机器人学导论作业5-6

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作业5-6，

1.

1(a)

$$g_{01}(\theta_1)=\begin{bmatrix} e^{\hat{z}\theta_1} & \begin{matrix}0\\0\\l_0\end{matrix} \\ \begin{matrix}0 & 0 & 0\end{matrix} & 1 \end{bmatrix}$$

$$g_{12}(\theta_2)=\begin{bmatrix} e^{\hat{z}\theta_2} & \begin{matrix}0\\0\\l_2\end{matrix} \\ \begin{matrix}0 & 0 & 0\end{matrix} & 1 \end{bmatrix}$$ （末端无关节角度）

$$g_{23}=\begin{bmatrix} I & \begin{matrix}0\\l_2\\0\end{matrix} \\ \begin{matrix}0 & 0 & 0\end{matrix} & 1 \end{bmatrix}$$

$$g_{03}(\theta)=g_{01}g_{12}g_{23}=\begin{bmatrix} e^{\hat{z}(\theta_1+\theta_2)} & \begin{matrix}-l_1s_1\\l_1c_1\\l_0+l_3\end{matrix} \\ \begin{matrix}0 & 0 & 0\end{matrix} & 1 \end{bmatrix} g_{23}=\begin{bmatrix} e^{\hat{z}(\theta_1+\theta_2)} & \begin{matrix}-s_{12}l_2-l_1s_1\\c_{12}l_2+l_1c_1\\l_0+l_3\end{matrix} \\ \begin{matrix}0 & 0 & 0\end{matrix} & 1 \end{bmatrix}$$

0, 1, 2, 3 → c_0, c_1, c_2, c_3

$c_{12}=\cos(\theta_1+\theta_2)$

$s_{12}=\sin(\theta_1+\theta_2)$

1(b)

$$V_{03}^s=Ag_{23}(V_{01}^s+Ag_{01}V_{12}^s)=Ag_{23}V_{02}^s$$

$V_{03}^s=V_{01}^s+Ad_{01}V_{12}^s$

$V_{12}^s=V_{12}^s+Ad_{12}V_{23}^s$

$V_{ab}^s=\xi\dot{\theta}$，$\xi=\begin{bmatrix}-\omega\times q\\ \omega\end{bmatrix}$

$$Adg_{ab}=\begin{bmatrix} R_{ab} & \hat{p}_{ab}R_{ab} \\ 0 & R_{ab}\end{bmatrix}$$

$$\omega_{01}=\omega_{12}=z=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$q_{01}=\begin{bmatrix}0\\0\\0\end{bmatrix},\quad q_{12}=\begin{bmatrix}0\\l_1\\0\end{bmatrix}$$

$$\xi_{01}=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\quad \xi_{12}=\begin{bmatrix}l_1\\0\\0\\0\\0\\1\end{bmatrix}$$

$$\Rightarrow V_{01}^s=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\dot{\theta}_1,\quad V_{02}^s=\begin{bmatrix}l_1\\0\\0\\0\\0\\1\end{bmatrix}\dot{\theta}_2$$

$$Adg_{01}=\begin{bmatrix} e^{\hat{z}\theta_1} & \begin{bmatrix}0&0&l_1\\0&0&0\\-l_1&0&0\end{bmatrix}e^{\hat{z}\theta_1} \\ 0 & e^{\hat{z}\theta_1}\end{bmatrix}=\begin{bmatrix} e^{\hat{z}\theta_1} & \begin{bmatrix}0&0&l_1\\0&0&0\\-l_1&l_1s_1&0\end{bmatrix} \\ 0 & e^{\hat{z}\theta_1}\end{bmatrix}$$

$$\Rightarrow V_{02}^s=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\dot{\theta}_1+\begin{bmatrix}l_1c_1\\l_1s_1\\0\\0\\0\\1\end{bmatrix}\dot{\theta}_2=\begin{bmatrix}l_1c_1\dot{\theta}_2\\l_1s_1\dot{\theta}_2\\0\\0\\0\\\dot{\theta}_1+\dot{\theta}_2\end{bmatrix}$$

$$Ag_{23}=\begin{bmatrix} I & \begin{bmatrix}0&0&l_2\\0&0&0\\-l_2&0&0\end{bmatrix} \\ 0 & I\end{bmatrix}$$

$$V_{03}^s=Ag_{23}V_{02}^s=\begin{bmatrix}l_1c_1\dot{\theta}_2+l_2(\dot{\theta}_1+\dot{\theta}_2)\\l_1s_1\dot{\theta}_2\\0\\0\\0\\\dot{\theta}_1+\dot{\theta}_2\end{bmatrix}$$

1(c) $V_{03}^s=Adg_{03}V_{03}^b$

$$\Rightarrow V_{03}^b=Adg_{03}^{-1}V_{03}^s$$

$$=Adg_{03}^{T}V_{03}^s$$

$$Adg_{03}^{-1}=\begin{bmatrix} R_{03}^T & -R_{03}^T\hat{p}_{03} \\ 0 & R_{03}^T\end{bmatrix}$$

$$-R_{03}^T\hat{p}_{03}=\begin{bmatrix}-(l_0+l_3)s_{12} & (l_1+l_3)c_{12} & -l_2\cos(2\theta_1+2\theta_2)-l_1\cos(2\theta_1+\theta_2)\\ -(l_0+l_3)c_{12} & -(l_0+l_3)s_{12} & l_2\sin(2\theta_1+2\theta_2)+l_1\sin(2\theta_1+\theta_2)\\ -l_1c_1-c_{12}l_2 & l_1s_1+l_2s_{12} & 0\end{bmatrix}$$

$$V_{03}^s=V_{02}^s+Adg_{02}V_{23}^s=V_{01}^s+Adg_{01}V_{12}^s+Adg_{02}V_{23}^s$$

$$V_{03}^{b} = \begin{bmatrix} v_{11} \\ v_{12} \\ 0 \\ 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$v_{11} = C_{12}[L_2(\dot{\theta}_1+\dot{\theta}_2) + \dot{\theta}_2 L_1 C_1] + \dot{\theta}_2 L_1 C_1 - [L_2 \cos(2\theta_1+2\theta_2) + L_1 \cos(2\theta_1+\theta_2)](\dot{\theta}_1+\dot{\theta}_2) + \dot{\theta}_2 L_1 S_{12} S_1$$

$$v_{12} = [L_2 \sin(2\theta_1+2\theta_2) + L_1 \sin(2\theta_1+\theta_2)](\dot{\theta}_1+\dot{\theta}_2) - S_{12}[L_2(\dot{\theta}_1+\dot{\theta}_2) + \dot{\theta}_2 L_1 C_1] + \dot{\theta}_2 L_1 C_{12} S_1$$

2.

$$R_{bc} = R_z(-70^\circ) R_y(110^\circ) = \begin{bmatrix} 0.94 & 0.34 & 0 \\ -0.34 & 0.94 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.34 & 0 & -0.94 \\ 0 & 1 & 0 \\ 0.94 & 0 & -0.34 \end{bmatrix} = \begin{bmatrix} -0.32 & 0.34 & -0.88 \\ 0.12 & 0.94 & 0.32 \\ 0.94 & 0 & -0.34 \end{bmatrix}$$

$$p_{bc} = [7 \quad -2 \quad 5]^T$$

$$\Rightarrow g_{bc} = \begin{bmatrix} R_{bc} & p_{bc} \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \bar{q}_b = g_{bc} \bar{q}_c$$

$$= \begin{bmatrix} -0.32 & 0.34 & -0.88 & 7 \\ 0.12 & 0.94 & 0.32 & -2 \\ 0.94 & 0 & -0.34 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.2 \\ 3.2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.092 \\ -0.728 \\ 4.382 \\ 1 \end{bmatrix}$$

3.

$$q_{a(0)} = g_{ab} q_{b(0)} = [-5.485, -6.5043, 7.8655, 1]^T$$

$$\bar{v}_{qa} = g_{ab} \bar{v}_{qb} = [0.1886, 1.7223, -0.8536, 0]^T$$

$$q_{a(5)} = q_{a(0)} + v_{qa} \cdot 5 = \begin{bmatrix} -4.8886 \\ 2.1073 \\ 3.5975 \end{bmatrix}$$

$g = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$

fix: $\omega_a \times p_a$, $R \to p$

body: $\omega_b \times p_b$, $p \to R$

坐标系变化（非点变换!!!）

先旋转再平移（可先旋转完再平移）

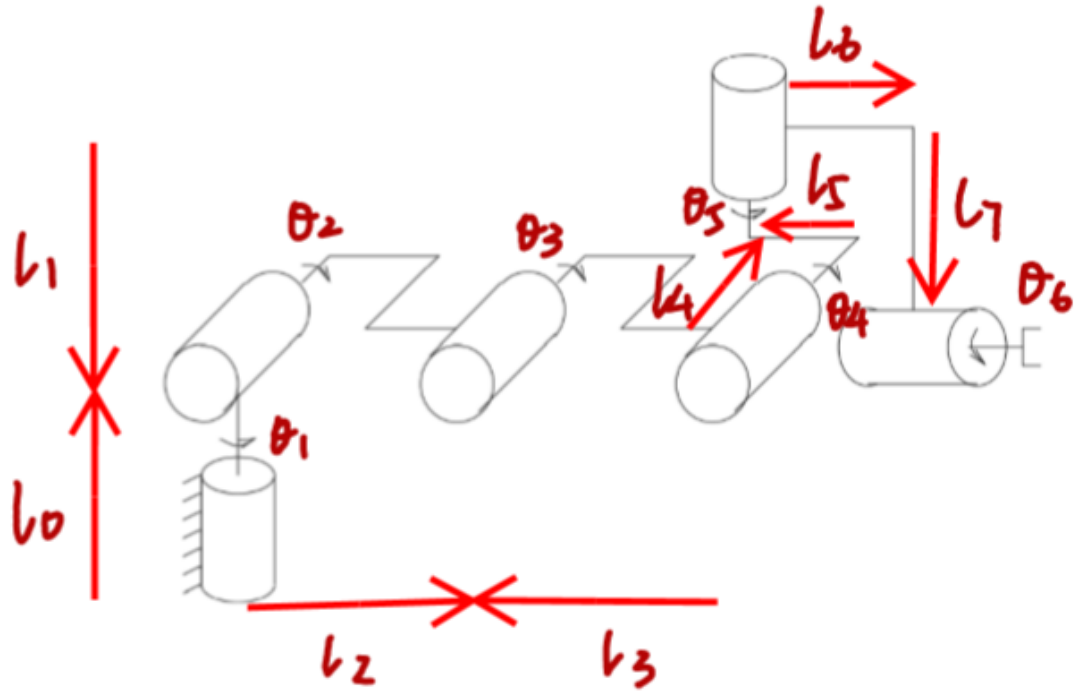
先平移再旋转（就是得按顺序）

∵ body frame 一直变，对应的 p_b 对于 frame a 也一直变

4.

(i)

假设变量如图



(i) Elbow manipulator

matlab

```
clear;
clc;
syms theta1 theta2 theta3 theta4 theta5 theta6;
syms l0 l1 l2 l3 l4 l5 l6 l7;

gst0=[rotx(-90),[0;l3+l4+l5;l1];
        0,0,0,1];

w1=[0;0;1];w5=w1;
w2=[1;0;0];w3=w2;w4=w2;
w6=[0;1;0];

q1=[0;0;0];
q2=[0;0;l0];
q3=[0;l2;l0];
q4=[0;l2+l3;l0];
q5=[-l4;l2+l3-l5;0];
q6=[-l4;l2+l3-l5+l6;l0+l1-l7];

v1 = -cross(w1,q1);
s1=[v1;w1];
v2 = -cross(w2,q2);
s2=[v2;w2];
v3 = -cross(w3,q3);
s3=[v3;w3];
v4 = -cross(w4,q4);
s4=[v4;w4];
v5 = -cross(w5,q5);
s5=[v5;w5];
v6 = -cross(w6,q6);
s6=[v6;w6];
```

```
expw1=angvec2r(theta1,w1);
expw2=angvec2r(theta2,w2);
expw3=angvec2r(theta3,w3);
expw4=angvec2r(theta4,w4);
expw5=angvec2r(theta5,w5);
expw6=angvec2r(theta6,w6);


exps1=[expw1,(eye(3)-expw1)*cross(w1,v1)+w1*w1'*v1*theta1;0,0,0,1];
exps2=[expw2,(eye(3)-expw2)*cross(w2,v2)+w2*w2'*v2*theta2;0,0,0,1];
exps3=[expw3,(eye(3)-expw3)*cross(w3,v3)+w3*w3'*v3*theta3;0,0,0,1];
exps4=[expw4,(eye(3)-expw4)*cross(w4,v4)+w4*w4'*v4*theta4;0,0,0,1];
exps5=[expw5,(eye(3)-expw5)*cross(w5,v5)+w5*w5'*v5*theta5;0,0,0,1];
exps6=[expw6,(eye(3)-expw6)*cross(w6,v6)+w6*w6'*v6*theta6;0,0,0,1];

gst_theta=exps1*exps2*exps3*exps4*exps5*exps6*gst0;
gst_theta=simplify(gst_theta)
```

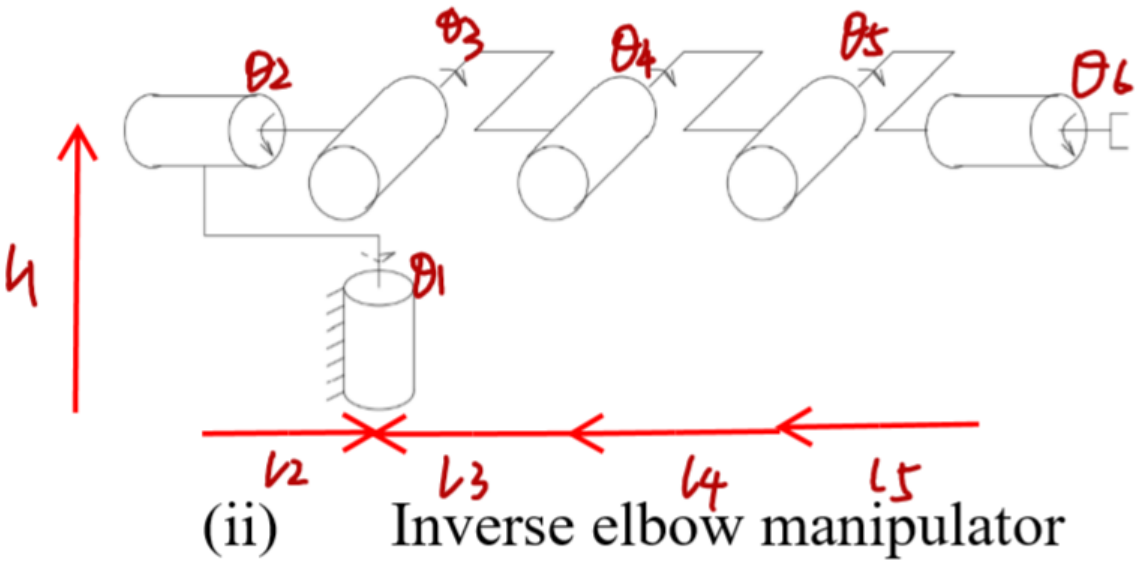
结果

```
gst_theta =

[ cos(theta6)*(cos(theta1)*cos(theta5) - cos(theta2 + theta3 +
theta4)*sin(theta1)*sin(theta5)) - sin(theta2 + theta3 +
theta4)*sin(theta1)*sin(theta6), - sin(theta6)*(cos(theta1)*cos(theta5) -
cos(theta2 + theta3 + theta4)*sin(theta1)*sin(theta5)) - sin(theta2 + theta3 +
theta4)*cos(theta6)*sin(theta1), - cos(theta1)*sin(theta5) - cos(theta2 + theta3
+ theta4)*cos(theta5)*sin(theta1), l1*(sin(theta2 + theta3 +
theta4)*cos(theta6)*sin(theta1) + cos(theta1)*cos(theta5)*sin(theta6) -
cos(theta2 + theta3 + theta4)*sin(theta1)*sin(theta5)*sin(theta6)) -
(cos(theta1)*sin(theta5) + cos(theta2 + theta3 +
theta4)*cos(theta5)*sin(theta1))*(l3 + l4 + l5) + (l4*(cos(theta6) - 1) -
sin(theta6)*(l0 + l1 - l7))*(cos(theta1)*cos(theta5) - cos(theta2 + theta3 +
theta4)*sin(theta1)*sin(theta5)) + cos(theta1)*(l4*(cos(theta5) - 1) +
sin(theta5)*(l2 + l3 - l5)) + cos(theta2 + theta3 + theta4)*sin(theta1)*
((cos(theta5) - 1)*(l2 + l3 - l5) - l4*sin(theta5)) - sin(theta2 + theta3 +
theta4)*sin(theta1)*((cos(theta6) - 1)*(l0 + l1 - l7) + l4*sin(theta6)) -
cos(theta2 + theta3)*sin(theta1)*(l2 + l3 - l2*cos(theta4) - l3*cos(theta4) +
l0*sin(theta4)) - sin(theta2 + theta3)*sin(theta1)*(l0*cos(theta4) - l0 +
l2*sin(theta4) + l3*sin(theta4)) - cos(theta2)*sin(theta1)*(l2 - l2*cos(theta3)
+ l0*sin(theta3)) - l0*sin(theta1)*sin(theta2) - sin(theta1)*sin(theta2)*
(l0*cos(theta3) - l0 + l2*sin(theta3))]
[ cos(theta6)*(cos(theta5)*sin(theta1) + cos(theta2 + theta3 +
theta4)*cos(theta1)*sin(theta5)) + sin(theta2 + theta3 +
theta4)*cos(theta1)*sin(theta6),   sin(theta2 + theta3 +
theta4)*cos(theta1)*cos(theta6) - sin(theta6)*(cos(theta5)*sin(theta1) +
cos(theta2 + theta3 + theta4)*cos(theta1)*sin(theta5)),   cos(theta2 + theta3 +
theta4)*cos(theta1)*cos(theta5) - sin(theta1)*sin(theta5), l1*
(cos(theta5)*sin(theta1)*sin(theta6) - sin(theta2 + theta3 +
theta4)*cos(theta1)*cos(theta6) + cos(theta2 + theta3 +
theta4)*cos(theta1)*sin(theta5)*sin(theta6)) - (sin(theta1)*sin(theta5) -
cos(theta2 + theta3 + theta4)*cos(theta1)*cos(theta5))*(l3 + l4 + l5) + (l4*
(cos(theta6) - 1) - sin(theta6)*(l0 + l1 - l7))*(cos(theta5)*sin(theta1) +
cos(theta2 + theta3 + theta4)*cos(theta1)*sin(theta5)) + sin(theta1)*(l4*
(cos(theta5) - 1) + sin(theta5)*(l2 + l3 - l5)) - cos(theta2 + theta3 +
theta4)*cos(theta1)*((cos(theta5) - 1)*(l2 + l3 - l5) - l4*sin(theta5)) +
sin(theta2 + theta3 + theta4)*cos(theta1)*((cos(theta6) - 1)*(l0 + l1 - l7) +
l4*sin(theta6)) + cos(theta2 + theta3)*cos(theta1)*(l2 + l3 - l2*cos(theta4) -
l3*cos(theta4) + l0*sin(theta4)) + sin(theta2 + theta3)*cos(theta1)*
(l0*cos(theta4) - l0 + l2*sin(theta4) + l3*sin(theta4)) +
cos(theta1)*cos(theta2)*(l2 - l2*cos(theta3) + l0*sin(theta3)) +
l0*cos(theta1)*sin(theta2) + cos(theta1)*sin(theta2)*(l0*cos(theta3) - l0 +
l2*sin(theta3))]
```

```
[                                                    sin(theta2 + theta3 +
theta4)*cos(theta6)*sin(theta5) - cos(theta2 + theta3 + theta4)*sin(theta6),
                                               - cos(theta2 + theta3 +
theta4)*cos(theta6) - sin(theta2 + theta3 + theta4)*sin(theta5)*sin(theta6),
                                   sin(theta2 + theta3 + theta4)*cos(theta5),


            l1*(cos(theta2 + theta3)*cos(theta4)*cos(theta6) - sin(theta2 +
theta3)*cos(theta6)*sin(theta4) + cos(theta2 +
theta3)*sin(theta4)*sin(theta5)*sin(theta6) + sin(theta2 +
theta3)*cos(theta4)*sin(theta5)*sin(theta6)) - cos(theta2 + theta3 + theta4)*
((cos(theta6) - 1)*(l0 + l1 - l7) + l4*sin(theta6)) - sin(theta2 + theta3 +
theta4)*((cos(theta5) - 1)*(l2 + l3 - l5) - l4*sin(theta5)) - cos(theta2 +
theta3)*(l0*cos(theta4) - l0 + l2*sin(theta4) + l3*sin(theta4)) - l0*
(cos(theta2) - 1) + sin(theta2 + theta3)*(l2 + l3 - l2*cos(theta4) -
l3*cos(theta4) + l0*sin(theta4)) + sin(theta2)*(l2 - l2*cos(theta3) +
l0*sin(theta3)) - cos(theta2)*(l0*cos(theta3) - l0 + l2*sin(theta3)) +
sin(theta2 + theta3 + theta4)*cos(theta5)*(l3 + l4 + l5) + sin(theta2 + theta3 +
theta4)*sin(theta5)*(l4*(cos(theta6) - 1) - sin(theta6)*(l0 + l1 - l7))]
[
                                                                      0,

                                                              0,
                                                                0,










                            1]
```

(ii)

假设变量如图



(ii) Inverse elbow manipulator

matlab

```
clear;
clc;
syms theta1 theta2 theta3 theta4 theta5 theta6;
syms l1 l2 l3 l4 l5 l6;

gst0=[rotx(-90),[0;l3+l4+l5;l1];
        0,0,0,1];

w1=[0;0;1];
w2=[0;1;0];w6=w2;
w3=[1;0;0];w4=w3;w5=w3;

q1=[0;0;0];
q2=[0;0;l1];
q3=[0;0;l1];
q4=[0;l3;l1];
q5=[0;l3+l4;l1];
q6=[0;0;l1];

v1 = -cross(w1,q1);
s1=[v1;w1];
v2 = -cross(w2,q2);
s2=[v2;w2];
v3 = -cross(w3,q3);
s3=[v3;w3];
v4 = -cross(w4,q4);
s4=[v4;w4];
v5 = -cross(w5,q5);
s5=[v5;w5];
v6 = -cross(w6,q6);
s6=[v6;w6];

expw1=angvec2r(theta1,w1);
expw2=angvec2r(theta2,w2);
expw3=angvec2r(theta3,w3);
expw4=angvec2r(theta4,w4);
expw5=angvec2r(theta5,w5);
```

```
expw6=angvec2r(theta6,w6);


exps1=[expw1,(eye(3)-expw1)*cross(w1,v1)+w1*w1'*v1*theta1;0,0,0,1];
exps2=[expw2,(eye(3)-expw2)*cross(w2,v2)+w2*w2'*v2*theta2;0,0,0,1];
exps3=[expw3,(eye(3)-expw3)*cross(w3,v3)+w3*w3'*v3*theta3;0,0,0,1];
exps4=[expw4,(eye(3)-expw4)*cross(w4,v4)+w4*w4'*v4*theta4;0,0,0,1];
exps5=[expw5,(eye(3)-expw5)*cross(w5,v5)+w5*w5'*v5*theta5;0,0,0,1];
exps6=[expw6,(eye(3)-expw6)*cross(w6,v6)+w6*w6'*v6*theta6;0,0,0,1];

gst_theta=exps1*exps2*exps3*exps4*exps5*exps6*gst0;
gst_theta=simplify(gst_theta)
```

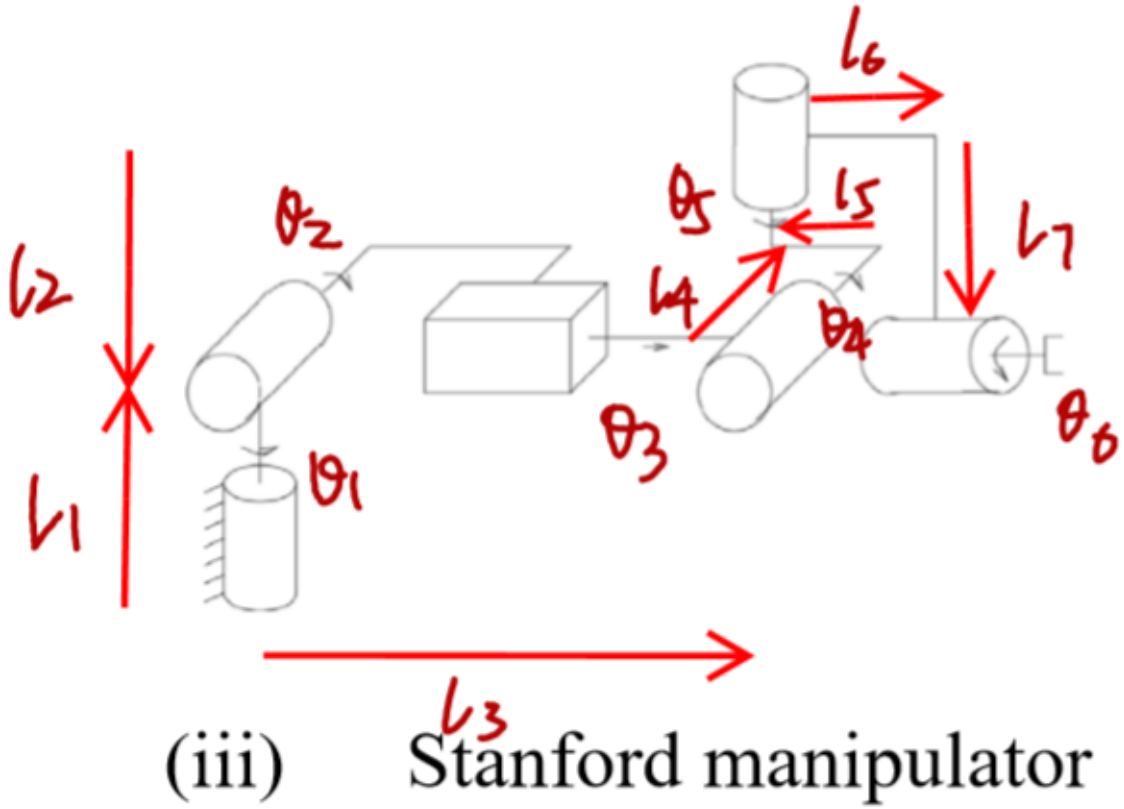
结果

```
gst_theta =

[ cos(theta1)*cos(theta2)*cos(theta6) - sin(theta6)*(cos(theta5)*(cos(theta4)*
(sin(theta1)*sin(theta3) + cos(theta1)*cos(theta3)*sin(theta2)) + sin(theta4)*
(cos(theta3)*sin(theta1) - cos(theta1)*sin(theta2)*sin(theta3))) + sin(theta5)*
(cos(theta4)*(cos(theta3)*sin(theta1) - cos(theta1)*sin(theta2)*sin(theta3)) -
sin(theta4)*(sin(theta1)*sin(theta3) + cos(theta1)*cos(theta3)*sin(theta2)))), -
cos(theta6)*(cos(theta5)*(cos(theta4)*(sin(theta1)*sin(theta3) +
cos(theta1)*cos(theta3)*sin(theta2)) + sin(theta4)*(cos(theta3)*sin(theta1) -
cos(theta1)*sin(theta2)*sin(theta3))) + sin(theta5)*(cos(theta4)*
(cos(theta3)*sin(theta1) - cos(theta1)*sin(theta2)*sin(theta3)) - sin(theta4)*
(sin(theta1)*sin(theta3) + cos(theta1)*cos(theta3)*sin(theta2)))) -
cos(theta1)*cos(theta2)*sin(theta6), sin(theta5)*(cos(theta4)*
(sin(theta1)*sin(theta3) + cos(theta1)*cos(theta3)*sin(theta2)) + sin(theta4)*
(cos(theta3)*sin(theta1) - cos(theta1)*sin(theta2)*sin(theta3))) - cos(theta5)*
(cos(theta4)*(cos(theta3)*sin(theta1) - cos(theta1)*sin(theta2)*sin(theta3)) -
sin(theta4)*(sin(theta1)*sin(theta3) + cos(theta1)*cos(theta3)*sin(theta2))),
l3*cos(theta1)*sin(theta2)*sin(theta3) - l4*cos(theta3)*cos(theta4)*sin(theta1)
- l3*cos(theta3)*sin(theta1) + l4*sin(theta1)*sin(theta3)*sin(theta4) -
l5*cos(theta3)*cos(theta4)*cos(theta5)*sin(theta1) +
l4*cos(theta1)*cos(theta3)*sin(theta2)*sin(theta4) +
l4*cos(theta1)*cos(theta4)*sin(theta2)*sin(theta3) +
l5*cos(theta3)*sin(theta1)*sin(theta4)*sin(theta5) +
l5*cos(theta4)*sin(theta1)*sin(theta3)*sin(theta5) +
l5*cos(theta5)*sin(theta1)*sin(theta3)*sin(theta4) +
l5*cos(theta1)*cos(theta3)*cos(theta4)*sin(theta2)*sin(theta5) +
l5*cos(theta1)*cos(theta3)*cos(theta5)*sin(theta2)*sin(theta4) +
l5*cos(theta1)*cos(theta4)*cos(theta5)*sin(theta2)*sin(theta3) -
l5*cos(theta1)*sin(theta2)*sin(theta3)*sin(theta4)*sin(theta5)]
[ sin(theta6)*(cos(theta5)*(cos(theta4)*(cos(theta1)*sin(theta3) -
cos(theta3)*sin(theta1)*sin(theta2)) + sin(theta4)*(cos(theta1)*cos(theta3) +
sin(theta1)*sin(theta2)*sin(theta3))) + sin(theta5)*(cos(theta4)*
(cos(theta1)*cos(theta3) + sin(theta1)*sin(theta2)*sin(theta3)) - sin(theta4)*
(cos(theta1)*sin(theta3) - cos(theta3)*sin(theta1)*sin(theta2)))) +
cos(theta2)*cos(theta6)*sin(theta1),   cos(theta6)*(cos(theta5)*(cos(theta4)*
(cos(theta1)*sin(theta3) - cos(theta3)*sin(theta1)*sin(theta2)) + sin(theta4)*
(cos(theta1)*cos(theta3) + sin(theta1)*sin(theta2)*sin(theta3))) + sin(theta5)*
(cos(theta4)*(cos(theta1)*cos(theta3) + sin(theta1)*sin(theta2)*sin(theta3)) -
sin(theta4)*(cos(theta1)*sin(theta3) - cos(theta3)*sin(theta1)*sin(theta2)))) -
cos(theta2)*sin(theta1)*sin(theta6), cos(theta5)*(cos(theta4)*
(cos(theta1)*cos(theta3) + sin(theta1)*sin(theta2)*sin(theta3)) - sin(theta4)*
(cos(theta1)*sin(theta3) - cos(theta3)*sin(theta1)*sin(theta2))) - sin(theta5)*
(cos(theta4)*(cos(theta1)*sin(theta3) - cos(theta3)*sin(theta1)*sin(theta2)) +
sin(theta4)*(cos(theta1)*cos(theta3) + sin(theta1)*sin(theta2)*sin(theta3))),
l3*cos(theta1)*cos(theta3) + l4*cos(theta1)*cos(theta3)*cos(theta4) -
l4*cos(theta1)*sin(theta3)*sin(theta4) + l3*sin(theta1)*sin(theta2)*sin(theta3)
+ l5*cos(theta1)*cos(theta3)*cos(theta4)*cos(theta5) -
l5*cos(theta1)*cos(theta3)*sin(theta4)*sin(theta5) -
l5*cos(theta1)*cos(theta4)*sin(theta3)*sin(theta5) -
l5*cos(theta1)*cos(theta5)*sin(theta3)*sin(theta4) +
l4*cos(theta3)*sin(theta1)*sin(theta2)*sin(theta4) +
l4*cos(theta4)*sin(theta1)*sin(theta2)*sin(theta3) +
l5*cos(theta3)*cos(theta4)*sin(theta1)*sin(theta2)*sin(theta5) +
l5*cos(theta3)*cos(theta5)*sin(theta1)*sin(theta2)*sin(theta4) +
l5*cos(theta4)*cos(theta5)*sin(theta1)*sin(theta2)*sin(theta3) -
l5*sin(theta1)*sin(theta2)*sin(theta3)*sin(theta4)*sin(theta5)]
```

```
[
                                                                  -
cos(theta6)*sin(theta2) - cos(theta3 + theta4 + theta5)*cos(theta2)*sin(theta6),

sin(theta2)*sin(theta6) - cos(theta3 + theta4 + theta5)*cos(theta2)*cos(theta6),

                                                  sin(theta3 + theta4 +
theta5)*cos(theta2),

                    l1 + l3*cos(theta2)*sin(theta3) +
l4*cos(theta2)*cos(theta3)*sin(theta4) + l4*cos(theta2)*cos(theta4)*sin(theta3)
+ l5*cos(theta2)*cos(theta3)*cos(theta4)*sin(theta5) +
l5*cos(theta2)*cos(theta3)*cos(theta5)*sin(theta4) +
l5*cos(theta2)*cos(theta4)*cos(theta5)*sin(theta3) -
l5*cos(theta2)*sin(theta3)*sin(theta4)*sin(theta5)]
[
                                                                  0,

                                                        0,

0,

        1]
```

(iii)

假设变量如图



(iii) Stanford manipulator

matlab

```
clear;
clc;
syms theta1 theta2 theta3 theta4 theta5 theta6;
syms l1 l2 l3 l4 l5 l6 l7;

gst0=[rotx(-90),[0;l3+l4+l5;l1];
        0,0,0,1];

w1=[0;0;1];w5=w1;
w2=[1;0;0];w4=w2;
w6=[0;1;0];
w3=[0;0;0];

q1=[0;0;0];
q2=[0;0;l1];
q4=[0;l3;l1];
q5=[-l4;l3-l5;l1+l2];
q6=[-l4;l3-l5+l6;l1+l2-l7];

v1 = -cross(w1,q1);
s1=[v1;w1];
v2 = -cross(w2,q2);
s2=[v2;w2];
v3 = [0;1;0];% 纯平移
s3=[v3;w3]
v4 = -cross(w4,q4);
s4=[v4;w4];
v5 = -cross(w5,q5);
s5=[v5;w5];
v6 = -cross(w6,q6);
```

```
s6=[v6;w6];

expw1=angvec2r(theta1,w1);
expw2=angvec2r(theta2,w2);
expw3=eye(3);% 纯平移
expw4=angvec2r(theta4,w4);
expw5=angvec2r(theta5,w5);
expw6=angvec2r(theta6,w6);

exps1=[expw1,(eye(3)-expw1)*cross(w1,v1)+w1*w1'*v1*theta1;0,0,0,1];
exps2=[expw2,(eye(3)-expw2)*cross(w2,v2)+w2*w2'*v2*theta2;0,0,0,1];
exps3=[expw3,(eye(3)-expw3)*cross(w3,v3)+v3*theta3;0,0,0,1];% 纯平移
exps4=[expw4,(eye(3)-expw4)*cross(w4,v4)+w4*w4'*v4*theta4;0,0,0,1];
exps5=[expw5,(eye(3)-expw5)*cross(w5,v5)+w5*w5'*v5*theta5;0,0,0,1];
exps6=[expw6,(eye(3)-expw6)*cross(w6,v6)+w6*w6'*v6*theta6;0,0,0,1];

gst_theta=exps1*exps2*exps3*exps4*exps5*exps6*gst0;
gst_theta=simplify(gst_theta)
```

结果

```
gst_theta =

[ cos(theta6)*(cos(theta1)*cos(theta5) + sin(theta5)*
(sin(theta1)*sin(theta2)*sin(theta4) - cos(theta2)*cos(theta4)*sin(theta1))) -
sin(theta6)*(cos(theta2)*sin(theta1)*sin(theta4) +
cos(theta4)*sin(theta1)*sin(theta2)), - sin(theta6)*(cos(theta1)*cos(theta5) +
sin(theta5)*(sin(theta1)*sin(theta2)*sin(theta4) -
cos(theta2)*cos(theta4)*sin(theta1))) - cos(theta6)*
(cos(theta2)*sin(theta1)*sin(theta4) + cos(theta4)*sin(theta1)*sin(theta2)),
cos(theta5)*(sin(theta1)*sin(theta2)*sin(theta4) -
cos(theta2)*cos(theta4)*sin(theta1)) - cos(theta1)*sin(theta5),
(cos(theta1)*cos(theta5) + sin(theta5)*(sin(theta1)*sin(theta2)*sin(theta4) -
cos(theta2)*cos(theta4)*sin(theta1)))*(l4*(cos(theta6) - 1) - sin(theta6)*(l1 +
l2 - l7)) - (cos(theta1)*sin(theta5) - cos(theta5)*
(sin(theta1)*sin(theta2)*sin(theta4) - cos(theta2)*cos(theta4)*sin(theta1)))*(l3
+ l4 + l5) - ((cos(theta5) - 1)*(l3 - l5) - l4*sin(theta5))*
(sin(theta1)*sin(theta2)*sin(theta4) - cos(theta2)*cos(theta4)*sin(theta1)) +
l1*(sin(theta6)*(cos(theta1)*cos(theta5) + sin(theta5)*
(sin(theta1)*sin(theta2)*sin(theta4) - cos(theta2)*cos(theta4)*sin(theta1))) +
cos(theta6)*(cos(theta2)*sin(theta1)*sin(theta4) +
cos(theta4)*sin(theta1)*sin(theta2))) + cos(theta1)*(l4*(cos(theta5) - 1) +
sin(theta5)*(l3 - l5)) - (cos(theta2)*sin(theta1)*sin(theta4) +
cos(theta4)*sin(theta1)*sin(theta2))*((cos(theta6) - 1)*(l1 + l2 - l7) +
l4*sin(theta6)) + cos(theta2)*sin(theta1)*(l3*(cos(theta4) - 1) -
l1*sin(theta4)) - sin(theta1)*sin(theta2)*(l1*(cos(theta4) - 1) +
l3*sin(theta4)) - theta3*cos(theta2)*sin(theta1) - l1*sin(theta1)*sin(theta2)]
[ cos(theta6)*(cos(theta5)*sin(theta1) - sin(theta5)*
(cos(theta1)*sin(theta2)*sin(theta4) - cos(theta1)*cos(theta2)*cos(theta4))) +
sin(theta6)*(cos(theta1)*cos(theta2)*sin(theta4) +
cos(theta1)*cos(theta4)*sin(theta2)),   cos(theta6)*
(cos(theta1)*cos(theta2)*sin(theta4) + cos(theta1)*cos(theta4)*sin(theta2)) -
sin(theta6)*(cos(theta5)*sin(theta1) - sin(theta5)*
(cos(theta1)*sin(theta2)*sin(theta4) - cos(theta1)*cos(theta2)*cos(theta4))), -
sin(theta1)*sin(theta5) - cos(theta5)*(cos(theta1)*sin(theta2)*sin(theta4) -
cos(theta1)*cos(theta2)*cos(theta4)), l1*(sin(theta6)*(cos(theta5)*sin(theta1) -
sin(theta5)*(cos(theta1)*sin(theta2)*sin(theta4) -
cos(theta1)*cos(theta2)*cos(theta4))) - cos(theta6)*
(cos(theta1)*cos(theta2)*sin(theta4) + cos(theta1)*cos(theta4)*sin(theta2))) -
(sin(theta1)*sin(theta5) + cos(theta5)*(cos(theta1)*sin(theta2)*sin(theta4) -
cos(theta1)*cos(theta2)*cos(theta4)))*(l3 + l4 + l5) + (cos(theta5)*sin(theta1)
- sin(theta5)*(cos(theta1)*sin(theta2)*sin(theta4) -
cos(theta1)*cos(theta2)*cos(theta4)))*(l4*(cos(theta6) - 1) - sin(theta6)*(l1 +
l2 - l7)) + (cos(theta1)*cos(theta2)*sin(theta4) +
cos(theta1)*cos(theta4)*sin(theta2))*((cos(theta6) - 1)*(l1 + l2 - l7) +
l4*sin(theta6)) + sin(theta1)*(l4*(cos(theta5) - 1) + sin(theta5)*(l3 - l5)) +
((cos(theta5) - 1)*(l3 - l5) - l4*sin(theta5))*
(cos(theta1)*sin(theta2)*sin(theta4) - cos(theta1)*cos(theta2)*cos(theta4)) -
cos(theta1)*cos(theta2)*(l3*(cos(theta4) - 1) - l1*sin(theta4)) +
cos(theta1)*sin(theta2)*(l1*(cos(theta4) - 1) + l3*sin(theta4)) +
theta3*cos(theta1)*cos(theta2) + l1*cos(theta1)*sin(theta2)]
```

```
[
                                                                sin(theta2 +
theta4)*cos(theta6)*sin(theta5) - cos(theta2 + theta4)*sin(theta6),

                                                 - cos(theta2 + theta4)*cos(theta6)
- sin(theta2 + theta4)*sin(theta5)*sin(theta6),
                                                      sin(theta2 +
theta4)*cos(theta5),




                                           sin(theta2)*(l3 - l3*cos(theta4) +
l1*sin(theta4)) - l1*(cos(theta2) - 1) + sin(theta2 + theta4)*(l3 - l5 -
l3*cos(theta5) + l5*cos(theta5) + l4*sin(theta5)) - cos(theta2 + theta4)*
((cos(theta6) - 1)*(l1 + l2 - l7) + l4*sin(theta6)) - cos(theta2)*
(l1*cos(theta4) - l1 + l3*sin(theta4)) + theta3*sin(theta2) + l1*(cos(theta2 +
theta4)*cos(theta6) + sin(theta2 + theta4)*sin(theta5)*sin(theta6)) + sin(theta2
+ theta4)*cos(theta5)*(l3 + l4 + l5) + sin(theta2 + theta4)*sin(theta5)*(l4*
(cos(theta6) - 1) - sin(theta6)*(l1 + l2 - l7))]
[
                                                          0,

                                       0,
                                                                  0,
```

```
                                                                  1]
```