Estimating Parametric Models

Given a description (3-10) and having observed the input-output data *u*, *y*, the (prediction) errors $e(t)$ in (3-10) can be computed as

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)] \tag{3-36}$$

These errors are, for given data *y* and *u*, functions of *G* and *H*. These in turn are parametrized by the polynomials in (3-14)-(3-19) or by entries in the state-space matrices defined in (3-26)-(3-29). The most common parametric identification method is to determine estimates of *G* and *H* by minimizing

$$V_N(G, H) = \sum_{t=1}^{N} e^2(t) \tag{3-37}$$

that is

$$[\hat{G}_N, \hat{H}_N] = argmin \sum_{t=1}^{N} e^2(t) \tag{3-38}$$

This is called a *prediction error method*. For Gaussian disturbances it coincides with the maximum likelihood method. (See Chapter 7 in Ljung (1999).)

A somewhat different philosophy can be applied to the ARX model (3-14). By forming filtered versions of the input

$$N(q)s(t) = M(q)u(t) \tag{3-39}$$

and by multiplying (3-14) with $s(t-k)$, $k = 1$, 2, $\cdots$, *na* and $u(t-nk+1-k)$, $k = 1$, 2, $\cdots$, *nb* and summing over *t*, the noise in (3-14) can be correlated out and solved for the dynamics. This gives the *instrumental variable* method, and $s(t)$ are called the instruments. (See Section 7.6 in Ljung (1999).)