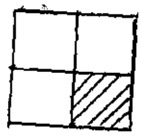
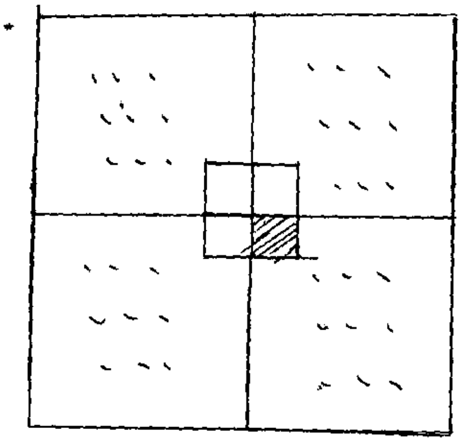
Jian Li

1. ① if $n=1$, the board is 2×2.



It is sure that 3-gnomon can cover the board no matter which square deleted.

② assume $2^n \cdot 2^n$ can be covered as required. Let's Prove $2^{n+1} \cdot 2^{n+1}$ can also be covered.



$2^{n+1} \cdot 2^{n+1}$ board is the combination of 4 $2^n \cdot 2^n$ boards. put a 3-gnomon in the center of $2^{n+1} \cdot 2^{n+1}$ as above. The square to be deleted is the one in shadow.

By doing this, each of the 4 $2^n \cdot 2^n$ boards deleted one square. Thus each of them can be covered as required according to assumption. So $2^{n+1} \cdot 2^{n+1}$ can be covered is proved.

2. in other files attached.

3. a. $f(n) = n^2 + f(\lceil \frac{3}{7} n \rceil)$.

$a=1$, $b=\frac{7}{3}$, $c=2$

$$p = \frac{\log 1}{\log \frac{7}{3}} = 0.$$

So $p < c$, $T(n) = \Theta(n^2)$

b. $g(n) = \lceil \sqrt{n} \rceil + 5g(\lfloor \frac{4}{7} n \rfloor) + 10g(\lfloor \frac{1}{5} n \rfloor)$,

Solve $5(\frac{4}{7})^p + 10 \times (\frac{1}{5})^p = 1$

using Matlab $p = 3.02 > \frac{1}{2}$

So $T(n) = \Theta(n^{3.02})$.

4. ① $10n^6+5^n \sim \Theta(5^n)$

② $5n^{-5}+4n^{-4} \sim \Theta(n^{-4})$

~~③ $50+\log\frac{n^2}{n}$~~

③ $50+\frac{\log n^2}{n} \sim \Theta(n^{-1}\log n)$

④ $n\sqrt{n} \sim \Theta(n^{\frac{3}{2}})$

⑤ $\log(n+\log n) \sim \Theta(\log n)$

⑥ $n^{0.2} \sim \Theta(n^{0.2})$

⑦ $(0.2)^n \sim \Theta(0.2^n)$

⑧ $\sqrt{n!} \sim \Theta(n^{\frac{2n+1}{4}}e^{-\frac{n}{2}}) \sim \Theta(n^{\frac{n}{2}})$

$\sqrt{n!}=\sqrt{n^n e^{-n}\sqrt{2\pi n}}=(n^n e^{-n}(2\pi n)^{\frac{1}{2}})^{\frac{1}{2}}$

$=n^{\frac{n}{2}}e^{-\frac{n}{2}}(2\pi n)^{\frac{1}{4}}$

⑨ $n^2\log n^2+n\log n \sim \Theta(n^2\log n)$

⑩ $\log 4^n \sim \Theta(n)$

⑪ $1.2^n \sim \Theta(1.2^n)$.

the decreasing order of growth is:

$\sqrt{n!} > 10n^6+5^n > 1.2^n > n^2\log n^2+n\log n$

$> n\sqrt{n} > \log 4^n > n^{0.2}$

$> \log(n+\log n) > 50+\frac{\log n^2}{n}$

$> 5n^{-5}+4n^{-4} > 0.2^n$

5. 1. $a=3 \quad b=8 \quad c=\frac{1}{2}$.

$p=\log_8 3 \quad c=\log_8 2\sqrt{2} \quad$ so $p>c$.

$T(n)=\Theta(n^{\log_8 3})$.

2. $a=6 \quad b=4 \quad c=1.3$.

$p=\log_4 6\approx 1.292 < c$.

$T(n)=\Theta(n^{1.3})$

3. ~~$a=5 \quad b=2$~~

formula: $5(\frac{1}{3})^p+2(\frac{1}{4})^p=1$. $c=3$

if $p=3$, $(\frac{1}{3})^3\times 5+(\frac{1}{4})^3\times 2<1$.

if $p=1$, $\frac{5}{3}+\frac{2}{4}>1$.

so $1<p<3$. thus $p<c$.

$T(n)=\Theta(n^3)$.

4. formula: $2(\frac{1}{5})^p+4(\frac{1}{3})^p=1$, $c=\frac{3}{2}$.

if $p=\frac{3}{2}$, $2\times(\frac{1}{5})^{\frac{3}{2}}+4\times(\frac{1}{3})^{\frac{3}{2}}\approx 0.94<1$.

$\therefore p<c$

$T(n)=\Theta(n^{\frac{3}{2}})$.

5. ~~$T(n)=3$~~

formula: $3(\frac{1}{5})^p+2\times(\frac{1}{4})^p+(\frac{1}{3})^p=1$

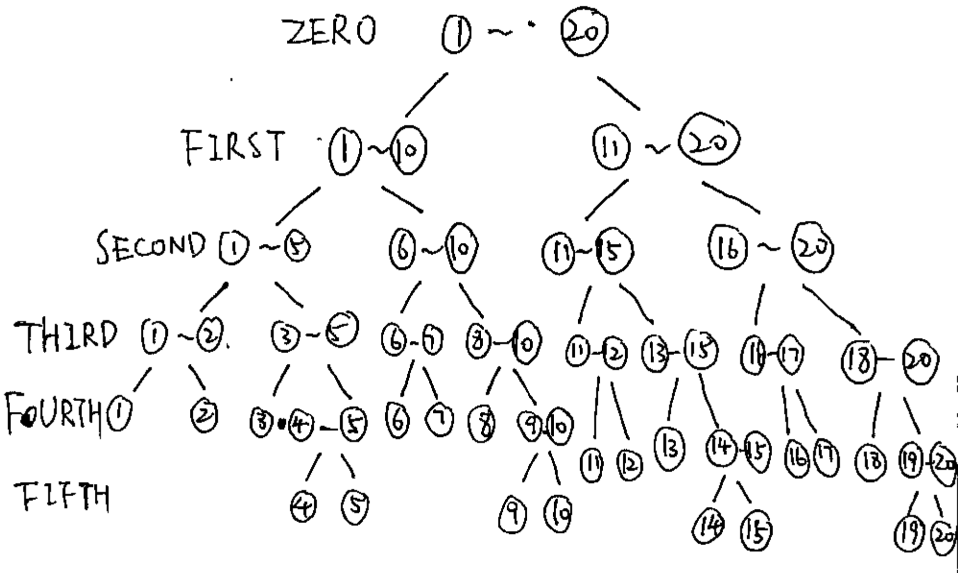
using matlab: $p=1.256$

According to Akara-Bazzi, $T(n)\in\Theta(n^{1.25}$

6. I use the index number in my representation ~~先~~ below. (Start from 1)

{19, 11, 13, 2, 28, 9, 46, 3, 27, 19, 31, 2, 21, 1, 1, 25, 47, 17, 11, 24}.

index: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20



depth	
0	Merge (1 ~ 20)
1	M (1 ~ 10)
2	M (1 ~ 5)
3	M (1 ~ 2)
4	M (1)
5	Return (1)
4	M (2)
5	Return (2)
4	Return (2 1)
3	M (3 − 5)
4	M (3)
5	Return (3)
4	M (4 − 5)
5	M (4)
6	Return (4)
5	M (5)
6	Return (5)
5	Return (4 5)
4	Return (4 3 5)
3	Return (4 2 3 1 5)
2	M (6 ~ 10)
3	M (6 − 7)
4	M (6)
5	Return (6)
4	M (7)
5	Return (7)
4	Return (6 7)

3 M (8 − 10)
4 M (8)
5 Return (8)
4 M (9 10)
5 M (9)
6 Return (9)
5 M (10)
6 Return (10)
5 Return (10 9)
4 Return (8 10 9)
3 Return (8 6 10 9 7)
2 Return (4 8 6 2 3 1 10 9 5 7)
1 M (11 − 20)
2 M (11 − 15)
3 M (11 − 12)
4 M (11)
5 R (11)

4 M (12)
5 Return (12)
4 Return (12 11)
3 M (13 − 15)
4 M (13)
5 Return (13)
4 M (14 15)
5 M (14)
6 Return (14)
5 M (15)
6 Return (15)
5 Return (14 15)
4 Return (14 15 13)
3 Return (14 15 12 13 11)
2 M (16 − 20)
3 M (16 − 17)
4 M (16)
5 Return (16)
4 M (17)

5 Return (17)
4 Return (16 17)
3 M (18 − 20)
4 M (18) 5 R (18)
4 M (19 20)
5 M (19) 6 R (19)
5 M (20) 6 R (20)
5 R (19 20)
4 R (19 18 20)
3 R (19 18 20 16 17)
2R (14 15 12 19 18 13 20 16 11 17)
1 R (14 15 4 12 8 6 2 19 3 18 1 10 13 20 16 9 5 11 7 17)