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Using Constrained Model Predictive Control to Control Two Quadrotors Transporting a Cable-Suspended Payload

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Abstract—This paper applies a constrained MPC controller to control two quadrotors which carry a cable-suspended payload together. The system dynamics is derived from the Euler-Lagrange equation. Given the dynamics complexity, a linear MPC controller is employed for a control task, which is to make the payload to track a desired trajectory while stabilising the two quadrotors. The constraints on quadrotor control signals and payload positions are taken into consideration. The constrained controller could be useful for practical control systems. The simulation results are provided to evaluate the control performance against an LQR controller.

I. INTRODUCTION

The research on Unmanned Aerial Vehicles (UAVs) transporting payloads by cables is growing worthily with a wide range of potential applications. In recent decades the maturation has been seen for civil and military applications in different research areas such as emergency rescue, reconnaissance, firefighting and freight transportation missions. The problem of multiple quadrotors transporting a cable-suspended payload is still challenging due to the complexity of systems.

Carrying a cable-suspended load by one, two, or multiple quadrotors have been investigated recently using various advanced control techniques such as linear and nonlinear controllers. A geometric controller was constructed to achieve the system stability of controlling a cable suspended payload with multiple quadrotors following a desired trajectory in [1], where a geometric feedback controller was designed initially for a reduced system then extended to the full system through singular perturbation method. Cooperative transportation of a cable suspended load was presented by the authors in [2] and [3] considering the tracking controller based on a coordinate-free model.

Those non-linear controllers are able to stabilise the control system but the dynamic performance cannot be optimised. Furthermore, the constraints on system states and input signals are not considered. In practical applications, it is necessary to handle these constraints for a control

system in order to achieve a better control performance. The constraints to be considered in such a system could include the payload angles, positions, or quadrotor thrusts.

Model Predictive Control (MPC) has emerged as one of the most popular and valuable control techniques since the 1980s. The strategy of an MPC controller is to find a sequence of optimal controlling signals in a finite optimization horizon first and then select the first one to control the system recursively at each time step. It can not only optimise the cost function but also take the state or input constraints into consideration.

Many MPC studies are proposed for single quadrotors in two or three dimensions to implement different tasks and improve the system performance. The simplified models led to the design of a robust MPC controller in [4] to control the attitude of single unmanned quadrotor helicopter in severe environmental conditions based on piecewise affine linear systems. The challenging issue is to consider the wind-disturbance when executing sudden manoeuvres. The constraint on input signals was taken into account of this system. In [5] two control algorithms LQR and MPC were presented based on linear quadratic technique and implemented on a quadrotor testbed (Qball-X4) to control against the loss of control effectiveness with a fault-tolerant control strategy. A linear MPC was proposed in [6] and the model was simplified with only two-dimensional movement based on the least square identification. An integral MPC controller was applied for the autonomous flight of single quadrotor to improve the translational motion tracking performance in [7]. The system was modelled based on a set of Piecewise Affine (PWA) systems and the physical constraint was considered. In manoeuvring over constrained environments, a robust MPC controller was presented in [8] for single quadrotor with a fixed payload to minimize the predicted tracking errors.

Cooperative UAV control in a form of multiple team formation was accomplished under a linear MPC control law [9] using the Qball-X4 quadrotor. This approach was implemented without constraints and no load considered.

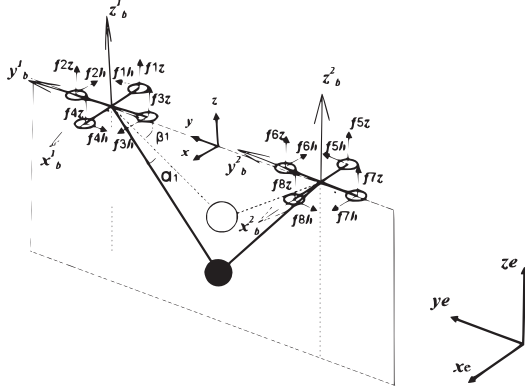


Fig. 1: Two quadrotors carrying a payload

In [10] an MPC controller was presented with PID for a quadrotor to track a predefined trajectory without constraints. An optimal flight control MPC for a quadrotor with a suspended load [11] was presented using visual feedback to compute the load position accurately. The state vector and control vector constraints were applied. In [12] an unconstrained MPC controller was proposed to track a trajectory for quadrotors based on three levels of control. By comparing the results of linear and nonlinear MPC control to track a trajectory using Micro Air Vehicles (MAVs) under wind disturbance [13], it shows that the difference between them is not significant.

Our previous works on single quadrotor in [14] and two quadrotors transporting a cable-suspended payload used an iterative LQR controller, which is an optimised solution for a non-linearity consideration, but without constraints. In this paper, we develop a constrained MPC controller for two quadrotors to transport a cable-suspended payload.

In the following, Section II presents the mathematical model of a cable suspended payload by two quadrotors. Section III illustrates the development of a constrained MPC controller. Section IV shows the simulation results. The conclusion and future work are given in Section V.

II. MODEL DESCRIPTION

The full system is presented in Fig. 1 including the inertial frame, intermediate frames, and body-fixed frames. The vertical and horizontal forces generated by each propeller and swing angles of the cable with respect to the intermediate frame are also shown where the intermediate frames are the translation results from the inertial frame to the centres of quadrotors. See table I for details. In order to simplify the problem, some reasonable assumptions are given as follows:

- 1) Each quadrotor is considered as a symmetrical rigid body.
- 2) The payload is considered as a point mass and is attached at the centre of each quadrotor.
- 3) The cable tension is always non-zero.
- 4) The air drag of the propellers is negligible.

TABLE I: Symbols and Definitions

Symbol	Description
$S_e : x_e y_e z_e$	Inertial frame
$S^k : x y z$	Intermediate frame: translation from S_e to the center of the i th quadrotors $k = 1, 2$
$S_b^k : x_b^k y_b^k z_b^k$	Body-fixed frame for quadrotor,
$E_i \in \mathbb{R}^3, i = 1, 2, 3$	Unit orthogonal vectors of S_e
$e_i^k \in \mathbb{R}^3, i = 1, 2, 3$	Unit orthogonal vectors of S_b^k
$\eta^k = [\phi, \theta, \psi]^T \in \mathbb{R}^3$	Euler angles of quadrotor defined in $Z - Y - X$
$T^k \in \mathbb{R}^{3 \times 3}$	Transformation matrix from S_e to S_b^k
$\Omega^k \in \mathbb{R}^3$	Angular velocity of quadrotor in S_b^k
m_Q^k	Mass of the quadrotor
m_P	Mass of the payload
$I_Q^k \in \mathbb{R}^3$	Inertial matrix of the quadrotor with respect to S_b^k
$\xi_Q^k \in \mathbb{R}^3$	Position of the center of quadrotor in S_e
$\xi_P \in \mathbb{R}^3$	Position of the payload in S_e
x_P, y_P, z_P	Three elements of ξ_P
L_r^k	Length of the rope
L_Q^k	Length of the quadrotor arm
$\alpha^k, \beta^k \in \mathbb{R}$	Angles of the rope with respect to S^k
$\rho^k \in \mathbb{R}^3$	Unit vector from the payload to the attached point
$f_{iz}^k, f_{ih}^k \in \mathbb{R}^3$	Vertical and horizontal forces generated by i th propeller, $i = 1, 2, 3, 4$
$k_F, k_M \in \mathbb{R}$	Propeller aerodynamic parameters

Some symbols and acronyms used in this paper are listed in table I. The following relationships are available.

$$\begin{aligned}
 \rho^k &= [-\sin(\beta^k), -\cos(\alpha^k)\cos(\beta^k), \sin(\alpha^k)\cos(\beta^k)]^T \\
 \xi_P &= x_P E_1 + y_P E_2 + z_P E_3 \\
 \xi_Q^k &= \xi_P + L_r^k \rho^k \\
 \Omega^k &= \begin{bmatrix} 1 & 0 & -\sin(\theta^k) \\ 0 & \cos(\phi^k) & \sin(\phi^k)\cos(\theta^k) \\ 0 & -\sin(\phi^k) & \cos(\phi^k)\cos(\theta^k) \end{bmatrix} \begin{bmatrix} \dot{\phi}^k \\ \dot{\theta}^k \\ \dot{\psi}^k \end{bmatrix}
 \end{aligned} \tag{1}$$

A. Euler-Lagrange Equation

The quadrotor-payload system has 13 degrees of freedom. Choosing $\mathbf{q} = [x_P, y_P, z_P, \alpha^1, \beta^1, \phi^1, \theta^1, \psi^1, \alpha^2, \beta^2, \phi^2, \theta^2, \psi^2]^T$ as the generalized coordinates will not only be convenient for controlling the trajectory of the payload but be helpful for extending to multi-vehicle situations. As a result, the Lagrangian \mathbf{L} is composed by subtracting the kinetic energy \mathbf{T} from the potential energy \mathbf{U} and clarified in the equations below

$$\begin{aligned}
 \mathbf{T} &= \frac{1}{2} m_P (\dot{\xi}_P)^T \cdot \dot{\xi}_P + \frac{1}{2} m_Q^1 (\dot{\xi}_Q^1)^T \cdot \dot{\xi}_Q^1 + \frac{1}{2} (\Omega^1)^T I_Q^1 \Omega^1 \\
 &\quad + \frac{1}{2} m_Q^2 (\dot{\xi}_Q^2)^T \cdot \dot{\xi}_Q^2 + \frac{1}{2} (\Omega^2)^T I_Q^2 \Omega^2 \\
 \mathbf{U} &= m_P g \xi_P \cdot E_3 + m_Q^1 g \xi_Q^1 \cdot E_3 + m_Q^2 g \xi_Q^2 \cdot E_3 \\
 \mathbf{L} &= \mathbf{T} - \mathbf{U}
 \end{aligned} \tag{2}$$

Then the Euler-Lagrange equation is

$$\frac{d}{dt} \left(\frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial \mathbf{L}}{\partial \mathbf{q}} = \mathbf{Q} \quad (3)$$

The generalized force \mathbf{Q} defined here is based on the choice of the generalized coordinates \mathbf{q} and the external conservative forces \mathbf{F}_i . For each quadrotor, the force \mathbf{F}_i consists of two complements \mathbf{f}_{iz} and \mathbf{f}_{ih} (equation (4)) which are related with the angular speed ω_i of i th propeller [14].

$$\begin{aligned} \mathbf{F}_i^k &= \mathbf{f}_{iz}^k + \mathbf{f}_{ih}^k, i = 1, 2, 3, 4 \\ \mathbf{f}_{iz}^k &= k_F \omega_i^2 \mathbf{e}_3^k, i = 1, 2, 3, 4 \\ \mathbf{f}_{1h}^k &= k_M \omega_1^2 \mathbf{e}_2^k \\ \mathbf{f}_{2h}^k &= k_M \omega_2^2 \mathbf{e}_1^k \\ \mathbf{f}_{3h}^k &= -k_M \omega_3^2 \mathbf{e}_2^k \\ \mathbf{f}_{4h}^k &= -k_M \omega_4^2 \mathbf{e}_1^k \end{aligned} \quad (4)$$

where $\mathbf{e}_i^k = \mathbf{T}_{e2b}^k \mathbf{E}_i$, $i = 1, 2, 3$.

As seen in Fig. 1, the point where the force \mathbf{F}_i^k is applied on is the centre of each propeller and the corresponding position vector is noted as $\boldsymbol{\xi}_i^k$ seen in equation (5).

$$\begin{aligned} \boldsymbol{\xi}_1^k &= \boldsymbol{\xi}_Q^k + L_Q \mathbf{e}_1^k \\ \boldsymbol{\xi}_2^k &= \boldsymbol{\xi}_Q^k + L_Q \mathbf{e}_2^k \\ \boldsymbol{\xi}_3^k &= \boldsymbol{\xi}_Q^k - L_Q \mathbf{e}_1^k \\ \boldsymbol{\xi}_4^k &= \boldsymbol{\xi}_Q^k - L_Q \mathbf{e}_2^k \end{aligned} \quad (5)$$

According to the principle of virtual work, the generalized forces are given by equation (6) [14].

$$Q_i = \sum_{j=1}^4 \sum_{k=1}^2 \frac{\partial (\mathbf{F}_j^k \cdot \boldsymbol{\xi}_j^k)}{\partial q_i}, i = 1, 2, \dots, 13 \quad (6)$$

Taking the generalized forces and equation (2) into equation (3), the Euler-Lagrange equation can be rewritten in

$$\mathbf{M} \ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}) \quad (7)$$

In the trajectory tracking control, the operating point is considered as the balance situation where the Euler angles of both quadrotors equal to zero and the cable angles equal to the designed constants. The state \mathbf{x} and control \mathbf{u} of the system are defined as below:

$$\begin{aligned} \mathbf{x} &= [x_P, \dot{x}_P, y_P, \dot{y}_P, z_P, \dot{z}_P, \alpha^1, \dot{\alpha}^1, \beta^1, \dot{\beta}^1, \phi^1, \dot{\phi}^1, \theta^1, \\ &\quad \dot{\theta}^1, \psi^1, \dot{\psi}^1, \alpha^2, \dot{\alpha}^2, \beta^2, \dot{\beta}^2, \phi^2, \dot{\phi}^2, \theta^2, \dot{\theta}^2, \psi^2, \dot{\psi}^2]^T \in \mathbb{R}^{26} \\ \mathbf{u} &= [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8]^T \in \mathbb{R}^8 \end{aligned}$$

The linearised model is obtained in equation (8).

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1} \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\mathbf{x}_0} \Delta \mathbf{x} + \mathbf{M}^{-1} \left. \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right|_{\mathbf{u}_0} \Delta \mathbf{u} \quad (8)$$

Furthermore, equation (8) can be transformed into a discrete form.

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \quad (9)$$

where $\mathbf{A}_k \in \mathbb{R}^{26 \times 26}$, $\mathbf{B}_k \in \mathbb{R}^{26 \times 8}$.

III. CONSTRAINED LINEAR MODEL PREDICTIVE CONTROL

Model predictive control (MPC) is a worth describing method based on the principle of prediction to a finite horizon at each sample starting from the current state. Subjected to the state and control constraints, MPC is solved to determine the optimal control input sequence over the finite horizon. Then, it implements the first control input in the optimised control input sequence. The discrete-time dynamical model description for the system with two quadrotors carrying a cable-suspended payload is shown in equation (9).

The finite horizon optimal control solves the following constrained optimisation problem at each time instant k to implement the MPC algorithm. The cost function is presented by:

$$\begin{aligned} J &= (\mathbf{x}_N - \mathbf{x}_N^*)^T \mathbf{Q}_f (\mathbf{x}_N - \mathbf{x}_N^*) \\ &+ \sum_{t=k}^{N+k-1} [(\mathbf{x}_t - \mathbf{x}_t^*)^T \mathbf{Q} (\mathbf{x}_t - \mathbf{x}_t^*) + \mathbf{u}_t^T \mathbf{R} \mathbf{u}_t] \end{aligned} \quad (10)$$

where the terminal state and its desired state are denoted by \mathbf{x}_N and \mathbf{x}_N^* , respectively, and the reference state is denoted by \mathbf{x}_k^* . The prediction horizon is denoted by N . \mathbf{Q}_f and \mathbf{Q} are presented by positive semidefinite state matrices and \mathbf{R} is presented by positive definite matrix.

The constrained optimisation problem (COP) at each time instant k is

$$\min_{\mathbf{u}_k} J$$

Subject to

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}_k \mathbf{x}_k + \mathbf{B}_k \mathbf{u}_k \\ \mathbf{x}_k &\in \mathbb{X}, \text{ the state constraints} \\ \mathbf{u}_k &\in \mathbb{U}, \text{ the control constraints} \end{aligned}$$

where the state and input constraints are presented as

$$\begin{aligned} \mathbb{X} &= \{\mathbf{x}_{kmin} \leq \mathbf{x}_k \leq \mathbf{x}_{kmax}\} \\ \mathbb{U} &= \{\mathbf{u}_{kmin} \leq \mathbf{u}_k \leq \mathbf{u}_{kmax}\} \end{aligned}$$

A quadratic programming (QP) algorithm can be used to solve the COP problem at each time instant k , which can produce an input sequence $\{\mathbf{u}_{k|k}, \mathbf{u}_{k+1|k}, \dots, \mathbf{u}_{k+N-1|k}\}$. Then the true input at k is $\mathbf{u}_k = \mathbf{u}_{k|k}$.

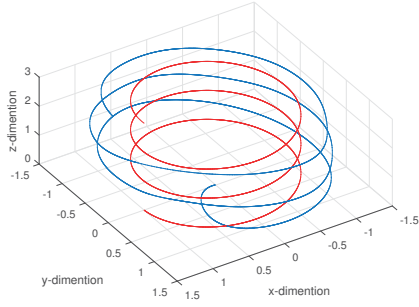


Fig. 3: 3D load position using the LQR controller with the control limitation

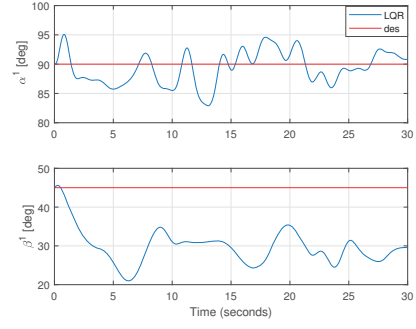


Fig. 6: The rope angles with the first quadrotor using the LQR controller

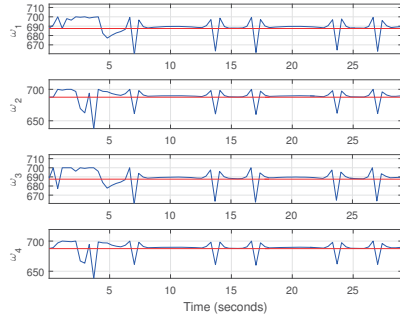


Fig. 4: The angular velocities of the first quadrotor using MPC controller

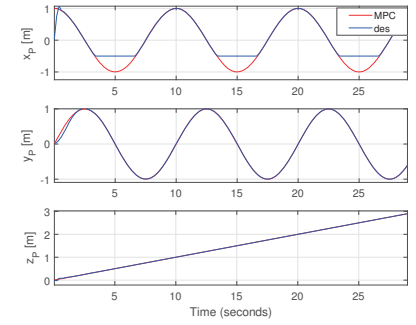


Fig. 7: The payload position using the MPC controller

V. CONCLUSION

In this paper, a constrained MPC controller is applied for tracking control of a cable suspended payload with two quadrotors. A nonlinear model of the system is developed and linearized. The constraints on state and control vectors are considered. We evaluate our model and the MPC controller using simulation and compare the performance of the constrained MPC controller and the LQR controller. The simulation results show the MPC controller is better in controlling the system than the

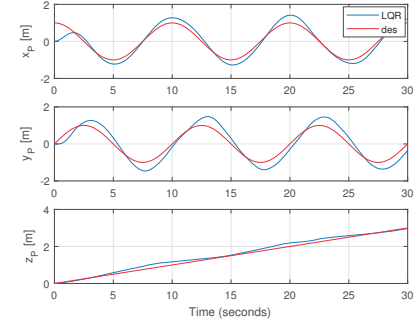


Fig. 8: The payload position using the LQR controller

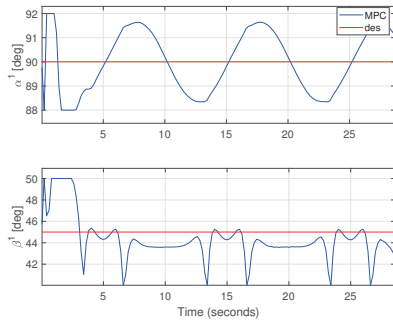


Fig. 5: The rope angles with the first quadrotor using the MPC controller

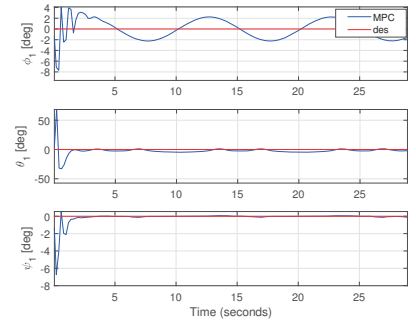


Fig. 9: The Euler angles of the first quadrotor using the MPC controller

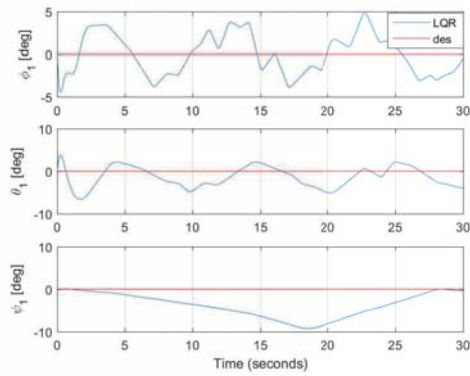


Fig. 10: The Euler angles of the first quadrotor using the LQR controller

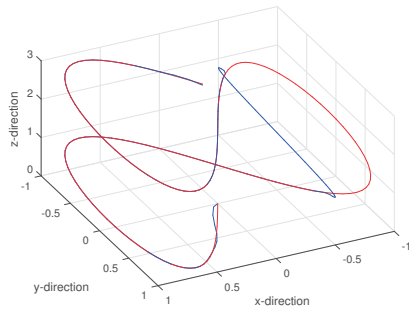


Fig. 11: 3D load position using the constrained MPC controller

LQR controller under the consideration of constraints. The constraints are an important factor in applying a controller to practical systems. Our next step work will be focused on the implementation of the constrained MPC controller on a practical system.

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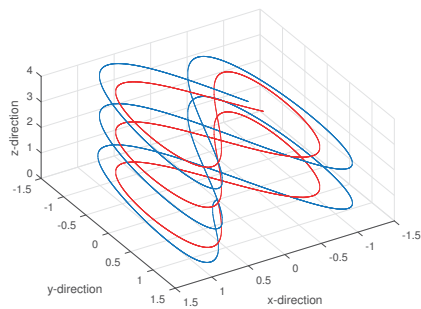


Fig. 12: 3D load position using the constrained LQR controller

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