Wasserstein-GAN

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Outline

- I. Introduction
- II. Different Distances
- III. Wasserstein GAN
- IV. Results

• I. Introduction

GAN

$$
\begin{aligned}
C(G) &= \max_D V(G,D) \\
&= \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}}[\log(1 - D_G^*(G(\boldsymbol{z})))] \\
&= \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}[\log D_G^*(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{x}\sim p_g}[\log(1 - D_G^*(\boldsymbol{x}))] \\
&= \mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\left[\log \frac{p_{\text{data}}(\boldsymbol{x})}{P_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right] + \mathbb{E}_{\boldsymbol{x}\sim p_g}\left[\log \frac{p_g(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x}) + p_g(\boldsymbol{x})}\right]
\end{aligned}
$$

$$
C(G) = -\log(4) + KL\left(p_{\text{data}} \middle\| \frac{p_{\text{data}} + p_g}{2}\right) + KL\left(p_g \middle\| \frac{p_{\text{data}} + p_g}{2}\right)
$$

$$
C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right)
$$

Different Distances

Total Variation(TV) distance

$$\delta(\mathbb{P}_r, \mathbb{P}_g) = \sup_{A \in \Sigma} |\mathbb{P}_r(A) - \mathbb{P}_g(A)|$$

Kullback-Leibler(KL) divergence

$$KL(\mathbb{P}_r \| \mathbb{P}_g) = \int \log \left(\frac{P_r(x)}{P_g(x)} \right) P_r(x) d\mu(x)$$

Jensen-Shannon(JS) divergence

$$JS(\mathbb{P}_r, \mathbb{P}_g) = KL(\mathbb{P}_r \| \mathbb{P}_m) + KL(\mathbb{P}_g \| \mathbb{P}_m)$$

Earth-Mover distance or Wasserstein-1

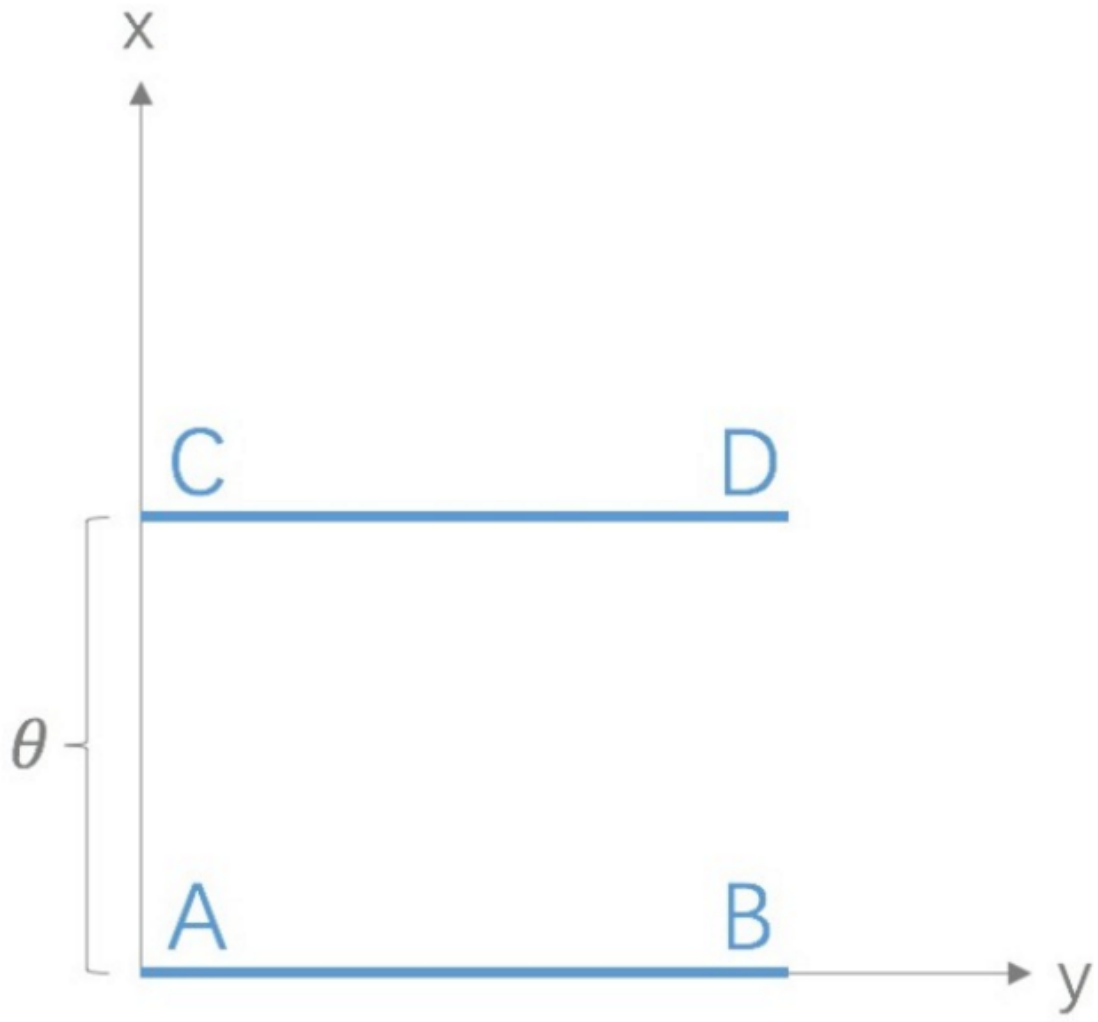
$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y) \sim \gamma} \left[\|x - y\| \right]$$

• II. Different Distances

Example 1 (Learning parallel lines). Let $Z \sim U[0,1]$ the uniform distribution on the unit interval. Let $\mathbb{P}_0$ be the distribution of $(0, Z) \in \mathbb{R}^2$ (a 0 on the x-axis and the random variable Z on the y-axis), uniform on a straight vertical line passing through the origin. Now let $g_\theta(z) = (\theta, z)$ with θ a single real parameter. It is easy to see that in this case,

- $JS(\mathbb{P}_0, \mathbb{P}_\theta) = \begin{cases} \log 2 & \text{if } \theta \neq 0\ , \\ 0 & \text{if } \theta = 0\ , \end{cases}$
- $KL(\mathbb{P}_\theta \| \mathbb{P}_0) = KL(\mathbb{P}_0 \| \mathbb{P}_\theta) = \begin{cases} +\infty & \text{if } \theta \neq 0\ , \\ 0 & \text{if } \theta = 0\ , \end{cases}$
- and $\delta(\mathbb{P}_0, \mathbb{P}_\theta) = \begin{cases} 1 & \text{if } \theta \neq 0\ , \\ 0 & \text{if } \theta = 0\ . \end{cases}$
- $W(\mathbb{P}_0, \mathbb{P}_\theta) = |\theta|,$

• II. Different Distances



• II. Different Distances

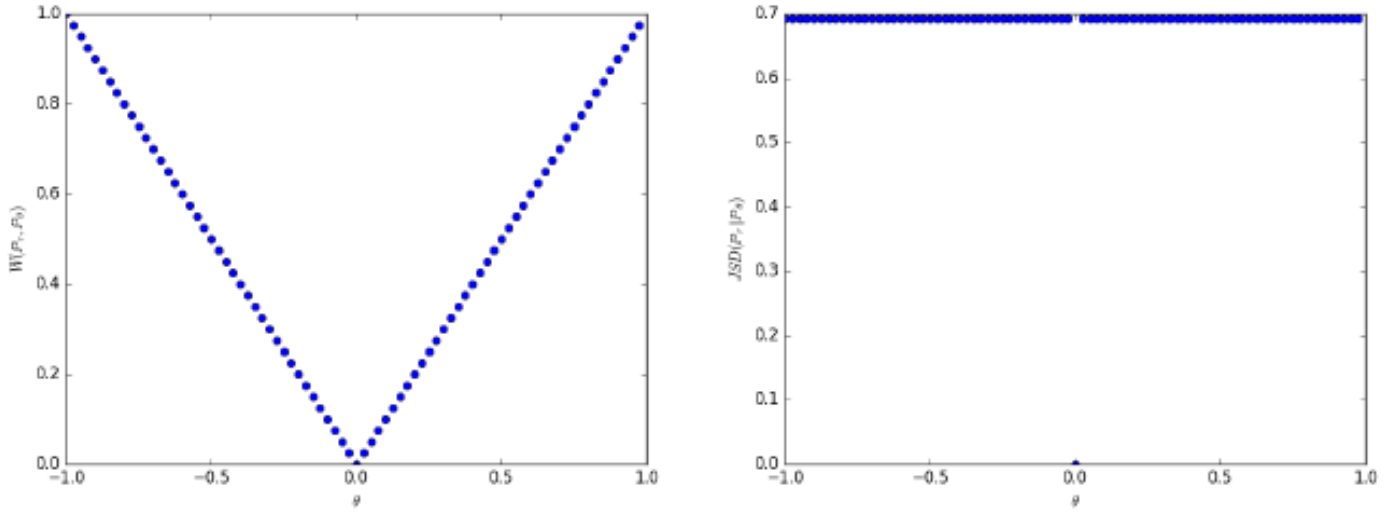


Figure 1: These plots show $\rho(\mathbb{P}_\theta, \mathbb{P}_0)$ as a function of θ when ρ is the EM distance (left plot) or the JS divergence (right plot). The EM plot is continuous and provides a usable gradient everywhere. The JS plot is not continuous and does not provide a usable gradient.

• III. Wasserstein GAN

$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y)\sim\gamma} \left[\|x - y\| \right]$$

$$W(\mathbb{P}_r, \mathbb{P}_\theta) = \sup_{\|f\|_L \leq 1} \mathbb{E}_{x\sim\mathbb{P}_r}[f(x)] - \mathbb{E}_{x\sim\mathbb{P}_\theta}[f(x)]$$

$$|f(x_1) - f(x_2)| \leq K|x_1 - x_2|$$

Discriminator loss

$$\mathbb{E}_{x\sim P_g}[f_w(x)] - \mathbb{E}_{x\sim P_r}[f_w(x)]$$

Generator loss

$$-\mathbb{E}_{x\sim P_g}[f_w(x)]$$

• III. Wasserstein GAN

W-GAN VS GAN

- No sigmoid
- No log
- Weight clipping
- RMSProp instead of Adam

• III. Wasserstein GAN

Algorithm 1 WGAN, our proposed algorithm. All experiments in the paper used the default values $\alpha = 0.00005$, $c = 0.01$, $m = 64$, $n_{\text{critic}} = 5$.

Require: : α, the learning rate. c, the clipping parameter. m, the batch size. n_{critic}, the number of iterations of the critic per generator iteration.

Require: : w_0, initial critic parameters. θ_0, initial generator's parameters.

while θ has not converged **do**
  for $t = 0, ..., n_{\text{critic}}$ **do**
    Sample $\{x^{(i)}\}_{i=1}^{m} \sim \mathbb{P}_r$ a batch from the real data.
    Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
    $g_w \leftarrow \nabla_w \left[\frac{1}{m}\sum_{i=1}^{m} f_w(x^{(i)}) - \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))\right]$
    $w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$
    $w \leftarrow \text{clip}(w, -c, c)$
  end for
  Sample $\{z^{(i)}\}_{i=1}^{m} \sim p(z)$ a batch of prior samples.
  $g_\theta \leftarrow -\nabla_\theta \frac{1}{m}\sum_{i=1}^{m} f_w(g_\theta(z^{(i)}))$
  $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, g_\theta)$
end while

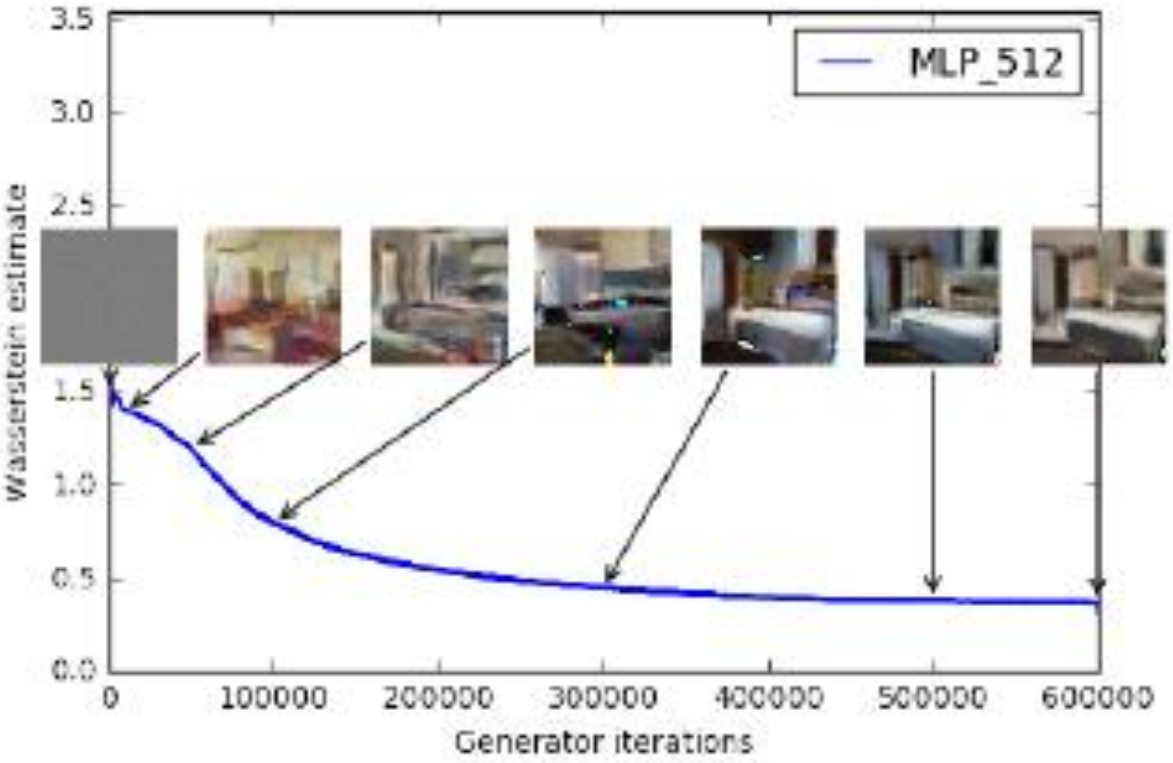
• IV. Results

Benefits:

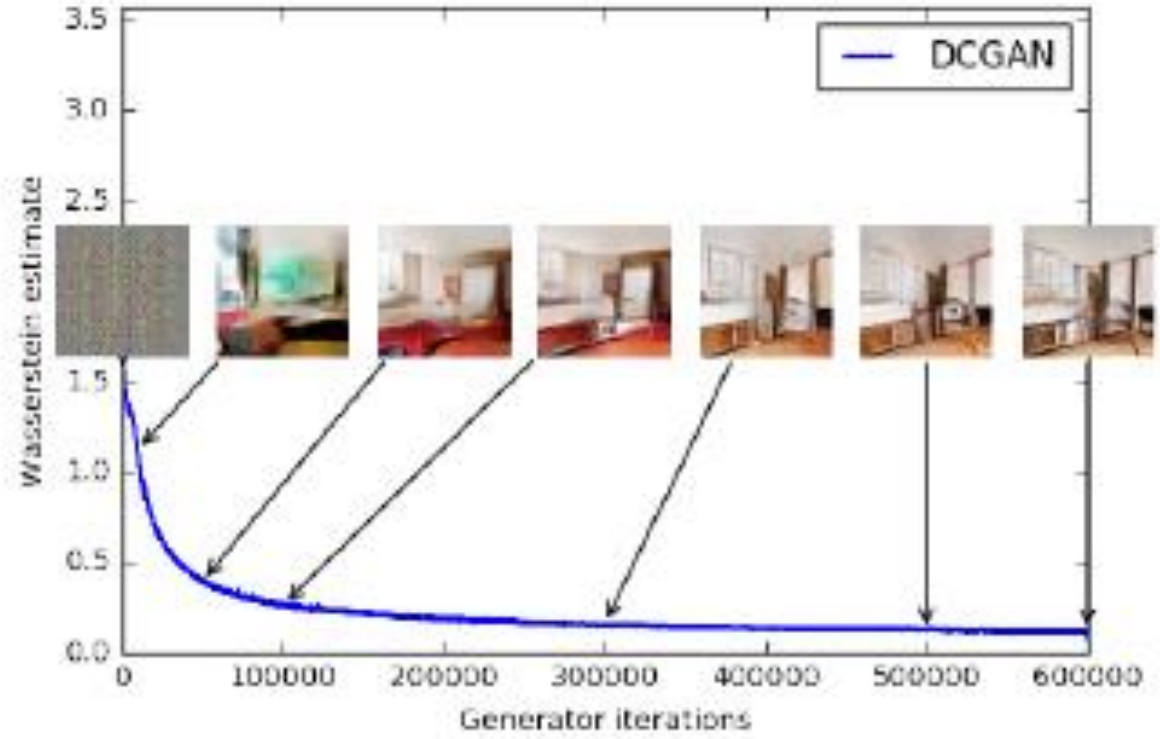
·a meaningful loss metric that correlates with the generator's convergence and sample quality

·improved stability of the optimization process

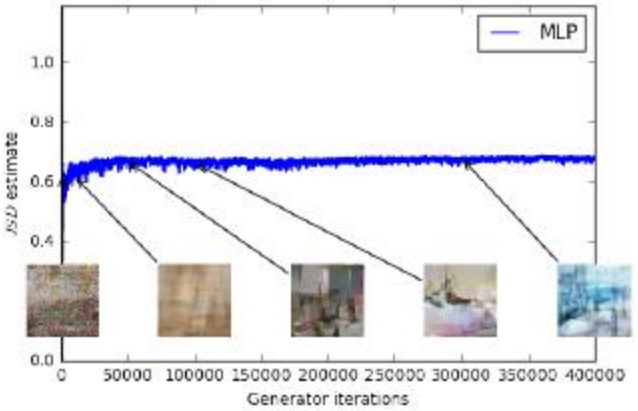
• IV. Results

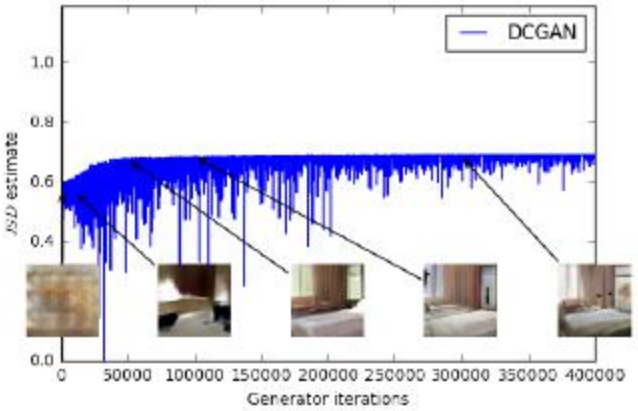

MLP_512
Wasserstein estimate
3.5
3.0
2.5
1.5
1.0
0.5
0.0
0
100000
200000
300000
400000
500000
600000
Generator iterations

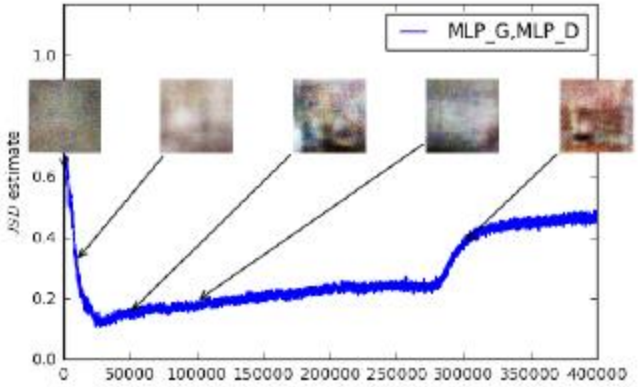
• IV. Results


DCGAN
Wasserstein estimate
3.5
3.0
2.5
1.5
1.0
0.5
0.0
0
100000
200000
300000
400000
500000
600000
Generator iterations

• IV. Results


MLP
JSD estimate
1.0
0.8
0.6
0.4
0.0
0
50000
100000
150000
200000
250000
300000
350000
400000
Generator iterations


DCGAN
JSD estimate
1.0
0.8
0.6
0.4
0.0
0
50000
100000
150000
200000
250000
300000
350000
400000
Generator iterations


MLP_G,MLP_D
JSD estimate
1.0
0.6
0.4
0.2
0.0
0
50000
100000
150000
200000
250000
300000
350000
400000

• IV. Results

Figure 5: Algorithms trained with a DCGAN generator. Left: WGAN algorithm. Right: standard GAN formulation. Both algorithms produce high quality samples.

Figure 6: Algorithms trained with a generator without batch normalization and constant number of filters at every layer (as opposed to duplicating them every time as in [18]). Aside from taking out batch normalization, the number of parameters is therefore reduced by a bit more than an order of magnitude. Left: WGAN algorithm. Right: standard GAN formulation. As we can see the standard GAN failed to learn while the WGAN still was able to produce samples.

• IV. Results

Figure 7: Algorithms trained with an MLP generator with 4 layers and 512 units with ReLU nonlinearities. The number of parameters is similar to that of a DCGAN, but it lacks a strong inductive bias for image generation. Left: WGAN algorithm. Right: standard GAN formulation. The WGAN method still was able to produce samples, lower quality than the DCGAN, and of higher quality than the MLP of the standard GAN. Note the significant degree of mode collapse in the GAN MLP.

Bibliography

- [1] Wasserstein GAN

Thanks !