机器人学导论作业11-12

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作业 11-12.

1.

建立广义坐标

设 m位于x得

$$r_m(x,\theta) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + l\sin\theta \\ -l\cos\theta \end{bmatrix} \qquad r_M(x) = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$\dot{r}_m = \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \dot{x} + l\cos\theta\cdot\dot{\theta} \\ l\sin\theta\cdot\dot{\theta} \end{bmatrix} \qquad \dot{r}_M = \begin{bmatrix} \dot{x} \\ 0 \end{bmatrix}$$

$$T = \frac{1}{2}M\|\dot{r}_M\|^2 + \frac{1}{2}m\|\dot{r}_m\|^2$$

$$= \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left[(\dot{x} + l\cos\theta\cdot\dot{\theta})^2 + (l\sin\theta\cdot\dot{\theta})^2\right] = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left[\dot{x}^2 + l^2\dot{\theta}^2 + 2l\cos\theta\dot{\theta}\dot{x}\right]$$

$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + ml\cos\theta\cdot\dot{\theta}\cdot\dot{x}$$

$$V = -mgl\cos\theta$$

$$L(q,\dot{q}) = T - V$$

$$= \frac{1}{2}(M+m)\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 + ml\cos\theta\cdot\dot{\theta}\cdot\dot{x} + mgl\cos\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{d}{dt}\left[(M+m)\dot{x} + ml\cos\theta\cdot\dot{\theta}\right] = (M+m)\ddot{x} + ml\cos\theta\cdot\ddot{\theta} - ml\sin\theta\cdot\dot{\theta}^2$$

$$\frac{\partial L}{\partial x} = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{d}{dt}(ml^2\dot{\theta} + ml\cos\theta\,\dot{x}) = ml^2\ddot{\theta} + ml\cos\theta\,\ddot{x} - ml\sin\theta\cdot\dot{\theta}\cdot\dot{x}$$

$$\frac{\partial L}{\partial \theta} = -ml\sin\theta\cdot\dot{\theta}\cdot\dot{x} - mgl\sin\theta$$

运动方程 $= \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}} - \dfrac{\partial L}{\partial q} = \tau$

$$\Rightarrow \begin{bmatrix} M+m & ml\cos\theta \\ ml\cos\theta & ml^2 \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} -ml\sin\theta\cdot\dot{\theta}^2 \\ mgl\sin\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2.

$$I=\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz}\end{bmatrix} = -\int_V \rho(r)\,\hat{r}^2\,dV$$

$\frac{\cancel{4\pi}}{\cancel{15}} \cdot \frac{3m}{\cancel{4\pi}}$
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$$\hat{r}^2=\begin{bmatrix} -(y^2+z^2) & xy & xz \\ xy & -(x^2+z^2) & yz \\ xz & yz & -(x^2+y^2)\end{bmatrix}$$

椭球方程：$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 　　椭球体积 $V=\frac{4}{3}\pi abc$

归一化令 $X=\frac{x}{a},\ Y=\frac{y}{b},\ Z=\frac{z}{c}$

球坐标下 $\begin{cases} X=r\sin\theta\sin t & =\frac{x}{a} \\ Y=r\sin\theta\cos t & =\frac{y}{b} \\ Z=r\cos\theta & =\frac{z}{c}\end{cases} \Rightarrow \begin{cases} x = ar\sin\theta\sin t & =aX \\ y = br\sin\theta\cos t & =bY \\ z = cr\cos\theta & =cZ\end{cases}$

$r\in[0,1],\ t\in[0,2\pi],\ \theta\in[0,\pi]$

$\Rightarrow dXdYdZ = r\sin\theta\,dt\cdot r\,d\theta\cdot dr = r^2\sin\theta\,dr\,dt\,d\theta \quad = \frac{1}{abc}\,dx\,dy\,dz$

$$\rho=\frac{m}{V}=\frac{3m}{4\pi abc}$$

$$I_{xx}=\int_V \rho\,(y^2+z^2)\,dx\,dy\,dz \quad = abc\int_V \rho\,(b^2Y^2+c^2Z^2)\,dX\,dY\,dZ$$

$$= abc\int_V \rho\,(b^2r^2\sin^2\theta\cos^2 t + c^2r^2\cos^2\theta)\,r^2\sin\theta\,dr\,dt\,d\theta$$

$$= \frac{3m}{4\pi}\int_0^{2\pi}\int_0^{\pi}\int_0^1 (b^2r^4\sin^3\theta\cos^2 t + c^2r^4\sin\theta\cos^2\theta)\,dr\,d\theta\,dt$$

$$= \frac{3m}{4\pi}\int_0^{2\pi}\int_0^{\pi} (b^2\sin^3\theta\cos^2 t + c^2\sin\theta\cos^2\theta)\,\frac{r^5}{5}\Big|_0^1\,d\theta\,dt$$

$$= \frac{3m}{20\pi}\int_0^{2\pi}\int_0^{\pi} (b^2\sin^3\theta\cos^2 t + c^2\sin\theta - c^2\sin^3\theta)\,d\theta\,dt$$

$$= \frac{3m}{20\pi}\int_0^{2\pi} [(b^2\cos^2 t - c^2)\frac{4}{3} + 2c^2]\,dt$$

$$= \frac{3m}{20\pi}\int_0^{4\pi} (b^2\,\frac{1+\cos 2t}{2} - c^2)\frac{4}{3}+2c^2)\,\frac{d2t}{2}$$

$$= \frac{3m}{20\pi}[(\frac{b^2}{2}-c^2)\frac{4}{3}4\pi + 8c^2\pi]\cdot\frac{1}{2}$$

$$= \frac{m}{5}(b^2+c^2)$$

$\int \sin^3\theta\cdot d\theta$
$=\int \sin^2\theta\cdot\sin\theta\cdot d\theta$
$=\int (1-\cos^2\theta)\cdot -d\cos\theta$
$=\int\cos^2\theta\,d\cos\theta - \int d\cos\theta$
$=\frac{1}{3}\cos^3\theta - \cos\theta$

同理

$$I_{yy} = \frac{m}{5}(a^2+c^2)$$

$$I_{zz} = \int_V \rho (x^2+y^2)\,dx\,dy\,dz = abc\int_V \rho (a^2X^2+b^2Y^2)\,dX\,dY\,dZ$$

$$= abc\int_V \rho (a^2r^2\sin^2\theta\sin^2t + b^2r^2\sin^2\theta\cos^2t)\,r^2\sin\theta\,dr\,dt\,d\theta$$

$$= \frac{3m}{20\pi}\int_0^{2\pi}\int_0^{\pi}(a^2\sin^2t+b^2\cos^2t)\sin^3\theta\,d\theta\,dt$$

$$= \frac{3m}{20\pi}\int_0^{2\pi}(a\sin^2t+b^2\cos^2t)\cdot\frac{4}{3}\,dt$$

$$= \frac{m}{5\pi}\int_0^{4\pi}\left[b^2+(a^2-b^2)\frac{1-\cos 2t}{2}\right]\frac{d2t}{2}$$

$$= \frac{m}{10\pi}\left[b^2 4\pi + (a^2-b^2)\frac{1}{2}\cdot 4\pi\right]$$

$$= \frac{m}{5}(a^2+b^2)$$

$$I_{xy} = \int_V \rho xy\,dx\,dy\,dz = abc\int_V \rho\, ab r^2\sin^2\theta\sin t\cos t\cdot r^2\sin\theta\,dr\,dt\,d\theta$$

$$= \frac{3mab}{4\pi}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\sin t\cos t\,d\theta\,dt$$

$$= \frac{3mab}{2\pi}\int_0^{4\pi}\frac{1}{2}\sin 2t\,\frac{d2t}{2}$$

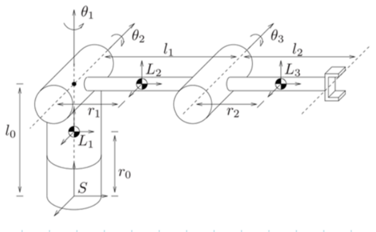
$$= 0$$

同理 $I_{xz} = I_{yz} = 0$

得椭球惯性张量

$$I = \frac{m}{5}\begin{bmatrix} b^2+c^2 & 0 & 0 \\ 0 & a^2+c^2 & 0 \\ 0 & 0 & a^2+b^2 \end{bmatrix}$$

3.



开链运动方程 $M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+N(\theta)=\tau$

① $M(\theta)=\sum_i J_i^T(\theta)M_i^b J_i(\theta)$

$$M_i=\begin{bmatrix} m_i & & & & & \\ & m_i & & & & \\ & & m_i & & & \\ & & & I_{xi} & & \\ & & & & I_{yi} & \\ & & & & & I_{zi} \end{bmatrix}$$

$J_i(\theta)=[\xi_1^\dagger,\cdots,\xi_i^\dagger,0,\cdots,0]=J_{sl_i}^b(\theta)$

$\omega_1=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $\omega_2=\begin{bmatrix}-1\\0\\0\end{bmatrix}=\omega_3$, $q_1=\begin{bmatrix}0\\0\\0\end{bmatrix}$, $q_2=\begin{bmatrix}0\\0\\l_0\end{bmatrix}$, $q_3=\begin{bmatrix}0\\l_1\\l_0\end{bmatrix}$

$\xi_1=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}$, $\xi_2=\begin{bmatrix}0\\-l_0\\0\\-1\\0\\0\end{bmatrix}$, $\xi_3=\begin{bmatrix}0\\-l_0\\l_1\\-1\\0\\0\end{bmatrix}$

$J_{sl_i}^b(\theta)=[\xi_1^\dagger,\xi_2^\dagger,\cdots,\xi_i^\dagger,0,\cdots,0]$

$\xi_j^\dagger=Ad^{-1}_{(e^{\hat{\xi}_{j+1}\theta_{j+1}}\cdots e^{\hat{\xi}_i\theta_i}g_{sl_i}(0))}\xi_i$, $i\le j$

$\Rightarrow J_{sl_1}^b(\theta)=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\0&0&0\\0&0&0\\1&0&0\end{bmatrix}$ $J_{sl_2}^b=\begin{bmatrix}-r_1c_2&0&0\\0&0&0\\0&-r_1&0\\0&-1&0\\-s_2&0&0\\c_2&0&0\end{bmatrix}$ $J_{sl_3}^b=\begin{bmatrix}-r_2c_3-l_1c_2&0&0\\0&l_1s_3&0\\0&-r_2-l_1c_3&-r_2\\0&0&-1\\-s_{23}&0&0\\c_{23}&0&0\end{bmatrix}$

$$M(\theta) = J_1^T M_1 J_1 + J_2^T M_2 J_2 + J_3^T M_3 J_3$$

$$= \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11} = I_{z1} + m_3(r_2 C_{23} + l_1 C_2)^2 + I_{z3} C_{23}^2 + I_{y3} S_{23}^2 + I_{z2} C_2^2 + I_{y2} S_2^2 + m_2 r_1^2 C_2^2$$

$$M_{22} = m_3 l_1^2 + 2m_3 C_3 l_1 r_2 + m_2 r_1^2 + m_3 r_2^2 + I_{x2} + I_{x3}$$

$$M_{33} = m_3 r_2^2 + I_{x3}$$

$$M_{23} = M_{32} = I_{x3} + m_3 r_2 (r_2 + l_1 C_3)$$

$$M_{12} = M_{21} = M_{31} = M_{13} = 0$$

② $$C_{ij}(\theta, \dot\theta) = \sum_{k=1}^{n} \Gamma_{ijk} \dot\theta_k = \frac{1}{2} \sum_{k=1}^{n} \left(\frac{\partial M_{ij}}{\partial \theta_k} + \frac{\partial M_{ik}}{\partial \theta_j} - \frac{\partial M_{kj}}{\partial \theta_i} \right) \dot\theta_k$$

非零的 Γ_{ijk}：

$$\Gamma_{112} = (I_{y2} - I_{z2} - m_2 r_1^2) C_2 S_2 + (I_{y3} - I_{z3}) C_{23} S_{23} - m_3 (l_1 C_2 + r_2 C_{23})(l_1 S_2 + r_2 S_{23})$$

$$\Gamma_{113} = (I_{y3} - I_{z3}) C_{23} S_{23} - m_3 r_2 S_{23} (l_1 C_2 + r_2 C_{23})$$

$$\Gamma_{121} = (I_{y2} - I_{z2} - m_2 r_1^2) C_2 S_2 + (I_{y3} - I_{z3}) C_{23} S_{23} - m_3 (l_1 C_2 + r_2 C_{23})(l_1 S_2 + r_2 S_{23})$$

$$\Gamma_{131} = (I_{y3} - I_{z3}) C_{23} S_{23} - m_3 r_2 S_{23} (l_1 C_2 + r_2 C_{23})$$

$$\Gamma_{211} = (I_{z2} - I_{y2} - m_2 r_1^2) C_2 S_2 + (I_{z3} - I_{y3}) C_{23} S_{23} + m_3 (l_1 C_2 + r_2 C_{23})(l_1 S_2 + r_2 S_{23})$$

$$\Gamma_{223} = -l_1 m_3 r_2 S_3$$

$$\Gamma_{232} = -l_1 m_3 r_2 S_3$$

$$\Gamma_{233} = -l_1 m_3 r_2 S_3$$

$$\Gamma_{311} = (I_{z3} - I_{y3}) C_{23} S_{23} + m_3 r_2 S_{23} (l_1 C_2 + r_2 C_{23})$$

$$\Gamma_{322} = l_1 m_3 r_2 S_3$$

③ $N(\theta)=\frac{\partial V}{\partial \theta}$

$V(\theta)=m_1 g h_1(\theta)+m_2 g h_2(\theta)+m_3 g h_3(\theta)$

$\because g_{sl_i}(\theta)=e^{\hat{\xi}_1\theta_1}\cdots e^{\hat{\xi}_i\theta_i}g_{sl_i}(0)$

$\Rightarrow h_1(\theta)=r_0$

$h_2(\theta)=l_0-r_1\sin\theta_2$

$h_3(\theta)=l_0-l_1\sin\theta_2-r_2\sin(\theta_2+\theta_3)$

$$\Rightarrow N(\theta)=\frac{\partial V}{\partial \theta}=\begin{bmatrix}0\\-(m_2 g r_1+m_3 g l_1)\cos\theta_2-m_3 g r_2\cos(\theta_2+\theta_3)\\-m_3 g r_2\cos(\theta_2+\theta_3)\end{bmatrix}$$

推导完成

综上 $M(\theta,\dot{\theta})\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+N(\theta)=\tau$

Matlab

```
syms l0 l1 l2 r0 r1 r2 theta1 theta2 theta3 real
syms m1 m2 m3 g Ix1 Iy1 Iz1 Ix2 Iy2 Iz2 Ix3 Iy3 Iz3 real

w1=[0;0;1];
w2=[-1;0;0];w3=w2;

q1=[0;0;0];
q2=[0;0;l0];
q3=[0;l1;l0];

v1 = -cross(w1,q1);
s1=[v1;w1]
v2 = -cross(w2,q2);
s2=[v2;w2]
v3 = -cross(w3,q3);
s3=[v3;w3]

expw1=angvec2r(theta1,w1);
expw2=angvec2r(theta2,w2);
expw3=angvec2r(theta3,w3);
```

```
exps1=[expw1,(eye(3)-expw1)*cross(w1,v1)+w1*w1'*v1*theta1;0,0,0,1];
exps2=[expw2,(eye(3)-expw2)*cross(w2,v2)+w2*w2'*v2*theta2;0,0,0,1];
exps3=[expw3,(eye(3)-expw3)*cross(w3,v3)+w3*w3'*v3*theta3;0,0,0,1];

gsl10=[eye(3),[0;0;r0];0,0,0,1];
s1_J1=Adjoint_ginv(gsl10)*s1;
J1 = [s1_J1,zeros(6,1),zeros(6,1)]

gsl20=[eye(3),[0;r1;l0];0,0,0,1];
s1_J2=Adjoint_ginv(exps2*gsl20)*s1;
s1_J2 = simplify(s1_J2);
s2_J2=Adjoint_ginv(gsl20)*s2;
J2 = [s1_J2,s2_J2,zeros(6,1)]

gsl30=[eye(3),[0;l1+r2;l0];0,0,0,1];
s1_J3=Adjoint_ginv(exps2*exps3*gsl30)*s1;
s1_J3 = simplify(s1_J3);
s2_J3=Adjoint_ginv(exps3*gsl30)*s2;
s2_J3=simplify(s2_J3);
s3_J3=Adjoint_ginv(gsl30)*s3;
s3_J3=simplify(s3_J3);
J3 = [s1_J3,s2_J3,s3_J3]

M1 = [m1,0,0,0,0,0;
    0,m1,0,0,0,0;
    0,0,m1,0,0,0;
    0,0,0,Ix1,0,0;
    0,0,0,0,Iy1,0;
    0,0,0,0,0,Iz1];
M2 = [m2,0,0,0,0,0;
    0,m2,0,0,0,0;
    0,0,m2,0,0,0;
    0,0,0,Ix2,0,0;
    0,0,0,0,Iy2,0;
    0,0,0,0,0,Iz2];
M3 = [m3,0,0,0,0,0;
    0,m3,0,0,0,0;
    0,0,m3,0,0,0;
    0,0,0,Ix3,0,0;
    0,0,0,0,Iy3,0;
    0,0,0,0,0,Iz3];

M = J1'*M1*J1 + J2'*M2*J2 + J3'*M3*J3;
M = simplify(M)

M = mat2cell(M,[1,1,1],[1,1,1]);

Gamma = sym(zeros(27,1));
t(1) = theta1;
t(2) = theta2;
t(3) = theta3;
for i=1:3
    for j=1:3
        for k=1:3
            n = (i-1)*9+(j-1)*3+(k-1)+1;
            fprintf("i:%i, j:%i, k:%i",i,j,k);
            Gamma(n,1) = simplify(1/2*(diff(M{i,j},t(k))+diff(M{i,k},t(j))-diff(M{k,j},t(i))));
        end
    end
end
Gamma

h1=r0;
h2=l0-r1*sin(theta2);
h3=l0-l1*sin(theta2)-r2*sin(theta2+theta3);
V = m1*g*h1+m2*g*h2+m3*g*h3;
N = [diff(V,theta1);diff(V,theta2);diff(V,theta3)]
```

输出

```
s1 = 6×1
     0
     0
     0
     0
     0
     1
```

s2 =

$$\begin{pmatrix} 0 \\ -l_0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

s3 =

$$\begin{pmatrix} 0 \\ -l_0 \\ l_1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

J1 =

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

J2 =

$$\begin{pmatrix} -r_1\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_1 & 0 \\ 0 & -1 & 0 \\ -\sin(\theta_2) & 0 & 0 \\ \cos(\theta_2) & 0 & 0 \end{pmatrix}$$

J3 =

$$\begin{pmatrix} -r_2\cos(\theta_2+\theta_3) - l_1\cos(\theta_2) & 0 & 0 \\ 0 & l_1\sin(\theta_3) & 0 \\ 0 & -r_2 - l_1\cos(\theta_3) & -r_2 \\ 0 & -1 & -1 \\ -\sin(\theta_2+\theta_3) & 0 & 0 \\ \cos(\theta_2+\theta_3) & 0 & 0 \end{pmatrix}$$

$M(\theta) =$

$$\left(\begin{matrix} \mathrm{Iz}_1 + m_3\left(r_2\cos\left(\theta_2+\theta_3\right) + l_1\cos\left(\theta_2\right)\right)^2 + \mathrm{Iz}_3\cos\left(\theta_2+\theta_3\right)^2 + \mathrm{Iy}_3\sin\left(\theta_2+\theta_3\right)^2 + \mathrm{Iz}_2\cos\left(\theta_2\right)^2 + \mathrm{Iy}_2\sin\left(\theta_2\right)^2 + m_2\,{r_1}^2\cos\left(\theta_2\right)^2 & \\ 0 & m_3\,{l_1}^2 + 2\,m_3\cos\left(\right. \\ 0 & \end{matrix}\right.$$

where

$$\sigma_1 = \mathrm{Ix}_3 + m_3\,r_2\left(r_2 + l_1\cos\left(\theta_3\right)\right)$$

$\Gamma(\theta)(n,1) =$

$$\left(\begin{array}{c} 0 \\ \sigma_3 \\ -\sigma_1 \\ \sigma_3 \\ 0 \\ 0 \\ -\sigma_1 \\ 0 \\ 0 \\ \sigma_6+\sigma_7+\sigma_5+\sigma_4-\frac{\mathrm{Iy}_3\,\sigma_8}{2}+\frac{\mathrm{Iz}_3\,\sigma_8}{2}-\frac{\mathrm{Iy}_2\,\sin(2\,\theta_2)}{2}+\frac{\mathrm{Iz}_2\,\sin(2\,\theta_2)}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_2 \\ 0 \\ \sigma_2 \\ \sigma_2 \\ \sigma_1 \\ 0 \\ 0 \\ 0 \\ l_1\,m_3\,r_2\,\sin\left(\theta_3\right) \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right) \quad **$$

where

$$\sigma_1=\sin\left(\theta_2+\theta_3\right)\,\left(m_3\cos\left(\theta_2+\theta_3\right)\,{r_2}^2+l_1\,m_3\cos\left(\theta_2\right)\,r_2-\mathrm{Iy}_3\cos\left(\theta_2+\theta_3\right)+\mathrm{Iz}_3\cos\left(\theta_2+\theta_3\right)\right)$$

$$\sigma_2=-l_1\,m_3\,r_2\,\sin\left(\theta_3\right)$$

$$\sigma_3=-\sigma_6-\sigma_7-\sigma_5-\sigma_4+\frac{\mathrm{Iy}_3\,\sigma_8}{2}-\frac{\mathrm{Iz}_3\,\sigma_8}{2}+\frac{\mathrm{Iy}_2\,\sin(2\,\theta_2)}{2}-\frac{\mathrm{Iz}_2\,\sin(2\,\theta_2)}{2}$$

$$\sigma_4=\frac{m_3\,\sigma_8\,{r_2}^2}{2}$$

$$\sigma_5=\frac{m_2\,\sin(2\,\theta_2)\,{r_1}^2}{2}$$

$$\sigma_6=\frac{m_3\,\sin(2\,\theta_2)\,{l_1}^2}{2}$$

$$\sigma_7=m_3\sin\left(2\,\theta_2+\theta_3\right)\,l_1\,r_2$$

$$\sigma_8=\sin\left(2\,\theta_2+2\,\theta_3\right)$$

`n = (i-1)*9+(j-1)*3+(k-1)+1`

$N(\theta)=$

$$\left(\begin{array}{c} 0 \\ -g\,m_3\,\left(r_2\cos\left(\theta_2+\theta_3\right)+l_1\cos\left(\theta_2\right)\right)-g\,m_2\,r_1\cos\left(\theta_2\right) \\ -g\,m_3\,r_2\cos\left(\theta_2+\theta_3\right) \end{array}\right)$$

经比对，matlab输出数据与结果一致