2. MODELING OF INDUSTRIAL CRANE AND ITS LOAD-SWAY PROBLEM

2.1. Introduction to gantry and overhead crane

Overhead crane is the type of cranes consists of: parallel runways, crane bridge connecting runways, hoist for lifting. As a result, overhead cranes can move load in 3 dimensions: up and down, along the runways and along the bridge. This type of crane is very popular inside factories due to its multi-directional movement ability.

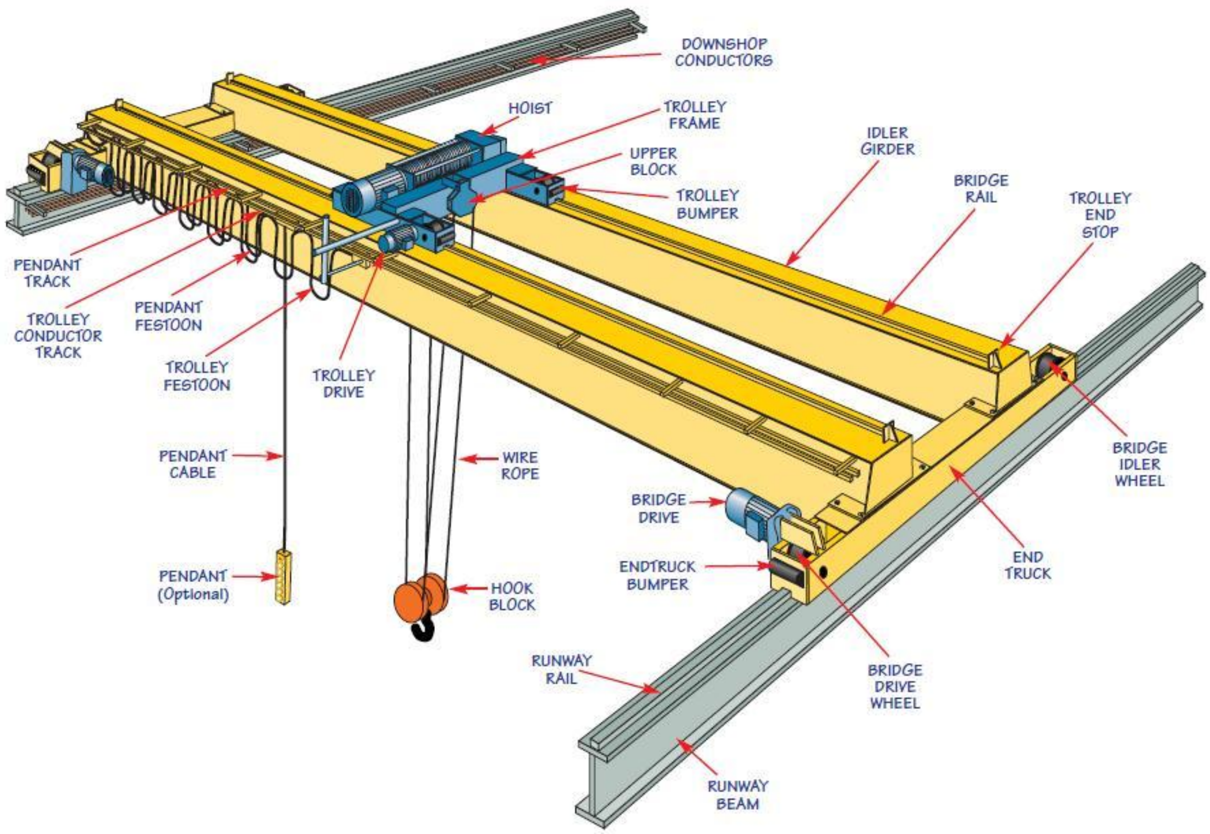

Figure 12. Crane structure (Baoding Weyou Technology Company, *EOT Crane*).

Gantry crane is very similar to overhead crane. However, it does not have the parallel runways which restrict the movement in 2 directions: up and down, along the bridge. Gantry cranes are very popular outside manufacturing factories.

The movement of cranes is enabled by motors. Motors are controlled by motor drive, also known as motor controller or frequency converter. Motor drive can control speed, acceleration, torque and various other attributes of a motor.

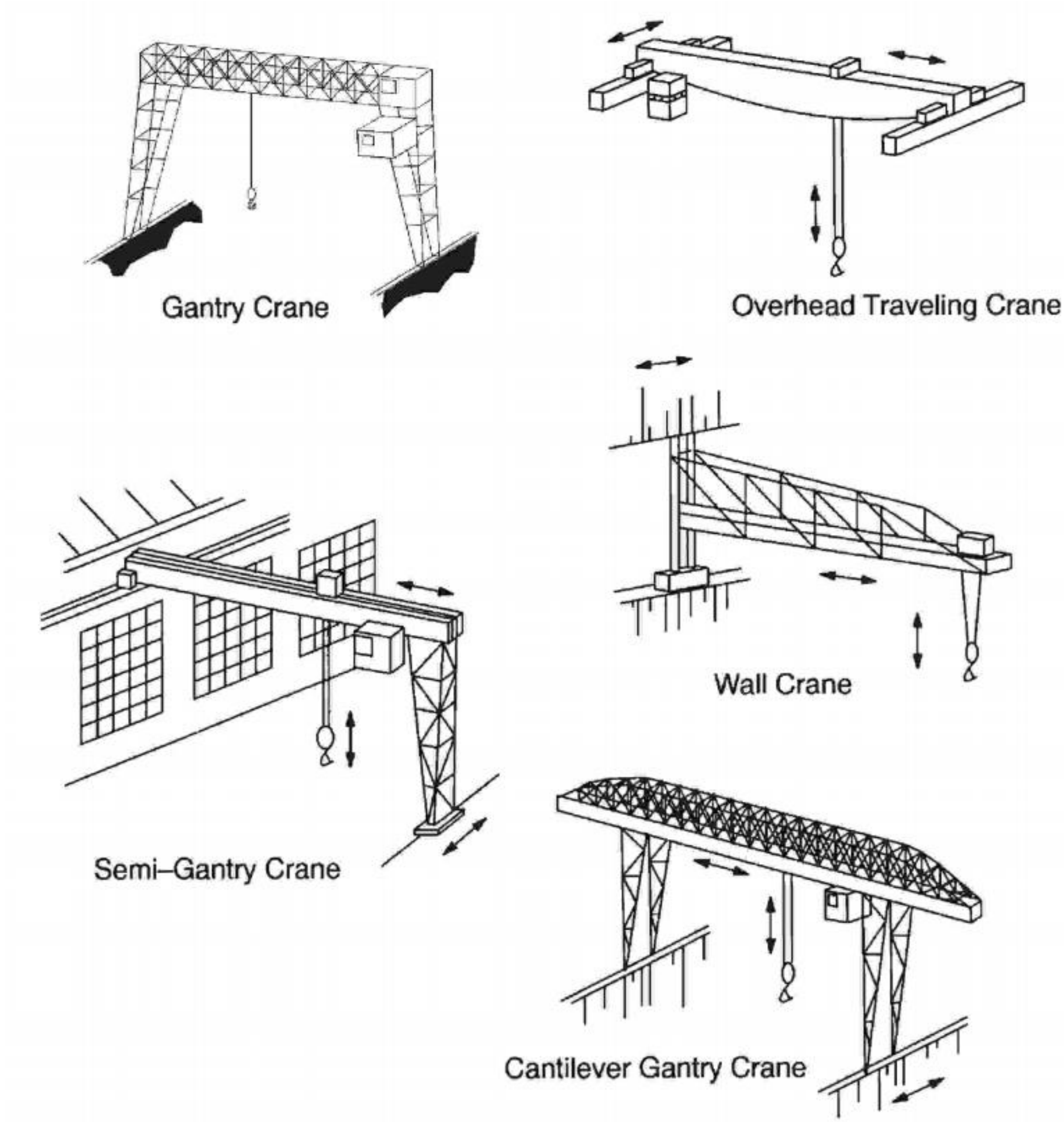

Figure 13. Different types of overhead and gantry crane (U.S. Department of Energy, 2007. *DOE Standard Hoisting and Rigging*).

2.2. Physics model of crane

Moving crane with heavy load can be modeled as a pendulum with movable pivot point.

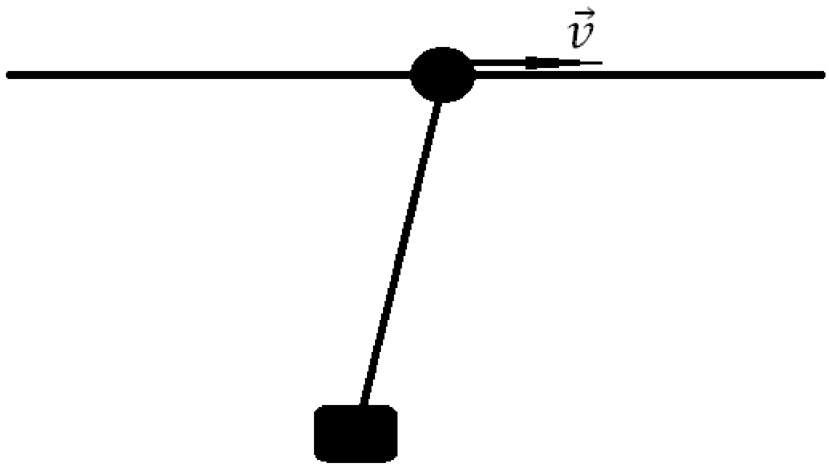

Figure 14. Moving crane physics model.

For small swing amplitude, we have following formulas for pendulum:

- Harmonic oscillation formula:

$$\theta(t) = \theta_0 \sin\left(\frac{2\pi}{T} t\right) \tag{2.1}$$

$\theta(t)$ is the sway angle from pendulum's equilibrium point at time t.

- Approximated formula of oscillation period can be expressed as:

$$T \approx 2\pi \sqrt{\frac{L}{g}}\ \textit{with the condition of}\ \theta_0 < 1\, rad \tag{2.2}$$

T is the period of oscillation.

L is the length of the rope that suspends the load.

g is the local strength of gravity.

θ_0 is the maximum sway angle from pendulum's equilibrium point.

Although above formula is standard for theoretical pendulum, it is not realistic to assume the condition of maximum swinging angle is met during crane's movement. Therefore, approximated formula of oscillation period is not applicable for crane.

However, the crane still possesses symmetric oscillation properties of the standard pendulum which we will exploit to solve the swinging problem of the crane.

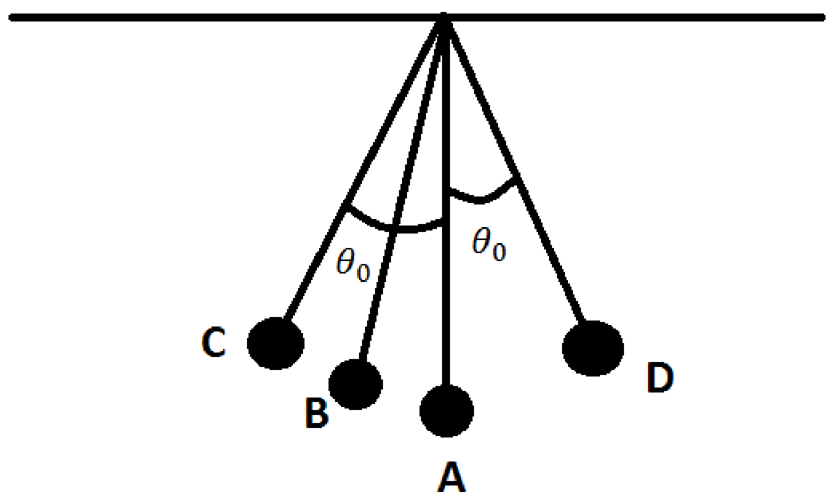

Figure 15. Symmetric oscillation of pendulum.

The symmetric oscillation property can be demonstrated with following examples (with reference to above figure):

- Time to oscillate from A to B will be same as from B to A.
- Time to oscillate from A to C will be same as from A to D, or from D to A, or from C to A.
- Time to oscillate from C to B will be same as from B to C.

Those are some typical examples to demonstrate symmetric oscillation property of crane models.

2.3. Observational frame of reference in crane physics model analysis

Throughout the analysis of crane's physics model, we will use crane's hoist as observational frame of reference. It means that we will observe the swinging phenomenon of the crane as if we are attached to the crane's hoist. Moreover, positive direction of coordinate system will be from left to right.

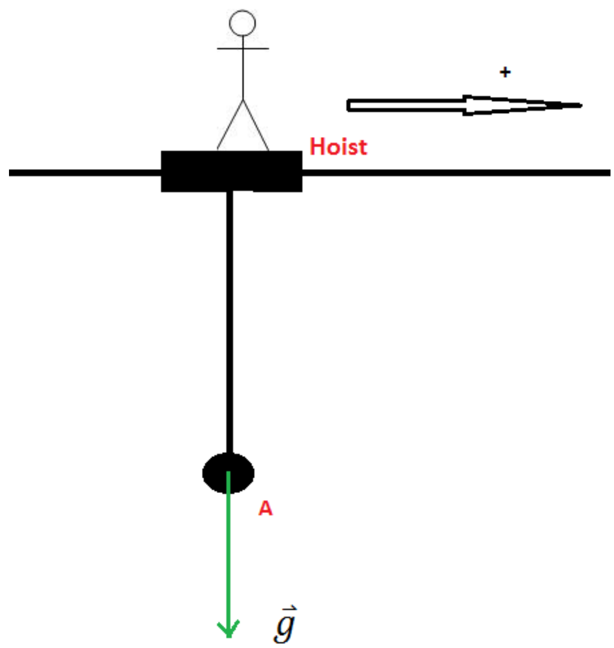

Figure 16. Observational frame of reference attached to the hoist and positive direction is from left to right.

The swinging phenomenon is actually the movement of the load in reference to the hoist. Therefore, the choice of using the hoist as reference coordinate system is logical.

It is worth noticing that: if the hoist is accelerating with acceleration $\vec{a}$ and we are using the hoist as observational frame, the load will look as if it suffers from a force $-m\vec{a}$ from our observational frame of reference (David Morin, 2008: 457 – 500).

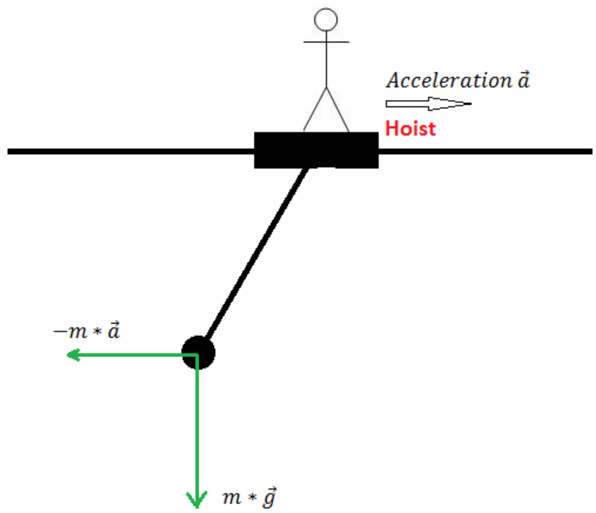

Figure 17. Observational frame of reference attached to the hoist.

The convention in which observational frame of reference is attached to crane's hoist will be used throughout this paper.

2.4. Sway analysis of moving cranes

The crane movement is divided into three separated periods:

- Ramping up period: during this period the motor will accelerate according to acceleration parameter configured in motor drive.
- Constant speed period: after ramping up period, the motor will reach maximum speed configured in motor drive. The motor will be kept at constant maximum speed until user decides to stop the movement.
- Ramping down period: this is stopping period of the crane. The crane speed will be decelerated according to deceleration parameter configured in motor drive.

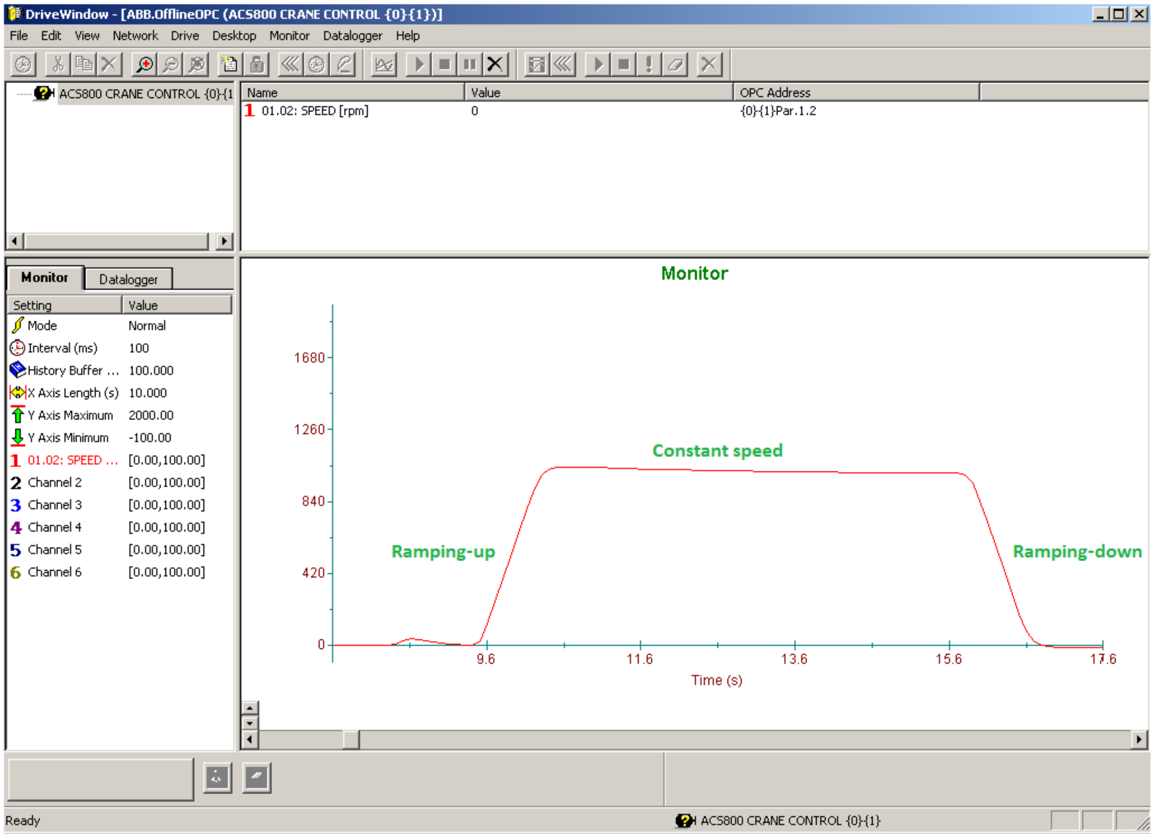

Figure 18. Speed diagram during crane's movement (taken from ABB ACS800 motor drive).

At beginning, let's assume that the load is at its normal equilibrium position as follow:

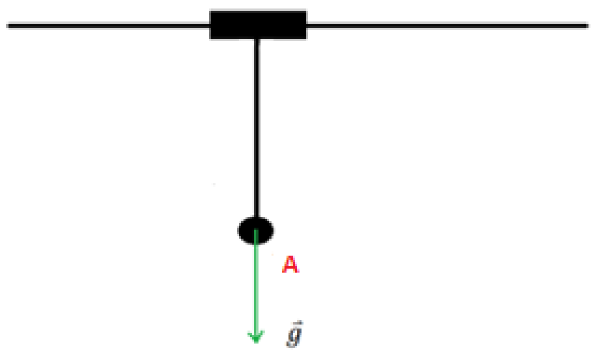

Figure 19. Crane at equilibrium position.

The load sway is initiated during the ramping up period.

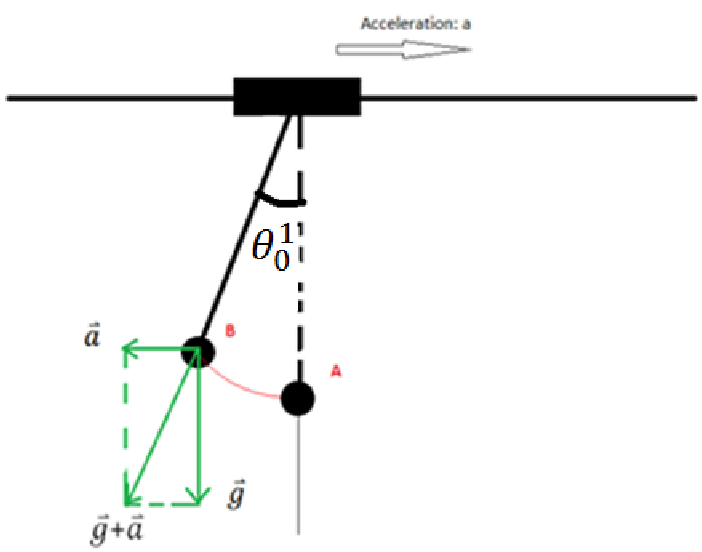

Figure 20. Crane during acceleration.

During ramping up period (with acceleration $\vec{a} > 0$), the equilibrium position is changed from A to B as above. As the result, crane's load will start to oscillate around new equilibrium point B (with maximum angle θ_0^1) during ramping up period.

Suppose that at the end of the ramping-up period (at the beginning of constant speed period), the load is at C position like in the figure below:

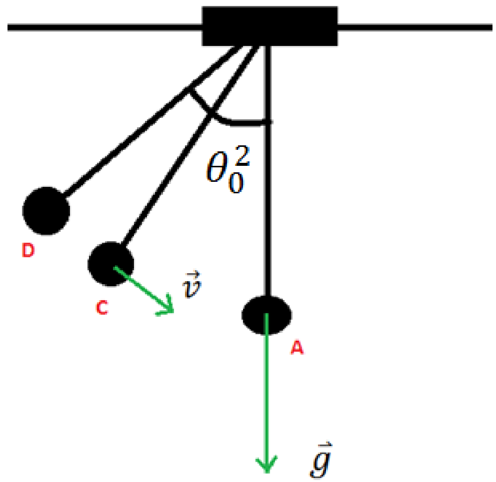

Figure 21. Load sway after acceleration.

Within the constant speed period ($\vec{a} = 0$), the equilibrium position is changed back to A. At C, the load has velocity resulted from ramping-up period's oscillation. Therefore the load will oscillate at even greater amplitude around A. As a result, the load will oscillate around A with maximum angle θ_0^2 for the rest of constant speed period.

Suppose that at the end of the constant speed period the load is at F position like the figure below:

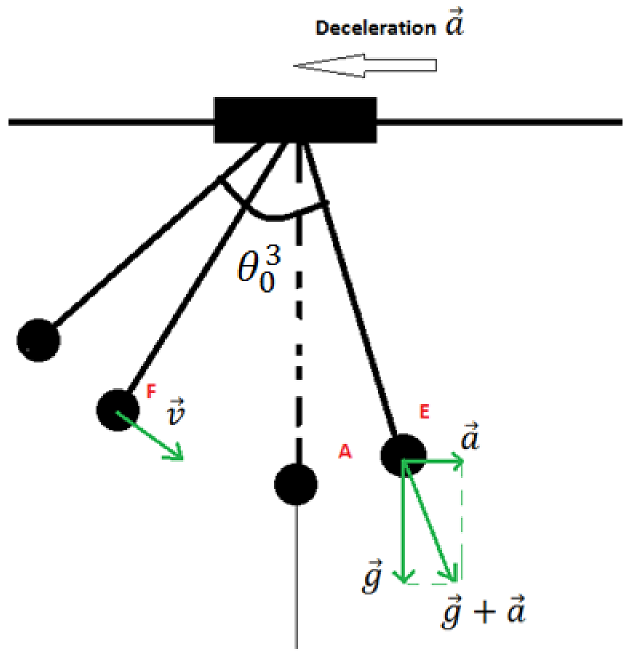

Figure 22. Load sway during deceleration.

In the ramping down period (with acceleration $\vec{a} < 0$), the equilibrium point is now E in figure above. The load, at the end of constant speed period is at F with certain velocity. As a result, during ramping down period, the load is oscillating around E with amplified maximum angle θ_0^3.

Suppose that at the end of the ramping down period, the load is at G position with certain velocity like in the figure below:

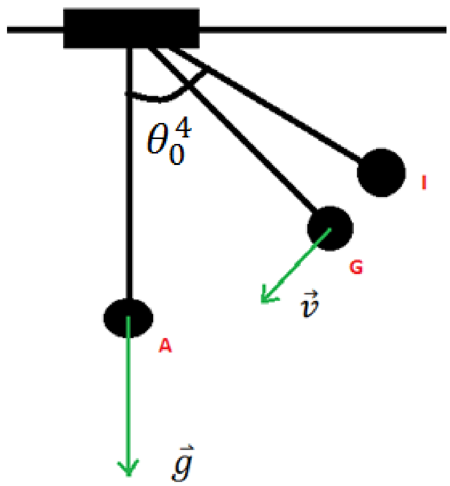

Figure 23. Load sway after deceleration.

When the crane is stopped, the pendulum's equilibrium position is once again changed back to A. At the end, the load will oscillate around A position with maximum angle θ_0^4.

In brief, during all periods of crane's movement, swing happens as the result of changing equilibrium position. Moreover, swing amplitude is magnified when crane is switching from one state to another (ramp-up → constant speed → ramp-down).

2.5. Conclusion

In this chapter, overhead crane as well as gantry crane were introduced and compared with each other. Both mentioned types of crane are affected by load swinging problem during operation. In order to analyze swinging problem, physics model of crane is presented based on a standard pendulum. However, small swing pendulum's calculations are not applicable to our crane model because it is not feasible to assume small swing during crane's movement. At the end, the analysis of crane motion demonstrates how swing happens and how swing amplitude is amplified during movement's process.

3. ANTI-SWAY ALGORITHM

3.1. Introduction and requirements of the anti-sway algorithm

The algorithm was developed in order to minimize swing of load for crane movement under the control of operator.

Algorithm‘s requirements are listed as follow:

- The speed control commands must be simple so that motor drive can perform effectively.
- The algorithm needs to take into account that crane might be carrying heavy load that it is not possible to change speed very frequently.
- Approximated formula such as $T \approx 2\pi\sqrt{\frac{L}{g}}$ is not applicable because small swing is not guaranteed.
- Rope’s length and load’s weight are not available for algorithm because we want algorithm to be able to perform well with variable load’s weight and rope’s length.
- All motor drive’s parameters are available to algorithm to utilize such as: current measurement of speed, speed settings (such as: step speed, max speed, min speed, etc), digital IOs, …
- Algorithm can send start/stop/set speed command to motor drive.
- The inclination (or angle) of the crane’s rope will be made available for no-swing algorithm by a sensor.
- The algorithm will be implemented using microcontroller.

3.2. Crane’s oscillation propositions

3.2.1. Oscillation when trolley has no acceleration

At normal condition (the trolley has no speed or constant speed), a pendulum will oscillate around the equilibrium position as following figure due to gravity:

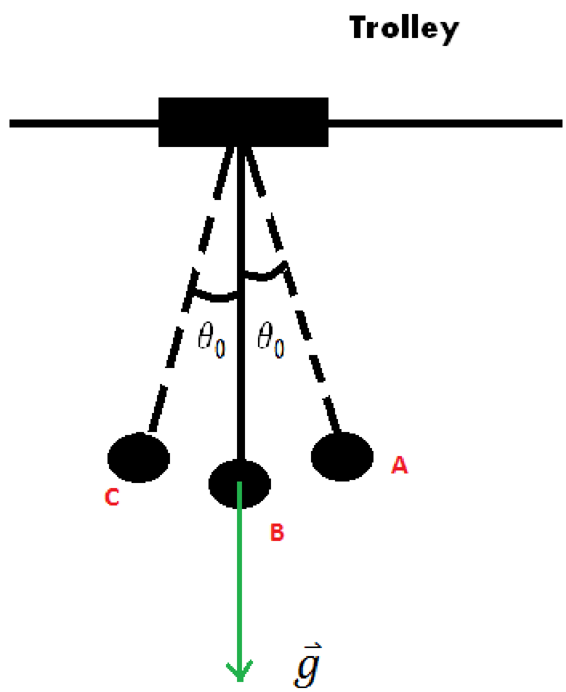

Figure 24. Crane oscillation during stable situation.

This condition is named as stable situation.

3.2.2. Oscillation during trolley acceleration/deceleration

In the situation which the trolley is accelerating with acceleration of $\vec{a}$, if we are taking the trolley as observational frame of reference (which basically means we are standing on the trolley), we will see the load swinging around another equilibrium position(named B in the figure) as follow:

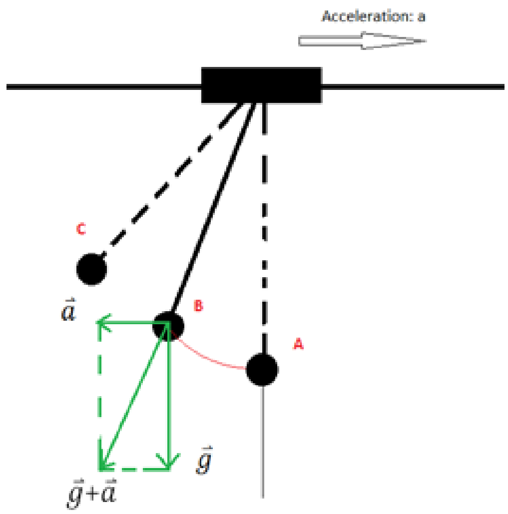

Figure 25. Crane oscillation during acceleration situation.

Moreover, if at beginning of the acceleration process, the load has no oscillation and velocity, the lowest point (named A in above figure) and highest point (named C in above figure) are θ_0 positions (maximum angle positions). As a result, velocity at A, $v_A = 0$ and velocity at C, $v_C = 0$ during oscillation period around B.

Let's name this accelerating situation. If we apply this with a=0, we will have situation described in "3.2.1. Oscillation when trolley has no acceleration", in which the trolley is moving at constant speed.

Same argument is applied for deceleration situation:

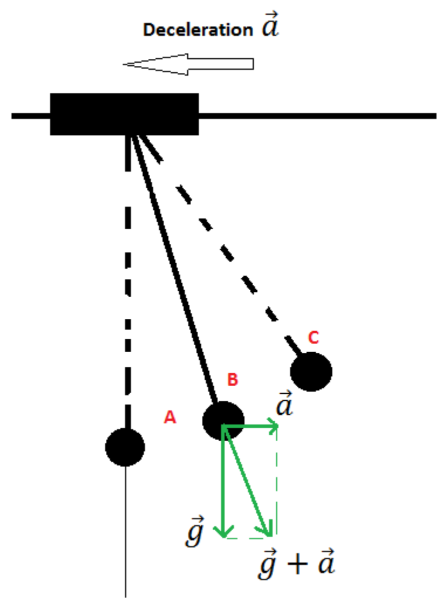

Figure 26. Crane oscillation during deceleration situation.

3.2.3. Symmetric property of oscillation

Oscillation is a function of *sine* or *cosine*. As a result, oscillation is a symmetric movement between two random positions. (Randall D. Knight, 2012: 378–406).

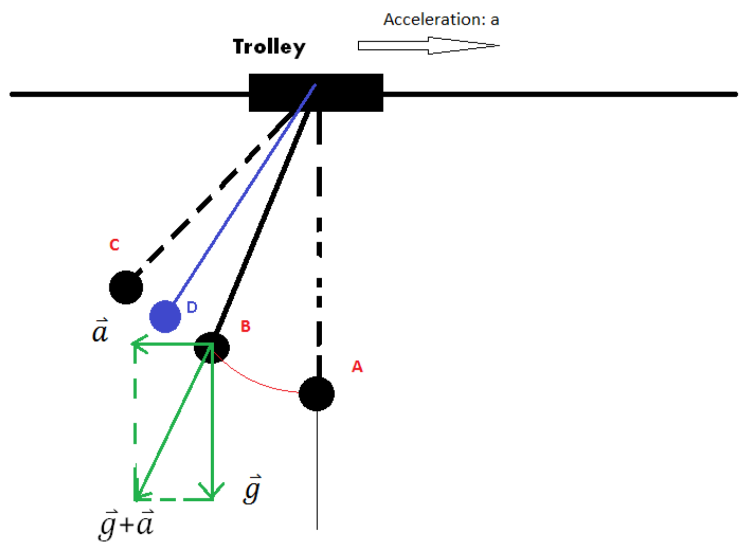

Figure 27. Symmetric oscillation around point B.

Therefore we will have following facts:

- Time taken to travel from A→B→D will be same as D→B→A
- Time taken to travel from A→B→D→C→D will be same as D→C→D→B→A.
- Velocity at certain position does not depend on moving direction. For example: velocity at point B when load is moving to the left is the same at velocity at point B when load is moving to the right.

In other words, time taken for travelling from one point to another is independent of moving direction during symmetric oscillation.

3.3. Anti-sway algorithm mechanism

3.3.1. Ramping up period

Based on theoretical analysis, following speed control pattern is proposed in order to reduce swing when ramping up.

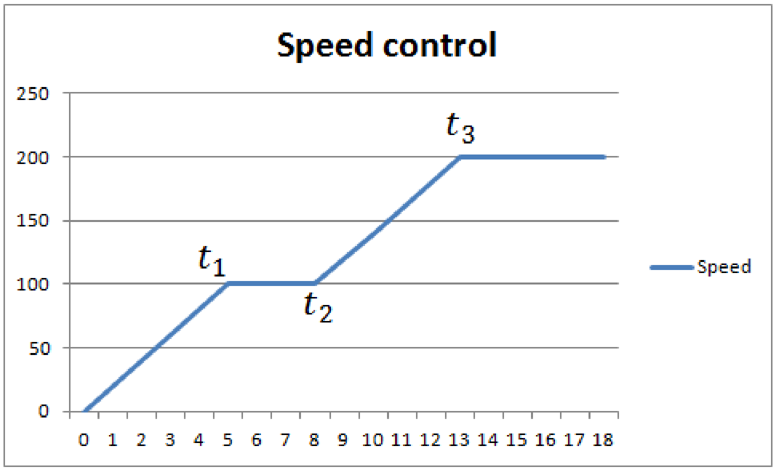

Figure 28. Speed control during ramping-up.

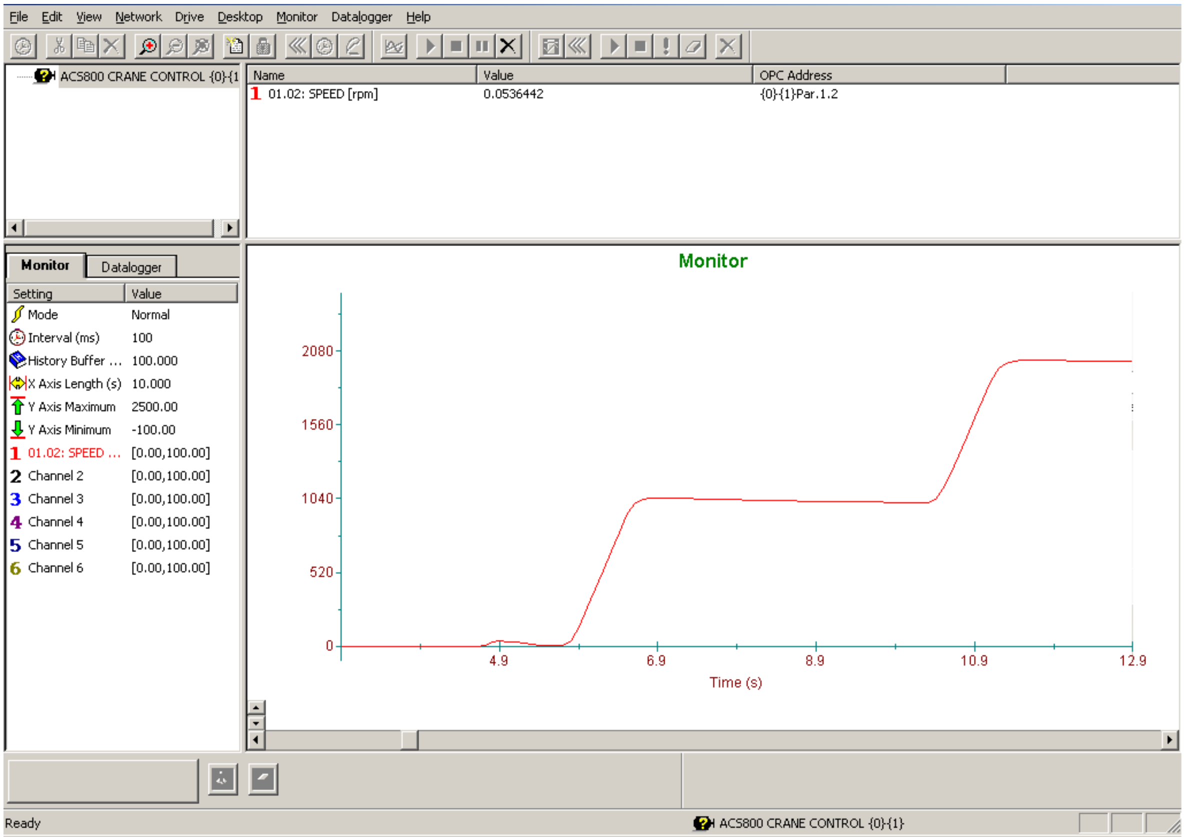

Figure 29. ABB ACS800 ramping-up.

The above figure shows the process when the motor is accelerating (ramping up). Motor deceleration (ramping down) will be controlled in same manner.

In this control pattern, we have three time marks: t_1, t_2 *and* t_3.

- From 0 to t_1 : motor will accelerate/ramp-up from 0 to half of target speed. The speed that the crane will achieve during constant speed period is called *target speed*.
- From t_1 to t_2: motor will keep constant speed.
- From t_2 to t_3: motor will accelerate/ramp-up to target speed.

The first period from 0 to t_1 is trivial. The algorithm will send a start command and set speed to half of target speed in motor drive. Then motor drive will ramp up its motor according to pre-defined acceleration parameter.

The third period from t_2 to t_3 is very similar to first period mentioned above. The algorithm will set speed to full target speed in motor drive. Then motor drive will ramp up its motor according to pre-defined acceleration parameter.

The critical period is from t_1 to t_2, the second period of the whole ramping-up process. In fact, the algorithm is all about deciding how long this period should be to effectively prevent oscillation/swing.

From 0 to t_1, the load will oscillate around point B with A and C as θ_0 positions (maximum angle positions). (Refer to “3.2.2. Oscillation during trolley acceleration/deceleration”).

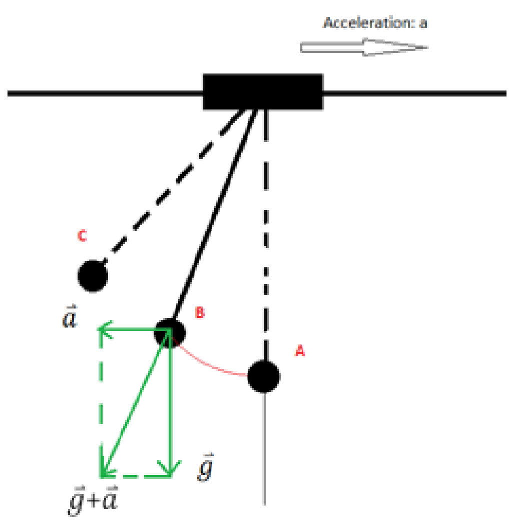

Figure 30. Oscillation around equilibrium point B during acceleration.

Let’s suppose at t_1, our load is at point D in the following figure:

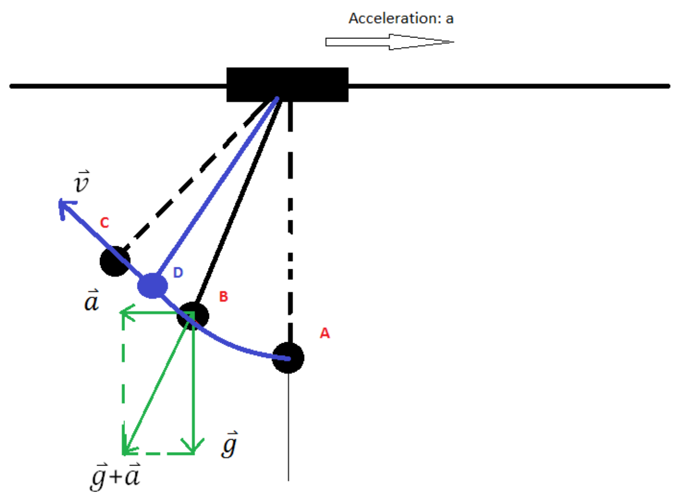

Figure 31. Load is at random position point D during acceleration.

After t_1, the motor will be kept running at constant speed. The load with its current speed will now oscillate around point A (refer to "3.2.1. Oscillation when trolley has no acceleration"). The load will go up a bit more and return to D with exactly same speed in reversed direction (refer to "3.2.3. Symmetric property of oscillation"). At the moment when the load is moving back to D again in reversed direction like following figure:

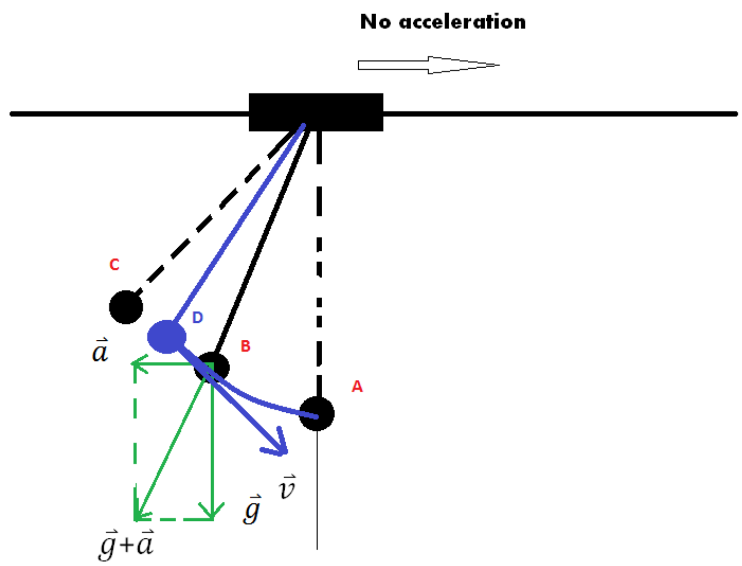

Figure 32. Load is at position point D again (in reverse direction).

Right at this moment (t_2), the motor drive‘s speed parameter will be set to full target speed. The motor will be ramped up to target speed.

When the crane reaches target speed (t_3), the load will be exactly at point A with zero velocity.

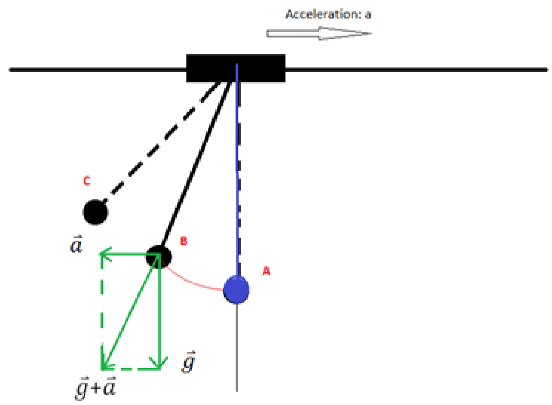

Figure 33. Load at position point A with zero velocity.

How do we know at t_3, the load will be at A? Because average acceleration $\vec{a}$ is constant, we have: $t_1 - 0 = t_2 - t_3$. (Time to accelerate from 0 to half of target speed is same as time to accelerate from half of target speed to full target speed).

$$\Delta t = \frac{\Delta v}{a} \tag{3.1}$$

Δt is duration of period.

Δv is change of speed.

a is acceleration.

While oscillating around B, it takes t_1 to travel from A→D, so D→A will take same time $(t_3 - t_2)$. Moreover, from t_2 to t_3, the load is oscillating around B with A as θ_0(maximum angle). Therefore, at A, velocity will equal zero (At maximum angle, oscillation has no speed).

At t_3 when the load is at A with zero speed, the crane will move with constant target speed. It means that the load oscillate around A (refer to "3.2.1. Oscillation when trolley has no acceleration") with both initial position and speed are zero. As the result, after t_3, all oscillation is killed.

Notes:

At t_1, the position of the load can be like following figure where the load has already reached C and it is going back towards A.

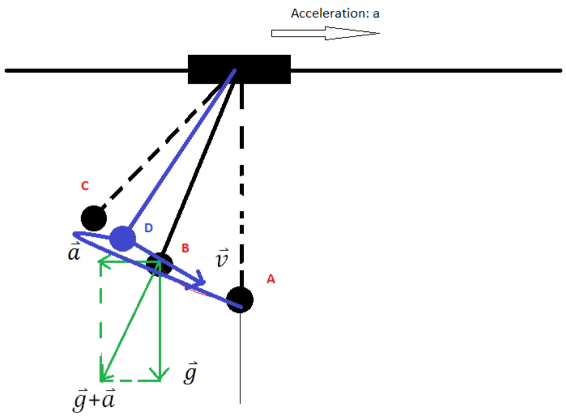

Figure 34. Load at position point D and going towards A (positive speed).

Then we need to keep the motor running at half of target speed until the load reaches D position in reversed direction again like following figure:

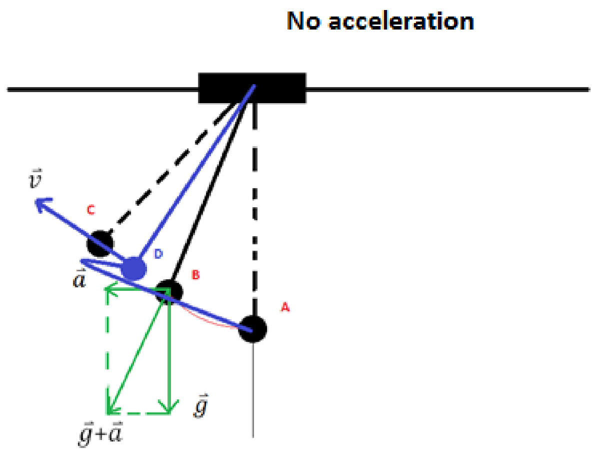

Figure 35. Load at position D and going towards C (negative speed).

When oscillating around B, time needs to travel A→B→C→D (t_1) will be same as D→C→B→A ($t_3 - t_2$).Therefore, at t_3 , the load will end up being at A with zero speed. Finally, after t_3 , all the oscillation will be killed.

In short, if at t_1 the load is at a particular point D, the motor will be kept running under constant speed (half of target speed) until the load is at D again in reversed direction (t_2). After that the crane will be accelerated up to target speed (t_3). Then, constant target speed is used for the rest of the movement. At the result of holding period (from t_1 to t_2), the oscillation will be fully eliminated during constant speed period.

3.3.2. Ramping down

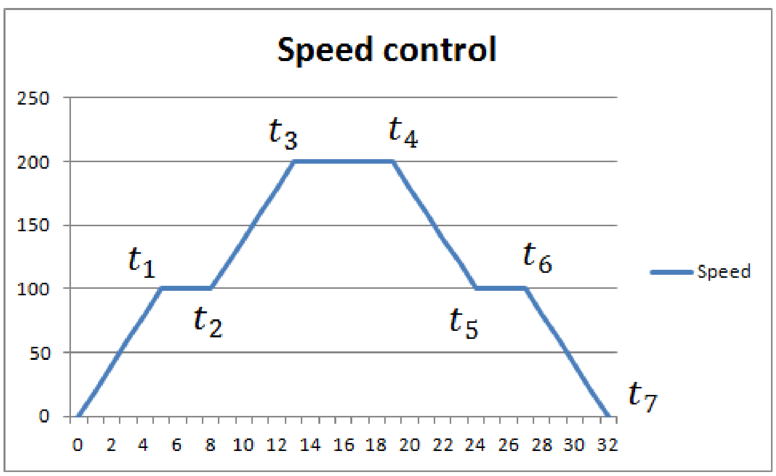

Figure 36. Anti-sway speed control.

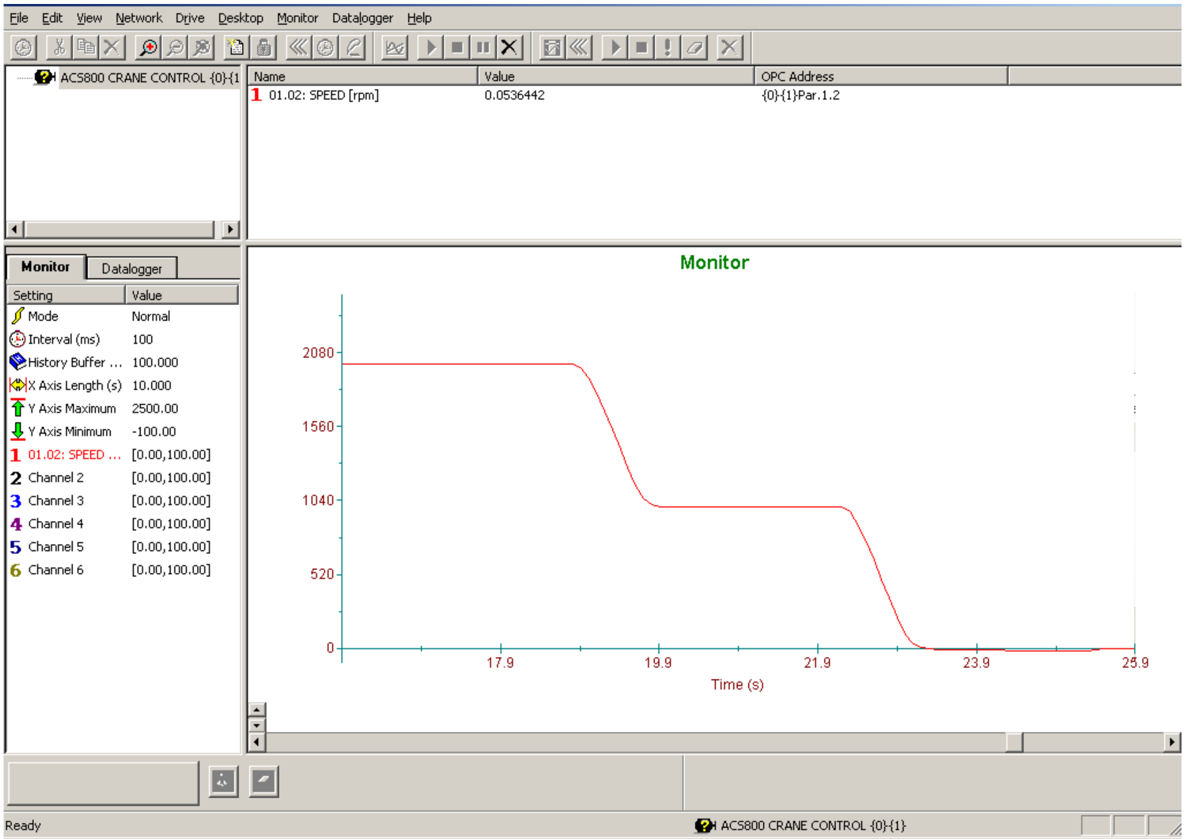

Figure 37.ABB ACS800 ramping-down.

The same principle is applied to ramp down / decelerate crane. After ramping up period (from 0 to t_3), all the oscillation is eliminated during constant speed period (from t_3 to t_4). Therefore, the situation right before t_4 is:

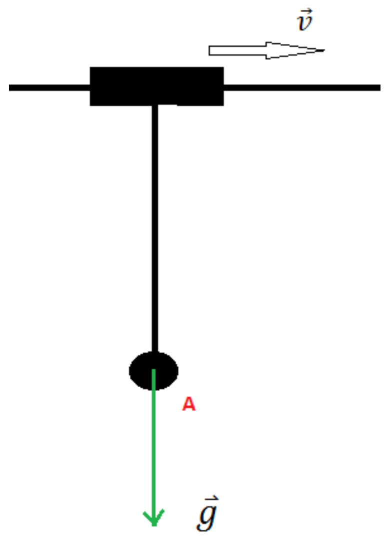

Figure 38. Crane with no oscillation before deceleration period.

Right before t_4, the oscillation's equilibrium position is A and there is no oscillation. At t_4, the speed parameter in motor drive is set to half of target speed. It causes the ramping down of motor speed. Because of deceleration $\vec{a}$, the equilibrium position is shifted to B as follow:

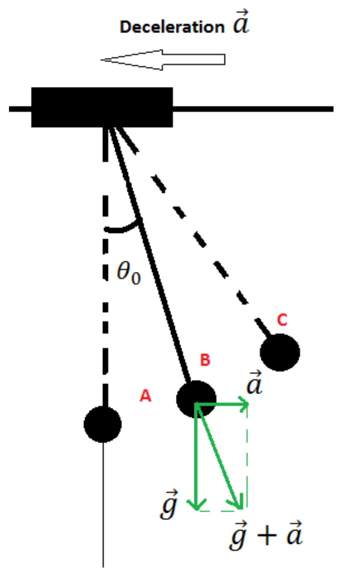

Figure 39. New equilibrium position point B during deceleration.

Because of the shift of equilibrium position, the load starts to oscillate around B with maximum angle (θ_0) at A and C. Suppose that the situation at t_5 is as follow:

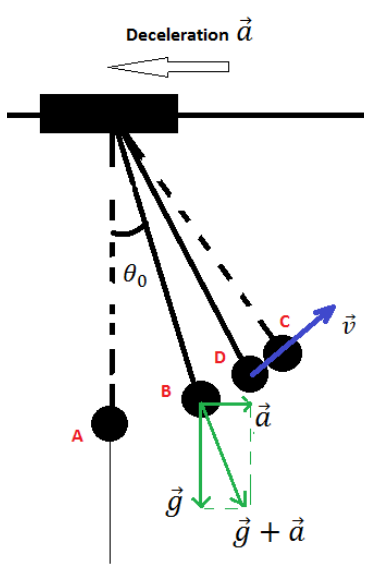

Figure 40. Load is at random position point D during deceleration.

At t_5, the load is at D with certain velocity $\vec{v}$. After t_5, motor is kept at constant speed (half of target speed). As a result, oscillation's equilibrium position is shifted back to A. Algorithm will wait until the load returns back to D with same velocity and in reversed direction as follow:

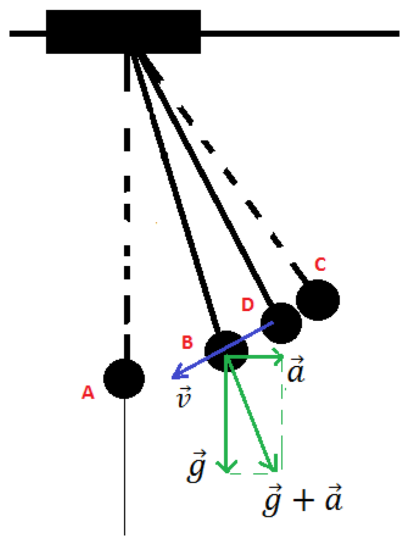

Figure 41. Load is at point D again (in reverse direction).

Right at above situation (t_6), algorithm sets speed parameter in motor drive to 0. The motor, then, will ramp down directly to zero in speed. During t_6 to t_7, the oscillation happens around equilibrium position B, with A as maximum angle.

According to formula, we can derive $(t_5 - t_4) = (t_7 - t_6)$ because we have same deceleration rate and change of speed during these periods. While oscillating around B, it takes $(t_5 - t_4)$ to travel from A→B→D, hence it will also take $(t_7 - t_6)$ to travel from D→B→A. As a result, at t_7 , the load is exactly at A (maximum angle position) with zero velocity, $v_A = 0$. After t_7 , the crane is stopped and no more oscillation is generated anymore. In other words, crane stops stably with no oscillation.

3.4. Anti-sway algorithm pseudo implementation

Let's denote that:

- $a(t_x)$ is swing direction at time t_x. $a(t_x)$ can either be 1 (swing in positive direction) or -1(swing in negative direction).
- $\theta(t_x)$ is vertical angle of the load at time t_x.
- $v(t_x)$ is crane's velocity at time t_x.
- v_{max} is crane's target speed.

The algorithm's logic is presented step by step as follow:

- At t = 0, the algorithm will set motor's speed reference to $\frac{v_{max}}{2}$ and start motor. Motor will ramp up.
- The algorithm will constantly poll actual speed value from motor drive. When actual speed reaches $\frac{v_{max}}{2}$, this moment is t_1. It means $v(t_1) = \frac{v_{max}}{2}$. Algorithm reads $a(t_1), \theta(t_1)$ from inclination sensor.
- The algorithm will constantly poll $a(t), \theta(t)$ until following two conditions is satisfied :
  - $\alpha(t) * \alpha(t_1) < 0$. This formula means the load is moving in opposite direction with $a(t_1)$.
  - $\alpha(t) * \theta(t) + \alpha(t_1) * \theta(t_1) < -\varepsilon, with\ \varepsilon\ is\ a\ very\ small\ number.$ This formula means the load is approaching $\theta(t_1)$ with very small difference ε or the load has already moved past $\theta(t_1)$.

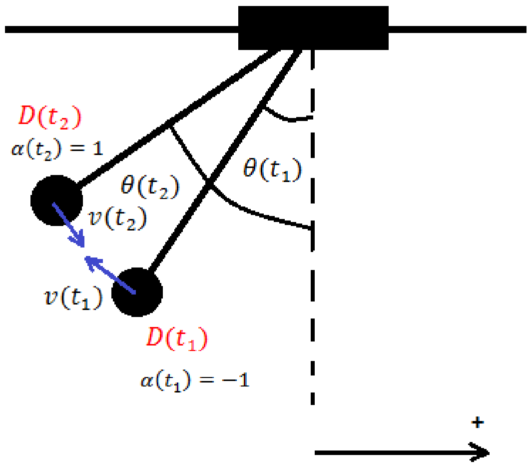

Figure 42. Crane oscillation variables.

If above two formulas are satisfied, the moment is t_2. The algorithm will set speed reference in motor drive to v_{max} causing motor to ramp up to full target speed.

- The algorithm waits until crane's operator presses button to stop the crane (t_4).
- At $t = t_4$, the algorithm will set motor's speed reference to $\frac{v_{max}}{2}$. Motor will ramp down from full speed v_{max} .
- The algorithm will constantly poll actual speed value from motor drive. When actual speed reaches $\frac{v_{max}}{2}$, this moment is t_5. It means $v(t_5) = \frac{v_{max}}{2}$. Algorithm reads $a(t_5), \theta(t_5)$ from inclination sensor.
- The algorithm will constantly poll $a(t), \theta(t)$ until following two conditions is satisfied :
  - $\alpha(t) * \alpha(t_5) < 0$. This formula means the load is moving in opposite direction with $a(t_5)$.
  - $\alpha(t) * \theta(t) + \alpha(t_5) * \theta(t_5) < -\varepsilon, with\ \varepsilon\ is\ a\ very\ small\ number.$ This formula means the load is approaching $\theta(t_5)$ with very small difference ε or the load has already moved past $\theta(t_5)$. Notice that this formula is exactly same as the formula for ramping up period.

- If above two formulas are satisfied, the moment is t_6. The algorithm will set speed reference in motor drive to 0 causing motor to ramp down to zero speed.

3.5. Conclusion

In this chapter, physics model of crane has been introduced and analyzed. Crane's hoist was chosen as observational frame of reference. Based on selected frame of reference, a detailed explanation of swinging phenomenon was presented. In normal crane's operation, swinging of crane is amplified during all periods of crane's movement: ramping up, constant speed and ramping down. In order to minimize swaying of crane, anti-sway algorithm was developed and proven using theoretical physics laws. Motor drive's parameters and inclination sensor's values are served as inputs for anti-sway algorithm. In next chapter, simulation tool will be used to simulate algorithm's behaviors as well as its effectiveness.