Ridge Regression

Here we will try to fit the dataset with a Ridge Regression model. The steps are

- Determine a class for the model supporting methods
  - fit
  - predict
  - score
- Search for hyperparameters through trial and error
  - evaluate the average training and validating error for each hyperparameter
- Plot the distributions of weight on the features
  - Does Ridge Regression give us sparsity
- Threshold the values to compare zero/non-zero against the weights of the target function

In [1]:

```
import numpy as np
import pandas as pd
import itertools
import matplotlib.pyplot as plt
%matplotlib inline

from scipy.optimize import minimize

from sklearn.base import BaseEstimator, RegressorMixin
from sklearn.linear_model import Ridge
from sklearn.model_selection import GridSearchCV, PredefinedSplit
from sklearn.model_selection import ParameterGrid
from sklearn.metrics import mean_squared_error, make_scorer
from sklearn.metrics import confusion_matrix

from load_data import load_problem

PICKLE_PATH = 'lasso_data.pickle'
```

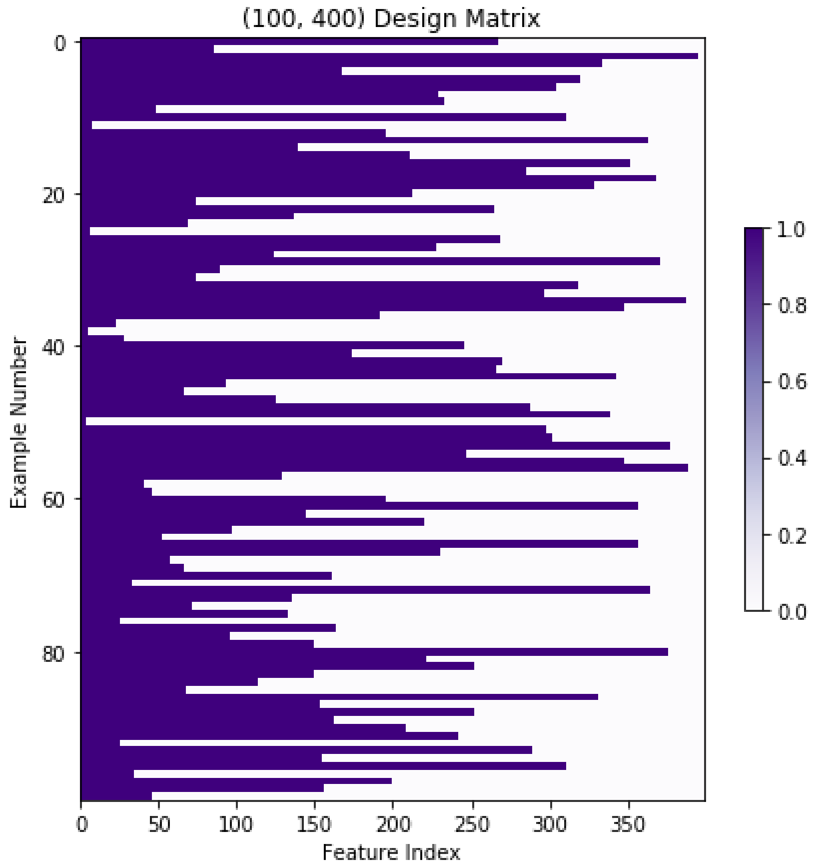
Dataset

In [2]:

```
#load data

x_train, y_train, x_val, y_val, target_fn, coefs_true, featurize = load_problem(
PICKLE_PATH)
X_train = featurize(x_train)
X_val = featurize(x_val)
```

In [6]:

```
#Visualize training data

fig, ax = plt.subplots(figsize = (7,7))
ax.set_title("({0}, {1}) Design Matrix".format(X_train.shape[0], X_train.shape[1
]))
ax.set_xlabel("Feature Index")
ax.set_ylabel("Example Number")
temp = ax.imshow(X_train, cmap=plt.cm.Purples, aspect="auto")
plt.colorbar(temp, shrink=0.5);
```



Class for Ridge Regression

In [12]:

```
class RidgeRegression(BaseEstimator, RegressorMixin):
    """ ridge regression"""

    def __init__(self, l2reg=1):
        if l2reg < 0:
            raise ValueError('Regularization penalty should be at least 0.')
        self.l2reg = l2reg

    def fit(self, X, y=None):
        n, num_ftrs = X.shape
        # convert y to 1-dim array, in case we're given a column vector
        y = y.reshape(-1)
        def ridge_obj(w):
            predictions = np.dot(X,w)
            residual = y - predictions
            empirical_risk = np.sum(residual**2) / n
            l2_norm_squared = np.sum(w**2)
            objective = empirical_risk + self.l2reg * l2_norm_squared
            return objective
        self.ridge_obj_ = ridge_obj

        w_0 = np.zeros(num_ftrs)
        self.w_ = minimize(ridge_obj, w_0).x
        return self

    def predict(self, X, y=None):
        try:
            getattr(self, "w_")
        except AttributeError:
            raise RuntimeError("You must train classifer before predicting data!")
        return np.dot(X, self.w_)

    def score(self, X, y):
        # Average square error
        try:
            getattr(self, "w_")
        except AttributeError:
            raise RuntimeError("You must train classifer before predicting data!")
        residuals = self.predict(X) - y
        return np.dot(residuals, residuals)/len(y)
```

We can compare to the `sklearn` implementation.

In [13]:

```
def compare_our_ridge_with_sklearn(X_train, y_train, l2_reg=1):

        # Fit with sklearn -- need to multiply l2_reg by sample size, since thei
r
        # objective function has the total square loss, rather than average squa
re
        # loss.
        n = X_train.shape[0]
        sklearn_ridge = Ridge(alpha=n*l2_reg, fit_intercept=False, normalize=Fal
se)
        sklearn_ridge.fit(X_train, y_train)
        sklearn_ridge_coefs = sklearn_ridge.coef_

        # Now run our ridge regression and compare the coefficients to sklearn's
        ridge_regression_estimator = RidgeRegression(l2reg=l2_reg)
        ridge_regression_estimator.fit(X_train, y_train)
        our_coefs = ridge_regression_estimator.w_

        print("Hoping this is very close to 0:{}".format(np.sum((our_coefs - skl
earn_ridge_coefs)**2)))
```

In [14]:

```
compare_our_ridge_with_sklearn(X_train, y_train, l2_reg=1.5)
```

```
Hoping this is very close to 0:4.6933160195738277e-11
```

Grid Search to Tune Hyperparameter

Now let's use sklearn to help us do hyperparameter tuning GridSearchCv.fit by default splits the data into training and validation itself; we want to use our own splits, so we need to stack our training and validation sets together, and supply an index (validation_fold) to specify which entries are train and which are validation.

In [15]:

```
default_params = np.unique(np.concatenate((10.**np.arange(-6,1,1), np.arange(1,3
,.3))))

def do_grid_search_ridge(X_train, y_train, X_val, y_val, params = default_params
):

        X_train_val = np.vstack((X_train, X_val))
        y_train_val = np.concatenate((y_train, y_val))
        val_fold = [-1]*len(X_train) + [0]*len(X_val) #0 corresponds to validati
on

        param_grid = [{'l2reg':params}]

        ridge_regression_estimator = RidgeRegression()
        grid = GridSearchCV(ridge_regression_estimator,
                                                param_grid,
                                                return_train_score=True,
                                                cv = PredefinedSplit(test_fold=v
al_fold),
                                                refit = True,
                                                scoring = make_scorer(mean_squar
ed_error,

greater_is_better = False))
        grid.fit(X_train_val, y_train_val)

        df = pd.DataFrame(grid.cv_results_)
        # Flip sign of score back, because GridSearchCV likes to maximize,
        # so it flips the sign of the score if "greater_is_better=FALSE"
        df['mean_test_score'] = -df['mean_test_score']
        df['mean_train_score'] = -df['mean_train_score']
        cols_to_keep = ["param_l2reg", "mean_test_score","mean_train_score"]
        df_toshow = df[cols_to_keep].fillna('-')
        df_toshow = df_toshow.sort_values(by=["param_l2reg"])
        return grid, df_toshow
```

In [16]:

```
grid, results = do_grid_search_ridge(X_train, y_train, X_val, y_val)
```

In [17]:

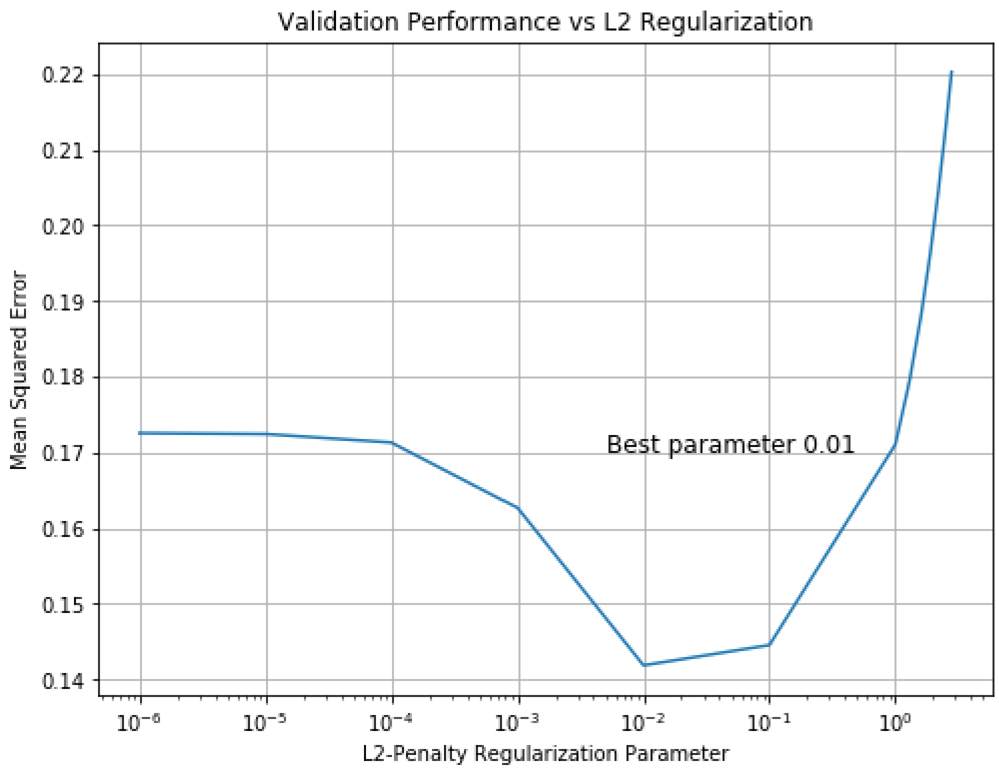
```
results
```

Out[17]:

	param_l2reg	mean_test_score	mean_train_score
0	0.000001	0.172579	0.006752
1	0.000010	0.172464	0.006752
2	0.000100	0.171345	0.006774
3	0.001000	0.162705	0.008285
4	0.010000	0.141887	0.032767
5	0.100000	0.144566	0.094953
6	1.000000	0.171068	0.197694
7	1.300000	0.179521	0.216591
8	1.600000	0.187993	0.233450
9	1.900000	0.196361	0.248803
10	2.200000	0.204553	0.262958
11	2.500000	0.212530	0.276116
12	2.800000	0.220271	0.288422

In [18]:

```
# Plot validation performance vs regularization parameter
fig, ax = plt.subplots(figsize = (8,6))
ax.grid()
ax.set_title("Validation Performance vs L2 Regularization")
ax.set_xlabel("L2-Penalty Regularization Parameter")
ax.set_ylabel("Mean Squared Error")
ax.semilogx(results["param_l2reg"], results["mean_test_score"])
ax.text(0.005,0.17,"Best parameter {0}".format(grid.best_params_['l2reg']), font
size = 12);
```



Comparing to the Target Function

Let's plot prediction functions and compare coefficients for several fits and the target function.

Let's create a list of dicts called `pred_fns` . Each dict has a "name" key and a "preds" key. The value corresponding to the "preds" key is an array of predictions corresponding to the input vector x. x_train and y_train are the input and output values for the training data

In [19]:

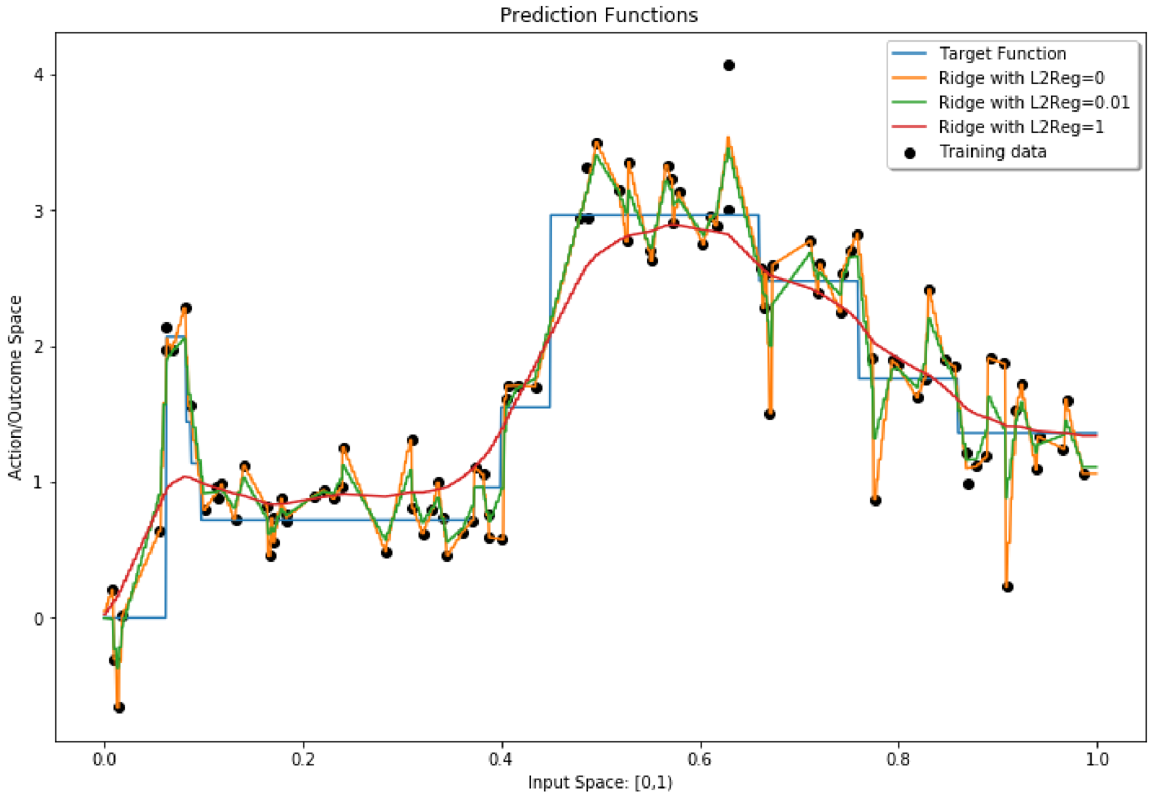
```
pred_fns = []
x = np.sort(np.concatenate([np.arange(0,1,.001), x_train]))

pred_fns.append({"name": "Target Function", "coefs": coefs_true, "preds": target
_fn(x)})

l2regs = [0, grid.best_params_['l2reg'], 1]
X = featurize(x)
for l2reg in l2regs:
    ridge_regression_estimator = RidgeRegression(l2reg=l2reg)
    ridge_regression_estimator.fit(X_train, y_train)
    name = "Ridge with L2Reg="+str(l2reg)
    pred_fns.append({"name":name,
                     "coefs":ridge_regression_estimator.w_,
                     "preds": ridge_regression_estimator.predict(X) })
```

In [20]:

```
def plot_prediction_functions(x, pred_fns, x_train, y_train, legend_loc="best"):

        fig, ax = plt.subplots(figsize = (12,8))
        ax.set_xlabel('Input Space: [0,1)')
        ax.set_ylabel('Action/Outcome Space')
        ax.set_title("Prediction Functions")
        plt.scatter(x_train, y_train, color="k", label='Training data')
        for i in range(len(pred_fns)):
                ax.plot(x, pred_fns[i]["preds"], label=pred_fns[i]["name"])
        legend = ax.legend(loc=legend_loc, shadow=True)
        return fig
```

In [21]:

```
plot_prediction_functions(x, pred_fns, x_train, y_train, legend_loc="best");
```



Visualizing the Weights

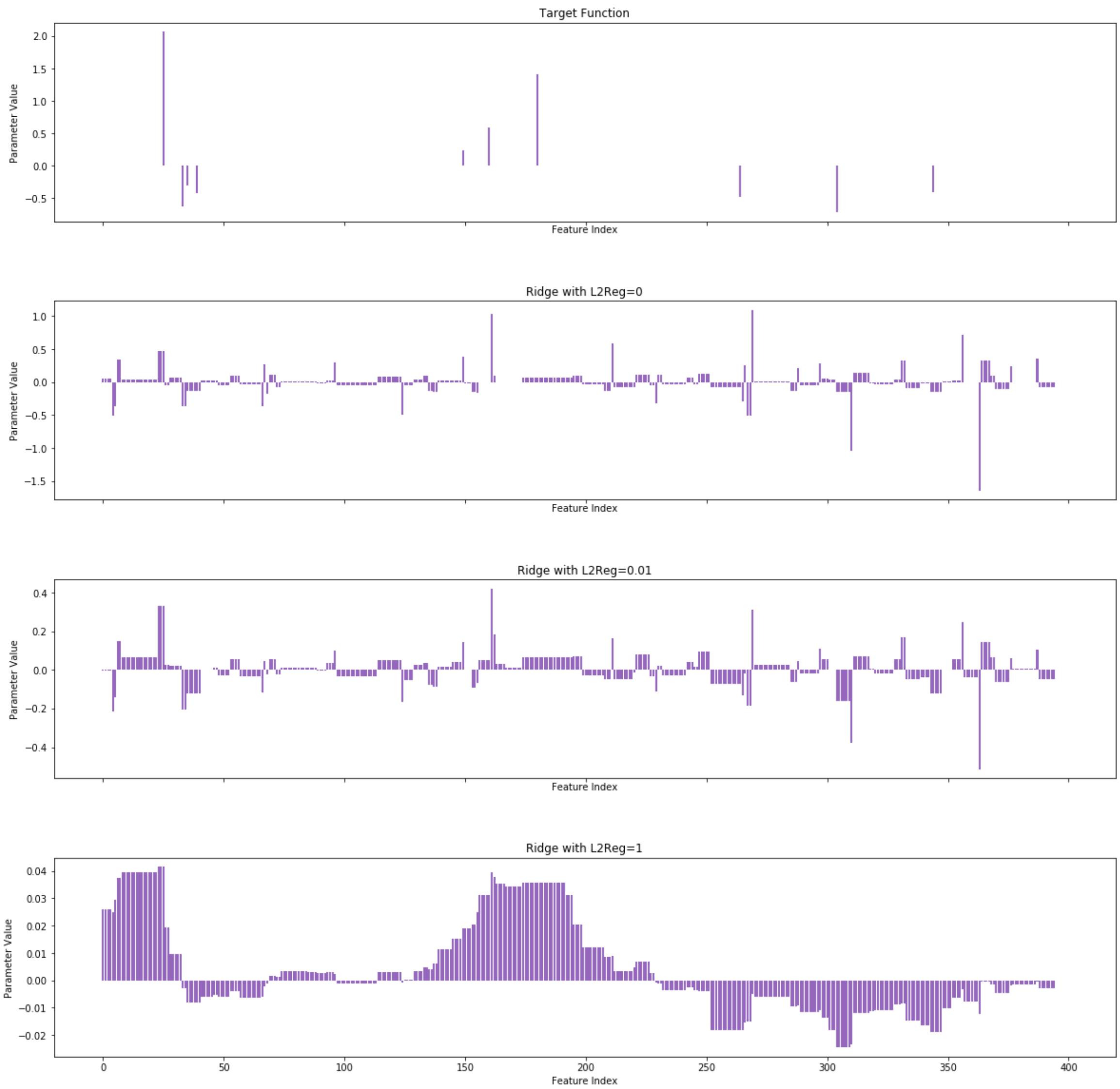
Using `pred_fns` let's try to see how sparse the weights are...

In [22]:

```
def compare_parameter_vectors(pred_fns):

        fig, axs = plt.subplots(len(pred_fns),1, sharex=True, figsize = (20,20))
        num_ftrs = len(pred_fns[0]["coefs"])
        for i in range(len(pred_fns)):
                title = pred_fns[i]["name"]
                coef_vals = pred_fns[i]["coefs"]
                axs[i].bar(range(num_ftrs), coef_vals, color = "tab:purple")
                axs[i].set_xlabel('Feature Index')
                axs[i].set_ylabel('Parameter Value')
                axs[i].set_title(title)

        fig.subplots_adjust(hspace=0.4)
        return fig
```

In [23]:

```
compare_parameter_vectors(pred_fns);
```



Confusion Matrix

We can try to predict the features with corresponding weight zero. We will fix a threshold `eps` such that any value between `-eps` and `eps` will get counted as zero. We take the remaining features to have positive value. These predictions of can be compared to the weights for the target function.

In [24]:

```
def plot_confusion_matrix(cm, title, classes):
        plt.imshow(cm, interpolation='nearest', cmap=plt.cm.Purples)
        plt.title(title)
        plt.colorbar()
        tick_marks = np.arange(len(classes))
        plt.xticks(tick_marks, classes, rotation=45)
        plt.yticks(tick_marks, classes)

        thresh = cm.max() / 2.
        for i, j in itertools.product(range(cm.shape[0]), range(cm.shape[1])):
                plt.text(j, i, format(cm[i, j], 'd'),
                                 horizontalalignment="center",
                                 color="white" if cm[i, j] > thresh else "blac
k")

        plt.tight_layout()
        plt.ylabel('True label')
        plt.xlabel('Predicted label')
```

Q1

In [62]:

```
lambdas = [1e-5,1e-4,1e-3,1e-2,1e-1,1,10]
losses = []
from tqdm import tqdm

for lambd in lambdas:
    model = RidgeRegression(lambd)
    model.fit(X_train,y_train)
    losses.append(model.score(X_val,y_val))
best_param = lambdas[np.argmin(np.array(losses))]
```

In [63]:

```
result = pd.DataFrame({"lambda":lambdas,"avg loss on val set":losses})
result
```

Out[63]:

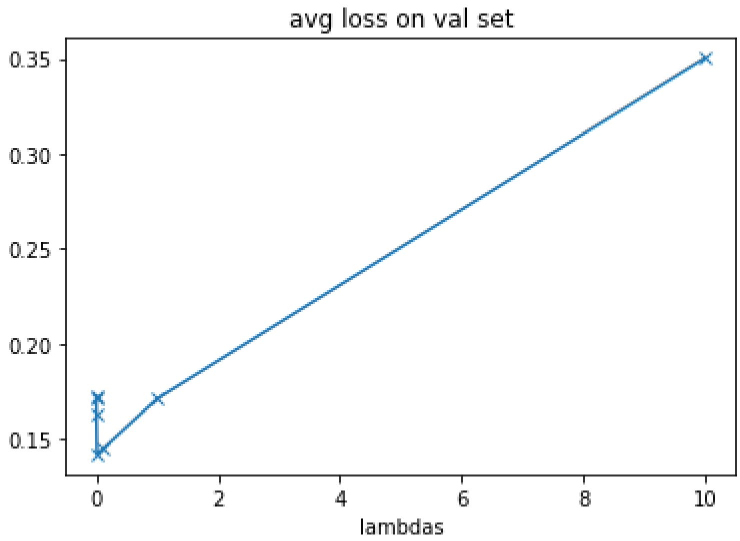
	lambda	avg loss on val set
0	0.00001	0.172464
1	0.00010	0.171345
2	0.00100	0.162705
3	0.01000	0.141887
4	0.10000	0.144566
5	1.00000	0.171068
6	10.00000	0.350321

In [64]:

```
plt.plot(lambdas,losses,"x-")
plt.xlabel("lambdas")
plt.title("avg loss on val set")
```

Out[64]:

```
Text(0.5, 1.0, 'avg loss on val set')
```



In [65]:

```
print("Best parameter is: ", best_param)
```

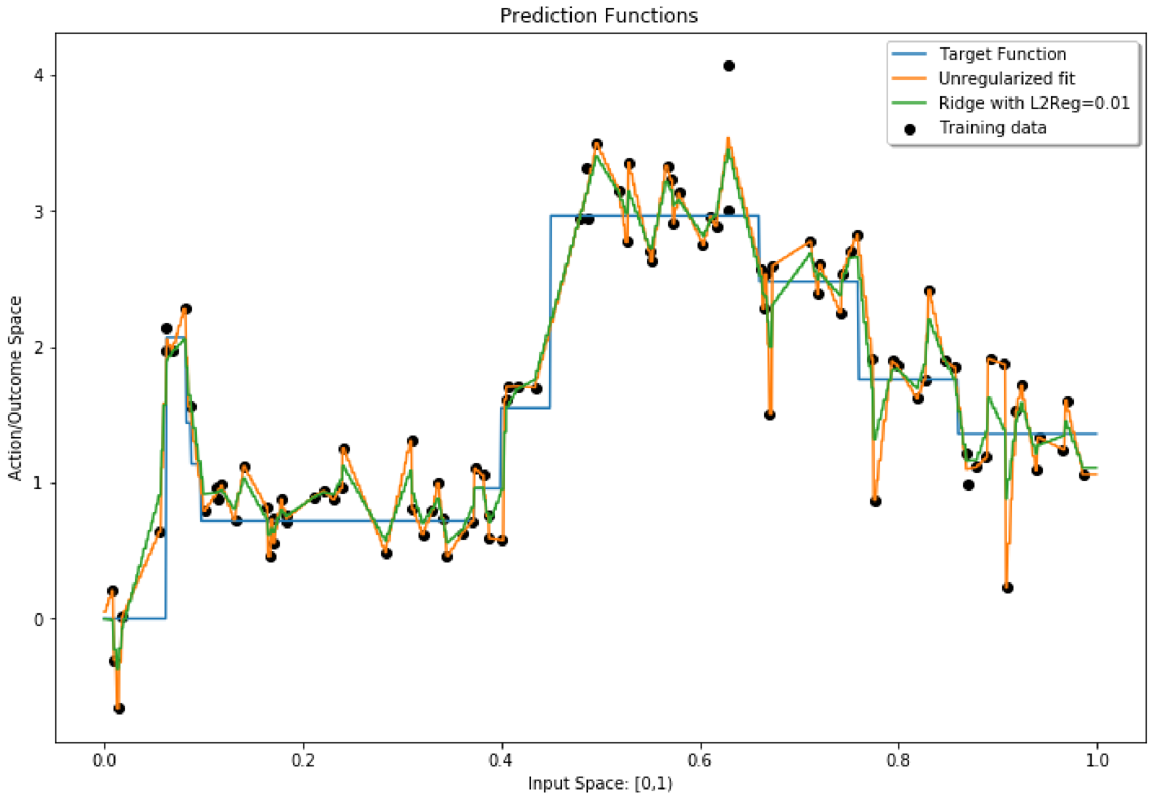
```
Best parameter is:  0.01
```

Q2

Plot the training data, the target function, an unregularized least squares fit (still using the featurizeddata), and the prediction function chosen in the previous proble

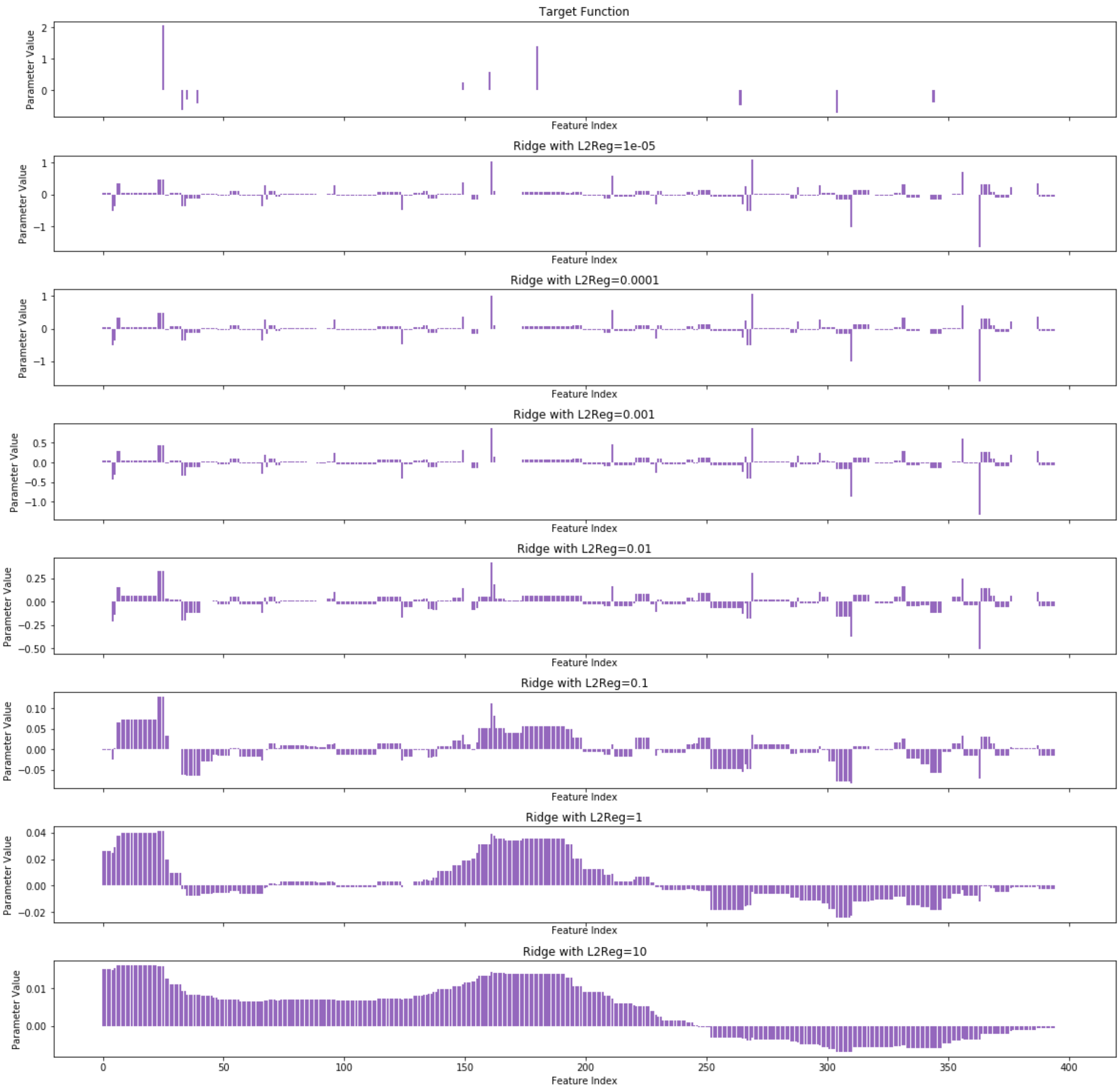
In [66]:

```
pred_fns = []
x = np.sort(np.concatenate([np.arange(0,1,.001), x_train]))

pred_fns.append({"name": "Target Function", "coefs": coefs_true, "preds": target
_fn(x)})

l2regs = [0, grid.best_params_['l2reg'], 1]
X = featurize(x)
lambdas = [0, best_param]
for l2reg in lambdas:
    ridge_regression_estimator = RidgeRegression(l2reg=l2reg)
    ridge_regression_estimator.fit(X_train, y_train)
    if l2reg != 0:
        name = "Ridge with L2Reg="+str(l2reg)
    else:
        name = "Unregularized fit"
    pred_fns.append({"name":name,
                     "coefs":ridge_regression_estimator.w_,
                     "preds": ridge_regression_estimator.predict(X) })
plot_prediction_functions(x, pred_fns, x_train, y_train, legend_loc="best");
```



In [70]:

```
pred_fns = []
pred_fns.append({"name": "Target Function", "coefs": coefs_true, "preds": target
_fn(x)})
lambdas = [1e-5,1e-4,1e-3,1e-2,1e-1,1,10]
for l2reg in lambdas:
    ridge_regression_estimator = RidgeRegression(l2reg=l2reg)
    ridge_regression_estimator.fit(X_train, y_train)
    if l2reg != 0:
        name = "Ridge with L2Reg="+str(l2reg)
    else:
        name = "Unregularized fit"
    pred_fns.append({"name":name,
                     "coefs":ridge_regression_estimator.w_,
                     "preds": ridge_regression_estimator.predict(X) })
compare_parameter_vectors(pred_fns);
```

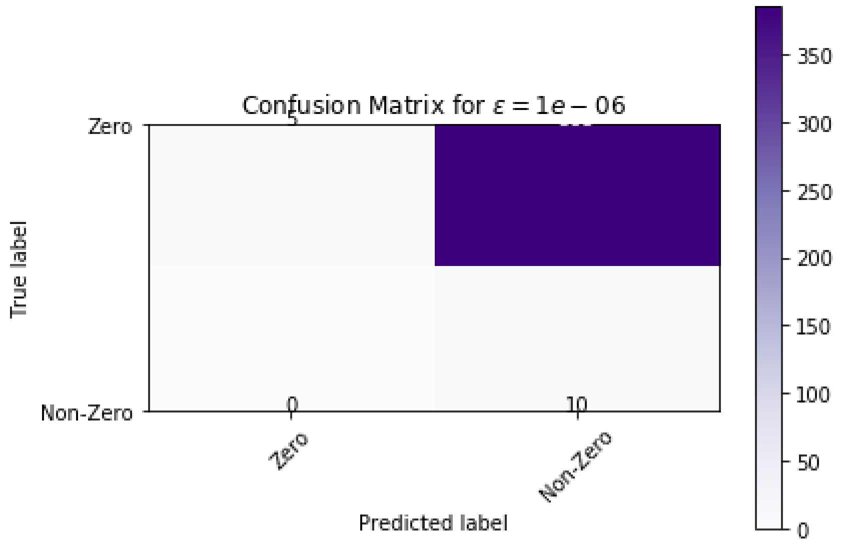


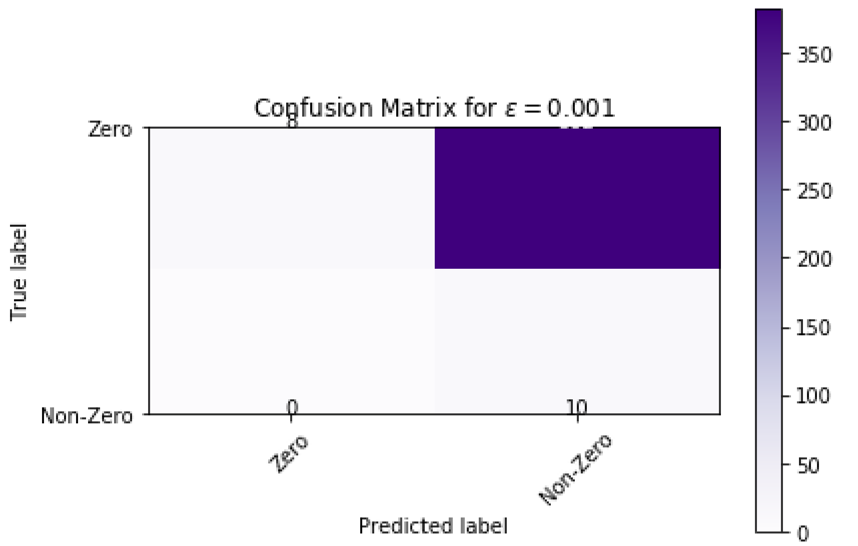
As we can see, as the regularization gets larger and larger, the coefficents will become smaller and smaller in absolute value. This confirms with how regularization works. Also, as regularization becomes smaller and closer to 0, the coefficients approach to those of the target function more and more.

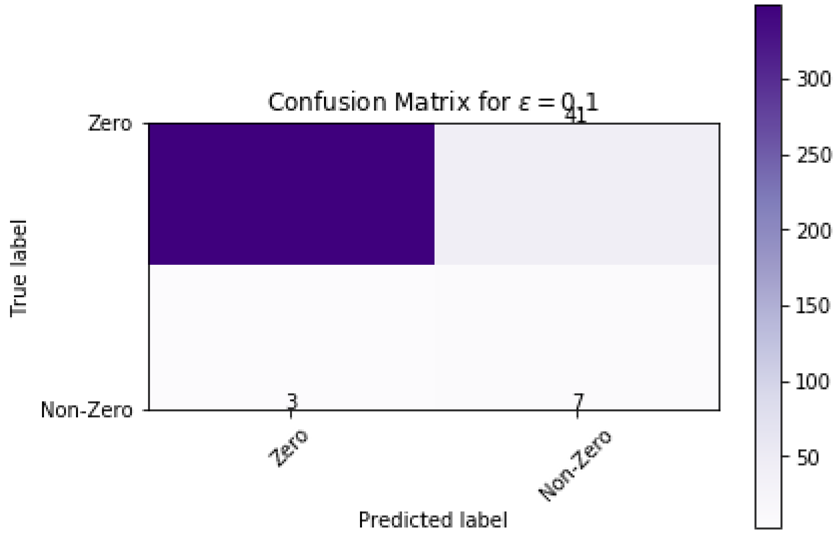
Q3

In [84]:

```
bin_coefs_true = coefs_true != 0  # your code goes here
eps_list = [1e-6,1e-3,1e-1]
ridge_regression_estimator = RidgeRegression(l2reg=best_param)
ridge_regression_estimator.fit(X_train, y_train)
w_tilde = ridge_regression_estimator.w_

for eps in eps_list:
    bin_coefs_estimated = np.abs(w_tilde) > eps  # your code goes here
    cnf_matrix = confusion_matrix(bin_coefs_true, bin_coefs_estimated)
    plt.figure()
    plot_confusion_matrix(cnf_matrix, title="Confusion Matrix for $\epsilon = {}
$".format(eps), classes=["Zero", "Non-Zero"])
```


Confusion Matrix for $\varepsilon = 1e-06$
True label
Zero
Non-Zero
5
0
10
Zero
Non-Zero
Predicted label
350
300
250
200
150
100
50
0


Confusion Matrix for $\varepsilon = 0.001$
True label
Zero
Non-Zero
8
0
10
Zero
Non-Zero
Predicted label
350
300
250
200
150
100
50
0



Confusion Matrix for $\varepsilon = 0.1$
41
3
7
Zero
Non-Zero
True label
Predicted label
300
250
200
150
100
50


```
In [ ]:
```

Lasso Regression

We will try to fit the dataset with a Lasso Regression model. The steps are

- Implement the Shooting Algorithm
  - allow for random or non-random order for the coordinates
  - allow for initial weights all zero or the corresponding solution to Ridge Regression
- Determine a class for the model supporting methods
  - fit
  - predict
  - score
- Tune hyperparameters
  - Search for hyperparameters through trial and error
  - Use upper bound on hyperparameter with warm start
- Plot the distributions of weight on the features
  - Does Lasso Regression give us sparsity
- Threshold the values to compare zero/non-zero against the weights of the target function
- Implement Projected Gradient Descent
  - Compare to Shooting Algorithm

In [1]:

```
import numpy as np
np.random.seed(42)
import pandas as pd
import itertools
import matplotlib.pyplot as plt
%matplotlib inline

from scipy.optimize import minimize

from sklearn.base import BaseEstimator, RegressorMixin
from sklearn.linear_model import Lasso
from sklearn.model_selection import GridSearchCV, PredefinedSplit
from sklearn.model_selection import ParameterGrid
from sklearn.metrics import mean_squared_error, make_scorer
from sklearn.metrics import confusion_matrix

from load_data import load_problem

PICKLE_PATH = 'lasso_data.pickle'
```

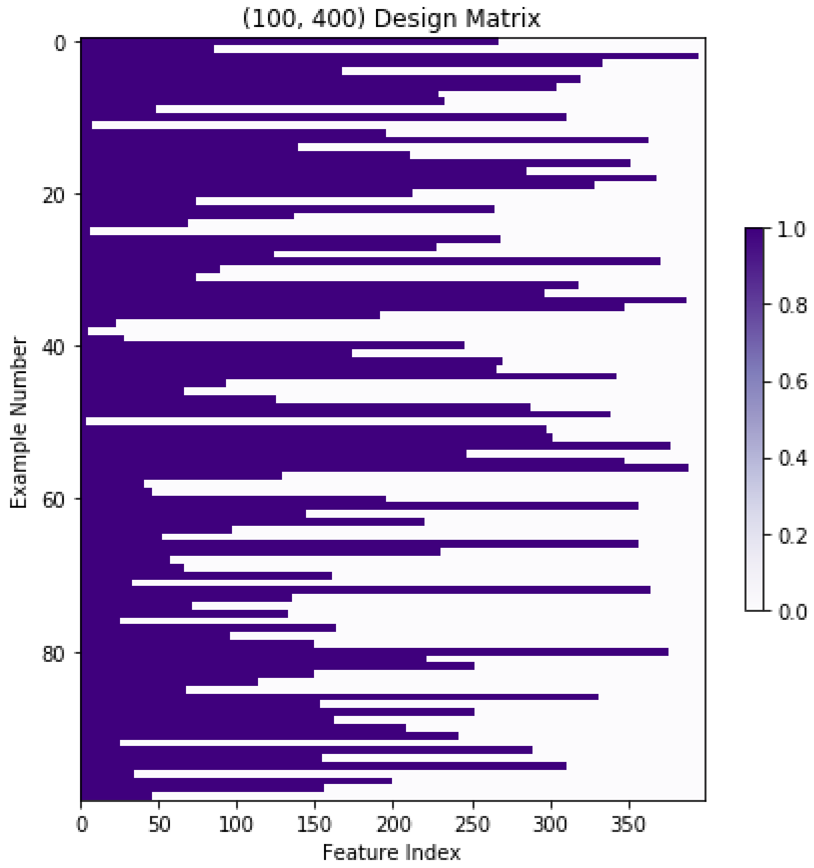
Dataset

In [2]:

```
#load data

x_train, y_train, x_val, y_val, target_fn, coefs_true, featurize = load_problem(
PICKLE_PATH)
X_train = featurize(x_train)
X_val = featurize(x_val)
```

In [3]:

```
#Visualize training data

fig, ax = plt.subplots(figsize = (7,7))
ax.set_title("({0}, {1}) Design Matrix".format(X_train.shape[0], X_train.shape[1
]))
ax.set_xlabel("Feature Index")
ax.set_ylabel("Example Number")
temp = ax.imshow(X_train, cmap=plt.cm.Purples, aspect="auto")
plt.colorbar(temp, shrink=0.5);
```



In [8]:

```
def soft_threshold(a, delta):
    if a < - delta:
        return (a + delta)
    elif a > delta:
        return (a - delta)
    else:
        return 0

def compute_sum_sqr_loss(X, y, w):
    return np.dot((np.dot(X,w)-y).T,np.dot(X,w)-y)

def compute_lasso_objective(X, y, w, l1_reg=0):
    return np.dot((np.dot(X,w)-y).T,np.dot(X,w)-y) + l1_reg*np.sum(np.abs(w))

def get_ridge_solution(X, y, l2_reg):
    return np.dot(np.linalg.inv(np.dot(X.T,X)+l2_reg*np.eye(X.shape[1])),np.dot(
X.T,y))
```

Remember that we should avoid loops in the implementation because we need to run the algorithm repeatedly for hyperparameter tuning.

Please see Lecture 4 notes for derivation of shooting algorithm.

Q1

$$a_j = 2X_{.j}^T X_{.j}$$
$$c_j = 2X_{.j}^T (y - Xw + w_j X_{.j})$$

Q2

In [3]:

```
def shooting_algorithm(X, y, w0=None, l1_reg = 1., max_num_epochs = 1000, min_ob
j_decrease=1e-8, random=False):
    if w0 is None:
        w = np.zeros(X.shape[1])
    else:
        w = np.copy(w0)
    d = X.shape[1]
    epoch = 0
    obj_val = compute_lasso_objective(X, y, w, l1_reg)
    obj_decrease = min_obj_decrease + 1.
    while (obj_decrease>min_obj_decrease) and (epoch<max_num_epochs):
        obj_old = obj_val
        # Cyclic coordinates descent
        coordinates = range(d)
        # Randomized coordinates descent
        if random:
            coordinates = np.random.permutation(d)
        for j in coordinates:
            aj = 2*np.dot(X[:,j].T,X[:,j])
            cj = 2*np.dot(X[:,j].T,y-np.dot(X,w)+w[j]*X[:,j])
            w[j] = soft_threshold(cj/aj,l1_reg/aj)
        obj_val = compute_lasso_objective(X, y, w, l1_reg)
        obj_decrease = obj_old - obj_val
        epoch += 1
    print("Ran for "+str(epoch)+" epochs. " + 'Lowest loss: ' + str(obj_val))
    return w, obj_val, epoch
```

In [6]:

```
randoms = [True,False]
regs = [1e-4,1e-2,1e-1,1,10,100]
ridge_solution = get_ridge_solution(X_train,y_train,l2_reg=0.01)
random_list = []
epoch_list = []
reg_list = []
loss_list = []
val_list = []
for random in randoms:
    for reg in regs:
        w, obj_val, epoch = shooting_algorithm(X_train, y_train, w0=None, l1_reg
=reg, max_num_epochs=1000, min_obj_decrease=1e-8, random=random)
        val_loss = compute_sum_sqr_loss(X_val,y_val,w)
        random_list.append(random)
        epoch_list.append(epoch)
        loss_list.append(obj_val)
        val_list.append(val_loss)
        reg_list.append(reg)
df = pd.DataFrame({"random":random_list,"l1_reg":reg_list,"epoch":epoch_list,"ob
j function":loss_list,"val loss":val_list})
df
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 1000 epochs. Lowest loss: 0.6798308598917894
Ran for 1000 epochs. Lowest loss: 1.085852696221787
Ran for 1000 epochs. Lowest loss: 3.97932878525749
Ran for 649 epochs. Lowest loss: 16.19773809673319
Ran for 568 epochs. Lowest loss: 56.81897672643175
Ran for 265 epochs. Lowest loss: 221.86396567518784
Ran for 1000 epochs. Lowest loss: 0.6789177209948681
Ran for 1000 epochs. Lowest loss: 1.042250022635014
Ran for 1000 epochs. Lowest loss: 3.9049775016337263
Ran for 824 epochs. Lowest loss: 16.19773797187303
Ran for 742 epochs. Lowest loss: 56.81897663333593
Ran for 732 epochs. Lowest loss: 221.86396593376537
```

Out[6]:

	random	l1_reg	epoch	obj function	val loss
0	True	0.0001	1000	0.679831	161.199238
1	True	0.0100	1000	1.085853	169.743510
2	True	0.1000	1000	3.979329	152.934274
3	True	1.0000	649	16.197738	137.242037
4	True	10.0000	568	56.818977	179.330099
5	True	100.0000	265	221.863966	757.209712
6	False	0.0001	1000	0.678918	205.219387
7	False	0.0100	1000	1.042250	202.856209
8	False	0.1000	1000	3.904978	185.525278
9	False	1.0000	824	16.197738	151.037437
10	False	10.0000	742	56.818977	186.912998
11	False	100.0000	732	221.863966	768.793780

As we can see from the table above, if we initialize w to be 0's, randomization with lambda=1 gives the best result

In [7]:

```
randoms = [True,False]
regs = [1e-4,1e-2,1e-1,1,10,100]
ridge_solution = get_ridge_solution(X_train,y_train,l2_reg=0.01)
random_list = []
epoch_list = []
reg_list = []
loss_list = []
val_list = []
for random in randoms:
    for reg in regs:
        w, obj_val, epoch = shooting_algorithm(X_train, y_train, w0=ridge_soluti
on, l1_reg=reg, max_num_epochs=1000, min_obj_decrease=1e-8, random=random)
        val_loss = compute_sum_sqr_loss(X_val,y_val,w)
        random_list.append(random)
        epoch_list.append(epoch)
        loss_list.append(obj_val)
        val_list.append(val_loss)
        reg_list.append(reg)
df = pd.DataFrame({"random":random_list,"l1_reg":reg_list,"epoch":epoch_list,"ob
j function":loss_list,"val loss":val_list})
df
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 579 epochs. Lowest loss: 0.6789110289549598
Ran for 534 epochs. Lowest loss: 1.0422438166565553
Ran for 1000 epochs. Lowest loss: 3.9049764100901316
Ran for 629 epochs. Lowest loss: 16.197738065777575
Ran for 682 epochs. Lowest loss: 56.81897673111054
Ran for 252 epochs. Lowest loss: 221.86396619400932
Ran for 486 epochs. Lowest loss: 0.6789105182730852
Ran for 571 epochs. Lowest loss: 1.0422432500360437
Ran for 880 epochs. Lowest loss: 3.9049739068639173
Ran for 720 epochs. Lowest loss: 16.197737980794297
Ran for 878 epochs. Lowest loss: 56.818976564156834
Ran for 836 epochs. Lowest loss: 221.8639658706465
```

Out[7]:

	random	l1_reg	epoch	obj function	val loss
0	True	0.0001	579	0.678911	155.311948
1	True	0.0100	534	1.042244	153.901668
2	True	0.1000	1000	3.904976	143.002751
3	True	1.0000	629	16.197738	112.229370
4	True	10.0000	682	56.818977	174.465633
5	True	100.0000	252	221.863966	761.061837
6	False	0.0001	486	0.678911	155.264592
7	False	0.0100	571	1.042243	154.268559
8	False	0.1000	880	3.904974	146.794280
9	False	1.0000	720	16.197738	111.072406
10	False	10.0000	878	56.818977	186.973830
11	False	100.0000	836	221.863966	766.225167

As we can see from the table above, if we initialize w to be the ridge solution, no randomization with lambda=1 gives the best result on the test set, and all losses seem to be lower compared to same setup but with $w_0 = \vec{0}$.

Q3

Class for Lasso Regression

In [8]:

```
class LassoRegression(BaseEstimator, RegressorMixin):
    """ Lasso regression"""
    def __init__(self, l1_reg=1.0, randomized=False):
        if l1_reg < 0:
            raise ValueError('Regularization penalty should be at least 0.')
        self.l1_reg = l1_reg
        self.randomized = randomized


    def fit(self, X, y, max_epochs = 1000, coef_init=None):
        # convert y to 1-dim array, in case we're given a column vector
        y = y.reshape(-1)
        if coef_init is None:
            coef_init = get_ridge_solution(X,y, self.l1_reg)

        self.w_, obj_val, epoch = shooting_algorithm(X, y, w0=coef_init, l1_reg=
self.l1_reg, max_num_epochs=max_epochs, min_obj_decrease=1e-8, random=self.rando
mized)
        return self

    def predict(self, X, y=None):
        try:
            getattr(self, "w_")
        except AttributeError:
            raise RuntimeError("You must train classifer before predicting dat
a!")

        return np.dot(X, self.w_)

    def score(self, X, y):
        try:
            getattr(self, "w_")
        except AttributeError:
            raise RuntimeError("You must train classifer before predicting dat
a!")

        return compute_sum_sqr_loss(X, y, self.w_)/len(y)
```

We can compare to the `sklearn` implementation.

In [9]:

```
def compare_our_lasso_with_sklearn(X_train, y_train, l1_reg=1):

    # Fit with sklearn -- need to multiply l1_reg by 2*sample size, since they
    # use a slightly different objective function.
    n = X_train.shape[0]
    sklearn_lasso = Lasso(alpha=2*n*l1_reg, fit_intercept=False, normalize=False
)
    sklearn_lasso.fit(X_train, y_train)
    sklearn_lasso_coefs = sklearn_lasso.coef_
    sklearn_lasso_preds = sklearn_lasso.predict(X_train)

    # Now run our lasso regression and compare the coefficients to sklearn's

    model = LassoRegression(randomized=True)
    model.fit(X_train, y_train,coef_init=None)
    our_coefs = model.w_
    lasso_regression_preds = model.predict(X_train)

    # Let's compare differences in predictions
    print("Hoping this is very close to 0 (predictions): {}".format( np.mean((sk
learn_lasso_preds - lasso_regression_preds)**2)))
    # Let's compare differences parameter values
    print("Hoping this is very close to 0: {}".format(np.sum((our_coefs - sklear
n_lasso_coefs)**2)))
```

In [10]:

```
compare_our_lasso_with_sklearn(X_train, y_train, l1_reg=1)
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 662 epochs. Lowest loss: 16.19773806585367
Hoping this is very close to 0 (predictions): 3.4346941284451464
Hoping this is very close to 0: 1.504649071501123
```

Grid Search to Tune Hyperparameter

Now let's use sklearn to help us do hyperparameter tuning GridSearchCv.fit by default splits the data into training and validation itself; we want to use our own splits, so we need to stack our training and validation sets together, and supply an index (validation_fold) to specify which entries are train and which are validation.

In [11]:

```
default_params = [0.000001, 0.00001, 0.0001, 0.001, 0.01, 0.1, 1., 1.3, 1.6, 1.9
, 2.2, 2.5, 2.8]

def do_grid_search_lasso(X_train, y_train, X_val, y_val,params = default_params
):
        X_train_val = np.vstack((X_train, X_val))
        y_train_val = np.concatenate((y_train, y_val))
        val_fold = [-1]*len(X_train) + [0]*len(X_val) #0 corresponds to validati
on

        param_grid = [{'l1_reg':params}]

        lasso_regression_estimator = LassoRegression(randomized=True)
        grid = GridSearchCV(lasso_regression_estimator,
                                                param_grid,
                                                return_train_score=True,
                                                cv = PredefinedSplit(test_fold=v
al_fold),
                                                refit = True,
                                                scoring = make_scorer(mean_squar
ed_error,

greater_is_better = False))
        grid.fit(X_train_val, y_train_val)

        df = pd.DataFrame(grid.cv_results_)
        # Flip sign of score back, because GridSearchCV likes to maximize,
        # so it flips the sign of the score if "greater_is_better=FALSE"
        df['mean_test_score'] = -df['mean_test_score']
        df['mean_train_score'] = -df['mean_train_score']
        cols_to_keep = ["param_l1_reg", "mean_test_score","mean_train_score"]
        df_toshow = df[cols_to_keep].fillna('-')
        df_toshow = df_toshow.sort_values(by=["param_l1_reg"])
        return grid, df_toshow
```

In [12]:

```
grid, results = do_grid_search_lasso(X_train, y_train, X_val, y_val)
```

```
Ran for 1 epochs. Lowest loss: 0.6752223426032413

/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars



Ran for 1 epochs. Lowest loss: 0.6755576054044078
Ran for 10 epochs. Lowest loss: 0.6789098853912399

/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 141 epochs. Lowest loss: 0.7123821721344291

/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 545 epochs. Lowest loss: 1.0422438085622936

/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 965 epochs. Lowest loss: 3.9049742926436295

/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 646 epochs. Lowest loss: 16.19773804234723
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
```

```
Ran for 663 epochs. Lowest loss: 18.20873769299461
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
```

```
Ran for 646 epochs. Lowest loss: 20.146332086459903
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
```

```
Ran for 674 epochs. Lowest loss: 22.016046580608936
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
```

```
Ran for 638 epochs. Lowest loss: 23.818255203736737
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
```

```
Ran for 658 epochs. Lowest loss: 25.559311005849157
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars
```

```
Ran for 643 epochs. Lowest loss: 27.242934055616693
Ran for 1000 epochs. Lowest loss: 92.81070262473658
```

In [13]:

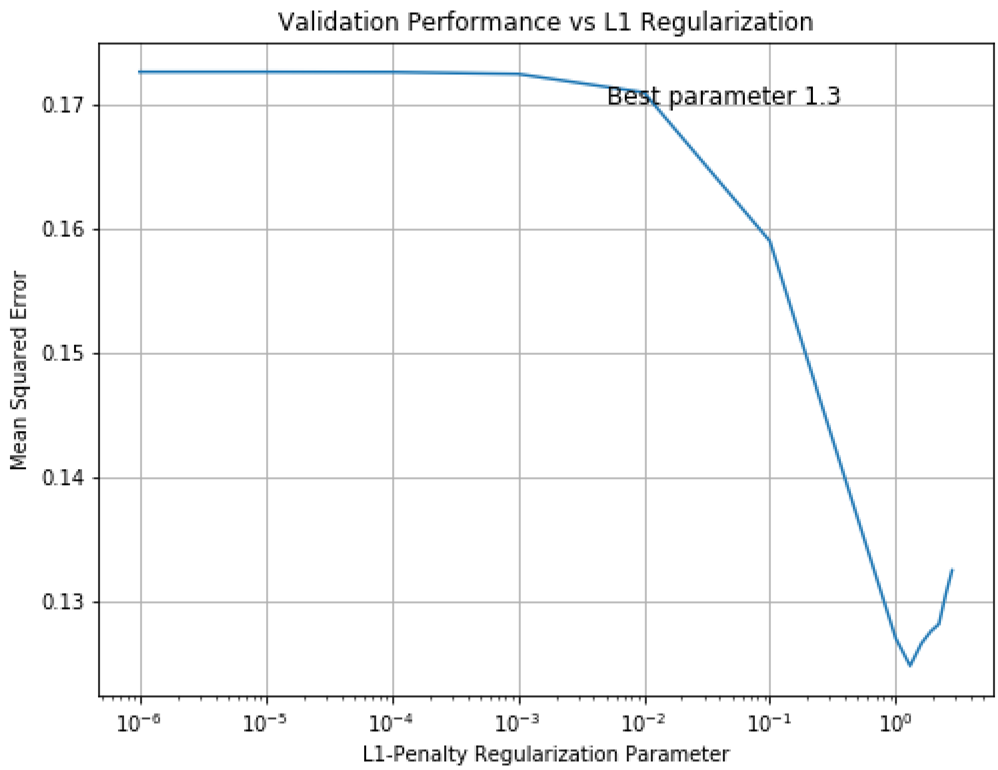
```
results
```

Out[13]:

	param_l1_reg	mean_test_score	mean_train_score
0	0.000001	0.172590	0.006752
1	0.000010	0.172589	0.006752
2	0.000100	0.172574	0.006752
3	0.001000	0.172423	0.006752
4	0.010000	0.170939	0.006805
5	0.100000	0.158959	0.011139
6	1.000000	0.126981	0.091950
7	1.300000	0.124774	0.096655
8	1.600000	0.126586	0.099935
9	1.900000	0.127556	0.103894
10	2.200000	0.128132	0.108347
11	2.500000	0.130583	0.112945
12	2.800000	0.132467	0.117907

In [14]:

```
# Plot validation performance vs regularization parameter

fig, ax = plt.subplots(figsize = (8,6))
ax.grid()
ax.set_title("Validation Performance vs L1 Regularization")
ax.set_xlabel("L1-Penalty Regularization Parameter")
ax.set_ylabel("Mean Squared Error")

ax.semilogx(results["param_l1_reg"], results["mean_test_score"])

ax.text(0.005,0.17,"Best parameter {0}".format(grid.best_params_['l1_reg']), fon
tsize = 12);
```



As we can see, the best parameter is 1 with ridge solution initialization and randomization.

Comparing to the Target Function

Let's plot prediction functions and compare coefficients for several fits and the target function.

Let's create a list of dicts called `pred_fns` . Each dict has a "name" key and a "preds" key. The value corresponding to the "preds" key is an array of predictions corresponding to the input vector x. x_train and y_train are the input and output values for the training data

In [15]:

```
pred_fns = []
x = np.sort(np.concatenate([np.arange(0,1,.001), x_train]))

pred_fns.append({"name": "Target Function", "coefs": coefs_true, "preds": target
_fn(x)})

l1regs = [0.1, grid.best_params_['l1_reg'], 1]
X = featurize(x)
for l1reg in l1regs:
    lasso_regression_estimator = LassoRegression(l1_reg=l1reg)
    lasso_regression_estimator.fit(X_train, y_train)
    name = "Lasso with L1Reg="+str(l1reg)
    pred_fns.append({"name":name,
                     "coefs":lasso_regression_estimator.w_,
                     "preds": lasso_regression_estimator.predict(X) })
```

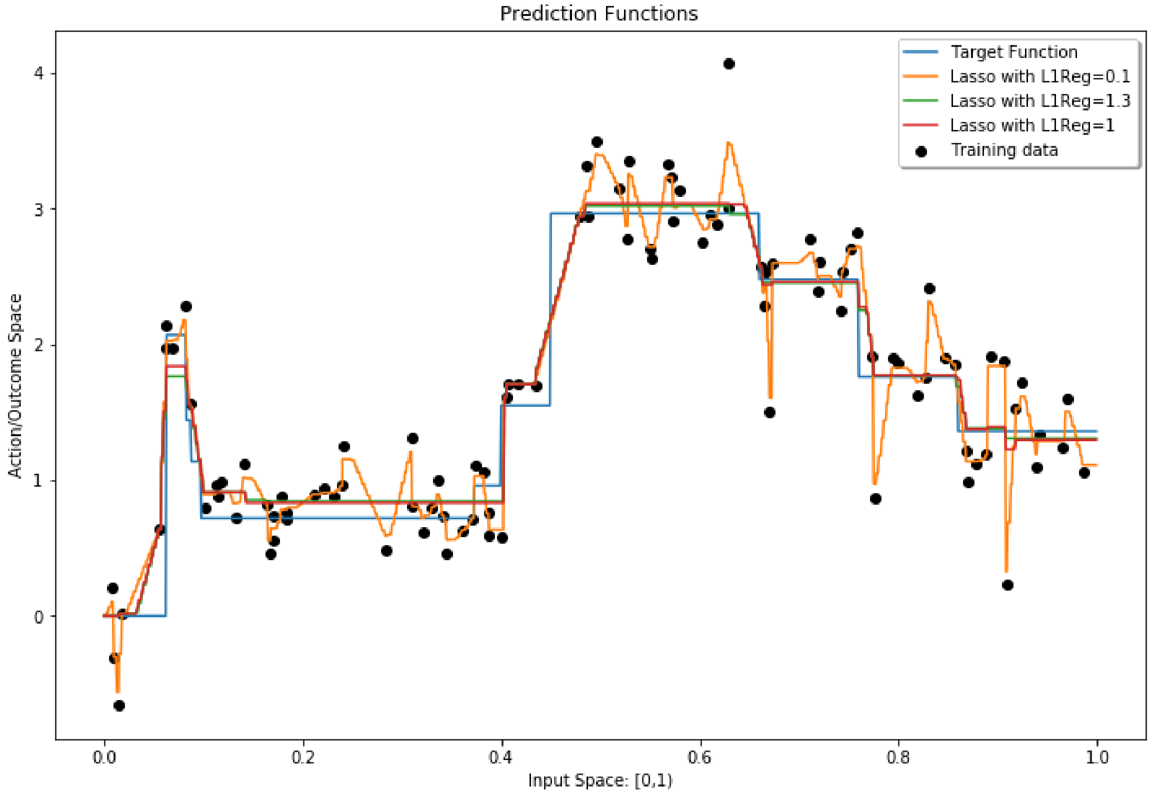
```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 830 epochs. Lowest loss: 3.9049738772510736
Ran for 717 epochs. Lowest loss: 18.208737675700142
Ran for 730 epochs. Lowest loss: 16.19773806144752
```

In [16]:

```
def plot_prediction_functions(x, pred_fns, x_train, y_train, legend_loc="best"):

        fig, ax = plt.subplots(figsize = (12,8))
        ax.set_xlabel('Input Space: [0,1)')
        ax.set_ylabel('Action/Outcome Space')
        ax.set_title("Prediction Functions")
        plt.scatter(x_train, y_train, color="k", label='Training data')
        for i in range(len(pred_fns)):
                ax.plot(x, pred_fns[i]["preds"], label=pred_fns[i]["name"])
        legend = ax.legend(loc=legend_loc, shadow=True)
        return fig
```

In [17]:

```
plot_prediction_functions(x, pred_fns, x_train, y_train, legend_loc="best");
```



Visualizing the Weights

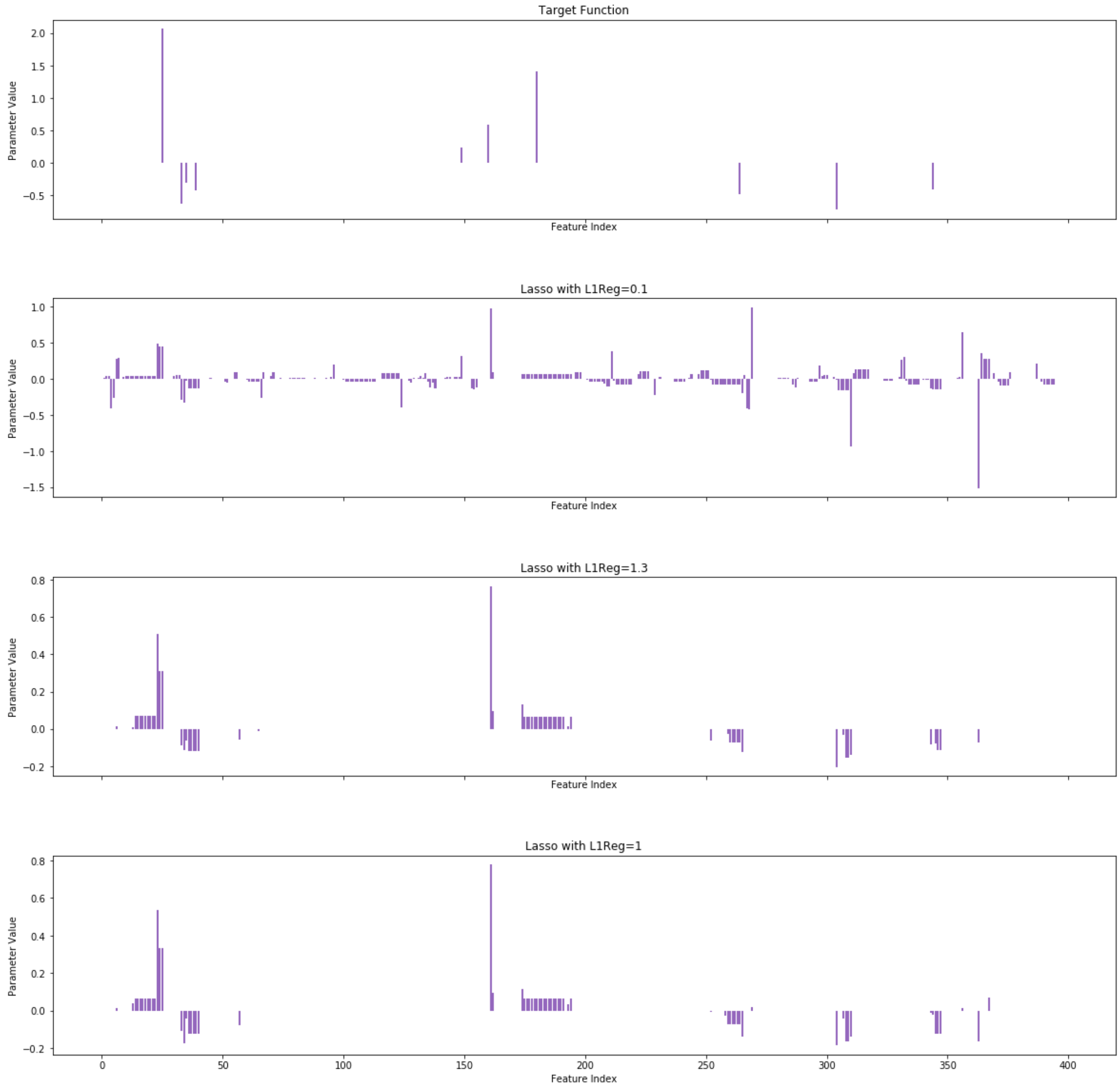
Using `pred_fns` let's try to see how sparse the weights are...

In [18]:

```
def compare_parameter_vectors(pred_fns):

        fig, axs = plt.subplots(len(pred_fns),1, sharex=True, figsize = (20,20))
        num_ftrs = len(pred_fns[0]["coefs"])
        for i in range(len(pred_fns)):
                title = pred_fns[i]["name"]
                coef_vals = pred_fns[i]["coefs"]
                axs[i].bar(range(num_ftrs), coef_vals, color = "tab:purple")
                axs[i].set_xlabel('Feature Index')
                axs[i].set_ylabel('Parameter Value')
                axs[i].set_title(title)

        fig.subplots_adjust(hspace=0.4)
        return fig
```

In [19]:

```
compare_parameter_vectors(pred_fns);
```



As we can see, as l1_reg becomes larger, more and more features become 0's. This confirms that lasso regularization can be used as feauture selection as it removes the weights of the useless feautures according to the regularization parameter.

Q4

Continuation Method

We compute the largest value of λ for which the weights can be nonzero.

In [6]:

```
def get_lambda_max_no_bias(X, y):
    return 2 * np.max(np.abs(np.dot(y, X)))
```

Use homotopy method to compute regularization path for LassoRegression.

In [21]:

```
from sklearn.base import clone

class LassoRegularizationPath:
    def __init__(self, estimator, tune_param_name):
        self.estimator = estimator
        self.tune_param_name = tune_param_name

    def fit(self, X, y, reg_vals, coef_init=None, warm_start=True):
        # reg_vals is a list of regularization parameter values to solve for.
        # Solutions will be found in the order given by reg_vals.

        #convert y to 1-dim array, in case we're given a column vector
        y = y.reshape(-1)

        if coef_init is not None:
            coef_init = np.copy(coef_init)

        self.results = []
        for reg_val in reg_vals:
            estimator = clone(self.estimator)
            w, obj_val, epoch = shooting_algorithm(X, y, w0=coef_init, l1_reg=re
g_val)
            self.results.append({"reg_val":reg_val, "estimator":estimator,"weigh
ts":w})

        return self

    def predict(self, X, y=None):
        predictions = []
        for i in range(len(self.results)):
            preds = self.results[i]["estimator"].predict(X)
            reg_val = self.results[i]["reg_val"]
            predictions.append({"reg_val":reg_val, "preds":preds})
        return predictions

    def score(self, X, y=None):
        scores = []
        for i in range(len(self.results)):
            score = self.results[i]["estimator"].score(X, y)
            reg_val = self.results[i]["reg_val"]
            scores.append({"reg_val":reg_val, "score":score})
        return scores
```

In [22]:

```
def do_grid_search_homotopy(X_train, y_train, X_val, y_val,
                            reg_vals=None, w0=None):
    if reg_vals is None:
        lambda_max = get_lambda_max_no_bias(X_train, y_train)
        reg_vals = [lambda_max * (.8**n) for n in range(0, 30)]


    estimator = LassoRegression()
    lasso_reg_path_estimator = LassoRegularizationPath(estimator, tune_param_nam
e="l1_reg")
    lasso_reg_path_estimator.fit(X_train, y_train,
                                 reg_vals=reg_vals[:], coef_init=w0,
                                 warm_start=True)
    return lasso_reg_path_estimator, reg_vals
```

In [23]:

```
lasso_reg_path_estimator, reg_vals = do_grid_search_homotopy(X_train,
                                                              y_train,
                                                              X_val,
                                                              y_val,
                                                              None)
```

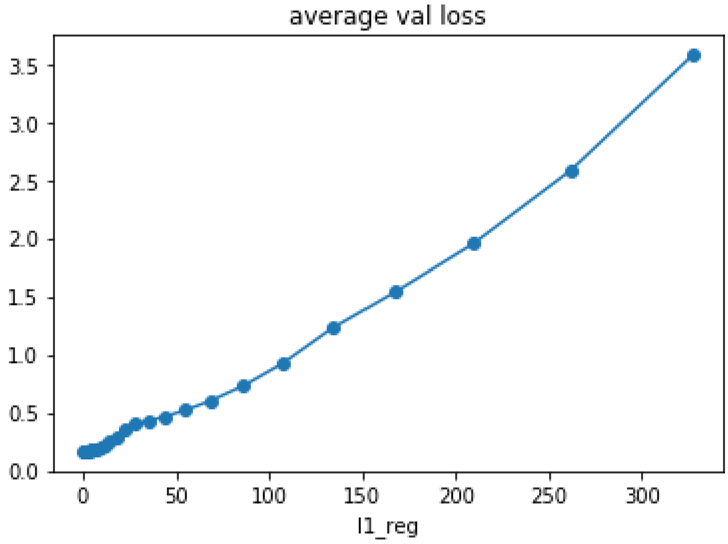
```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 1 epochs. Lowest loss: 359.6674002813196
Ran for 594 epochs. Lowest loss: 348.52108633392805
Ran for 653 epochs. Lowest loss: 323.5371648261513
Ran for 582 epochs. Lowest loss: 293.22926498817293
Ran for 732 epochs. Lowest loss: 262.2363733469605
Ran for 731 epochs. Lowest loss: 231.30364707665956
Ran for 733 epochs. Lowest loss: 202.0174855998118
Ran for 734 epochs. Lowest loss: 175.6829689439503
Ran for 736 epochs. Lowest loss: 152.73952248856375
Ran for 737 epochs. Lowest loss: 133.12872423056882
Ran for 737 epochs. Lowest loss: 116.62091784825118
Ran for 738 epochs. Lowest loss: 102.89040535085165
Ran for 739 epochs. Lowest loss: 91.40127989084868
Ran for 740 epochs. Lowest loss: 80.59513459352864
Ran for 741 epochs. Lowest loss: 70.6513115923515
Ran for 742 epochs. Lowest loss: 61.83896929177382
Ran for 743 epochs. Lowest loss: 54.081080014686314
Ran for 740 epochs. Lowest loss: 47.326669350039296
Ran for 731 epochs. Lowest loss: 41.56428608924523
Ran for 765 epochs. Lowest loss: 36.697129451829106
Ran for 820 epochs. Lowest loss: 32.323174118580575
Ran for 838 epochs. Lowest loss: 28.43476228579073
Ran for 834 epochs. Lowest loss: 25.071530450949826
Ran for 794 epochs. Lowest loss: 22.211091306666702
Ran for 809 epochs. Lowest loss: 19.79969729994602
Ran for 818 epochs. Lowest loss: 17.789508949588427
Ran for 824 epochs. Lowest loss: 16.121413410130746
Ran for 926 epochs. Lowest loss: 14.563979329023843
Ran for 932 epochs. Lowest loss: 12.993126648493345
Ran for 935 epochs. Lowest loss: 11.487542080929062
```

In [24]:

```
lasso_reg_path_estimator.results[0]
rs = []
losses = []
for dic in lasso_reg_path_estimator.results:
    rs.append(dic["reg_val"])
    losses.append(compute_sum_sqr_loss(X_val,y_val,dic["weights"])/X_val.shape[0
])

plt.plot(rs,losses,"-o")
plt.xlabel("l1_reg")
plt.title("average val loss")
```

Out[24]:

```
Text(0.5, 1.0, 'average val loss')
```



Q5

In [25]:

```
# Add an unregularized bias term
X_train = np.hstack((X_train, np.ones((X_train.shape[0], 1))))
X_val = np.hstack((X_val, np.ones((X_val.shape[0], 1))))
```

In [26]:

```
l1_reg = 1.
l2_reg = 0.1

lasso = LassoRegression(l1_reg=l1_reg,randomized=True)
lasso.fit(X_train,y_train)
print("Avg val loss for L1: ", lasso.score(X_val,y_val))

ridge_sol = get_ridge_solution(X_train,y_train,l2_reg=l2_reg)
print("Avg val loss for L2: ", compute_sum_sqr_loss(X_val,y_val,ridge_sol)/X_val
.shape[0])
```

```
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 651 epochs. Lowest loss: 16.197738066251986
Avg val loss for L1:  0.12473404392083155
Avg val loss for L2:  0.1627421890227181
```

We can see that adding an unregularized bias term by padding 1's is having the same effect in terms of avg val loss value comparing to our original setup. This confirms that we have to “pay for” this offset using our penalized parameters.

In [27]:

```
ridge_sol[-1]
```

Out[27]:

```
0.03160551307882997
```

For example in the ridge solution, we can see that the last weight (w_{d+1}) corresponding to the unregularized bias term is nonzero.

Projected SGD

Q1

In [4]:

```
def projection_SGD_split(X, y, theta_positive_0, theta_negative_0, lambda_reg = 
1.0, alpha = 0.1, num_iter = 1000):
    m, n = X.shape
    theta_positive = np.zeros(n)
    theta_negative = np.zeros(n)
    theta_positive[0:n] = theta_positive_0
    theta_negative[0:n] = theta_negative_0
    times = 0
    theta = theta_positive - theta_negative
    loss = compute_sum_sqr_loss(X, y, theta)/X.shape[0]
    loss_change = 1.
    while (loss_change>1e-6) and (times<num_iter):
        loss_old = loss
        for i in range(m):
            for j in range(n):
                if theta_positive[j]:
                    theta_positive[j] -= alpha*2*((np.dot(theta_positive-theta_n
egative,X[i])-y[i])*X[i][j]+lambda_reg)
                else:
                    theta_positive[j] -= alpha*2*((np.dot(theta_positive-theta_n
egative,X[i])-y[i])*X[i][j])
                if theta_negative[j]:
                    theta_negative[j] -= alpha*2*((np.dot(theta_positive-theta_n
egative,X[i])-y[i])*(-X[i][j])+lambda_reg)
                else:
                    theta_negative[j] -= alpha*2*((np.dot(theta_positive-theta_n
egative,X[i])-y[i])*(-X[i][j]))

            #theta_positive -= alpha*2*(np.dot(theta_positive-theta_negative,X
[i])-y[i])*X[i]
            #theta_negative -= alpha*2*(np.dot(theta_positive-theta_negative,X
[i])-y[i])*-(X[i])
            np.clip(theta_positive, 0., None)
            np.clip(theta_negative, 0., None)
            theta = theta_positive - theta_negative
        loss = compute_sum_sqr_loss(X, y, theta)/X.shape[0]
        #print(loss)
        loss_change = loss_old - loss#np.abs(loss - loss_old)
        times +=1

    print('(SGD) Ran for {} epochs. Loss:{} Lambda: {}'.format(times,loss,lambda
_reg))
    return theta



#theta_positive += -alpha*(2*np.dot(X.T,np.dot(X,theta_positive-theta_negative)-
y)/X.shape[0] + np.array([lambda_reg if i else 0. for i in theta_positive]))
#theta_negative += -alpha*(2*np.dot(-X.T,np.dot(X,theta_positive-theta_negative)
-y)/X.shape[0] + np.array([lambda_reg if i else 0. for i in theta_negative]))
#print(theta_positive)
#print(theta_negative)
#np.clip(theta_positive, 0., None)
#np.clip(theta_negative, 0., None)
#theta = theta_positive - theta_negative
#print(theta)
```

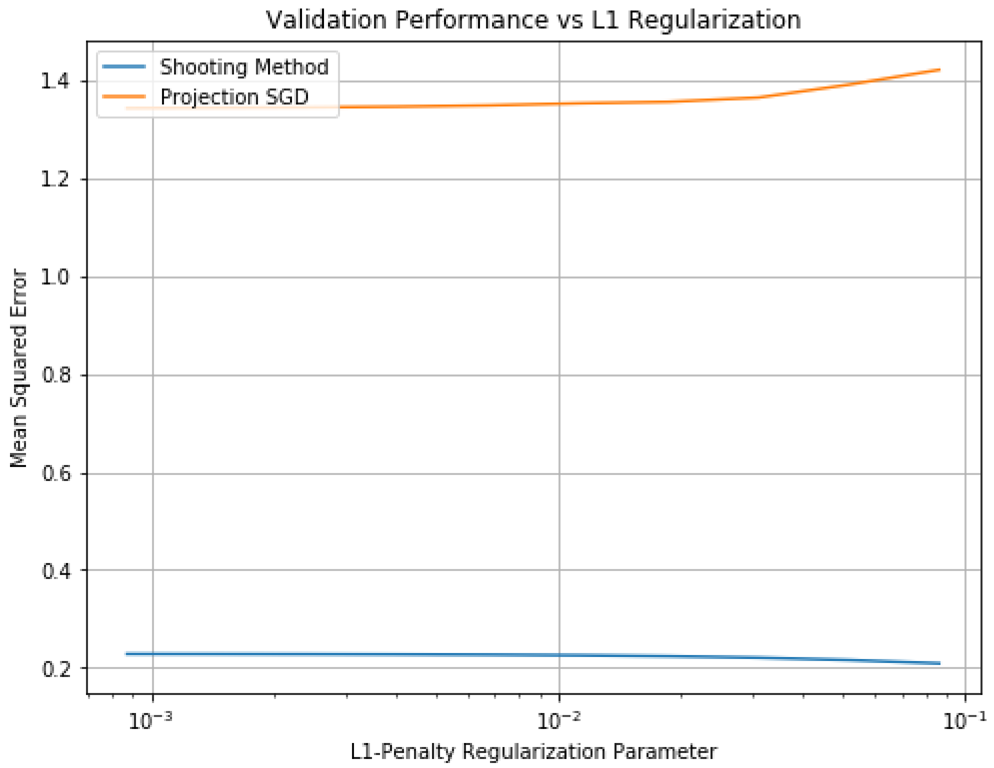
In [9]:

```
x_training, y_training, x_validation, y_validation, target_fn, coefs_true, featu
rize = load_problem(PICKLE_PATH)
X_training = featurize(x_training)
X_validation = featurize(x_validation)
D = X_training.shape[1]

lambda_max = get_lambda_max_no_bias(x_training, y_training)
reg_vals = [lambda_max * (.6**n) for n in range(15, 25)]

loss_SGD_list = []
loss_shooting = []
loss_GD_list = []

for lambda_value in reg_vals:
    theta_projected =  projection_SGD_split(X_training, y_training, np.zeros(X_t
raining.shape[1]), np.zeros(X_training.shape[1]), lambda_reg = lambda_value, alp
ha = 0.1, num_iter = 1000)
    loss_SGD_list.append(compute_sum_sqr_loss(X_validation,y_validation,theta_pr
ojected)/X_validation.shape[0])
    theta_shooting, _, _ = shooting_algorithm(X_training, y_training, w0=None, l
1_reg = lambda_value, max_num_epochs = 1000, min_obj_decrease=1e-8, random=False
)
    loss_shooting.append(compute_sum_sqr_loss(X_validation,y_validation,theta_sh
ooting)/X_validation.shape[0])
```

```
(SGD) Ran for 30 epochs. Loss:1.402028509912859 Lambda: 0.0863912469
3688116

/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: invalid value encountered in double_scal
ars
/opt/conda/envs/dsga-1003/lib/python3.7/site-packages/ipykernel_laun
cher.py:20: RuntimeWarning: divide by zero encountered in double_sca
lars

Ran for 1000 epochs. Lowest loss: 3.5178841123598166
(SGD) Ran for 31 epochs. Loss:1.3746442831273407 Lambda: 0.051834748
16212869
Ran for 1000 epochs. Lowest loss: 2.4648530491515035
(SGD) Ran for 31 epochs. Loss:1.349870066287359 Lambda: 0.0311008488
9727721
Ran for 1000 epochs. Lowest loss: 1.781887325644532
(SGD) Ran for 31 epochs. Loss:1.3416890233973284 Lambda: 0.018660509
33836633
Ran for 1000 epochs. Lowest loss: 1.3515108201528356
(SGD) Ran for 32 epochs. Loss:1.339546440418353 Lambda: 0.0111963056
03019797
Ran for 1000 epochs. Lowest loss: 1.0854433720625416
(SGD) Ran for 32 epochs. Loss:1.3354908978891746 Lambda: 0.006717783
361811878
Ran for 1000 epochs. Lowest loss: 0.9229608309275833
(SGD) Ran for 32 epochs. Loss:1.3331566689934982 Lambda: 0.004030670
017087127
Ran for 1000 epochs. Lowest loss: 0.8244451040361851
(SGD) Ran for 32 epochs. Loss:1.3317868975355374 Lambda: 0.002418402
0102522756
Ran for 1000 epochs. Lowest loss: 0.7649593498421838
(SGD) Ran for 32 epochs. Loss:1.330934159108682 Lambda: 0.0014510412
061513652
Ran for 1000 epochs. Lowest loss: 0.7291304075451188
(SGD) Ran for 32 epochs. Loss:1.330209314507957 Lambda: 0.0008706247
236908191
Ran for 1000 epochs. Lowest loss: 0.7075834082913903
```

In [10]:

```
# Plot validation performance vs regularization parameter

fig, ax = plt.subplots(figsize = (8,6))
ax.grid()
ax.set_title("Validation Performance vs L1 Regularization")
ax.set_xlabel("L1-Penalty Regularization Parameter")
ax.set_ylabel("Mean Squared Error")

plt.semilogx(reg_vals, loss_shooting, label = 'Shooting Method')
plt.semilogx(reg_vals, loss_SGD_list, label = 'Projection SGD')
plt.legend(loc='upper left')
plt.show();
```



In [11]:

```
# Report the best

lambda_best_SGD = reg_vals[np.argmin(loss_SGD_list)]
theta_lasso_SGD_best = projection_SGD_split(X_training, y_training, np.zeros(X_t
raining.shape[1]), np.zeros(X_training.shape[1]), lambda_reg=lambda_best_SGD, al
pha = 0.01)
print('Best lambda for SGD is {0} with loss {1}'.format(lambda_best_SGD, np.min(
loss_SGD_list)))
```

```
(SGD) Ran for 1000 epochs. Loss:0.24696250032547792 Lambda: 0.000870
6247236908191
Best lambda for SGD is 0.0008706247236908191 with loss 1.34340705377
63425
```

In [ ]:

```
```