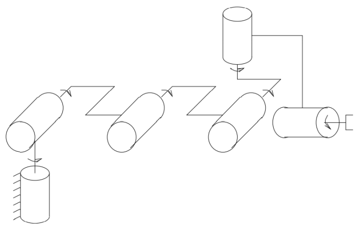
Homework 9-10

(Due time: 24:00, Apr. 23, 2020)

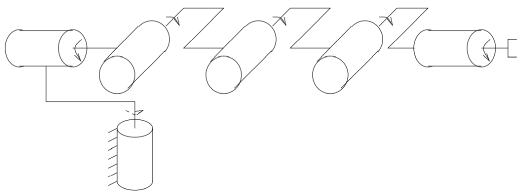
1. For each of the manipulators shown schematically in Figure 1:

   (a) Derive the spatial and body Jacobians.

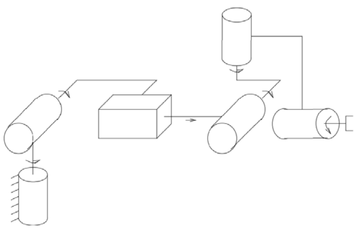
   (b) Give a geometric description of the singular configurations.



(i) Elbow manipulator



(ii) Inverse elbow manipulator



(iii) Stanford manipulator

Fig 1. Sample manipulators. Revolute joints are represented by cylinders; prismatic joints are represented by rectangular boxes.

2. Euler angles can be used to represent rotations via the product of exponentials formula. If we think of (α, β, γ) as joints angles of a robot manipulator, then we can find the singularities of an Euler angle parameterization by calculating the Jacobian of the "forward kinematics", where we are concerned only with the rotation portion of the forward kinematics map. Use this point of view to find singularities for the following classes of Euler angles:

   (a) ZYZ Euler angles

   (b) ZYX Euler angles

3. *Kinematic singularity: four coplanar revolute joints*

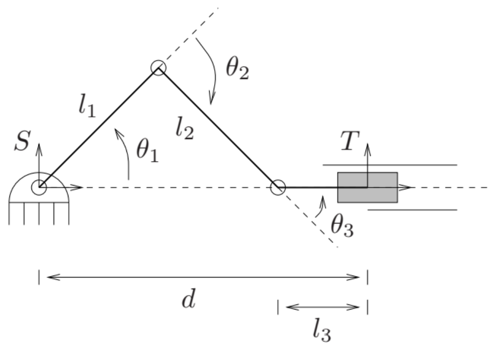
   Four revolute joint axes with twists $\xi_i = (q_i \times \omega_i, \omega_i), i = 1, \cdots, 4$, are said to be coplanar if there exists a plane with unit normal n such that:

   (a) Each axis direction is orthogonal to n: $n^T \omega_i = 0, i = 1, \cdots, 4$.

   (b) The vector from q_i to q_j is orthogonal to n: $n^T (q_i - q_j) = 0,\ i = 1, \cdots, 4$.

   Show that when four of its revolute joint axes are coplanar, a six degree of freedom manipulator is at a singular configuration. Give an example of a manipulator exhibiting such a singularity.

4. Consider the slider-crank mechanism shown below:



   (a) Calculate the structure equations for the mechanism.

   (b) Calculate the Jacobian of the structure equations; give explicit expressions for the instantaneous twists for each of the joints.

   (c) Find the singular configurations of the mechanism if d is the active variable.

   (d) Find the singular configurations if θ_1 is treated as the active variable. Under what conditions (on l_1, l_2, l_3) do singular configurations exist?