Appendix A

For the sake of analysis, nodes in the distribution network with radial topology are numbered from the source node. The node incidence matrix $\boldsymbol{A}^{N_\mathrm{n}\times N_\mathrm{n}}$ is defined as follows.

$$A(i,j)=\begin{cases}1, & \text{if node } j \text{ belongs to the sub-tree of node } i\\ 0, & \text{if node } j \text{ does not belong to the sub-tree of node } i\end{cases} \tag{A1.a}$$

It is specified that branch l with node k as the downstream node is numbered as L_{k-1}. Node k and the nodes belonging to node k's sub-tree ($A(k,i)=1$) are the general downstream nodes of branch l. Then, the general branch-node incidence matrix $\boldsymbol{M}^{N_\mathrm{b}\times N_\mathrm{n}}$ is defined as follows.

$$M_{l-i}=A(k,i),\ i\in\{1,2,\dots,N_\mathrm{n}\}$$

$$M_{l-i}=\begin{cases}1, & \text{if node } i \text{ is the general downstream node of branch } l\\ 0, & \text{if node } i \text{ is not the general downstream node of branch } l\end{cases} \tag{A1.b}$$

Appendix B

Operation constraints of devices

(1) Source side

Operation constraints of DG are shown as follows.

$$\begin{aligned}&P_{i,t}^{\mathrm{DG}}=P_{i,t}^{\mathrm{DG,ref}}\\&(P_{i,t}^{\mathrm{DG}})^2+(Q_{i,t}^{\mathrm{DG}})^2\le(S_i^{\mathrm{DG}})^2\\&Q_{i,t}^{\mathrm{DG}\prime}\ge Q_{i,t}^{\mathrm{DG}},Q_{i,t}^{\mathrm{DG}\prime}\ge -Q_{i,t}^{\mathrm{DG}}\end{aligned} \tag{B1.a}$$

where $Q_{i,t}^{\mathrm{DG}\prime}$ indicates the absolute value of reactive power of DG at node i in time t.

(2) Network side

a) Operation constraints of SOP

$$P_{i,t}^{\mathrm{SOP}}+P_{j,t}^{\mathrm{SOP}}+P_{i,t}^{\mathrm{SOP,L}}+P_{j,t}^{\mathrm{SOP,L}}=0,\ \forall i,j\in\Omega_{\mathrm{SOP}} \tag{B1.b}$$

$$\alpha_{c,0}P_{i,t}^{\mathrm{SOP}}+\alpha_{c,1}Q_{i,t}^{\mathrm{SOP}}+\alpha_{c,2}S_i^{\mathrm{SOP}}\le 0,\forall c\in\{1,2,\dots,12\} \tag{B1.c}$$

$$\alpha_{c,0}P_{i,t}^{\mathrm{SOP}}+\alpha_{c,1}Q_{i,t}^{\mathrm{SOP}}+\alpha_{c,2}\frac{P_{i,t}^{\mathrm{SOP,L}}}{A_i^{\mathrm{S,L}}}\le 0,\forall c\in\{1,2,\dots,12\} \tag{B1.d}$$

$$\underline{P}_i^{\mathrm{SOP}}\le P_{i,t}^{\mathrm{SOP}}\le \bar{P}_i^{\mathrm{SOP}},\underline{Q}_i^{\mathrm{SOP}}\le Q_{i,t}^{\mathrm{SOP}}\le \bar{Q}_i^{\mathrm{SOP}} \tag{B1.e}$$

where (B1.b) is the power balance constraint. (B1.c) is the linearized SOP capacity constraint. (B1.d) is the operational loss constraint after convex relaxation [5] and linearization. Note that the non-linear capacity constraints and loss constraints of devices are linearized through the approximate polygon method.

b) Operation constraints of ESS

$$E_{i,t}^{\mathrm{ESS}}=E_{i,t-1}^{\mathrm{ESS}}-(P_{i,t}^{\mathrm{ESS}}+P_{i,t}^{\mathrm{ESS,L}})\Delta t,\ \forall i\in\Omega_{\mathrm{ESS}} \tag{B1.f}$$

$$\underline{E}_i^{\mathrm{ESS}}\le E_{i,t}^{\mathrm{ESS}}\le \bar{E}_i^{\mathrm{ESS}} \tag{B1.g}$$

$$E_{i,\mathrm{T}}^{\mathrm{ESS}}=E_{i,0}^{\mathrm{ESS}} \tag{B1.h}$$

$$\underline{P}_i^{\mathrm{ESS}}\le P_{i,t}^{\mathrm{ESS}}\le \bar{P}_i^{\mathrm{ESS}},\underline{Q}_i^{\mathrm{ESS}}\le Q_{i,t}^{\mathrm{ESS}}\le \bar{Q}_i^{\mathrm{ESS}} \tag{B1.i}$$

$$\alpha_{c,0}P_{i,t}^{\mathrm{ESS}}+\alpha_{c,1}Q_{i,t}^{\mathrm{ESS}}+\alpha_{c,2}S_i^{\mathrm{ESS}}\le 0,\forall c\in\{1,2,\dots,12\} \tag{B1.j}$$

$$\alpha_{c,0}P_{i,t}^{\mathrm{ESS}}+\alpha_{c,1}Q_{i,t}^{\mathrm{ESS}}+\alpha_{c,2}\frac{P_{i,t}^{\mathrm{ESS,L}}}{A_i^{\mathrm{E,L}}}\le 0,\forall c\in\{1,2,\dots,12\} \tag{B1.k}$$

where (B1.f)-(B1.h) are the SOC constraints. (B1.j) and (B1.k) are the linearized capacity constraint and loss constraint, respectively.

(3) Demand side

a) Operation constraints of DL

The power consumption of DL can respond to TOU and flexibility requirements as long as the charging demand can be satisfied within its available time. Electric vehicles and data centers are typical of DLs.

$$0\le P_{i,t}^{\mathrm{DL}}\le \bar{P}_i^{\mathrm{DL}} \tag{B2.a}$$

$$\underline{E}_i^{\mathrm{DL}}\le E_{i,t}^{\mathrm{DL}}\le \bar{E}_i^{\mathrm{DL}},t\ge \mathrm{T}_{i,\mathrm{start}}^{\mathrm{DL}} \tag{B2.b}$$

$$E_{i,t}^{\mathrm{DL}}\ge E_i^{\mathrm{DL,req}},t\ge \mathrm{T}_{i,\mathrm{req}}^{\mathrm{DL}} \tag{B2.c}$$

$$E_{i,t}^{\mathrm{DL}}=E_{i,t-1}^{\mathrm{DL}}+P_{i,t}^{\mathrm{DL}}\Delta t \tag{B2.d}$$

$$0\le Q_{i,t}^{\mathrm{DL}}\le P_{i,t}^{\mathrm{DL}}\cdot\tan(\cos^{-1}\vartheta_i^{\mathrm{DL}}) \tag{B2.e}$$

where $\vartheta_i^{\mathrm{DL}}$ is the power factor of DL at node i.

b) Operation constraints of TL

The power consumption of TL must be satisfied at once based on the pre-determined schedule.

$$P_{i,t}^{\mathrm{TL}}=P_{i,t}^{\mathrm{TL,ref}} \tag{B2.f}$$

$$Q_{i,t}^{\mathrm{TL}}=Q_{i,t}^{\mathrm{TL,ref}} \tag{B2.g}$$