Problem 1

a.

The transposition of c and n will reverse the mappings of **$\pi(c)=k$ and $\pi(n)=q$.**

The relevant bigrams that occur in the encrypted message are

ln,nm,ln,nu,hc,cl,lc,ce

These are decrypted to the bigrams

hq, qy, hq, qj, ik, kh, hk, kz

with frequencies **0 , 0 , 0, 0, 0, 1, 0, 0.**

The transposed decrypted bigrams are **hk, ky, hk, kj, iq, qh, hq, qz.**

with frequencies **0, 2, 0, 0, 0, 0, 0, 0**

Thus $\alpha(\pi,\pi') = 2/1 = 2 > 0.5$.

Thus the transposition was likely to be accepted. The decryption will change:

	a	b	c	d	e	f	g	h	I	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
l	m	v	**q**	t	z	p	a	i	o	g	x	h	y	**k**	e	s		b	n	u	j	r	d	f	c	w

b.

The transposition of a and z will reverse the mappings of **$\pi(a)=m$ and $\pi(z)=w$.**

The relevant bigrams that occur in the encrypted message are

vz, za, iz, zl, vz, za, iz, zl, rz , zh, yz, zl, iz, zl, lz, zq, iz, zq, iz, zq.

za, ao, za, ao, qa,al,qa,al,la,ab,la,ae.

These are decrypted to the bigrams

rw, wm, ow, wh, rw, wm, ow, wh, bw, wi, cw, wh, ow, wh, hw, w , ow, w .

wm,me,wm,me, m,mh, m,mh,hm,mv,hm,mz

with frequencies **7,0,182,67,7,0,182,67,0,99,0,67,182,67,1,90,182,90.**

0, 0, 0, 0, 153, 0, 153, 0, 4,0,4, 0

The transposed decrypted bigrams are

rm,mm,om,mh, rm, mm, om,mh, bm, mj, cm, mh, om, mh,hm, m , om, m .

ww,we,ww,we, w,wh, w,wh,hw,wv,hw,wz

with frequencies **28,7,61,0,28,7,61,0,0,46,0,0,61,0,4,59,61,59.**

0,1,0,1,275, 67, 275, 67,1, 0, 1,0

Thus α(π,π') =

(0.1*0.1*28*7*61*28*7*61*46*61*4*59*61*59*1*1*275*67*275*67*1*1)/

(7*182*67*7*182*67*99*67*182*67*1*90*1*182*90*153*153*4*4)

= 7.25e-08 <0.5

Thus the transposition was unlikely to be accepted. The decryption map stay the same:

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
l	m	v	k	t	z	p	a	i	o	g	x	h	y	q	e	s		b	n	u	j	r	d	f	c	w

Problem 2

There are eight states in this Markov Chain, the Elicitation and Analysis of phase Requirements(R-E,R-A for short); System and Detailed Design of Design(D-S, D-D); Coding and Integration of Implementation(I-C, I-I); System Testing and Acceptance Testing of Testing (T-S and T-A).

Each state has a transition to itself. The probability for these transitions are 0.9, 0.8, 0.9, 0.95, 0.9, 14/15, 0.8, 0.8 respectively. The transport probability of each activity is calculated as the problem requires.

The stochastic matrix is:

	R-E	**R-A**	**D-S**	**D-D**	**I-C**	**I-I**	**T-S**	**T-A**
R-E	**0.9**	**0.1**	**0**	**0**	**0**	**0**	**0**	**0**
R-A	**0.1**	**0.8**	**0.1**	**0**	**0**	**0**	**0**	**0**
D-S	**0**	**0**	**0.9**	**0.1**	**0**	**0**	**0**	**0**
D-D	**0**	**0**	**0.01**	**0.95**	**0.04**	**0**	**0**	**0**
I-C	**0**	**0**	**0**	**0**	**0.9**	**0.1**	**0**	**0**
I-I	**0**	**0**	**0**	**0**	**0.02**	**14/15**	**7/150**	**0**
T-S	**0**	**0**	**0**	**0**	**0**	**0**	**0.8**	**0.2**
T-A	**0.12**	**0**	**0**	**0**	**0**	**0**	**0.08**	**0.8**

To calculate invariant distribution for the matrix above, we have

x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] -1 = 0

-0.1*x[1] + 0.1*x[2] + 0.12*x[8] = 0

0.1*x[1] - 0.2*x[2] = 0

0.1*x[2] - 0.1*x[3] + 0.01*x[4] = 0

0.1*x[3] - 0.05*x[4] = 0

0.04*x[4] - 0.1*x[5] + 0.02*x[6] = 0

0.1*x[5] - 1/15*x[6] = 0

7/150*x[6] - 0.2*x[7] + 0.08*x[8] = 0

0.2*x[7] - 0.2*x[8] = 0

The final answer is:

x1-x8

(0.16683217 0.08341609 0.10427011 0.20854022 0.11916584 0.17874876 0.06951341 0.06951341)

Problem 3

As the problem requires, we have,

$$(v1, v2, v3, v4) \begin{pmatrix} 0.3 & 0.1 & 0.2 & 0.4 \\ 0.1 & 0.3 & 0.6 & 0 \\ 0.4 & 0.4 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.5 & 0.1 \end{pmatrix} = (v1\ v2\ v3\ v4)$$

thus,

-0.7*x[1] + 0.1*x[2] + 0.4*x[3] + 0.2*x[4] = 0

0.2*x[1] + 0.6*x[2] - 0.9*x[3] + 0.5*x[4] = 0

0.4*x[1] + 0 * x[2] + 0.1*x[3] -0.9*x[4] = 0

additionaly,

x[1] + x[2] + x[3] + x[4] -1 = 0

So the invariant distribution is

(0.26, 0.26, 0.32, 0.15)

The next 3 states of the Markov chain are:

(0.1, 0.3, 0.6, 0)

(0.3, 0.34, 0.26, 0.10)

(0.248, 0.256, 0.34, 0.156)

Problem 4

$\pi = (0.1, 0.3, 0.6, 0)$

1. $\alpha(1,2) = \min\{1, \pi(2) * q21/\pi(1) * q12\} = 1$.

so probability does not change. It remains same. p12= 0.1

2. $\alpha(1,3) = \min\{1, \pi(3) * q31/\pi(1) * q13\} = 1$

so probability does not change. It remains same. p13= 0.2

3. $\alpha(1,4) = \min\{1, \pi(4) * q41/\pi(1) * q14\} = 0$

so no transition occurs. p14 = 0

p11 = 1 − p12 − p13 − p14 = 0.7

4. $\alpha(2,1) = \min\{1, \pi(1) * q12 / \pi(2) * q21\} = 1/3$

so the probability changes. p21 = q21$*\alpha(2, 1)$ = 1/30

5. $\alpha(2, 3) = \min\{1, \pi(3) * q32 / \pi(2) * q23\} = 1$

so probability does not change. It remains same. p23 = 0.6

6. $\alpha(2, 4) = \min\{1, \pi(4) * q42 / \pi(2) * q24\} = 0$

so no transition occurs. p24 = 0

p22 = 1 − p21 − p23 − p24 = 11/30

7. $\alpha(3, 1) = \min\{1, \pi(1)* q13 / \pi(3)* q31\} = 1/12$

so the probability changes. p31 = q31$*\alpha(3, 1)$ = 1/30

8. $\alpha(3, 2) = \min\{1, \pi(2)* q23 / \pi(3)* q32\} = 3/4$

so the probability changes. p32 = q32$*\alpha(3, 2)$ = 3/10

9. $\alpha(3, 4) = \min\{1, \pi(4)* q43 / \pi(3)* q34\} = 0$

so no transition occurs.p34 = 0

p33 = 1 − p31 − p32 − p34 = 2/3

10. $\alpha(4, 1) = \min\{1, \pi(1)* q14 / \pi(4)* q41\} = 1$

so probability does not change. It remains same. p41= 1/5

11. $\alpha(4, 2) = \min\{1, \pi(2)* q24 / \pi(4)* q42\} = 1$

so probability does not change. It remains same. p42 = 1/5

12. $\alpha(4, 3) = \min\{1, \pi(3)* q34 / \pi(4)* q43\} = 1$

so probability does not change. It remains same. p43 = 1/2

p44 = 1 − p31 − p32 − p34 = 0.1

The modified matrix is :

7/10	1/10	1/5	0
1/30	11/30	0.6	0
1/30	3/10	2/3	0
1/5	1/5	1/2	1/10

Problem 5

	effect	non-effect
effect	0.5	0.1
non-effect	0.5	0.9

We can formalize this problem as a imperfect test.

The probability table is above.

As bi-normial distribution . If we want to have a confidence of 95%, the upper and lower boundary of it is $np - 1.96(np(1-p))^{1/2}$, $np + 1.96(np(1-p))^{1/2}$ respectively.

we now have two distributions here(with p=0.5 and p = 0.1). In order to have the correct effect testing result, we need to make the upper bound of p=0.1 equals the lower bound when p =0.5 to make these two non-overlap

Thus, we have

$n*0.5 - 1.96(n*0.5(1-0.5))^{1/2} = n*0.1 + 1.96(n*0.1*(1-0.1))^{1/2}$

so we can have 15 or 16 patients to be treated.