Type I error and power simulation

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Simulation Set-up

$$\begin{bmatrix} Y_i \\ \hat{Y}_i \end{bmatrix} \mid Z_{ik} \sim N\left(\begin{bmatrix} \beta_G G + \beta X_i \\ \alpha X_i \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

- $G \sim Bin(2, maf)$
- $maf = 0.25, X_i \sim N(0, 1)$
- $\alpha = \beta = 0.11, \, \beta_g = 0.11575982$
- missing rate $\in \{0, 0.25, 0.5, 0.75\}$
- $\rho \in \{0, 0.25, 0.5, 0.75\}$

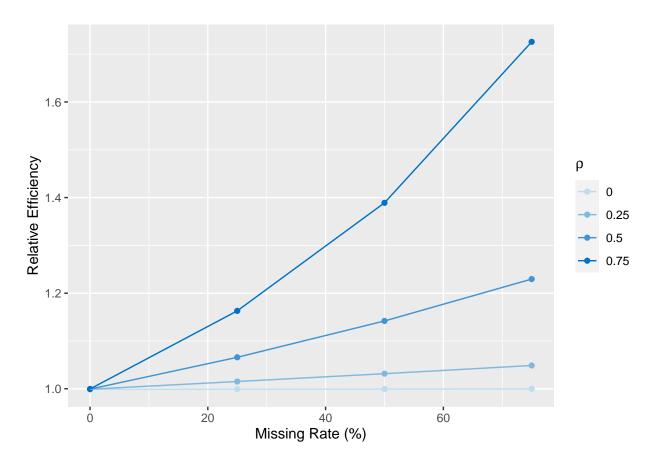
I do not think these parameters match whats in the simulation file, please confirm.

Type I error

Table 1: Proportion of test making type I error

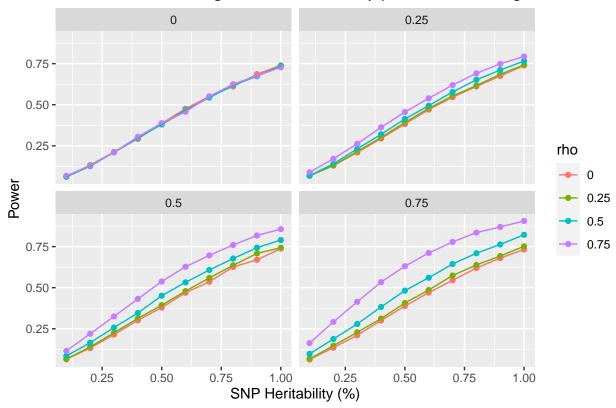
| missing | rho | t1e rejection | Chisq |
|---------|------|---------------|-------|
| 0.00 | 0.00 | 0.05 | 0.97 |
| 0.00 | 0.25 | 0.05 | 1.00 |
| 0.00 | 0.50 | 0.05 | 0.98 |
| 0.00 | 0.75 | 0.05 | 0.98 |
| 0.25 | 0.00 | 0.05 | 1.00 |
| 0.25 | 0.25 | 0.06 | 1.03 |
| 0.25 | 0.50 | 0.05 | 1.02 |
| 0.25 | 0.75 | 0.05 | 1.00 |
| 0.50 | 0.00 | 0.05 | 1.00 |
| 0.50 | 0.25 | 0.05 | 1.00 |
| 0.50 | 0.50 | 0.05 | 1.00 |
| 0.50 | 0.75 | 0.05 | 1.00 |
| 0.75 | 0.00 | 0.05 | 0.99 |
| 0.75 | 0.25 | 0.05 | 1.00 |
| 0.75 | 0.50 | 0.05 | 1.01 |
| 0.75 | 0.75 | 0.05 | 1.01 |

Increased Power relative to baseline GWAS

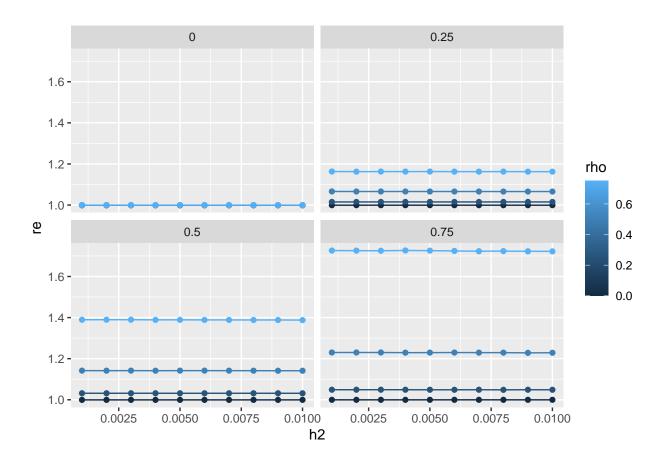


Power as a function of heritability

Power of bivariate regression, stratified by percent of missing lables



Unlike the previous plot, SNP heritability plays no role in relative efficiency.



Tables and Figures for the Manuscript

```
## % latex table generated in R 4.1.2 by xtable 1.8-4 package
## % Mon Aug 1 12:35:30 2022
## \begin{table}[ht]
## \centering
## \begin{tabular}{rrrrrr}
##
     \hline
   & Missing Rate (\%) & \rho Type I Error & \phi Power \\
##
##
     \hline
## 1 & 0.00 & 0.00 & 0.05 & 0.97 & 0.39 \\
     2 & 0.00 & 0.25 & 0.05 & 1.00 & 0.38 \\
##
##
     3 & 0.00 & 0.50 & 0.05 & 0.98 & 0.38 \\
##
     4 & 0.00 & 0.75 & 0.05 & 0.98 & 0.39 \\
##
     5 & 0.25 & 0.00 & 0.05 & 1.00 & 0.38 \\
##
     6 & 0.25 & 0.25 & 0.06 & 1.03 & 0.39 \\
##
     7 & 0.25 & 0.50 & 0.05 & 1.02 & 0.41 \\
##
     8 & 0.25 & 0.75 & 0.05 & 1.00 & 0.46 \\
     9 & 0.50 & 0.00 & 0.05 & 1.00 & 0.38 \\
##
##
     10 & 0.50 & 0.25 & 0.05 & 1.00 & 0.39 \\
##
     11 & 0.50 & 0.50 & 0.05 & 1.00 & 0.45 \\
     12 & 0.50 & 0.75 & 0.05 & 1.00 & 0.54 \\
##
     13 & 0.75 & 0.00 & 0.05 & 0.99 & 0.39 \\
##
     14 & 0.75 & 0.25 & 0.05 & 1.00 & 0.41 \\
```

```
## 15 & 0.75 & 0.50 & 0.05 & 1.01 & 0.48 \\
## 16 & 0.75 & 0.75 & 0.05 & 1.01 & 0.63 \\
## \hline
## \end{tabular}
## \end{table}
```