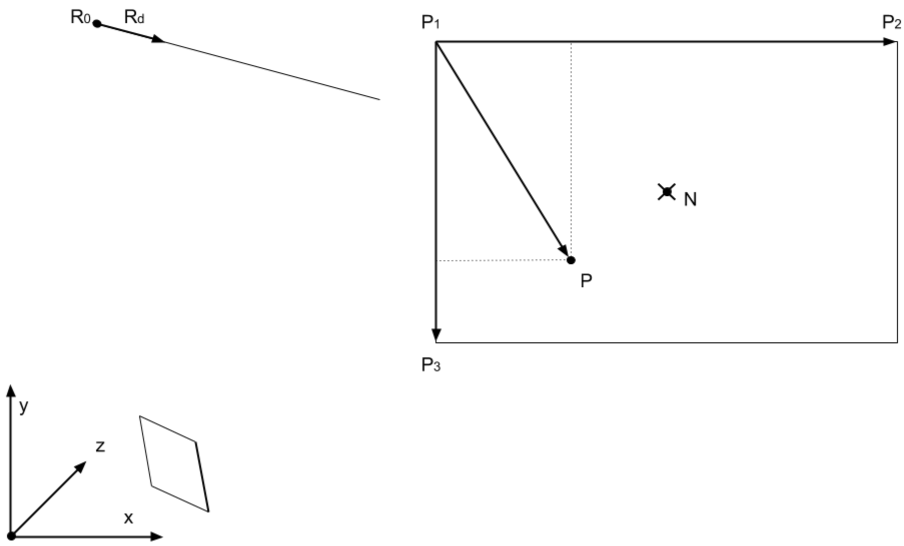

FIGURE 1.2: Ray-Plane intersection

If a point P on the plane and also belongs to the ray, we have quadric equation:

$$\begin{cases} (\overrightarrow{P} - \overrightarrow{P_1}) \cdot \overrightarrow{N} = 0 \\ \overrightarrow{P} = \overrightarrow{R_0} + \overrightarrow{R_d} \cdot t \end{cases} \tag{1.4}$$

Solution for the t is:

$$t = \begin{cases} \frac{-\overrightarrow{N} \cdot (\overrightarrow{R_0} - \overrightarrow{P_1})}{\overrightarrow{N} \cdot \overrightarrow{R_d}} & \text{if } \overrightarrow{N} \cdot \overrightarrow{R_d} \nsim 0 \\ \varnothing & \text{if } \overrightarrow{N} \cdot \overrightarrow{R_d} \sim 0 \end{cases}$$

At last, we have to verify if the collision is inside of the quadrangle by putting t back to 1.4, and the t is valid only if:

$$\mu = \sqrt{(\overrightarrow{P} - \overrightarrow{P_1}) \cdot (\overrightarrow{P_2} - \overrightarrow{P_1}))} \in [0, |\overrightarrow{P_2} - \overrightarrow{P_1}|]$$
$$\nu = \sqrt{(\overrightarrow{P} - \overrightarrow{P_1}) \cdot (\overrightarrow{P_3} - \overrightarrow{P_1}))} \in [0, |\overrightarrow{P_3} - \overrightarrow{P_1}|]$$

1.1.3 Ray-Box

There is a octree implementation in the VR 3D world that separate the 3D world to 3D boxes to avoid unnecessary ray-object collision detection. In this section, I am going to first explain Ray-Box-2D collision detection, then derive out Ray-Box-3D intersection.

Ray-Box-2D

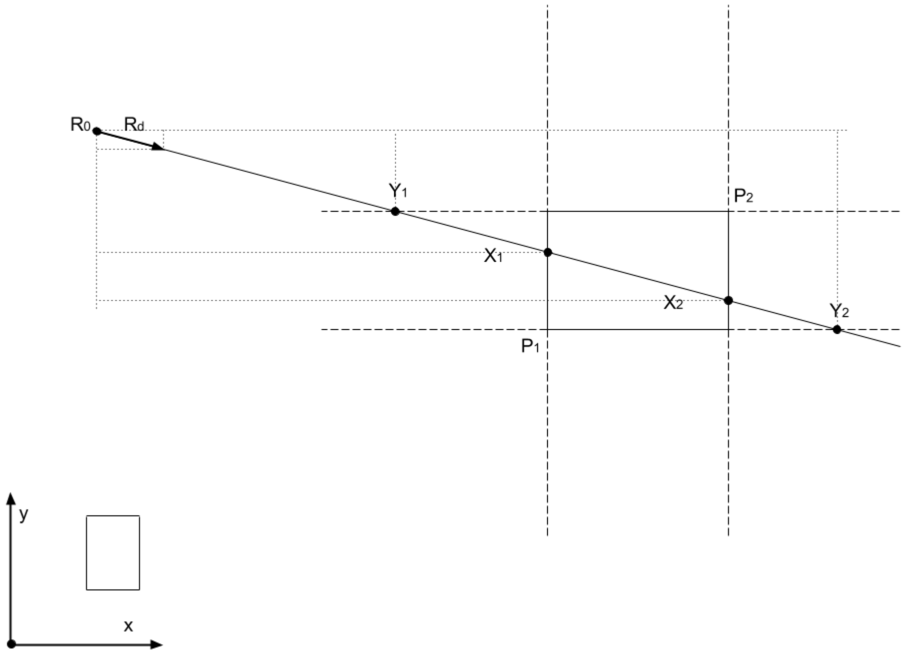

FIGURE 1.3: Ray-Box-2D intersection

$\because$ Known R_0, R_d, P_1, P_2

$$X_1 = \begin{cases} x_{P_1} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_2} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases} \qquad Y_1 = \begin{cases} y_{P_1} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_2} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$X_2 = \begin{cases} x_{P_2} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_1} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases} \qquad Y_2 = \begin{cases} y_{P_2} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_1} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$t_{X_1} = \frac{X_1}{x_{R_d}} \qquad t_{Y_1} = \frac{Y_1}{y_{R_d}}$$
$$t_{X_2} = \frac{X_2}{x_{R_d}} \qquad t_{Y_2} = \frac{Y_2}{y_{R_d}}$$

$\&$ When collision happens, we have formula

$$\begin{cases} t_{X_1} < t_{X_2} \\ t_{Y_1} < t_{Y_2} \end{cases}$$

$\therefore$ Which is

$$max(t_{X_1}, t_{Y_1}) < min(t_{X_2}, t_{Y_2}) \tag{1.5}$$

Ray-Box-3D

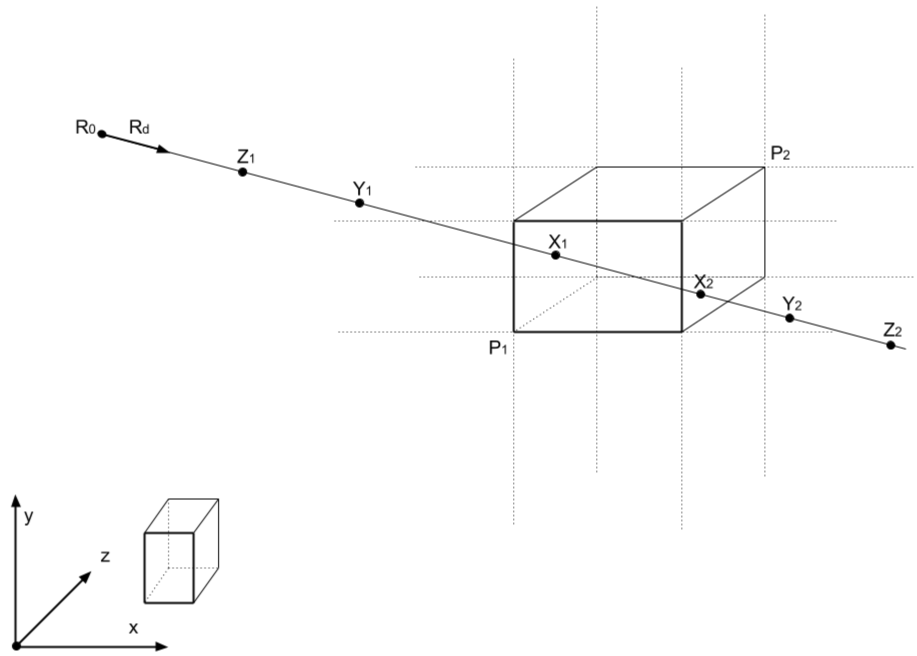

FIGURE 1.4: Ray-Box-3D intersection

$\because$ Known R_0, R_d, P_1, P_2

$$X_1 = \begin{cases} x_{P_1} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_2} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases}$$

$$X_2 = \begin{cases} x_{P_2} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_1} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases}$$

$$t_{X_1} = \frac{X_1}{x_{R_d}}$$
$$t_{X_2} = \frac{X_2}{x_{R_d}}$$

$$Y_1 = \begin{cases} y_{P_1} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_2} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$Y_2 = \begin{cases} y_{P_2} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_1} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$t_{Y_1} = \frac{Y_1}{y_{R_d}}$$
$$t_{Y_2} = \frac{Y_2}{y_{R_d}}$$

$$Z_1 = \begin{cases} z_{P_1} - z_{R_0} & \text{if } z_{R_d} > 0 \\ z_{P_2} - z_{R_0} & \text{if } z_{R_d} < 0 \end{cases}$$

$$Z_2 = \begin{cases} z_{P_2} - z_{R_0} & \text{if } z_{R_d} > 0 \\ z_{P_1} - z_{R_0} & \text{if } z_{R_d} < 0 \end{cases}$$

$$t_{Z_1} = \frac{Z_1}{z_{R_d}}$$
$$t_{Z_2} = \frac{Z_2}{z_{R_d}}$$

$\&$ When collision happens, we have formula

$$\begin{cases} t_{X_1} < t_{X_2} \\ t_{Y_1} < t_{Y_2} \\ t_{Z_1} < t_{Z_2} \end{cases}$$

$\therefore$ Which is

$$max(t_{X_1}, t_{Y_1}, t_{Z_1}) < min(t_{X_2}, t_{Y_2}, t_{Z_2}) \tag{1.6}$$

1.2 Camera Movement

A Appendix Title Here

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