CS5800 Algorithms Spring 2015 Assignment #5

1. (20 points) Run the Kruskal algorithm on the following undirected graph:

Edge	Length	Edge	Length	Edge	Length	Edge	Length
{1,5}	5.4	{1,6}	5.0	{1,8}	5.4	{2,3}	8.0
{2,4}	7.5	{2,9}	3.7	{3,4}	3.5	{3,5}	2.3
{3,7}	6.8	{4,5}	3.7	{4,6}	2.8	{4,8}	5.3
{5,6}	6.5	{5,7}	5.9	{5,9}	4.9	{6,7}	6.7
{6,9}	4.8	{7,8}	5.8				

Show the method calls during each step as well as the union-find structure after each step. It is not necessary to show the steps for any sorting that is done. For the union-find algorithm use these optimizations: path compression and rank minimization.

2. Let G be a weighted undirected graph (V,E) Let T be a MST of G. Prove the following:

a. (8 points) If e is an edge in T, then T-{e} (i.e., remove e from T) has exactly two connected components.

b. (12 points) Let C be one of the disconnected components of T-{e}, and let e' an edge that crosses the cut C (i.e., has one vertex in C and one vertex not in C), and has the same weight as e. Show that T-{e}u{e'} (i.e., remove e and add e') is a MST.

3. (20 points) Run the Jarník algorithm on the following undirected graph:

Edge	Length	Edge	Length	Edge	Length	Edge	Length
{1,3}	2	{1,4}	7	{1,7}	8	{2,3}	10
{2,4}	4	{2,5}	8	{3,4}	4	{3,6}	1
{3,9}	5	{4,5}	5	{4,7}	9	{4,9}	2
{5,6}	9	{5,8}	5	{6,7}	2	{6,8}	3
{7,8}	5	{7,9}	5				

The initial node is node 5, and only show the steps until there are 3 nodes (2 edges) in the tree. Use a binary heap for the priority queue.

Points will be deducted if more steps are shown than required.

4. (24 points) One of the duties of an operating system is to schedule the execution of tasks. Suppose that we have a set of N tasks { w_1, w_2, ..., w_N } such that each task w_i requires time t_i to perform. The total amount of time required is the sum of the t_i's no matter what order they are performed. The operating system should schedule the tasks so that the total amount of time each task waits until it is finished is minimized. For example, if one has two tasks w_1 and w_2 that require t_1=15 seconds and t_2=10 seconds, then if they are executed in the order w_1 followed by w_2, then w_1 takes 15 seconds to complete and w_2 takes 25 seconds to complete (including the time for w_1 to complete) for a total of 40 seconds. If they are executed in the other order then the total is 35 seconds, so they should be executed in the order w_2 followed by w_1.

Develop and algorithm for determining the best order for executing the tasks. Your algorithm must have time complexity no worse than O(n log n) where n is the number of tasks. Prove that your algorithm finds the best order, and prove that your algorithm has the required time complexity.

5. (16 points) Find the optimal variable-length bit encoding for this sequence of characters:

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HABBIEIHIHAAHAEBGCHBBHICDHGCHIHBBGBBBBGGIGACHABHACIBCBGIIBBAIBAIABBBAIGEBBCBCBCBDACBBBBFAAGAAAAAFAFF
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Show your encoding using the linear format. For example, the encoding W=0, X=10, Y=110, Z=111 is specified with W,(X,(Y,Z)).

How many bits are required for your encoding? How many bits are required for the encoding using ASCII bytes?