Problem 2.

a. Use Inductive method.

Inductive Hypothesis: Assume G be a weighted undirected graph (V, E), T be a MST of G, if e is an edge in T, then $T-\{e\}$ has exactly two connected components.

Base: If there is one edge e in T, $T-\{e\}$ will be empty, i.e., exist two individual nodes, absolutely two connected part.

Maintenance:

hypothesis: if e is an edge in T, $|T|=K$, $T-\{e\}$ has two connected compnonents.

Let's prove, $|T|=K+1$, $T-\{e\}$ has two connected components.

As the "cut" defined in textbook, cut on T, thus $T=\{X\}+\{T-X\}$. (X is a node of T). According to textbook, $\{X\}$, $\{T-X\}$ are two connected components separetly. ⟨1⟩ if $e=X$, thus $\{e\}$, $\{T-e\}$ are two separately connected components ⟨2⟩ if $e \neq X$, according to our hypothesis, $\{T-e\}$ and $\{e\}$ are two connected components, however, X must connects to one of $\{T-e\}$, $\{e\}$, thus there are still two connected components.

b. ~~Assume T_1, T_2 are two connected components of $T-\{e\}$. (according to cut in textbook). N_1, N_2 are the node set of T_1, T_2. P 136.~~

Assume C and C' are two connected components of $T-\{e\}$. (according to what proved in a, there only exist two part).

Because e' is an edge that crosses the cut C, which means the other one vertex of e' must be in C', for there are only two connected components ~~of $T-\{e\}$~~ exists (conclusion a.). Besides, e' and e has the same weight, so $T-\{e\} \cup \{e'\}$ is also a MST.