

MASSEY UNIVERSITY

---

# Geographic Data Visualization

---

*Author*  
Yang JIANG

*Supervisor*  
Dr Arno LEIST

*A thesis submitted in fulfillment of the requirements  
for the degree of **Master of Information Sciences**  
in the*

**Software Engineering  
159.888 Computer Science Professional Project**

October 6, 2016

## *Abstract*

Visualization has proven effective for not only presenting essential information in vast amounts of data but also driving complex analyses.

\*\*\*\*\*

overview

propose

What was done?

Why was it done?

How was it done?

What was found?

What is the significance of the findings in short?

**Keywords:** Keywords, for, your, thesis

# Contents

<b>Abstract</b>	<b>i</b>
<b>Contents</b>	<b>ii</b>
<b>List of Figures</b>	<b>iv</b>
<b>List of Tables</b>	<b>v</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background and Overview	1
1.2 Literature Review	1
1.2.1 Current Landscape	1
1.2.2 Respective Limitations	1
1.3 Aims and Objectives	1
<b>2 Technology</b>	<b>2</b>
2.1 Android Phone	2
2.1.1 Mobile Device	2
2.1.2 Google Cardboard	2
2.2 OpenGL ES	2
2.3 Keyhole Markup Language	2
2.4 Network Data	3
<b>3 Implementation</b>	<b>4</b>
3.1 Google VR SDK	4
3.2 OpenGL ES	4
3.3 File Server	4
3.3.1 Assets	4
3.3.2 Patch	4
3.4 Scene	4
3.4.1 Keyhole Markup Language	5
3.4.2 Octree	5
3.5 Earth	7
3.6 Placemark	8
3.6.1 Geographic Coordinate System	11
3.6.2 Description	13
3.6.3 OBJ Model	14
3.7 Information Display	15
3.8 Camera Movement	15
3.9 Ray Intersection	16
3.9.1 Ray-Sphere	17
3.9.2 Ray-Plane	19
3.9.3 Ray-Box	19
Ray-Box-2D	20
Ray-Box-3D	21
<b>4 Discussion</b>	<b>23</b>
<b>5 Conclusion</b>	<b>24</b>

<b>A Appendix A</b>	<b>25</b>
<b>Bibliography</b>	<b>26</b>

# List of Figures

2.1	kml-schema	3
3.1	kml-parser-simple	5
3.2	octree-split	6
3.3	uv-sphere-mapping	7
3.4	uv-sphere-vertex	8
3.5	icosahedron-rectangles	9
3.6	icosphere-subdivide	10
3.7	icosphere-refinement	10
3.8	ecef	11
3.9	ellipsoid-parameters	12
3.10	lla2ecef	13
3.11	description-analysis	14
3.12	camera-movement	16
3.13	ray-sphere-intersection	17
3.14	ray-plane-intersection	19
3.15	ray-box-2d-intersection	20
3.16	ray-box-3d-intersection	21

# List of Tables

3.1	Octree Octant	6
3.2	Rounding Icosphere	11
3.3	WGS 84 parameters	12

# 1 Introduction

\*\*\*\*\*  
Why am I doing it?

## 1.1 Background and Overview

\*\*\*\*\*  
VR

## 1.2 Literature Review

\*\*\*\*\*  
What is known?  
What is unknown?

### 1.2.1 Current Landscape

\*\*\*\*\*  
Review of research

### 1.2.2 Respective Limitations

\*\*\*\*\*  
Identifying gaps

## 1.3 Aims and Objectives

\*\*\*\*\*  
What do I hope to discover?  
What is expected in a thesis?

## 2 Technology

\*\*\*\*\*

How am I going to discover it?  
.... why is best suited to my aims, because ...

### 2.1 Android Phone

\*\*\*\*\*

#### 2.1.1 Mobile Device

\*\*\*\*\*

Sensor

#### 2.1.2 Google Cardboard

\*\*\*\*\*

Google VR SDK

### 2.2 OpenGL ES

\*\*\*\*\*

OpenGL ES 2.0, 3.0, 3.1, 3.1 ext

### 2.3 Keyhole Markup Language

We were looking for a simple markup languages that we can publish and consume data in interoperable formats without the need for technical assistance.

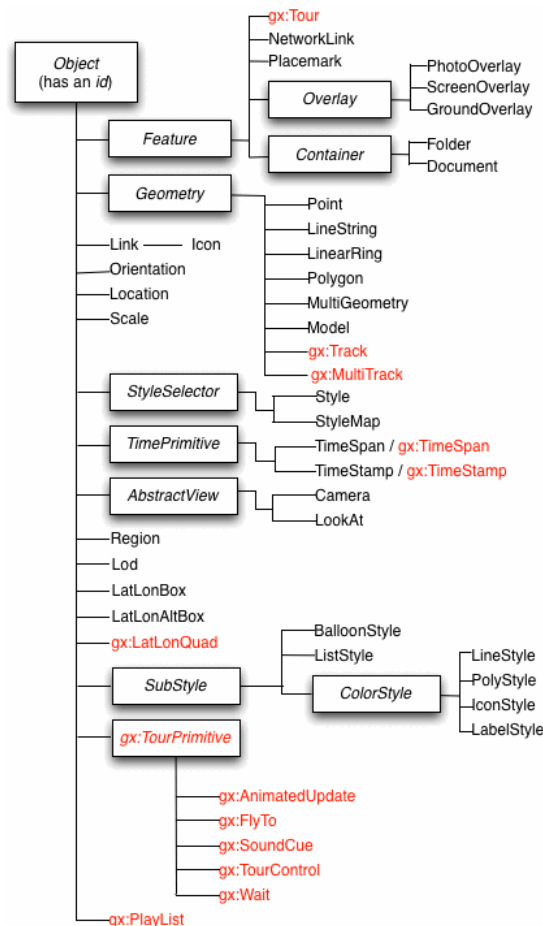
In this section, we present Keyhole Markup Language (KML) (Google, 2016c), a file format to display geographic data (Note that KML files can be combined with other supporting files such as imagery in a zip archive, producing a KMZ file). It is an international standard maintained by the Open Geospatial Consortium, Inc. (OGC), and also it is supported by many Virtual Globes (VG) applications and other GIS systems and is therefore already becoming a de facto standard. Such as the most wellknown VG application Google Earth that has the largest community, NASA World Wind has more focus toward scientific users, and ArcGIS Explorer that is a lightweight client to the ArcGIS Server (Blower et al., 2007).



From an environmental science point of view, KML is a somewhat limited language. It describes only simple geometric shapes on the globe (points, lines and polygons) and is not extensible. It is, in many respects, analogous to Geography Markup Language (GML) 3.0+ is much more sophisticated and allows the rich description of geospatial features such as weather fronts and radiosonde profiles. So, KML is currently not suitable as a fully-featured, general-purpose environmental data exchange format.

Figure 2.1 shows the KML schema. From the point of view of usability, KML spans a gap between very simple (e.g. GeoRSS) and more complex (GML) formats, that makes it easy for non-technical scientists to share and visualize simple geospatial information which can then be manipulated in other applications if required. Also it makes it easier for user to visualize their data quickly using a single, simple data file. Moreover, its rapidly-growing adoption by scientists, and it is important to be aware of that virtual geographic data visualization (and KML) do not attempt to replace more sophisticated systems.

FIGURE 2.1: KML schema (Google, 2016c)



## 2.4 Network Data

\*\*\*\*\*

File server - golang  
Client - volley, Jsoup

## 3 Implementation

### 3.1 Google VR SDK

\*\*\*\*\*

### 3.2 OpenGL ES

\*\*\*\*\*

Pass Perspective matrix, View matrix, and Model matrix to shader to avoid calculate MV or MVP in cpu

Perspective: eye.getPerspective(float zNear, float zFar)

View: eye.getEyeView() \* camera.matrix

camera.matrix: build by position, lookAt, up

Model: translation \* scale \* rotation \* mat(1)

### 3.3 File Server

\*\*\*\*\*

Web server - golang - <https://golang.org/pkg/net/http/>

[http.FileServer](http://FileServer)

<https://github.com/ant0ine/go-json-rest>

Go-Json-Rest is a thin layer ("Keep it simple, stupid") on top of net/http that helps building RESTful JSON APIs easily.

Volley - an HTTP library that makes networking for Android apps easier and most importantly, faster.

<https://developer.android.com/training/volley/index.html>

#### 3.3.1 Assets

\*\*\*\*\*

static, layer, kml, resource,model

#### 3.3.2 Patch

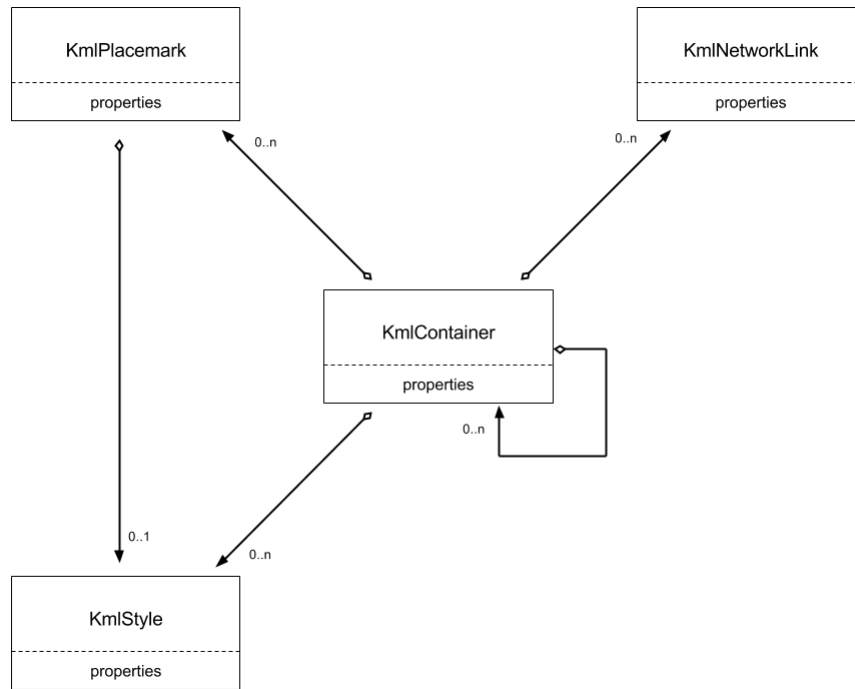
### 3.4 Scene

\*\*\*\*\*

### 3.4.1 Keyhole Markup Language

In this project, we only take use of few feature of KML 2.1: Container, Style, Placemark, and NetworkLink. The KML parser we are using is based on the open-source library `android-maps-utils` (Google, 2016a) (NetworkLink is one of the unsupported feature in the library). Main modifications are getting rid of `com.google.android.gms.maps.GoogleMap` dependency, and extending NetworkLink feature support in accordance with the current design pattern.

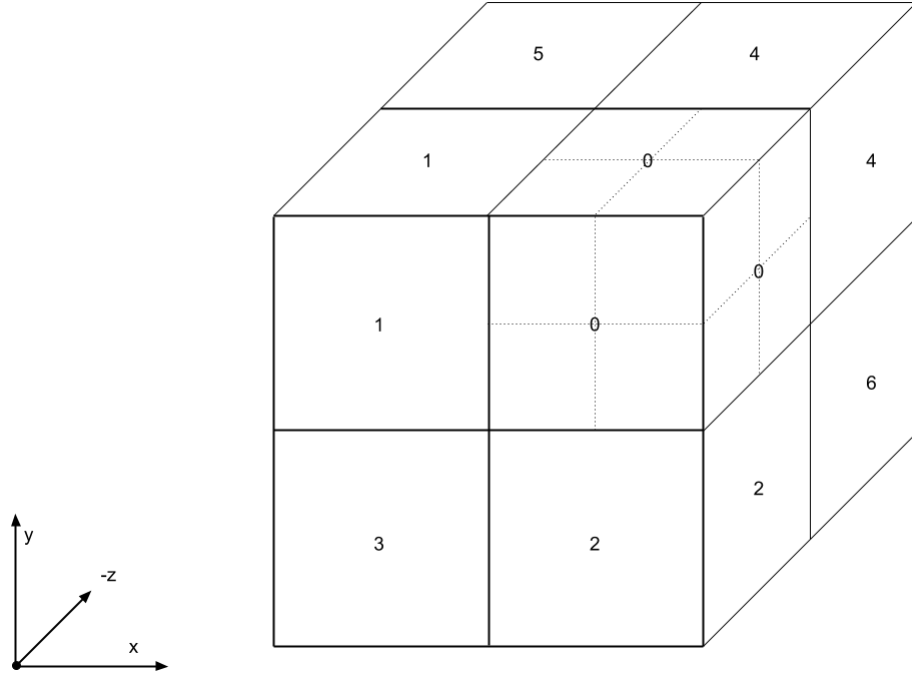
FIGURE 3.1: kML parser simple



### 3.4.2 Octree

To reduce the ray-object intersection tests, space partitioning is needed. The main requirement is not to use a spatial data structure for an ray intersect with irregular geometry, but to determine what objects are in the same cell to avoid doing an  $n^2$  check on all objects. In this case, it is spherical placemark. Therefore, we don't need overlapped volumes, and contained objects don't need to be cut on volume boundaries. It is actually 3D space partitioning process with a predefined restricted maximum number of objects in the same cell. A simple axis-aligned Octree is fully satisfy in here 3.2.

FIGURE 3.2: Octree split



For each box splitting process, we also generate eight indexes to indicate the relative position inside the box. These indexes are important for the next partition, where we might need to relink contained objects to the new corresponding box. To insert a object into the box only if the existed contained number of objects is less then the predefined constant value, otherwise splitting the box then relink existed object and insert the new object again.

Integer indexes of box is defined by three boolean valuse that indicates three axis-relative value:

Any position  $P$  in the box with known center  $O$ :

$$dx = P_x - O_x$$

$$dy = P_y - O_y$$

$$dz = P_z - O_z$$

TABLE 3.1: Octree Octant

Index	Octant	Geometric Meaning
0x00000000	T, T, T	$dx > 0, dy > 0, dz > 0$
0x00000001	F, T, T	$dx < 0, dy > 0, dz > 0$
0x00000010	T, F, T	$dx > 0, dy < 0, dz > 0$
0x00000011	F, F, T	$dx < 0, dy < 0, dz > 0$
0x00000100	T, T, F	$dx > 0, dy > 0, dz < 0$
0x00000101	F, T, F	$dx < 0, dy > 0, dz < 0$
0x00000110	T, F, F	$dx > 0, dy < 0, dz < 0$
0x00000111	F, F, F	$dx < 0, dy < 0, dz < 0$

Octant solution:

```
octant[] = (index & 1, index & 2, index & 4)
```

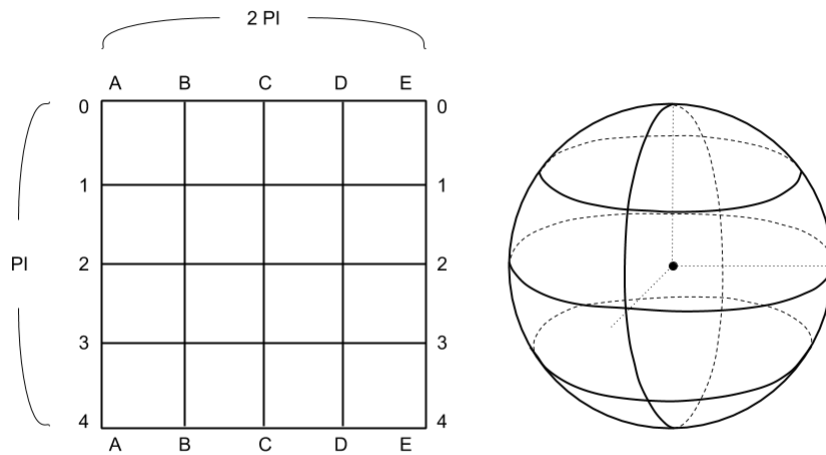
Index solution:

```
For Each oction[i]:
    index |= (1 << i)
```

### 3.5 Earth

The Earth is created as a UV Sphere, which somewhat like latitude and longitude lines of the earth, uses rings and segments. Near the poles (both on the Z-axis with the default orientation) the vertical segments converge on the poles. UV spheres are best used in situations where you require a very smooth, symmetrical surface.

FIGURE 3.3: UV sphere mapping

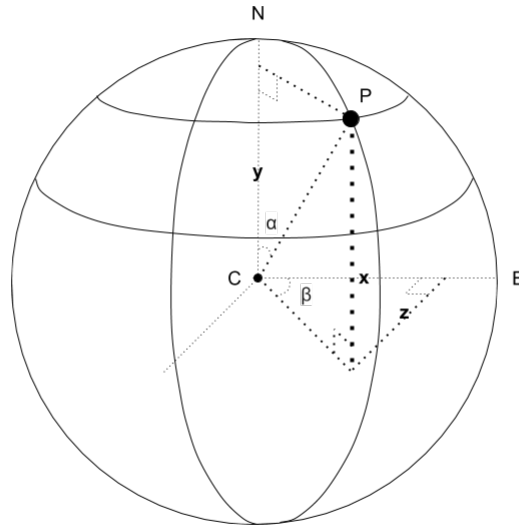


As we can see the mapping from 3.3. Vertex  $A_0, A_1, A_2, A_3, A_4$  and  $E_0, E_1, E_2, E_3, E_4$  are duplicated, and  $A_0, B_0, C_0, D_0, E_0$  converge together as well as  $A_4, B_4, C_4, D_4, E_4$ . So we can simply define it as a UV sphere has 5 rings and 4 segments. Also be noticed that each ring spans  $2\pi$  radians, but each segment spans  $\pi$  radians in the sphere mapping.

The total vertex number is:

$$Vertices = Rings \times Segments \quad (3.1)$$

FIGURE 3.4: UV sphere vertex



For each vertex  $P$  on sphere from ring  $r$  and segment  $s$ , we have:

$$\begin{aligned} v &= r \times \frac{1}{rings-1} \\ u &= s \times \frac{1}{segments-1} \\ \angle\alpha &= v \times \pi \\ \angle\beta &= u \times 2\pi \end{aligned}$$

$\therefore P(x, y, z)$

$$\begin{aligned} x &= (\sin(\alpha) \times radius) \times \cos(\beta) \\ y &= \cos(\alpha) \times radius \\ z &= (\sin(\alpha) \times radius) \times \sin(\beta) \end{aligned}$$

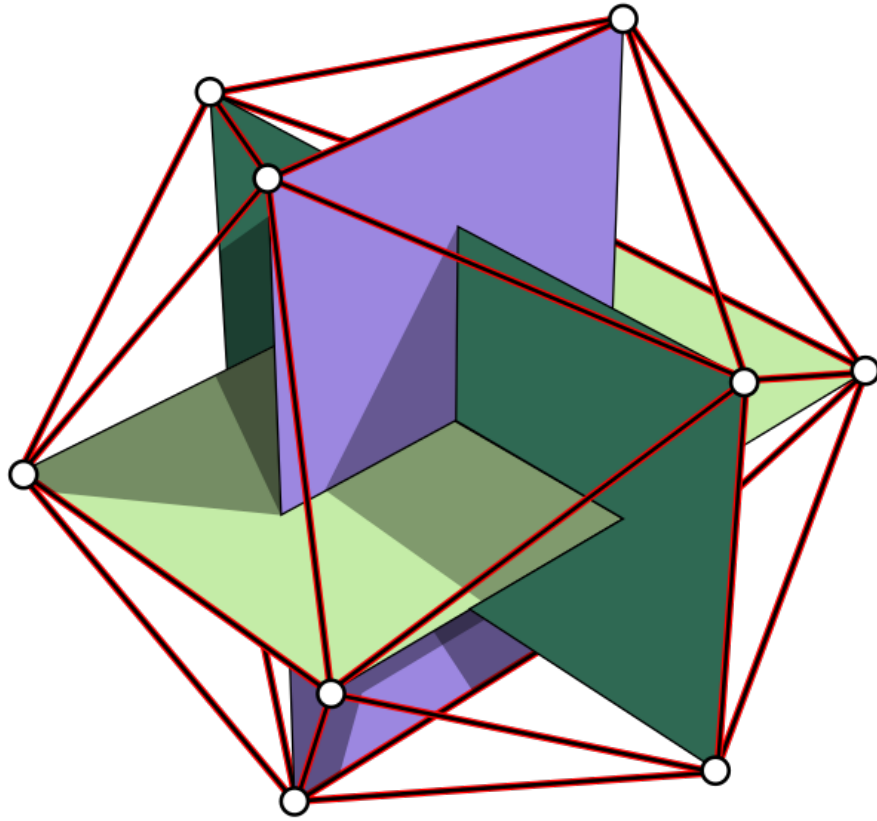
& 2D Texture (x, y) mapping for vertex  $P$  is:

$$\begin{aligned} x &= u \\ y &= v \end{aligned}$$

### 3.6 Placemark

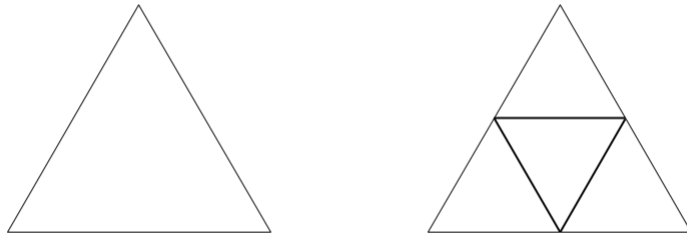
Generation of vertices for placemark is a recursion process of subdividing icosphere. Figure 3.5 shows that the initial vertices of an icosahedron are the corners of three orthogonal rectangles.

FIGURE 3.5: Icosahedron rectangles (Fropuff, 2006)



---

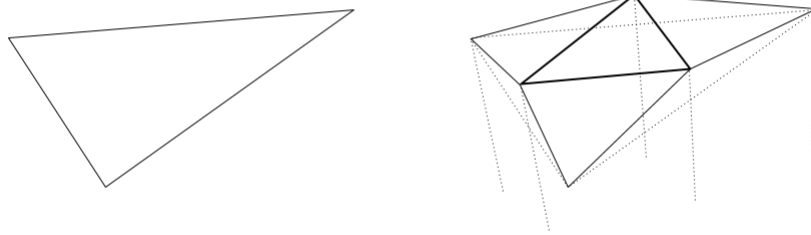
Rounding icosphere by subdividing a face to an arbitrary level of resolution. One face can be subdivided into four by connecting each edge's midpoint.



---

FIGURE 3.6: Icosphere subdivide

Then, push edge's midpoints to surface of the sphere.



---

FIGURE 3.7: Icosphere refinement



TABLE 3.2: Rounding Icosphere

Recursion Level	Vertex Count	Face Count	Edge Count
0	12	20	30
1	42	80	120
2	162	320	480
3	642	1280	1920

### 3.6.1 Geographic Coordinate System

A geographic coordinate system is a coordinate system that enables every location on the Earth to be specified by a set of numbers or letters, or symbols (Wikipedia, 2016c). A common geodetic-mapping coordinates is latitude, longitude and altitude (LLA), which also is the raw location data read from KML.

We introduce ECEF ("earth-centered, earth-fixed") coordinate system for converting LLA coordinates to position coordinates. According to, the z-axis is pointing towards the north but it does not coincide exactly with the instantaneous earth rotational axis. The x-axis intersects the sphere of the earth at 0 latitude and 0 longitude (Wikipedia, 2016b).

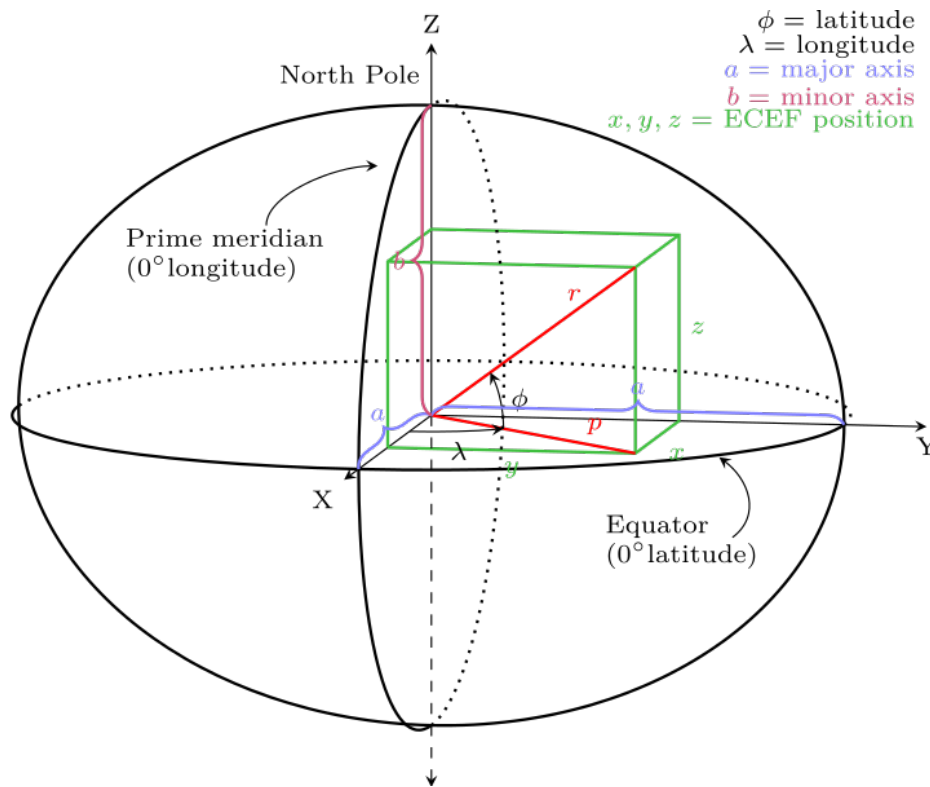


FIGURE 3.8: earth-centered, earth-fixed (Wikipedia, 2016b)

The ECEF coordinates are expressed in a reference system that is related to mapping representations. Because the earth has a complex shape, a simple, yet accurate, method to approximate the earth's shape is required. The use of a reference ellipsoid allows for the conversion between ECEF and LLA (blox, 1999).

A reference ellipsoid can be described by a series of parameters that define its shape and which include a semi-major axis ( $a$ ), a semi-minor axis ( $b$ ), its first eccentricity ( $e_1$ ) and its second eccentricity ( $e_2$ ) as shown in Table 3.3.

TABLE 3.3: WGS 84 parameters

Parameter	Notation	Value
Reciprocal of flattening	$1/f$	298.257 223 563
Semi-major axis	$a$	6 378 137 m
Semi-minor axis	$b$	$a (1 - f)$
First eccentricity squared	$e_1^2$	$1 - b^2/a^2 = 2 f - f^2$
Second eccentricity squared	$e_2^2$	$a^2/b^2 - 1 = f (2 - f)/(1 - f)^2$

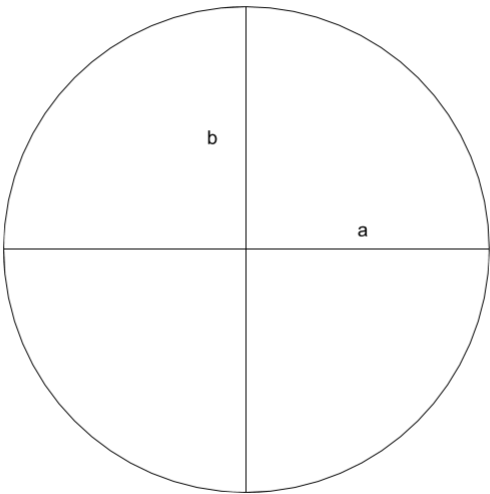


FIGURE 3.9: Ellipsoid Parameters

The conversion from LLA to ECEF is shown below.

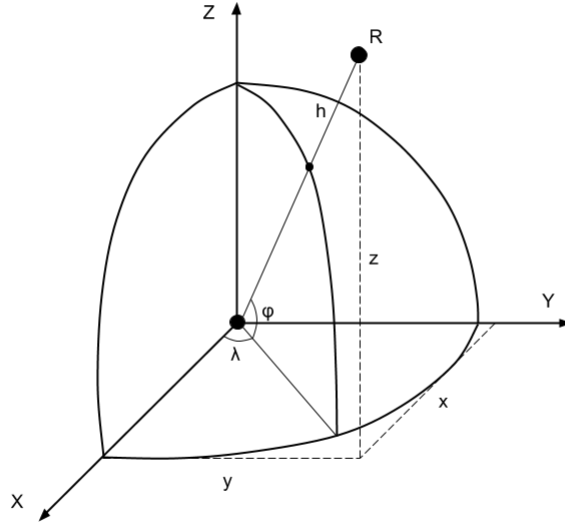


FIGURE 3.10: LLA to ECEF

$$\begin{aligned}
 x &= (N + h) \cos(\varphi) \cos(\lambda) \\
 y &= (N + h) \cos(\varphi) \sin(\lambda) \\
 z &= \left(\frac{b^2}{a^2} N + h\right) \sin(\varphi)
 \end{aligned}$$

Where

$\varphi$  = latitude

$\lambda$  = longitude

$h$  = height above ellipsoid (meters)

$N$  = Radius of Curvature (meters), defined as:

$$= \frac{a}{\sqrt{1 - e^2 \sin^2(\varphi)}}$$

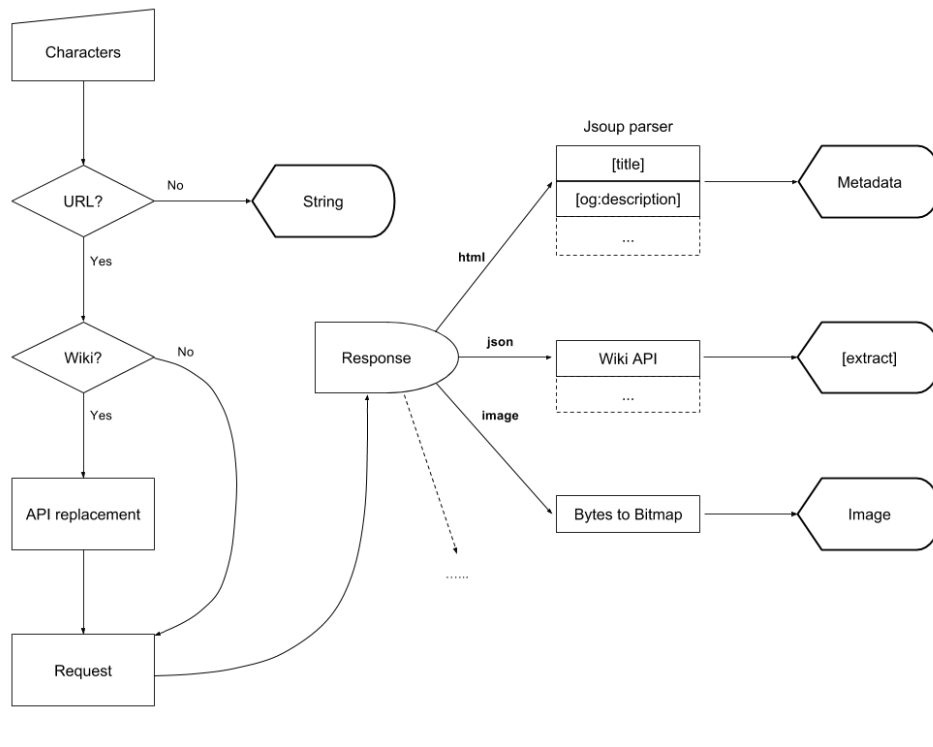
At last, for this project usage, where high accuracy is not required,  $a$  equals to  $b$ . And also the ECEF coordinate system is y-east, z-north (up), and x points to 0 latitude and 0 longitude, but for project specific, we still need to convert ECEF to x-east, y-north (up), and x points to 0 latitude and 180 longitude.

### 3.6.2 Description

Description of placemark requires an appropriate analysis for display. The raw data of description is a set of characters that could be a normal text, an image URL, an URL returns different type of content, or maybe just some meaningless characters.

Although the implementation of analysis in this project did not cover every situation, but it is flexible and extendable for more functionality.

FIGURE 3.11: Description Analysis



In order to get an extracted content from a wikipedia page, we can transform the URL to a Wiki-API based open-search url (Wikipedia, 2016a), which will returns a json format raw data that we can easily get what we need from different json tags.

Replace `.wikipedia.org/wiki/`  
To `.wikipedia.org/w/api.php?APIs`

Where *APIs* is:

```

format=json
&action=query
&redirects=1
&prop=extracts
&exintro=
&explaintext=
&indexpageids=
&titles=

```

For *html* parser, we introduced jsoup (it is a Java library for working with real-world HTML (jsoup, 2016)), to get the basic information we need, such as *title*, and some other metadata. In this project, I am also use *og : description* (one of the open graph meta tags (ogp, 2014)) from the html source if it exist.

### 3.6.3 OBJ Model

A simple and common OBJ format model can be loaded as an extra model for the placemaker.

```
*****
vfn
```

### 3.7 Information Display

A textfield is a a rectangle vertices based renderable component to display text on a flat plane. Since it is a GL scene, the actual text will be drawn as a texture. By a constant width and native `android.text.StaticLayout` support, the height of the texture can be calculated.

A menu contains multi-textfield can be seen as an empty textfield based which texture is fill full a pure background color, and several textfields are laid out on the top of it with a certain vertical dimension.

A head rotation matrix (quaternion matrix (Verth, 2013)) is required for locating object in front of camera (mathworks, 2016) .

```
*****
quaternions
```

### 3.8 Camera Movement

In general, there are two sensors can be useful to manager camera movement: ACCELEROMETER (API level 3), LINEAR\_ACCELERATION (API level 9) and STEP\_DETECTOR (API level 19).

LINEAR\_ACCELERATION is same as ACCELERATION which measures the acceleration force in meter per second repeatedly, except linear acceleration sensor is a synthetic sensor with gravity filtered out.

$$\begin{aligned} \text{LinearAcceleration} &= \text{AccelerometerData} - \text{Gravity} \\ v &= \int a \, dt \\ x &= \int v \, dt \end{aligned}$$

First of all, we take the acclerometer data and remove gravity that is called gravity compensation, whatever is left is linear movement. Then we have to integrate it one to get velocity, integrated again to get position, which is called double integral. Now if the first integral creates drift, double integrals are really nasty that they create horrible drift. Because of these noise, using acceleration data it isn't so accurate, it is really hard to do any kind of linear movement (GoogleTechTalks, 2010).

On the other hand, use step counter from STEP\_DETECTOR, and pedometer algorithm for pedestrian navigation, that in fact works very well for this project.

$$\begin{aligned} p_1 &= p_0 + v_0 \times dt \\ v_1 &= v_0 + a \times dt \end{aligned}$$

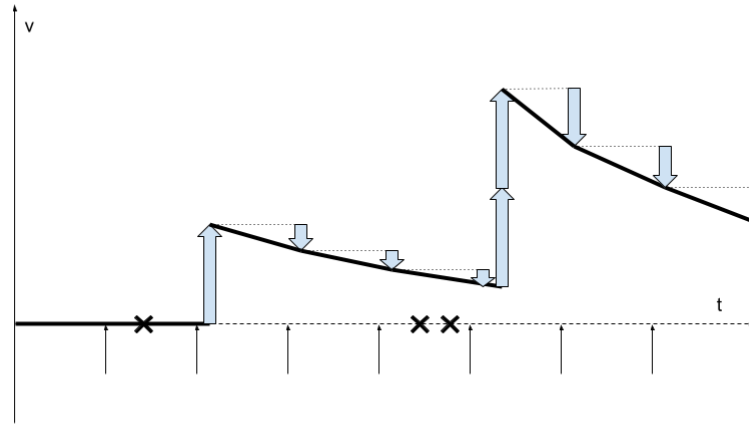
The accuracy of this depends on how precision we can get for changing velocity. Considering that velocity is made of 3-axis directions, the current heading direction is required for a correct velocity calculation. Since the frame life cycle is implemented based on (Google, 2016b), which provide the heading direction in each frame callback. So I collect everything I need from the last frame to new frame, and update both velocity and position for each new frame.

For updating process, first of all,

First of all, damping is required. I reduce velocity by a percentage. It is simply for avoiding that camera taking too long to stop. Damping by percentage can stable and stop the camera in a certain of time that won't be affected by the current camera speed.

Secondly, a constant value in head forwarding direction is been used as a pulse for each step. Because a step is happening instantaneously which implies  $a \, dt$  made by each step is actually can be replace by a constant value.

FIGURE 3.12: Camera movement



For each new frame:

$$\begin{aligned}\vec{V}_0 &= \vec{V}_0 \cdot Damping \\ \vec{P}_1 &= \vec{P}_0 + \vec{V}_0 \cdot dt \\ \vec{V}_1 &= \vec{V}_0 + \vec{Forwarding} \cdot Pulse \cdot Steps \\ Damping &\in [0, 1] \\ Pulse &\in [0, \infty)\end{aligned}$$

### 3.9 Ray Intersection

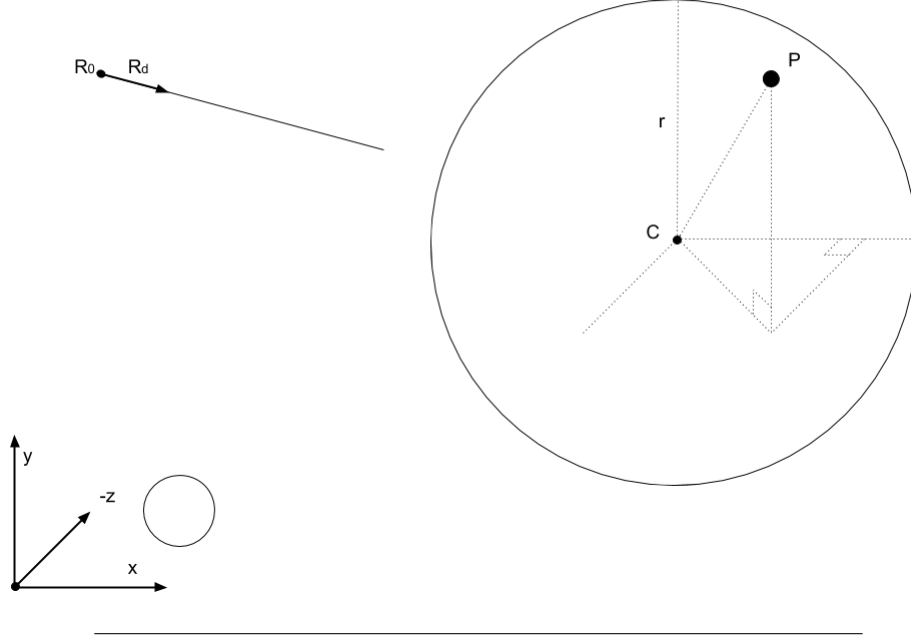
Detect collisions between ray and models is the key to allow user selecting objects in the VR world, which is one of the important experience for user interaction.

A ray can be describe in a equation with known ray start position  $\vec{R}_0$  and ray direction  $\vec{R}_d$ .

$$\vec{R}(t) = \vec{R}_0 + \vec{R}_d \cdot t \quad (3.2)$$

### 3.9.1 Ray-Sphere

FIGURE 3.13: Ray-Sphere intersection



A point  $P$  on the surface of sphere should match the equation:

$$(x_p - x_c)^2 + (y_p - y_c)^2 + (z_p - z_c)^2 = r^2 \quad (3.3)$$

If the ray intersects with the sphere at any position  $P$  must match the equation 3.2 and 3.3. Therefore the solution of  $t$  in the cointegrate equation implies whether or not the ray will intersect with the sphere:

$$\begin{aligned} (x_{R_0} + x_{R_d} \cdot t - x_c)^2 + (y_{R_0} + y_{R_d} \cdot t - y_c)^2 + (z_{R_0} + z_{R_d} \cdot t - z_c)^2 &= r^2 \\ \vdots \\ x_{R_d}^2 t^2 + (2 x_{R_d} (x_{R_0} - x_c)) t + (x_{R_0}^2 - 2 x_{R_0} x_c + x_c^2) \\ + y_{R_d}^2 t^2 + (2 y_{R_d} (y_{R_0} - y_c)) t + (y_{R_0}^2 - 2 y_{R_0} y_c + y_c^2) \\ + z_{R_d}^2 t^2 + (2 z_{R_d} (z_{R_0} - z_c)) t + (z_{R_0}^2 - 2 z_{R_0} z_c + z_c^2) &= r^2 \end{aligned}$$

It can be seen as a quadratic formula:

$$a t^2 + b t + c = 0 \quad (3.4)$$

At this point, we are able to solve the  $t$ :

$$t = \begin{cases} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} & \text{if } b^2 - 4ac > 0 \\ \frac{-b}{2a} & \text{if } b^2 - 4ac = 0 \\ \emptyset & \text{if } b^2 - 4ac < 0 \end{cases}$$

Then, I take a further step to get rid of formula complexity.

∴ Equation 3.3, 3.4

$$\begin{aligned} a &= x_{R_d}^2 + y_{R_d}^2 + z_{R_d}^2 \\ b &= 2(x_{R_d}(x_{R_0} - x_c) + y_{R_d}(y_{R_0} - y_c) + z_{R_d}(z_{R_0} - z_c)) \\ c &= (x_{R_0} - x_c)^2 + (y_{R_0} - y_c)^2 + (z_{R_0} - z_c)^2 - r^2 \end{aligned}$$

&

$$\begin{aligned} |\vec{R_d}| &= \sqrt{x_{R_d}^2 + y_{R_d}^2 + z_{R_d}^2} = 1 \\ \vec{V_{c\_R_0}} &= \vec{R_0} - \vec{C} = (x_{R_0} - x_c, y_{R_0} - y_c, z_{R_0} - z_c) \end{aligned}$$

∴

$$\begin{aligned} a &= 1 \\ b &= 2 \cdot \vec{R_d} \cdot \vec{V_{c\_R_0}} \\ c &= \vec{V_{c\_R_0}} \cdot \vec{V_{c\_R_0}} \cdot r^2 \end{aligned}$$

∴ The formula for  $t$  can also be optimized

$$\begin{aligned} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} &= -\alpha \pm \sqrt{\beta} \\ \alpha &= \frac{1}{2}b \\ \beta &= \alpha^2 - c \end{aligned}$$

∴ The final solution for  $t$

$$t = \begin{cases} -\alpha \pm \sqrt{\beta} & \text{if } \beta > 0 \\ -\alpha & \text{if } \beta = 0 \\ \emptyset & \text{if } \beta < 0 \end{cases}$$

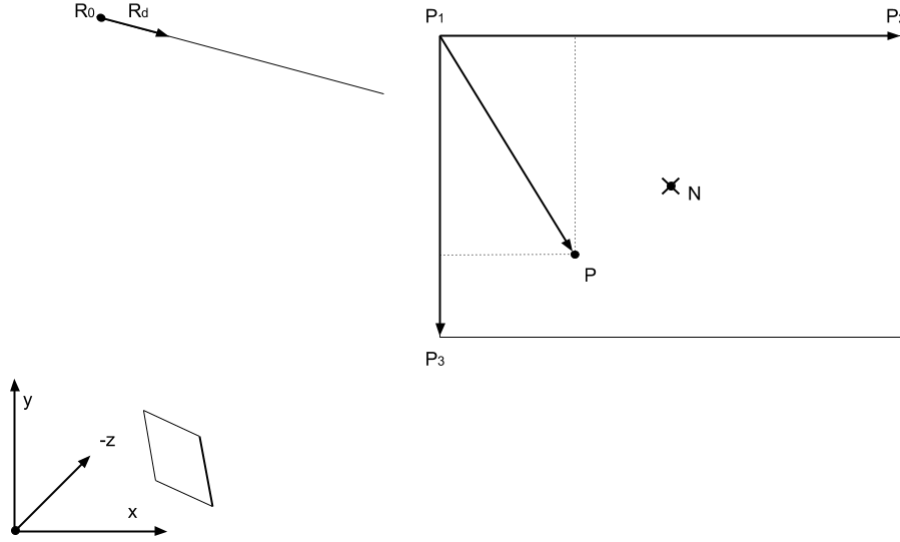
And the collision position for each  $t$  is:

$$\vec{P} = \vec{R_0} + \vec{R_d} \cdot t$$



### 3.9.2 Ray-Plane

FIGURE 3.14: Ray-Plane intersection



If a point  $P$  on the plane and also belongs to the ray, we have quadric equation:

$$\begin{aligned} (\vec{P} - \vec{P}_1) \cdot \vec{N} &= 0 \\ \vec{P} &= \vec{R}_0 + \vec{R}_d \cdot t \end{aligned} \quad (3.5)$$

Solution for the  $t$  is:

$$t = \begin{cases} \frac{-\vec{N} \cdot (\vec{R}_0 - \vec{P}_1)}{\vec{N} \cdot \vec{R}_d} & \text{if } \vec{N} \cdot \vec{R}_d \neq 0 \\ \emptyset & \text{if } \vec{N} \cdot \vec{R}_d \approx 0 \end{cases}$$

At last, we have to verify if the collision is inside of the quadrangle by putting  $t$  back to 3.5, (user3146587, 2014) the  $t$  is valid only if:

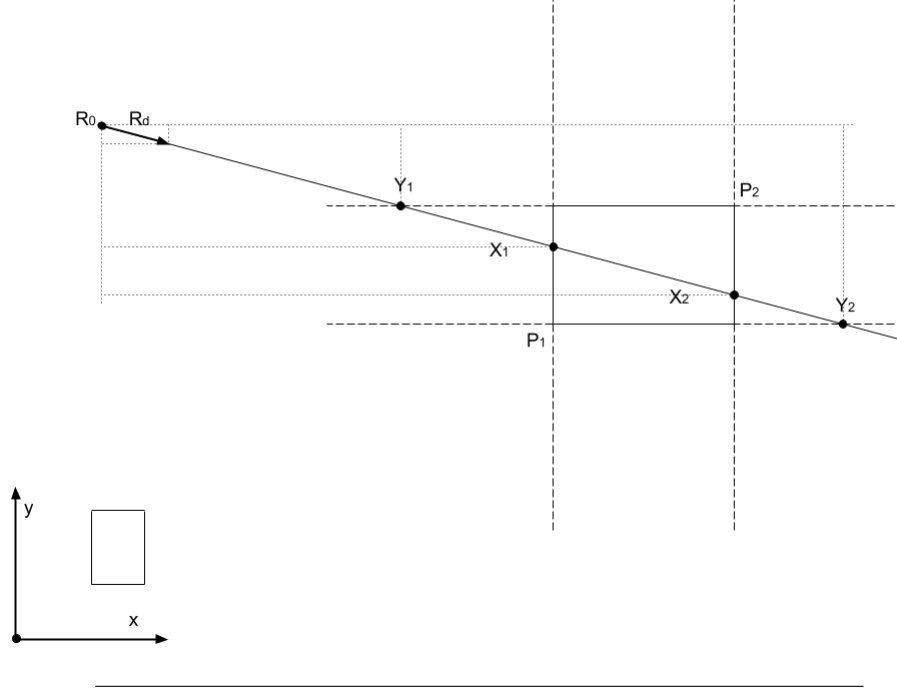
$$\begin{aligned} \mu &= \sqrt{(\vec{P} - \vec{P}_1) \cdot (\vec{P}_2 - \vec{P}_1)} \in [0, |\vec{P}_2 - \vec{P}_1|] \\ \nu &= \sqrt{(\vec{P} - \vec{P}_1) \cdot (\vec{P}_3 - \vec{P}_1)} \in [0, |\vec{P}_3 - \vec{P}_1|] \end{aligned}$$

### 3.9.3 Ray-Box

There is a octree implementation 3.4.2 in the VR 3D world that separate the 3D world to invisible 3D boxes that each box contains a certain number of other models. It is to avoid unnecessary ray-object collision detection. In this section, I am going to first explain Ray-Box-2D collision detection (Barnes, 2011), then derive out Ray-Box-3D intersection.

**Ray-Box-2D**

FIGURE 3.15: Ray-Box-2D intersection



$\therefore$  Known  $R_0, R_d, P_1, P_2$

$$X_1 = \begin{cases} x_{P_1} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_2} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases} \quad Y_1 = \begin{cases} y_{P_1} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_2} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$X_2 = \begin{cases} x_{P_2} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_1} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases} \quad Y_2 = \begin{cases} y_{P_2} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_1} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$t_{X_1} = \frac{X_1}{x_{R_d}} \quad t_{Y_1} = \frac{Y_1}{y_{R_d}}$$

$$t_{X_2} = \frac{X_2}{x_{R_d}} \quad t_{Y_2} = \frac{Y_2}{y_{R_d}}$$

& When collision happens, we have formula:

$$t_{X_1} < t_{X_2}$$

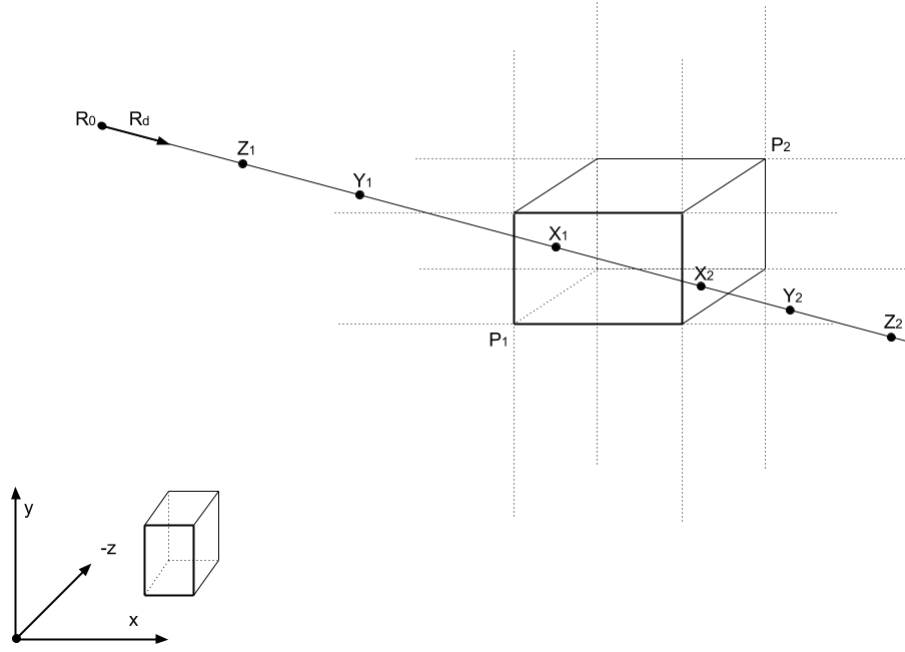
$$t_{Y_1} < t_{Y_2}$$

$\therefore$  Which is

$$\max(t_{X_1}, t_{Y_1}) < \min(t_{X_2}, t_{Y_2}) \quad (3.6)$$

**Ray-Box-3D**

FIGURE 3.16: Ray-Box-3D intersection



$\therefore$  Known  $R_0, R_d, P_1, P_2$

$$X_1 = \begin{cases} x_{P_1} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_2} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases} \quad Y_1 = \begin{cases} y_{P_1} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_2} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$X_2 = \begin{cases} x_{P_2} - x_{R_0} & \text{if } x_{R_d} > 0 \\ x_{P_1} - x_{R_0} & \text{if } x_{R_d} < 0 \end{cases} \quad Y_2 = \begin{cases} y_{P_2} - y_{R_0} & \text{if } y_{R_d} > 0 \\ y_{P_1} - y_{R_0} & \text{if } y_{R_d} < 0 \end{cases}$$

$$t_{X_1} = \frac{X_1}{x_{R_d}} \quad t_{Y_1} = \frac{Y_1}{y_{R_d}}$$

$$t_{X_2} = \frac{X_2}{x_{R_d}} \quad t_{Y_2} = \frac{Y_2}{y_{R_d}}$$

$$Z_1 = \begin{cases} z_{P_1} - z_{R_0} & \text{if } z_{R_d} > 0 \\ z_{P_2} - z_{R_0} & \text{if } z_{R_d} < 0 \end{cases}$$

$$Z_2 = \begin{cases} z_{P_2} - z_{R_0} & \text{if } z_{R_d} > 0 \\ z_{P_1} - z_{R_0} & \text{if } z_{R_d} < 0 \end{cases}$$

$$t_{Z_1} = \frac{Z_1}{z_{R_d}} \quad t_{Z_2} = \frac{Z_2}{z_{R_d}}$$

& When collision happens, we have formula:

$$\begin{cases} t_{X_1} < t_{X_2} \\ t_{Y_1} < t_{Y_2} \\ t_{Z_1} < t_{Z_2} \end{cases}$$

$\therefore$  Which is

$$\max(t_{X_1}, t_{Y_1}, t_{Z_1}) < \min(t_{X_2}, t_{Y_2}, t_{Z_2}) \quad (3.7)$$

## 4 Discussion

\*\*\*\*\*

compare to others. etc. this allows to do similar things, google earth etc...  
this, strength, limitation

## 5 Conclusion

\*\*\*\*\*

outcomes; findings; pass on to ...

2d and 3d env...

vr can .... it explores.....

might apply to other data, not only earth geo d. eg, other natural sys..

# A Appendix A

Write your Appendix content here.

# Bibliography

- Barnes, Tavian (2011). *FAST, BRANCHLESS RAY/BOUNDING BOX INTERSECTIONS*. URL: <https://tavianator.com/fast-branchless-raybounding-box-intersections>.
- Blower, Jonathan David et al. (2007). "Sharing and visualizing environmental data using Virtual Globes". In:
- blox, u (1999). *Datum Transformations of GPS Positions*. URL: [http://www.nalresearch.com/files/Standard%20Modems/A3LA-XG/A3LA-XG%20SW%20Version%201.0.0/GPS%20Technical%20Documents/GPS.G1-X-00006%20\(Datum%20Transformations\).pdf](http://www.nalresearch.com/files/Standard%20Modems/A3LA-XG/A3LA-XG%20SW%20Version%201.0.0/GPS%20Technical%20Documents/GPS.G1-X-00006%20(Datum%20Transformations).pdf).
- Fropuff, Mysid (2006). *Icosahedron Golden Rectangles*. URL: <https://commons.wikimedia.org/wiki/File:Icosahedron-golden-rectangles.svg>.
- Google (2016a). *android-maps-utils*. URL: <https://github.com/googlemaps/android-maps-utils/tree/master/library/src/com/google/maps/android/kml>.
- (2016b). *Google VR SDK for Android*. URL: <https://developers.google.com/vr/android/>.
- (2016c). *Keyhole Markup Language*. URL: <https://developers.google.com/kml/>.
- GoogleTechTalks (2010). *Sensor Fusion on Android Devices: A Revolution in Motion Processing*. URL: <https://www.youtube.com/watch?v=C7JQ7Rpwn2k&feature=youtu.be&t=23m21s>.
- jsoup (2016). *jsoup: Java HTML Parser*. URL: <https://jsoup.org/>.
- mathworks (2016). *Quaternion Rotation*. URL: <http://au.mathworks.com/help/aeroblks/quaternionrotation.html>.
- ogp (2014). *The Open Graph protocol*. URL: <http://ogp.me/>.
- user3146587 (2014). URL: <http://stackoverflow.com/questions/21114796/3d-ray-quad-intersection-test-in-java>.
- Verth, Jim Van (2013). *Understanding Quaternions*. URL: [http://www.essentialmath.com/GDC2013/GDC13\\_quaternions\\_final.pdf](http://www.essentialmath.com/GDC2013/GDC13_quaternions_final.pdf).
- Wikipedia (2016a). *API*. URL: <https://www.mediawiki.org/wiki/API:Opensearch>.
- (2016b). *ECEF*. URL: <https://en.wikipedia.org/wiki/ECEF>.
- (2016c). *Geographic coordinate system*. URL: [https://en.wikipedia.org/wiki/Geographic\\_coordinate\\_system](https://en.wikipedia.org/wiki/Geographic_coordinate_system).