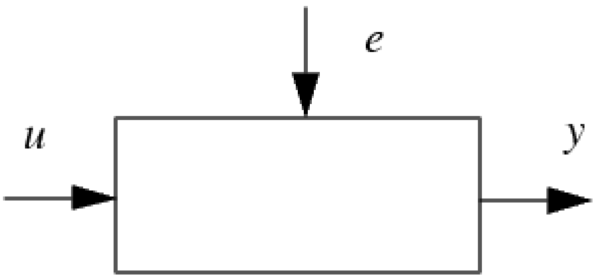
The System Identification Problem

This section discusses different basic ways to describe linear dynamic systems and also the most important methods for estimating such models.

Impulse Responses, Frequency Functions, and Spectra



The basic input-output configuration is depicted in the figure above. Assuming unit sampling interval, there is an input signal

$$u(t); \qquad t = 1, 2, \ldots, N$$

and an output signal

$$y(t); \qquad t = 1, 2, \ldots, N$$

Assuming the signals are related by a linear system, the relationship can be written

$$y(t) = G(q)u(t) + v(t) \tag{3-1}$$

where q is the shift operator and $G(q)u(t)$ is short for

$$G(q)u(t) = \sum_{k=1}^{\infty} g(k)u(t-k) \tag{3-2}$$

and

$$G(q) = \sum_{k=1}^{\infty} g(k)q^{-k}; \qquad q^{-1}u(t) = u(t-1) \tag{3-3}$$

The numbers $\{g(k)\}$ are called the *impulse response* of the system. Clearly, $g(k)$ is the output of the system at time k if the input is a single (im)pulse at time zero. The function $G(q)$ is called the *transfer function* of the system. This function evaluated on the unit circle $(q = e^{i\omega})$ gives the *frequency function*

$$G(e^{i\omega}) \tag{3-4}$$

In (3-1) $v(t)$ is an additional, unmeasurable disturbance (noise). Its properties can be expressed in terms of its (power) spectrum

$$\Phi_v(\omega) \tag{3-5}$$

which is defined by

$$\Phi_v(\omega) = \sum_{\tau=-\infty}^{\infty} R_v(\tau)e^{-i\omega\tau} \tag{3-6}$$

where $R_v(\tau)$ is the covariance function of $v(t)$

$$R_v(\tau) = Ev(t)v(t-\tau) \tag{3-7}$$

and *E* denotes mathematical expectation. Alternatively, the disturbance $v(t)$ can be described as filtered white noise

$$v(t) = H(q)e(t) \tag{3-8}$$

where $e(t)$ is white noise with variance λ and

$$\Phi_v(\omega) = \lambda\left|H(e^{i\omega})\right|^2 \tag{3-9}$$

Equations (3-1) and (3-8) together give a *time domain description* of the system

$$y(t) = G(q)u(t) + H(q)e(t) \tag{3-10}$$

where *G* is the *transfer function* of the system. Equations (3-4) and (3-5) constitute a *frequency domain description*.

$$G(e^{i\omega}); \qquad \Phi_v(\omega) \tag{3-11}$$

The impulse response (3-3) and the frequency domain description (3-11) are called *nonparametric model descriptions* since they are not defined in terms of a finite number of parameters. The basic description (3-10) also applies to the multivariable case; i.e., to systems with several (say *nu*) input signals and several (say *ny*) output signals. In that case $G(q)$ is an *ny*-by-*nu* matrix while $H(q)$ and $\Phi_v(\omega)$ are *ny*-by-*ny* matrices.

An Introductory Example to Command Mode

Polynomial Representation of Transfer Functions