机器人学导论作业3-4

SZ170320207

刘健恒

1、由题知对应 XYZ（固定坐标系模型）

$$v' = R_{ab} v$$

$$= R_{XA}(\varphi) R_{ZA}(\theta) v$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\varphi & -S_\varphi \\ 0 & S_\varphi & C_\varphi \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} v$$

$$= \begin{bmatrix} C_\theta & -S_\theta & 0 \\ C_\varphi S_\theta & C_\varphi C_\theta & -S_\varphi \\ S_\varphi S_\theta & S_\varphi C_\theta & C_\varphi \end{bmatrix} v$$

$$\Rightarrow R_{ab} = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ C_\varphi S_\theta & C_\varphi C_\theta & -S_\varphi \\ S_\varphi S_\theta & S_\varphi C_\theta & C_\varphi \end{bmatrix}$$

$$\theta = 30, \quad \varphi = 45 \Rightarrow \begin{cases} S_\theta = \frac{1}{2} \\ C_\theta = \frac{\sqrt{3}}{2} \end{cases}, \begin{cases} S_\varphi = \frac{\sqrt{2}}{2} \\ C_\varphi = \frac{\sqrt{2}}{2} \end{cases}$$

$$\Rightarrow R_{ab} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

2、(ZYX) 欧拉角模型

$$R_{ab} = R_{ZB}(\theta) R_{XB}(\varphi)$$

$$R_{XB}(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\varphi & -S_\varphi \\ 0 & S_\varphi & C_\varphi \end{bmatrix}$$

$$R_{ZB}(\theta) = \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{ab} = \begin{bmatrix} C_\theta & -S_\theta C_\varphi & S_\theta S_\varphi \\ S_\theta & C_\theta C_\varphi & -C_\theta S_\varphi \\ 0 & S_\varphi & C_\varphi \end{bmatrix}$$

当 $\theta = 30, \varphi = 45^\circ \Rightarrow \begin{cases} S_\theta = \frac{1}{2} \\ C_\theta = \frac{\sqrt{3}}{2} \end{cases}, \begin{cases} S_\varphi = \frac{\sqrt{2}}{2} \\ C_\varphi = \frac{\sqrt{2}}{2} \end{cases}$

$$R_{ab} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\ \frac{1}{2} & \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

3、

$$R_{ab} = R_{XA}(-90^\circ) R_{ZA}(90^\circ)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$p_{ab} = [3, 7, 9]^T$$

$\Rightarrow$ 齐次变换矩阵：

$$g_{ab} = \begin{bmatrix} R_{ab} & p_{ab} \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 7 \\ -1 & 0 & 0 & 9 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

4、由齐次变换矩阵知

$$g = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix} \Rightarrow d = 0$$

又：$R \in SO(3)$

$\therefore \det R = b = 1$

$$R^T R = \begin{bmatrix} a & 1 & c \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 & -1 \\ 1 & 0 & 0 \\ c & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+1+c^2 & -c & -a \\ -c & 1 & 0 \\ -a & 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow \begin{cases} a = 0 \\ c = 0 \end{cases}$$

$$\therefore g = \left[\begin{array}{ccc|c} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

5. 欧拉角模型

$R_{ab} = R_{XB}(-90^\circ) R_{ZB}(90^\circ)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

$P_{ab} = [3, 7, 9]^T$

$$\Rightarrow g_{ab} = \begin{bmatrix} R_{ab} & P_{ab} \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 7 \\ -1 & 0 & 0 & 9 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

6.

$$g_{cb} = g_{cn} g_{na} g_{ab}$$

$$= g_{cn} g_{na} (g_{ba})^{-1}$$

又∵ $g^{-1} = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix}$

$$\therefore g_{cb} = \begin{bmatrix} R_{cn} & P_{cn} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{na} & P_{na} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{ba}^T & -R_{ba}^T P_{ba} \\ 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & -3 \\ \frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{1}{2} & -3 \\ \frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{2} & 3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 11 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -1 \\ 0 & 0 & 1 & 8 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & -5\sqrt{3}+10 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 5+10\sqrt{3} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & \frac{11}{2}\sqrt{3}-\frac{5}{2} \\ \frac{6}{8} & \frac{\sqrt{3}}{4} & -\frac{1}{2} & \frac{11}{4}\sqrt{3}-\frac{31}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & \frac{\sqrt{3}}{2} & \frac{5}{4}\sqrt{3}+\frac{23}{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & -5\sqrt{3}+10 \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 5+10\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} \frac{1}{2} & -\frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{1}{2}\sqrt{3}+5 \\ \frac{3}{4} & \frac{7}{8} & -\frac{\sqrt{3}}{8} & \frac{1}{4}\sqrt{3}-14 \\ \frac{\sqrt{3}}{4} & -\frac{\sqrt{3}}{8} & \frac{7}{8} & 5\sqrt{3}+\frac{93}{4} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

7. (a) $R_{ab} = R_{ZB}(180^\circ)$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P_{ab} = [3, 0, 0]^T$

$$g_{ab} = \left[\begin{array}{ccc|c} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(b) $R_{ac} = R_{ZC}(30^\circ) R_{YC}(90^\circ)$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -1 & 0 & 0 \end{bmatrix}$$

$P_{ac} = [3, 0, 2]^T$

$$g_{ac} = \left[\begin{array}{ccc|c} 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 3 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -1 & 0 & 0 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(c) $R_{bc} = R_{YB}(90^\circ) R_{XB}(150^\circ)$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -1 & 0 & 0 \end{bmatrix}$$

$P_{bc} = [0, 0, 2]^T$

$$g_{bc} = \left[\begin{array}{ccc|c} 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -1 & 0 & 0 & 2 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

(d)

$$R_{ca} = R_{XA}(30^\circ) R_{YA}(-90^\circ)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt3}{2} & -\frac12 \\ 0 & \frac12 & \frac{\sqrt3}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ -\frac12 & \frac{\sqrt3}{2} & 0 \\ \frac{\sqrt3}{2} & \frac12 & 0 \end{bmatrix}$$

$$p_{ca} = \left[2, \frac32, -\frac{3\sqrt3}{2}\right]^T$$

$$g_{ca} = \left[\begin{array}{ccc:c} 0 & 0 & -1 & 2 \\ -\frac12 & \frac{\sqrt3}{2} & 0 & \frac32 \\ \frac{\sqrt3}{2} & \frac12 & 0 & -\frac{3\sqrt3}{2} \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

8.

$$g = e^{\hat\xi\theta} = \begin{bmatrix} e^{\hat\omega\theta} & (I-e^{\hat\omega\theta})\hat\omega v + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}$$

由Rodrigues公式知 ($V_\theta = 1-\cos\theta$)

$$e^{\hat\omega\theta} = I + \hat\omega\sin\theta + \hat\omega^2(1-\cos\theta)$$

$$= \begin{bmatrix} \omega_1^2 V_\theta + C_\theta & \omega_1\omega_2 V_\theta - \omega_3 S_\theta & \omega_1\omega_3 V_\theta + \omega_2 S_\theta \\ \omega_1\omega_2 V_\theta + \omega_3 S_\theta & \omega_2^2 V_\theta + C_\theta & \omega_2\omega_3 V_\theta - \omega_1 S_\theta \\ \omega_1\omega_3 V_\theta - \omega_2 S_\theta & \omega_2\omega_3 V_\theta + \omega_1 S_\theta & \omega_3^2 V_\theta + C_\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = R$$

又∵ $tr(R) = 1 + 2\cos\theta = 0$

$\Rightarrow \cos\theta = -\frac12 \Rightarrow \theta = 120^\circ = \frac23\pi$

$$\Rightarrow \omega = \frac{1}{2S_\theta}\begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix} = \frac{\sqrt3}{3}\begin{bmatrix} 0-(-1) \\ 0-(-1) \\ 0-1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt3}{3} \\ \frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} \end{bmatrix}$$

又∵ $(I-e^{\hat\omega\theta})\hat\omega v + \omega\omega^T v\theta = [10, 20, 1]^T$

$= Av$

$(A = (I-e^{\hat\omega\theta})\hat\omega + \omega\omega^T\theta)$

$$A = (I-e^{\hat\omega\theta})\hat\omega + \omega\omega^T\theta$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & \frac{\sqrt3}{3} & \frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} & 0 & -\frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} & \frac{\sqrt3}{3} & 0 \end{bmatrix} + \begin{bmatrix} \frac{\sqrt3}{3} \\ \frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} \end{bmatrix}\begin{bmatrix} \frac{\sqrt3}{3} & \frac{\sqrt3}{3} & -\frac{\sqrt3}{3} \end{bmatrix}\frac{2\pi}{3}$$

$$= \begin{bmatrix} \frac{\sqrt3}{3} & \frac{\sqrt3}{3} & \frac23\sqrt3 \\ -\frac23\sqrt3 & \frac{\sqrt3}{3} & -\frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} & \frac23\sqrt3 & \frac{\sqrt3}{3} \end{bmatrix} + \begin{bmatrix} \frac29\pi & \frac29\pi & -\frac29\pi \\ \frac29\pi & \frac29\pi & -\frac29\pi \\ -\frac29\pi & -\frac29\pi & \frac29\pi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt3}{3}+\frac29\pi & \frac{\sqrt3}{3}+\frac29\pi & \frac23\sqrt3-\frac29\pi \\ -\frac23\sqrt3+\frac29\pi & \frac{\sqrt3}{3}+\frac29\pi & -\frac{\sqrt3}{3}-\frac29\pi \\ -\frac{\sqrt3}{3}-\frac29\pi & \frac23\sqrt3-\frac29\pi & \frac{\sqrt3}{3}+\frac29\pi \end{bmatrix}$$

∵ $p = Av \Rightarrow v = A^{-1}p$

$$\Rightarrow v = \begin{bmatrix} \frac{29}{2\pi} - \frac{31}{3\sqrt3} \\ \frac{32}{3\sqrt3} + \frac{29}{2\pi} \\ \frac{1}{3\sqrt3} - \frac{29}{2\pi} \end{bmatrix}$$

$$\Rightarrow \xi = \left[\begin{array}{c} \frac{29}{2\pi} - \frac{31}{3\sqrt3} \\ \frac{32}{3\sqrt3} + \frac{29}{2\pi} \\ \frac{1}{3\sqrt3} - \frac{29}{2\pi} \\ \hdashline \frac{\sqrt3}{3} \\ \frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} \end{array}\right]$$

∵ $\hat\omega v = \omega \times v = q$

$$\Rightarrow q = \begin{bmatrix} 0 & \frac{\sqrt3}{3} & \frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} & 0 & -\frac{\sqrt3}{3} \\ -\frac{\sqrt3}{3} & \frac{\sqrt3}{3} & 0 \end{bmatrix}\begin{bmatrix} \frac{29}{2\pi} - \frac{31}{3\sqrt3} \\ \frac{32}{3\sqrt3} + \frac{29}{2\pi} \\ \frac{1}{3\sqrt3} - \frac{29}{2\pi} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{3} \\ \frac{10}{3} \\ 7 \end{bmatrix}$$

$v = -\omega \times q$

令 $q = [a, b, c]^T$

$\Rightarrow$ 代入 $v = -\omega \times q$

无解

$q = -\hat\omega^{-1} v$

9.

(a)

zyz2exp

```
function [axis,theta] = zyz2exp(angleZ1,angleY2,angleZ3)
% please input angle in rad
% number behind represents order
```

```
RZ1 = [cos(angleZ1), -sin(angleZ1),0;
       sin(angleZ1), cos(angleZ1), 0;
       0,            0,            1];
RY2 = [cos(angleY2), 0,            sin(angleY2);
       0,            1,            0;
       -sin(angleY2),0,            cos(angleY2)];
RZ3 = [cos(angleZ3),-sin(angleZ3),0;
       sin(angleZ3),cos(angleZ3), 0;
       0,           0,            1];
R = RZ1*RY2*RZ3
trR = R(1,1)+R(2,2)+R(3,3);
% acos->arccos
theta = acos((trR-1)/2);
axis = [R(3,2)-R(2,3);
        R(1,3)-R(3,1);
        R(2,1)-R(1,2)];
axis = axis/(2*sin(theta));
theta= theta/pi*180;

if trR == 3
    fprintf("R=I\n");
elseif trR==-1
    fprintf("Singularity!\n")
end

end
```

实验例子

1)

```
>> [axis,theta]=zyz2exp(0,0,pi/3)

R =

    0.5000   -0.8660         0
    0.8660    0.5000         0
         0         0    1.0000


axis =

         0
         0
    1.0000


theta =

   60.0000
```

2)

```
>> [axis,theta]=zyz2exp(pi/4,pi/3,pi/6)

R =
```

```
   -0.0474   -0.7891    0.6124
    0.6597    0.4356    0.6124
   -0.7500    0.4330    0.5000


axis =

   -0.0898
    0.6823
    0.7256


theta =

   93.2037
```

exp2zyz

```
function [angleZ1,angleY2,angleZ3] = exp2zyz(axis,theta)
% please input angle in rad
% angleZ1->gama,angleY2->beta,angleZ3->alpha
% number behind represents order
axis_h=[0,       -axis(3),axis(2);
        axis(3), 0,       -axis(1);
        -axis(2),axis(1), 0];
R=eye(3)+axis_h*sin(theta)+(axis_h^2)*(1-cos(theta))

angleY2 = atan2(sqrt(R(3,1)^2+R(3,2)^2),R(3,3));
angleZ3 = atan2(R(2,3)/sin(angleY2),R(1,3)/sin(angleY2));
angleZ1 = atan2(R(3,2)/sin(angleY2),-R(3,1)/sin(angleY2));

angleY2 = angleY2/pi*180;
angleZ3 = angleZ3/pi*180;
angleZ1 = angleZ1/pi*180;

if angleY2==0 || angleY2==180
    fprintf("Singularity!\n")
end

end
```

实验例子

1)

```
>> [angleZ1,angleY2,angleZ3]=exp2zyz([0;0;1],pi/3)

R =

    0.5000   -0.8660         0
    0.8660    0.5000         0
         0         0    1.0000

Singularity!
```

```
angleZ1 =

   NaN


angleY2 =

     0


angleZ3 =

   NaN
```

2)

```
>> [angleZ1,angleY2,angleZ3]=exp2zyz([-0.0898;0.6823;0.7256],93.2037/180*pi)

R =

   -0.0475   -0.7892    0.6124
    0.6598    0.4356    0.6124
   -0.7500    0.4331    0.4999


angleZ1 =

   44.9986


angleY2 =

   60.0051


angleZ3 =

   30.0030
```

结果分析

分别使用`zyz2exp`和`exp2zyz`分别对两组实验数据进行验证，两函数所得结果相符，并且能够顺利识别出奇异角。

(b)

rpy2exp

```
function [axis,theta] = rpy2exp(angleR1,angleP2,angleY3)
% please input angle in rad
% R->X, P->Y, Y->Z
% number behind represents order
RR1 = [1,              0,              0;
       0,              cos(angleR1), -sin(angleR1);
       0,              sin(angleR1), cos(angleR1)];
RY2 = [cos(angleP2), 0,              sin(angleP2);
```

```
       0,             1,            0;
       -sin(angleP2),0,             cos(angleP2)];
RY3 = [cos(angleY3),-sin(angleY3),0;
       sin(angleY3),cos(angleY3), 0;
       0,            0,            1];
R = RY3*RY2*RR1
trR = R(1,1)+R(2,2)+R(3,3);
% acos->arccos
theta = acos((trR-1)/2);
axis = [R(3,2)-R(2,3);
        R(1,3)-R(3,1);
        R(2,1)-R(1,2)];
axis = axis/(2*sin(theta));
theta= theta/pi*180;

if trR == 3
    fprintf("R=I\n");
elseif trR==-1
    fprintf("Singularity!\n")
end

end
```

实验例子

1)

```
>> [axis,theta]=rpy2exp(0,0,pi/3)

R =

    0.5000   -0.8660         0
    0.8660    0.5000         0
         0         0    1.0000


axis =

         0
         0
    1.0000


theta =

   60.0000
```

2)

```
>> [axis,theta]=rpy2exp(pi/4,pi/3,pi/6)

R =

    0.4330    0.1768    0.8839
    0.2500    0.9186   -0.3062
   -0.8660    0.3536    0.3536
```

```
axis =

    0.3525
    0.9350
    0.0391


theta =

   69.3559
```

exp2rpy

```
function [angleR1,angleP2,angleY3] = exp2rpy(axis,theta)
% please input angle in rad
% angleR1->Roll(X),angleP2->Pitch(Y),angleY3->Yaw(Z)
% number behind represents order
axis_h=[0,       -axis(3),axis(2);
        axis(3), 0,       -axis(1);
        -axis(2),axis(1), 0];
R=eye(3)+axis_h*sin(theta)+(axis_h^2)*(1-cos(theta))

angleP2 = atan2(-R(3,1),sqrt(R(3,2)^2+R(3,3)^2));
angleY3 = atan2(R(2,1)/cos(angleP2),R(1,1)/cos(angleP2));
angleR1 = atan2(R(3,2)/cos(angleP2),R(3,3)/cos(angleP2));

angleP2 = angleP2/pi*180;
angleY3 = angleY3/pi*180;
angleR1 = angleR1/pi*180;

if angleP2==90 || angleP2==-90
    fprintf("Singularity!\n")
end

end
```

实验例子

1)

```
>> [angleR1,angleP2,angleY3] = exp2rpy([0;0;1],60/180*pi)

R =

    0.5000   -0.8660         0
    0.8660    0.5000         0
         0         0    1.0000


angleR1 =

     0


angleP2 =
```

```
     0


angleY3 =

    60
```

2)

```
>> [angleR1,angleP2,angleY3] = exp2rpy([0.3525;0.9350;0.0391],69.3559/180*pi)

R =

    0.4330    0.1768    0.8839
    0.2500    0.9186   -0.3062
   -0.8660    0.3535    0.3535


angleR1 =

   44.9991


angleP2 =

   60.0013


angleY3 =

   29.9982
```

结果分析

分别使用`rpy2exp`和`exp2rpy`分别对两组实验数据进行验证，两函数所得结果相符，并且能够顺利识别出奇异角。