Problem 1

time 1

BFS from node 8	max flow to node	parent with max flow
8 -> 3	4	8
8 -> 4	3	8
8 -> 5	7	8
8 -> 6	7	8
3 -> 7	7	3
3 -> 8	21	3
7 -> 2	21	7

The path is 8,3,7,2 with flow 4

Residual graph edge	Weight
(1,5)	9
(1,8)	25
(2,4)	21
(2,6)	21
(2,7)	**11**
(3,4)	5
(3,6)	24
(3,7)	**3**
(3,8)	**25**
(4,3)	24
(4,5)	12
(4,7)	15
(4,8)	24
(5,3)	13
(5,6)	12
(6,3)	21
(6,4)	2
(6,5)	11

(6,8)	19
(7,2)	**17**
(7,3)	**4**
(8,4)	3
(8,5)	7
(8,6)	7

time 2

BFS from node 8	max flow to node	parent with max flow
8 -> 4	3	8
8 -> 5	7	8
8 -> 6	7	8
4 -> 3	24	4
4 -> 7	15	4
4 -> 8	24	4
7 -> 2	17	7

The path is 8,4,7,2 with flow 3

Residual graph edge	Weight
(1,5)	9
(1,8)	25
(2,4)	21
(2,6)	21
(2,7)	**14**
(3,4)	5
(3,6)	24
(3,7)	3
(3,8)	25
(4,3)	24
(4,5)	12
(4,7)	**12**

(4,8)	**27**
(5,3)	13
(5,6)	12
(6,3)	21
(6,4)	2
(6,5)	11
(6,8)	19
(7,2)	**14**
(7,3)	4
(7,4)	**3**
(8,5)	7
(8,6)	7

time 3

BFS from node 8	max flow to node	parent with max flow
8 -> 5	7	8
8 -> 6	7	8
5 -> 3	13	5
6 -> 4	2	6
6 -> 8	19	6
3 -> 7	3	3
7 -> 2	14	7

The path is 8,5,3,7,2 with flow 3

Residual graph edge	Weight
(1,5)	9
(1,8)	25
(2,4)	21
(2,6)	21
(2,7)	**17**
(3,4)	5

(3,5)	**3**
(3,6)	24
(3,8)	25
(4,3)	24
(4,5)	12
(4,7)	12
(4,8)	27
(5,3)	**10**
(5,6)	12
(5,8)	**3**
(6,3)	21
(6,4)	2
(6,5)	11
(6,8)	19
(7,2)	**11**
(7,3)	**7**
(7,4)	3
(8,5)	**4**
(8,6)	7

time 4

BFS from node 8	max flow to node	parent with max flow
8 -> 5	4	8
8 -> 6	7	8
5 -> 3	10	5
5 -> 8	3	5
6 -> 4	2	6
4 -> 7	12	4
7 -> 2	11	7

The path is 8,6,4,7,2 with flow 2

Residual graph edge	Weight
(1,5)	9
(1,8)	25
(2,4)	21
(2,6)	21
(2,7)	**19**
(3,4)	5
(3,5)	3
(3,6)	24
(3,8)	25
(4,3)	24
(4,5)	12
(4,6)	**2**
(4,7)	**10**
(4,8)	27
(5,3)	10
(5,6)	12
(5,8)	3
(6,3)	21
(6,5)	11
(6,8)	**21**
(7,2)	**9**
(7,3)	7
(7,4)	**5**
(8,5)	4
(8,6)	**5**

time 5

BFS from node 8	max flow to node	parent with max flow
8 -> 5	4	8
8 -> 6	5	8

6 -> 3	21	6
3 -> 4	5	3
4 -> 7	10	4
7 -> 2	9	7

The path is 8,6,3,4,7,2 with flow 5

Residual graph edge	Weight
(1,5)	9
(1,8)	25
(2,4)	21
(2,6)	21
(2,7)	**24**
(3,5)	3
(3,6)	**29**
(3,8)	25
(4,3)	**29**
(4,5)	12
(4,6)	2
(4,7)	**5**
(4,8)	27
(5,3)	10
(5,6)	12
(5,8)	3
(6,3)	**16**
(6,5)	11
(6,8)	**26**
(7,2)	**4**
(7,3)	7
(7,4)	**10**
(8,5)	4

time 6

BFS from node 8	max flow to node	parent with max flow
8 -> 5	4	8
5 -> 3	10	5
5 -> 6	12	5

This does not reach node 2, so it is a min cut.

The max flow is 17 .

The min cut consists of { 3, 5, 6, 8 }

The edges across the cut are

(3 ,4) with weight 5

(3 ,7) with weight 7

(6 ,4) with weight 2

(8 ,4) with weight 3

Problem 2

	x1	x2	x3	x4	x5	s1	s2	s3	s4	s5	constant
	8.00	0.00	3.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	1.00	3.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	4.00
2	3.00	4.00	8.00	2.00	7.00	0.00	1.00	0.00	0.00	0.00	4.00
3	0.00	1.00	8.00	6.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
4	7.00	0.00	1.00	0.00	3.00	0.00	0.00	0.00	1.00	0.00	2.00
5	0.00	5.00	0.00	1.00	5.00	0.00	0.00	0.00	0.00	1.00	7.00

Variable	Index	Best Constraint	Function Increase
x1	1	Row 4	16/7
x3	3	Row 3	3/8
x5	5	Row 2	8/7

Pivoting on basic variable s4 and non-basic variable x1

The following is the current tableau:

	x1	x2	x3	x4	x5	s1	s2	s3	s4	s5	constant
	0. 00	0. 00	1. 86	0. 00	−1. 43	0. 00	0. 00	0. 00	−1. 14	0. 00	−2. 29
1	0. 00	1. 00	3. 00	0. 00	0. 00	1. 00	0. 00	0. 00	0. 00	0. 00	4. 00
2	0. 00	4. 00	7. 57	2. 00	5. 71	0. 00	1. 00	0. 00	−0. 43	0. 00	3. 14
3	0. 00	1. 00	8. 00	6. 00	1. 00	0. 00	0. 00	1. 00	0. 00	0. 00	1. 00
4	1. 00	0. 00	0. 14	0. 00	0. 43	0. 00	0. 00	0. 00	0. 14	0. 00	0. 29
5	0. 00	5. 00	0. 00	1. 00	5. 00	0. 00	0. 00	0. 00	0. 00	1. 00	7. 00

Variable	Index	Best Constraint	Function Increase
x3	3	Row 3	3/8

Pivot on non-basic variable x3 and basic variable s3

The following is the current tableau:

	x1	x2	x3	x4	x5	s1	s2	s3	s4	s5	constant
	0. 00	−0. 23	0. 00	−1. 40	−1. 66	0. 00	0. 00	−0. 23	−1. 14	0. 00	−2. 52
1	0. 00	0. 63	0. 00	−2. 25	−0. 38	1. 00	0. 00	−0. 38	0. 00	0. 00	3. 63
2	0. 00	3. 05	0. 00	−3. 68	4. 77	0. 00	1. 00	−0. 95	−0. 43	0. 00	2. 20
3	0. 00	0. 13	1. 00	0. 75	0. 13	0. 00	0. 00	0. 13	0. 00	0. 00	0. 13
4	1. 00	−0. 02	0. 00	−0. 11	0. 41	0. 00	0. 00	−0. 02	0. 14	0. 00	0. 27
5	0. 00	5. 00	0. 00	1. 00	5. 00	0. 00	0. 00	0. 00	0. 00	1. 00	7. 00

The maximum value of the objective function has been found.

The maximum value of the objective function is: 2.52

The maximum point is

(x1 =0.27, x2=0.00, x3 = 0.13, x4 =0.00, x5=0.00)

Problem 3

	a	b	c	d	s1	s2	s3	s4	s5	constant
	8.00	7.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	7.00	0.00	1.00	0.00	0.00	0.00	0.00	4.00
2	3.00	0.00	6.00	2.00	0.00	1.00	0.00	0.00	0.00	8.00
3	0.00	7.00	0.00	4.00	0.00	0.00	1.00	0.00	0.00	0.00
4	4.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	8.00
5	3.00	8.00	0.00	2.00	0.00	0.00	0.00	0.00	1.00	7.00

Variable	Index	Best Constraint	Function Increase
a	1	row 4	16
b	2	row 3	0
c	3	row 1	4/7

Pivoting on basic variable s4 and non-basic variable a

The following is the current tableau:

	a	b	c	d	s1	s2	s3	s4	s5	constant
	0.00	7.00	1.00	−2.00	0.00	0.00	0.00	−2.00	0.00	−16.00
1	0.00	0.00	7.00	0.00	1.00	0.00	0.00	0.00	0.00	4.00
2	0.00	0.00	6.00	1.25	0.00	1.00	0.00	−0.75	0.00	2.00
3	0.00	7.00	0.00	4.00	0.00	0.00	1.00	0.00	0.00	0.00
4	1.00	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00	2.00
5	0.00	8.00	0.00	1.25	0.00	0.00	0.00	−0.75	1.00	1.00

Variable	Index	Best Constraint	Function Increase
b	2	row 3	0
c	3	row 2	1/3

Pivoting on basic variable s2 and non-basic variable c

The following is the current tableau:

	a	b	c	d	s1	s2	s3	s4	s5	constant
	0.00	7.00	0.00	−2.21	0.00	−0.17	0.00	−1.88	0.00	−16.33
1	0.00	0.00	0.00	−1.46	1.00	−1.17	0.00	0.88	0.00	1.67
2	0.00	0.00	1.00	0.21	0.00	0.17	0.00	−0.13	0.00	0.33
3	0.00	7.00	0.00	4.00	0.00	0.00	1.00	0.00	0.00	0.00
4	1.00	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00	2.00
5	0.00	8.00	0.00	1.25	0.00	0.00	0.00	−0.75	1.00	1.00

Variable	Index	Best Constraint	Function Increase
b	2	row 3	0

Pivoting on basic variable s3 and non-basic variable b

The following is the current tableau:

	a	b	c	d	s1	s2	s3	s4	s5	constant
	0.00	0.00	0.00	−6.21	0.00	−0.17	−1.00	−1.88	0.00	−16.33
1	0.00	0.00	0.00	−1.46	1.00	−1.17	0.00	0.88	0.00	1.67
2	0.00	0.00	1.00	0.21	0.00	0.17	0.00	−0.13	0.00	0.33
3	0.00	1.00	0.00	0.57	0.00	0.00	0.14	0.00	0.00	0.00
4	1.00	0.00	0.00	0.25	0.00	0.00	0.00	0.25	0.00	2.00
5	0.00	0.00	0.00	−3.32	0.00	0.00	−1.14	−0.75	1.00	1.00

The maximum value of the objective function has been found.

The maximum value of the objective function is: 16.33

The solution is a = 2.00, b = 0.00, c = 0.33, d = 0.00

Problem 4

formulate this problem to a max flow problem

Construct a DAG with

- one node as the source,
- one node as the target,
- and nodes for the students and dorms.

There are edges from the source to each student, from each dorm to the target, and from each student to each dorm who want to live in that dorm. Note that the edges could all be reversed.

The weight on the edge from a dorm to the target is the maximum number of students that the dorm can accommodate. The weight on the edge from the source to a student is one. Other edges have infinite (unlimited) weight. A maximum flow from the source to the target gives the maximum number of students that can be accommodated subject to the constraints.

<u>Algorithm</u>

Input: the graph with weights on its edges

Output: the max flow of this graph

```
while    find one flow from source to end:
         keep track of the flow;
         compute the residual graph;
return flow
```

Compute the residual graph:

c^f:

$$\begin{cases} c_{uv} - f_{uv} & \textbf{if } (u,v) \in E \textbf{ and } f_{uv} < c_{uv} \\ f_{vu} & \textbf{if } (v,u) \in E \textbf{ and } f_{vu} > 0 \end{cases}$$

Prove

using contradiction

Assume the solution our algorithm works out is not the max flow. Thus, we can have another flow in the residual graph flows form source to the end. However, according our algorithm, it terminates until there is not a path from the source th the end in the residual graph, this raises a contradiction. So the solution must be the optimal one.