(d)

$$R_{ca} = R_{X_A}(30^\circ) R_{Y_A}(-90^\circ)$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$p_{ca} = \left[2, \frac{3}{2}, -\frac{3\sqrt{3}}{2}\right]^T$$

$$g_{ca} = \left[\begin{array}{ccc:c} 0 & 0 & -1 & 2 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{3}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & -\frac{3\sqrt{3}}{2} \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]$$

8.

$$g = e^{\hat{\xi}\theta} = \begin{bmatrix} e^{\hat{\omega}\theta} & (I - e^{\hat{\omega}\theta})\hat{\omega} v + \omega\omega^T v\theta \\ 0 & 1 \end{bmatrix}$$

由Rodrigues公式知 ($v_\theta = 1-\cos\theta$)

$$e^{\hat{\omega}\theta} = I + \hat{\omega}\sin\theta + \hat{\omega}^2(1-\cos\theta)$$

$$= \begin{bmatrix} \omega_1^2 v_\theta + c_\theta & \omega_1\omega_2 v_\theta - \omega_3 s_\theta & \omega_1\omega_3 v_\theta + \omega_2 s_\theta \\ \omega_1\omega_2 v_\theta + \omega_3 s_\theta & \omega_2^2 v_\theta + c_\theta & \omega_2\omega_3 v_\theta - \omega_1 s_\theta \\ \omega_1\omega_3 v_\theta - \omega_2 s_\theta & \omega_2\omega_3 v_\theta + \omega_1 s_\theta & \omega_3^2 v_\theta + c_\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = R$$

又∵ $tr(R) = 1 + 2\cos\theta = 0$

$$\Rightarrow \cos\theta = -\frac{1}{2} \quad \Rightarrow \theta = 120^\circ = \frac{2}{3}\pi$$

$$\Rightarrow \omega = \frac{1}{2s_\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} = \frac{\sqrt{3}}{3}\begin{bmatrix} 0-(-1) \\ 0-(-1) \\ 0-1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{bmatrix}$$

又∵ $(I - e^{\hat{\omega}\theta})\hat{\omega} v + \omega\omega^T v\theta = [10, 20, 1]^T$

$= Av$

$(A = (I - e^{\hat{\omega}\theta})\hat{\omega} + \omega\omega^T\theta)$

$$A = (I - e^{\hat{\omega}\theta})\hat{\omega} + \omega\omega^T\theta$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & 0 & -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 0 \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \end{bmatrix} \frac{2\pi}{3}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 0 \\ 0 & \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & \frac{2}{3}\sqrt{3} & -\frac{\sqrt{3}}{3} \end{bmatrix} + \begin{bmatrix} \frac{2}{9}\pi & \frac{2}{9}\pi & -\frac{2}{9}\pi \\ \frac{2}{9}\pi & \frac{2}{9}\pi & -\frac{2}{9}\pi \\ -\frac{2}{9}\pi & -\frac{2}{9}\pi & \frac{2}{9}\pi \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{3} + \frac{2}{9}\pi & \frac{\sqrt{3}}{3} + \frac{2}{9}\pi & -\frac{2}{9}\pi \\ \frac{2}{9}\pi & \frac{\sqrt{3}}{3} + \frac{2}{9}\pi & -\frac{\sqrt{3}}{3} - \frac{2}{9}\pi \\ \frac{\sqrt{3}}{3} - \frac{2}{9}\pi & \frac{2}{3}\sqrt{3} - \frac{2}{9}\pi & -\frac{\sqrt{3}}{3} + \frac{2}{9}\pi \end{bmatrix}$$

$$\because \quad p = Av \quad \Rightarrow v = A^{-1}p$$

$$\Rightarrow v = \begin{bmatrix} \frac{21}{\pi} - \frac{29}{6}\sqrt{3} \\ \frac{31}{\sqrt{3}} \\ -\frac{21}{\pi} - \frac{29}{2\sqrt{3}} \end{bmatrix}$$

$$\Rightarrow \xi = \left[\begin{array}{c} \frac{87}{2\pi} - \frac{31}{\sqrt{3}} \\ \frac{32}{\sqrt{3}} - \frac{87}{2\pi} \\ \frac{1}{\sqrt{3}} - \frac{87}{2\pi} \\ \hdashline \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{array}\right]$$

轴上一点q取

$$q = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{bmatrix}$$