**Problem 1**

**a.**

The transposition of c and n will reverse the mappings of **π(c)=k and π(n)=q.**

The relevant bigrams that occur in the encrypted message are

**ln,nm,ln,nu,hc,cl,lc,ce**

These are decrypted to the bigrams

**hq, qy, hq, qj**, **ik, kh, hk, kz**

with frequencies **0 , 0 , 0, 0, 0, 1, 0, 0.**

The transposed decrypted bigrams are **hk, ky, hk, kj,** **iq, qh, hq, qz.**

with frequencies **0, 2, 0, 0, 0, 0, 0, 0**

Thus α(π,π') = 2/1 =2 > 0.5.

Thus the transposition was likely to be accepted. The decryption will change:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | a | b | c | d | e | f | g | h | I | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
| l | m | v | **q** | t | z | p | a | i | o | g | x | h | y | **k** | e | s |  | b | n | u | j | r | d | f | c | w |

**b.**

The transposition of a and z will reverse the mappings of **π(a)=m and π(z)=w.**

The relevant bigrams that occur in the encrypted message are

**vz, za, iz, zl, vz, za, iz, zl, rz , zh, yz, zl, iz, zl, lz, zq, iz, zq, iz, zq.**

**za, ao, za, ao, qa,al,qa,al,la,ab,la,ae.**

These are decrypted to the bigrams

**rw, wm, ow, wh, rw, wm, ow, wh, bw, wi, cw, wh, ow, wh, hw, w , ow, w .**

**wm,me,wm,me, m,mh, m,mh,hm,mv,hm,mz**

with frequencies **7,0,182,67,7,0,182,67,0,99,0,67,182,67,1,90,182,90.**

**0, 0, 0, 0, 153, 0, 153, 0, 4,0,4, 0**

The transposed decrypted bigrams are

**rm,mm,om,mh, rm, mm, om,mh, bm, mj, cm, mh, om, mh,hm, m , om, m .**

**ww,we,ww,we, w,wh, w,wh,hw,wv,hw,wz**

with frequencies **28,7,61,0,28,7,61,0,0,46,0,0,61,0,4,59,61,59.**

**0,1,0,1,275, 67, 275, 67,1, 0, 1,0**

Thus α(π,π') =

(0.1\*0.1\*28\*7\*61\*28\*7\*61\*46\*61\*4\*59\*61\*59\*1\*1\*275\*67\*275\*67\*1\*1)/

(7\*182\*67\*7\*182\*67\*99\*67\*182\*67\*1\*90\*1\*182\*90\*153\*153\*4\*4)

= 7.25e-08 <0.5

Thus the transposition was unlikely to be accepted. The decryption map stay the same:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o | p | q | r | s | t | u | v | w | x | y | z |
| l | m | v | k | t | z | p | a | i | o | g | x | h | y | q | e | s |  | b | n | u | j | r | d | f | c | w |

**Problem 2**

There are eight states in this Markov Chain, the Elicitation and Analysis of phase Requirements(R-E,R-A for short); System and Detailed Design of Design(D-S, D-D); Coding and Integration of Implementation( I-C, I-I); System Testing and Acceptance Testing of Testing (T-S and T-A).

Each state has a transition to itself. The probability for these transitions are 0.9, 0.8, 0.9, 0.95, 0.9, 14/15, 0.8, 0.8 respectively. The transport probability of each activity is calculated as the problem requires.

The stochastic matrix is:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **R-E** | **R-A** | **D-S** | **D-D** | **I-C** | **I-I** | **T-S** | **T-A** |
| **R-E** | **0.9** | **0.1** | **0** | **0** | **0** | **0** | **0** | **0** |
| **R-A** | **0.1** | **0.8** | **0.1** | **0** | **0** | **0** | **0** | **0** |
| **D-S** | **0** | **0** | **0.9** | **0.1** | **0** | **0** | **0** | **0** |
| **D-D** | **0** | **0** | **0.01** | **0.95** | **0.04** | **0** | **0** | **0** |
| **I-C** | **0** | **0** | **0** | **0** | **0.9** | **0.1** | **0** | **0** |
| **I-I** | **0** | **0** | **0** | **0** | **0.02** | **14/15** | **7/ 150** | **0** |
| **T-S** | **0** | **0** | **0** | **0** | **0** | **0** | **0.8** | **0.2** |
| **T-A** | **0.12** | **0** | **0** | **0** | **0** | **0** | **0.08** | **0.8** |

To calculate invariant distribution for the matrix above, we have

x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] -1 = 0

-0.1\*x[1] + 0.1\*x[2] + 0.12\*x[8] = 0

0.1\*x[1] - 0.2\*x[2] = 0

0.1\*x[2] - 0.1\*x[3] + 0.01\*x[4] = 0

0.1\*x[3] - 0.05\*x[4] = 0

0.04\*x[4] - 0.1\*x[5] + 0.02\*x[6] = 0

0.1\*x[5] - 1/15\*x[6] = 0

7/150\*x[6] - 0.2\*x[7] + 0.08\*x[8] = 0

0.2\*x[7] - 0.2\*x[8] = 0

**The final answer is:**

**x1-x8**

**(0.16683217 0.08341609 0.10427011 0.20854022 0.11916584 0.17874876 0.06951341 0.06951341)**

**Problem 3**

As the problem requires, we have,

( 0.3 0.1 0.2 0.4

(v1, v2, v3,v4) 0.1 0.3 0.6 0 = (v1 v2 v3 v4)

0.4 0.4 0.1 0.1

0.2 0.2 0.5 0.1)

thus,

-0.7\*x[1] + 0.1\*x[2] + 0.4\*x[3] + 0.2\*x[4] = 0

0.2\*x[1] + 0.6\*x[2] - 0.9\*x[3] + 0.5\*x[4] = 0

0.4\*x[1] + 0 \* x[2] + 0.1\*x[3] -0.9\*x[4] = 0

additionaly,

x[1] + x[2] + x[3] + x[4] -1 = 0

**So** **the invariant distribution is**

(0.26, 0.26, 0.32, 0.15)

**The next 3 states of the Markov chain are:**

(0.1, 0.3, 0.6, 0)

(0.3, 0.34, 0.26, 0.10)

(0.248, 0.256, 0.34, 0.156)

**Problem 4**

π = (0.1, 0.3, 0.6, 0)

1. α(1,2) = min{1, π(2) ∗ q21/π(1) ∗ q12 } =1 .

so probability does not change. It remains same. p12= 0.1

1. α(1,3) = min{1, π(3) ∗ q31/π(1) ∗ q13} =1

so probability does not change. It remains same. p13= 0.2

1. α(1,4) = min{1, π(4) ∗ q41/π(1) ∗ q14 } =0

so no transition occurs. p14 = 0

p11 = 1 − p12 − p13 − p14 = 0.7

1. α(2,1) = min{1, π(1) ∗ q12／π(2) ∗ q21} =1/3

so the probability changes. p21 = q21∗α(2, 1) = 1/30

1. α(2, 3) = min{1, π(3) ∗ q32／π(2) ∗ q23} =1

so probability does not change. It remains same. p23 = 0.6

1. α(2, 4) = min{1, π(4) ∗ q42／π(2) ∗ q24} =0

so no transition occurs. p24 = 0

p22 = 1 − p21 − p23 − p24 = 11/30

1. α(3, 1) = min{1, π(1)∗ q13／π(3)∗ q31} = 1/12

so the probability changes. p31 = q31∗α(3, 1) = 1/30

1. α(3, 2) = min{1, π(2)∗ q23／π(3)∗ q32}= 3/4

so the probability changes. p32 = q32∗α(3, 2) = 3/10

9. α(3, 4) = min{1, π(4)∗ q43／π(3)∗ q34}=0

so no transition occurs.p34 = 0

p33 = 1 − p31 − p32 − p34 = 2/3

1. α(4, 1) = min{1, π(1)∗ q14／π(4)∗ q41} = 1

so probability does not change. It remains same. p41= 1/5

1. α(4, 2) = min{1, π(2)∗ q24／π(4)∗ q42} = 1

so probability does not change. It remains same. p42 = 1/5

12. α(4, 3) = min{1, π(3)∗ q34／π(4)∗ q43} = 1

so probability does not change. It remains same. p43 = 1/2

p44 = 1 − p31 − p32 − p34 = 0.1

**The modified matrix is :**

|  |  |  |  |
| --- | --- | --- | --- |
| 7/10 | 1/10 | 1/5 | 0 |
| 1/30 | 11/30 | 0.6 | 0 |
| 1/30 | 3/10 | 2/3 | 0 |
| 1/5 | 1/5 | 1/2 | 1/10 |

**Problem 5**

|  |  |  |
| --- | --- | --- |
|  | effect | non-effect |
| effect | 0.5 | 0.1 |
| non-effect | 0.5 | 0.9 |

We can formalize this problem as a imperfect test.

The probability table is above.

As bi-normial distribution . If we want to have a confidence of 95%, the upper and lower boundary of it is np – 1.96(np(1-p))1/2, np + 1.96(np(1-p))1/2 respectively.

we now have two distributions here(with p=0.5 and p = 0.1). In order to have the correct effect testing result, we need to make the upper bound of p=0.1 equals the lower bound when p =0.5 to make these two non-overlap

Thus, we have

n\*0.5 – 1.96(n\*0.5(1-0.5))1/2 = n\*0.1 + 1.96(n\*0.1\*(1-0.1))1/2

so we can have 15 or 16 patients to be treated.