CS5800 Algorithms Spring 2015 Assignment #7

1. An advertising agency is planning an advertising campaign. It has M advertising outlets that are suitable for the campaign and N different advertisements that can be used. Each combination of advertisement and outlet has a different cost and impact. Each outlet has a limit on how many advertisements of each kind that it will accept for each monthly advertising period, as well as a limit on the total number of advertisements of all kinds in this campaign. Both of these are average values over time. The campaign will continue for an extended period of time, so it is meaningful to speak of having a fractional number of advertisements. The agency wants to maximize the impact of the campaign within a fixed average monthly budget.

(a) (10 points) Show how to solve this problem in general.

(b) (15 points) Solve the following specific case:

Monthly Costs					
	Dine	Gourmet	Gastronomica	Taste of Home	Cooking Light
Menu Ad	500	400	350	100	50
Celebrity Ad	1000	1500	500	200	80
Environment Ad	200	400	500	100	300

The monthly budget is 5000.

Impacts					
	Dine	Gourmet	Gastronomica	Taste of Home	Cooking Light
Menu Ad	3	4	2	5	4
Celebrity Ad	4	5	2	1	2
Environment Ad	1	3	6	5	2

Limits					
	Dine	Gourmet	Gastronomica	Taste of Home	Cooking Light
Menu Ad	1	1	1	2	1
Celebrity Ad	2	2	1	1	2
Environment Ad	2	1	2	3	4
Total Ads	3	3	3	4	4

2. (20 points) Maximize $4.9\ x_2 + 0.7\ x_3 + 1.7\ x_4 + 7.1\ x_6$ subject to

$3.6\ x_1 + 0.6\ x_2 + 5.9\ x_3 + 4.5\ x_4 + 0.4\ x_5 \leqslant 7.4$

$4.5\ x_2 + 1.8\ x_4 + 0.5\ x_6 \leqslant 4.3$

$8.1\ x_1 + 3.3\ x_2 + 7.5\ x_4 \leqslant 0.9$

$5.8\ x_1 + 5.6\ x_2 + 4.8\ x_6 \leqslant 2$

$1.5\ x_1 + 0.9\ x_3 + 1.8\ x_5 + 8.6\ x_6 \leqslant 2.6$

$3.1\ x_1 + 5.3\ x_2 + 1.1\ x_4 + 8\ x_5 \leqslant 8.1$

$9.9\ x_2 + 5.1\ x_4 + 7.9\ x_6 \geqslant -3$

$0.5\ x_2 + 5.3\ x_3 + 0.2\ x_4 + 8.6\ x_5 + 5.1\ x_6 \geqslant -8.5$

and such that all variables are nonnegative. Show that your solution satisfies the constraints.

3. (20 points) Maximize p + 2q + 3r + 4s such that

a. 2p + 4r is at least -10.
b. 5q + 3r is at least -12.
c. 8p + 2q is at most 5.
d. p + 6s is at most 2.
e. q + 7r is at most 1.
f. 3q + 5s is at most 8.

Show that your solution satisfies the constraints.

4. (15 points) Maximize $1.6\ x_1 + 1.7\ x_2 + 0.6\ x_3 + 9.3\ x_4$ such that:

$1.7\ x_1 + 4.7\ x_2 - 1.1\ x_3 + 2.7\ x_4 \leqslant 3.5$

$8.4\ x_1 + 5.9\ x_2 - 0.4\ x_3 \leqslant 5$

$-1.4\ x_2 + 9.6\ x_3 + 7\ x_4 \leqslant 7.9$

$5\ x_1 + 0.3\ x_2 + 8.9\ x_3 - 1.5\ x_4 \leqslant 9.9$

$-1.1\ x_2 + 8.1\ x_4 \leqslant 1.3$

and such that all variables are nonnegative. Show that your solution satisfies the constraints.

5. (20 points) Minimize -5a + 3b + 5c - 3d - 4e - 9f such that

$-6a + 3b + 5c + 2d + 5e + 6f \leqslant 1$

$10b + 5c + 5d - 5e + 8f \leqslant 4$

$8a - 4b + 7c + 4d - 5e + 3f \leqslant 10$

$-4a - 3b - 5c - 2d + 9e - 5f \geqslant -8$

$3a + 8b + 4c - d - 4e + 2f \geqslant -4$

and such that all variables are nonnegative.

Express this problem in standard form, and find the solution to the dual problem. Prove your solution is correct using the solution to the dual problem.