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Citation: using R package lpsolveAPI to solve the lp part of my assignment in all problems.

Problem 1

this problem is a typical Linear programming problem.

(a)

Variables

one ad variable X for each kind of ad of each outlet.

In this problem, X11-X15 is the number of menu ad for Dine, Gourment, Gastronomica, Taste of Home, Cooking Light. X21-X25, X31-X35 are number of celebrity and environment ad of them.

Constraints

1. the totoal costs of ads should be less or equal to the budget 5000
2. each ad of a oulet should be less or equal to the limits (15 constraints in total)

   details of these constraints are represented in (b)

Objective Funtion

Max the total impact function, i.e., the X (matrix of all kinds of ad) * Transpose of Impact(impact of all kinds of ad)

max(X * Impacts^T) .

(b)

In this problem, we maximize $c^T x$ subject to $Ax \leqslant b$ and $x \geqslant 0$ where

$C = (3,4,2,5,4,4,5,2,1,2,1,3,6,5,2)^T$

$X = (x11, x12, x13, x14, x15, x21, x22, x23, x24, x25, x31, x32, x33, x34, x35)^T$

A =(

500,	400,	350,	100,	50,	1000,	1500,	500,	200,	80,	200,	400,	500,	100,	300,
1,	0,	0,	0,	0,	1,	0,	0,	0,	0,	1,	0,	0,	0,	0,
0,	1,	0,	0,	0,	0,	1,	0,	0,	0,	0,	1,	0,	0,	0,
0,	0,	1,	0,	0,	0,	0,	1,	0,	0,	0,	0,	1,	0,	0,
0,	0,	0,	1,	0,	0,	0,	0,	1,	0,	0,	0,	0,	1,	0,
0,	0,	0,	0,	1,	0,	0,	0,	0,	1,	0,	0,	0,	0,	1,

)

B = $(5000,3,3,3,4,4)^T$

Other limits constraints I made a bounder on X in R package IpsolveAPI.

After using lpsolve tool,

The optimal solution is p = 59.9

attained at X = $(1.0\ 1.0\ 1.0\ 1.0\ 1.0\ 1.3\ 0.0\ 0.0\ 0.0\ 2.0\ 0.7\ 1.0\ 2.0\ 3.0\ 1.0)^T$

Checking the constraints:

1. $c^Tx = 5000 <= 5000$
2. $c^Tx = 3 <= 3$
3. $c^Tx = 2 <= 3$
4. $c^Tx = 3 <= 3$
5. $c^Tx = 4 <= 4$
6. $c^Tx = 4 <= 4$

Problem 2

In this problem, we maximize c^Tx subject to $Ax \leqslant b$ and $x \geqslant 0$ where

C = $(0, 4.9, 0.7, 1.7, 0, 7.1)^T$

X = $(x1, x2, x3, x4, x5, x6)^T$

A =(

3.6, 0.6, 5.9, 4.5, 0.4, 0.0,

0.0, 4.5, 0.0, 1.8, 0.0, 0.5,

8.1, 3.3, 0.0, 7.5, 0.0, 0.0,

5.8, 5.6, 0.0, 0.0, 0.0, 4.8,

1.5, 0.0, 0.9, 0.0, 1.8, 8.6,

3.1, 5.3, 0.0, 1.1, 8.0, 0.0,

0.0, 9.9, 0.0, 5.1, 0.0, 7.9,

0.0, 0.5, 5.3, 0.2, 8.6, 5.1

)

B = (7.4, 4.3, 0.9, 2, 2.6, 8.1, -3, -8.5)^T

After using lpsolve tool,

The optimal solution is p =3.16,

attained at X = (0.00, 0.21, 1.21, 0.03, 0.00, 0.18)^T

Checking the constraints:

1. c^Tx = 7.40 <= 7.4
2. c^Tx = 1.07 <= 4.3
3. c^Tx = 0.90 <=0.9
4. c^Tx = 2.00 <= 2
5. c^Tx = 2.60 <= 2.6
6. c^Tx = 1.13 <= 8.1
7. c^Tx = 3.58 >= -3
8. c^Tx = 7.42 >= -8.5

Problem 3

In this problem, we maximize c^Tx subject to $Ax \leqslant b$ and $x \geqslant 0$ where

Considering non-negative, replace p = x1 –x2, q = x3 –x4, r= x5 – x6, s = x7 – x8.

C = (1, -1, 2, -2, 3, -3 , 4 ,-4)^T

X = (x1, x2, x3, x4, x5, x6, x7, x8)^T

A =(
2, 0, 4, 0,
0, 5, 3, 0,
8, 2, 0, 0,
1, 0, 0, 6,
0, 1, 7, 0,
0, 3, 0, 5
)

B = (-10, -12, 5, 2, 1, 8)^T

After using lpsolve tool,

The optimal solution is p =5.15

attained at X = (0.09 0.00 2.14 0.00 0.00 0.16 0.32 0.00)^T

Checking the constraints:

1. $c^Tx = -0.47 >= -10$
2. $c^Tx = 10.19 >= -12$
3. $c^Tx = 5 <= 5$
4. $c^Tx = 2 <= 2$
5. $c^Tx = 1 <= 1$
6. $c^Tx = 8 <= 8$

Problem 4

In this problem, we maximize c^Tx subject to $Ax \leqslant b$ and $x \geqslant 0$ where

C = (1.6, 1.7, 0.6, 9.3)^T

X = (x1, x2, x3, x4)^T

A =(
1.7, 4.7, -1.1, 2.7,
8.4, 5.9, -0.4, 0.0,
0.0, -1.4, 9.6, 7.0,

5.0, 0.3, 8.9,-1.5,

0.0,-1.1, 0.0, 8.1

)

$B = (3.5, 5, 7.9, 9.9, 1.3)^T$

After using lpsolve tool,

The optimal solution is p =4.28

attained at $X = (0.12, 0.73, 0.74, 0.26)^T$

Checking the constraints:

1. $c^Tx = 3.5 <= 3.5$
2. $c^Tx = 5 <= 5$
3. $c^Tx = 7.90 <= 7.9$
4. $c^Tx = 7.02 <= 9.9$
5. $c^Tx = 1.30 <= 1.3$

Problem 5

In this problem, we minimize c^Tx subject to $Ax \leqslant b$ and $x \geqslant 0$ where

$C = (-5, 3, 5, -3, -4, -9)^T$

$X = (a, b, c, d, e, f)^T$

A =(

-6, 3, 5, 2, 5, 6,

0, 10, 5, 5, -5, 8,

8, -4, 7, 4, -5, 3,

-4, -3, -5, -2, 9,-5,

3, 8, 4, -1, -4, 2

)

$B = (1,4,10,-8,-4)^T$

After using lpsolve tool,

The optimal solution is p = -54.35

attained at X = (5.85 0.71 0.00 0.00 6.80 0.00)^T

Checking the constraints:

1. $c^Tx = 1 <= 1$
2. $c^Tx = -26.95 <= 4$
3. $c^Tx = 10 <= 10$
4. $c^Tx = 35.67 >= -8$
5. $c^Tx = -4 >= -4$

<u>dual problem</u>

we minimize b^Ty subject to $A^Ty \geqslant c$ and $y \geqslant 0$

The optimal solution is p = -54.35

attained at Y =(5.61 0 4.85 0 0.05)^T

Thus, the optimal solution of dual equals the original results.

So, original solution is optimal.