# Homework 3-4

## (Due time: 24:00, Apr. 4, 2020)

Note: If there is no extra statement, we assume that Frame $\{A\}:\{X_A, Y_A, Z_A\}$ is a spatial frame, and Frame $\{B\}:\{X_B, Y_B, Z_B\}$ is a body frame attached to a rigid body.

1. A vector $v$ is rotated about $Z_A$ by $\theta$ degrees and is subsequently rotated about $X_A$ by $\varphi$ degrees. Give the rotation matrix $R_{ab}$ that accomplished these rotations in the given order. If $\theta = 30$ degrees and $\varphi = 45$ degrees, compute the rotation matrix.

2. A frame $\{B\}$ is located initially coincident with a frame $\{A\}$. We rotate $\{B\}$ about $Z_B$ by $\theta$ degrees, and then we rotate the resulting frame about $X_B$ by $\varphi$ degrees. Find the rotation matrix $R_{ab}$ that will change the description of vectors from $\{B\}$ to $\{A\}$. If $\theta = 30$ degrees and $\varphi = 45$ degrees, calculate the rotation matrix.

3. Find the homogeneous transformation matrix $g_{ab}$ of the frame $\{B\}$ with respect to the frame $\{A\}$, which is generated by the following sequence of rigid motion.
   (1) Rotation about $Z_A$ by 90 degrees;
   (2) Rotation about $X_A$ by -90 degrees;
   (3) Translation to the vector $(3, 7, 9)^T$.

4. The following matrix represents a homogeneous transformation matrix. Find the four unknown entries in the first column, $a, b, c, d$.

$$g = \begin{bmatrix} a & 0 & -1 & 0 \\ b & 0 & 0 & 1 \\ c & -1 & 0 & 2 \\ d & 0 & 0 & 1 \end{bmatrix}$$

5. Find the homogeneous transformation matrix $g_{ab}$ of the frame $\{B\}$ with respect to the frame $\{A\}$, which is generated by the following sequence of rigid motion.
   (1) Translation along the vector $(3, 7, 9)^T$;
   (2) Rotated about $X_B$ by -90 degrees;
   (3) Rotated about $Z_B$ by 90 degrees.

6. The following frame definitions are given as known

$$g_{ua} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & 11 \\ 1/2 & \sqrt{3}/2 & 0 & -1 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{ba} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 10 \\ 0 & 1/2 & \sqrt{3}/2 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{cu} = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 & -3 \\ \sqrt{3}/4 & 3/4 & -1/2 & -3 \\ 1/4 & \sqrt{3}/4 & \sqrt{3}/2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solve for $g_{cb}$.

7. (a) Referring to Fig. 2, give the value of $g_{ab}$;
   (b) Referring to Fig. 2, give the value of $g_{ac}$;
   (c) Referring to Fig. 2, give the value of $g_{bc}$;
   (d) Referring to Fig. 2, give the value of $g_{ca}$.

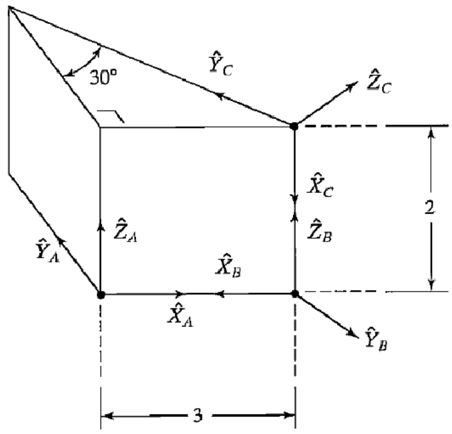


Fig.2. Frames at the corners of a wedge.

8. Given a homogeneous transformation matrix

$$g = \begin{bmatrix} 0 & 1 & 0 & 10 \\ 0 & 0 & -1 & 20 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

Find the equivalent rotational axis $\omega$, rotation angle $\theta$, $\xi = [v^T, \omega^T]^T$, and a point $q$ on the axis.

**9. Matlab programing**

(a) Write a Matlab program function that transforms rigid transformation matrices from the **ZYZ convention** to the **exponential** representation, named **zyz2exp**; and write a Matlab program function that transforms rigid transformation matrices from the **exponential representation** to the **ZYZ** convention, named **exp2zyz**. Submit your Matlab code and give two examples to verify your program.

(b) Write a Matlab program function that transforms rigid transformation matrices from the **RPY convention** to the **exponential** representation, named **rpy2exp**; and write a program function that transforms rigid transformation matrices from the **exponential representation** to the **RPY** convention, named **exp2rpy**. Submit your Matlab code and give two examples to verify your program.