Subject:	Review_request LIU AK11884 Valtierra
From:	pra@aps.org Oct 15, 2019 2:33:39 AM
To:	jianliupku@pku.edu.cn

Re: AK11884
Quasiprobability currents on the sphere
by Iv\'an F. Valtierra, Andrei B. Klimov, Gerd Leuchs, et al.

Dear Prof. Liu,

We would appreciate your review of this manuscript, which has been submitted to Physical Review A.

Comments from the editor:

Does this manuscript contain enough new and significant physics to warrant publication in the Physical Review?

Thank you for your help.

Yours sincerely,

Frank Narducci
Associate Editor
Physical Review A
Email: pra@aps.org
https://journals.aps.org/pra/

We ask that you download the manuscript and return your report via:

https://referees.aps.org/r/AK11884

ABSTRACT:

We present analytic expressions for the s-parametrized currents on the sphere for both unitary and dissipative evolutions. We examine the spatial distribution of the flow generated by these currents for quadratic Hamiltonians. The results are applied for the study of the quantum dissipative dynamics of the time-honored Kerr and Lipkin models, exploring the appearance of the semiclassical limit in stable, unstable and tunnelling regimes.

FORMS AND MEMOS:

Please see the following:

https://journals.aps.org/pra/referees/guidelines-for-referees
Advice to referees for Physical Review

Alternatively, you may send your completed Referee Response Form by email to pra@aps.org. If you use email, either reply to this message or give as the subject "Report LIU AK11884 Valtierra".

Physical Review - REFEREE RESPONSE FORM

Insert X in the parentheses in the response form as appropriate.

Referee: Prof. Liu,

Manuscript Number: AK11884

Author: Iv\'an F. Valtierra, Andrei B. Klimov, Gerd Leuchs, et al.

Title: Quasiprobability currents on the sphere

1. Please summarize your assessment of the paper:

	YES	MAYBE	NO
Does the paper contain enough significant new physics to warrant publication in the Physical Review?	( )	( )	( )
Is the paper scientifically sound and not misleading?	( )	( )	( )
Is the paper well organized and clearly written?	( )	( )	( )
Are the subject matter and style of presentation appropriate for the Physical Review?	( )	( )	( )
Is the length appropriate?	( )	( )	( )

2. Please evaluate quality of Research and Presentation:

( )	Excellent	( )
( )	Good	( )
( )	Average	( )
( )	Marginal	( )
( )	Poor	( )

3. Recommendation:

( ) Publish without change (please give reasons in report).
( ) Publish after authors have considered the optional revisions mentioned in the report.
( ) Publish after authors have made the revisions mentioned in the report. (I do not need to see the manuscript again.)
( ) Revisions are necessary. Return to me on resubmittal.
( ) Revisions are necessary. On resubmittal, send to:

(Use as much space as needed to list names of alternative reviewers.)

( ) Manuscript is more appropriate for another
( ) journal (specify):
or ( ) section (specify):
( ) Do not publish; see report.
( ) Other; see report.

****
begin_report (use as much space as is needed):

end_report
****

Subject:	Resub_review_request LIU AK11884 Valtierra
From:	pra@aps.org Dec 11, 2019 4:04:18 AM
To:	jianliupku@pku.edu.cn
Attachments:	ak11884_changes.pdf

Re: AK11884
Quasiprobability currents on the sphere
by Iv\'an F. Valtierra, Andrei B. Klimov, Gerd Leuchs, et al.

Dear Prof. Liu,

We would appreciate your review of this manuscript, which has been submitted to Physical Review A.

Comments from the editor:

We append copies of reports from you and the other referee on the previous version of the manuscript and of the author's response. Is the response of the authors to all the previous recommendations satisfactory? We are returning the manuscript only to you, so please consider the response of the authors to all the referee reports.

Thank you for your help.

Yours sincerely,

Thomas Pattard
Managing Editor
Physical Review A
Email: pra@aps.org
https://journals.aps.org/pra/

We ask that you download the manuscript and return your report via:

https://referees.aps.org/r/AK11884

ABSTRACT:

We present analytic expressions for the s-parametrized currents on the sphere for both unitary and dissipative evolutions. We examine the spatial distribution of the flow generated by these currents for quadratic Hamiltonians. The results are applied for the study of the quantum dissipative dynamics of the time-honored Kerr and Lipkin models, exploring the appearance of the semiclassical limit in stable, unstable and tunnelling regimes.

Report of the First Referee -- AK11884/Valtierra

Phase-space representations, over decades of their continuing study, have proven an indispensable tool of quantum mechanics, in particular of systems in the border region to classical mechanics. However, on he face of it, they just provide a static picture. The time evolution of phase-space representations and semiclassical approximations to them therefore occupy a similarly important position in the theoretical toolbox. A third element, which has turned out to be extremely useful in the analysis of complex dynamics on the classical level, are probability currents, as they reveal features of dynamical systems not easily accessible otherwise. The conception and study of their quantum-mechanical counterparts is a rather more recent development that still leaves many questions open.

Defining representations of quantum mechanics on spaces as similar as possible to classical phase space is not a trivial task and generally has no unique solution. In particular, these definitions depend strongly on the topology of the relevant classical phase-space manifold (real 2f-dimensional space, torus, sphere ...). The present manuscript is dedicated to defining quantum analogues to classical probability currents, i.e., in particular quasi-probability currents associated to the Wigner function, on spherical phase spaces as they arise above all in systems where the modulus of the angular momentum is a constant of the motion.

In this weighty article, the authors achieve satisfactory solutions of most of the basic questions implied by this formidable task. I am sure it will play a long-lasting role as reference on its subject and therefore strongly recommend publishing it in Physical Review A.

Notwithstanding this very positive general impression, I found a number of points where I think the paper deserves being improved. They concern above all its general conception:

- The authors introduce the concept of quasiprobability currents already in the introduction, in Eq. (1.2). It comes, however, without even a minimum of further explanation. Since this is still a recent development, this leaves unprepared readers with a host of doubts, e.g., what is the underlying quantum phase-space velocity that imparts its vector character to the current. I think such questions should be answered at least qualitatively, already in this early stage of the article.

- I find it far from obvious how the concept of a current applies to a quantity, such as the quasiprobability, that can take negative values. Do interference patterns, where such negative regions occur, just move with the current? Certainly the comparison with currents of positive vs. negative electric charges is not appropriate here.

- In the sections on numerical results, the authors introduce and discuss what they call "stagnation lines", i.e., manifolds along which the azimuthal or the polar component of the quasiprobability current vanish. Evidently, at intersections of these lines, this current vanishes completely. Does that mean that at these zeros, vortices of the current appear? More generally, the vorticity of the current (i.e., rot j) would be a relevant quantity to discuss.

- In systems with dissipation, there must be sinks of the energy flow. They could be reflected in similar singularities of the quasiprobability current. In any case, just as the curl, also the divergence of the quasiprobability current should bear important information on the underlying dynamics. One expects to see it defined and maybe discussed in this paper.

Furthermore, there are a few technical bugs in the presentation that should be amended:

- In the line following Eq. (1.1), the authors say that "... W_H^(s)(Omega) is the symbol of the Hamiltonian ...". I guess this should read "... is the *Weyl* symbol ...".

- The notation "(Omega|t)" for the combination of spatial angle and time as arguments (if this interpretation is correct) is at least unconventional and should be explained or replaced by "(Omega,t)".

- In section V.A, the terms "cat time" and "best squeezing time" are used. Most readers will not be familiar with this terminology.

- Shortly later, the authors refer to the Bloch sphere, but it enters somewhat through the back door, at a stage where this concept is in the background implicitly already through several paragraphs. Why not introduce it from the moment on when the density operator is introduced as a function of azimuth and polar angle.

In Figs. 2 and 4, the stagnation lines are barely visible, the arrows indicating the quasiprobability flow are practically indiscernible. These figures should be enlarged, or else the current should be represented by bolder arrows.

Could the authors please consider these comments in preparing a final version.

Report of the Second Referee -- AK11884/Valtierra

The manuscript "Quasiprobability currents on the sphere" by Valtierra et al implements the Wigner function as an example to describe the quasiprobability current on the sphere for the Kerr and LMG models.

Here are some specific comments:

1. It has been an old idea to use the Wigner or Husimi function to illustrate the current or the evolution of the density operator. In addition to Wigner, P, Q functions, one can design other quantum phase space representations to show this, as described in Session 2 of J. Chem. Phys. 134, 104101 (2011), as well as in J. Math. Phys. 7, 781 (1966). Section II of the present manuscript could have been more complete in presenting a summary on the quasiprobability distribution.

2. The present paper does not suggest any useful numerical methodologies in solving the exact evolution for the mapping quantum density on the sphere when the Wigner function is employed for general Hamiltonians for realistic systems. Most challenging examples are beyond quadratic Hamiltonians. Provided that exact quantum evoluation results are available, the visuallization in the Wigner phase space is relatively trivial. Any suggestion in practically solving the evolution of the quasiprobabilty distribution for general systems?

3. The definition of the quantum current in Eq. (1.2) in the phase space is NOT unique, although the classical limit is easy to obtain. E.g., see the discussion in J. Chem. Phys., 134, 194110 (2011). The authors can certainly make one choice on the sphere, but there are various other options. More disucssion on this point will be necessary.

4. I certainly understand that the Wigner function has both positive and negative values while the P or Husimi function is always positive-definite. But the quantum nature should be there even when the P or Husimi function is used.

5. When the system Hamiltonian involve multi dimensions or many spins, the so-called stagnation points will be multi-dimensional contours. How useful will the Wigner function be useful or convenient remains a question. It will be important for the authors to discuss it.

Resubmission letter from author -- AK11884/Valtierra

See Attachment: ak11884_changes.pdf

FORMS AND MEMOS:

Please see the following:

https://journals.aps.org/pra/referees/guidelines-for-referees
Advice to referees for Physical Review

Alternatively, you may send your completed Referee Response Form by email to pra@aps.org. If you use email, either reply to this message or give as the subject "Report LIU AK11884 Valtierra".

Physical Review - REFEREE RESPONSE FORM

Insert X in the parentheses in the response form as appropriate.

Referee: Prof. Liu,

Manuscript Number: AK11884

Author: Iv\'an F. Valtierra, Andrei B. Klimov, Gerd Leuchs, et al.

Title: Quasiprobability currents on the sphere

1. Please summarize your assessment of the paper:

	YES	MAYBE	NO
Does the paper contain enough significant new physics to warrant publication in the Physical Review?	( )	( )	( )
Is the paper scientifically sound and not misleading?	( )	( )	( )
Is the paper well organized and clearly written?	( )	( )	( )
Are the subject matter and style of presentation appropriate for the Physical Review?	( )	( )	( )
Is the length appropriate?	( )	( )	( )

2. Please evaluate quality of Research and Presentation:

( )	Excellent	( )
( )	Good	( )
( )	Average	( )
( )	Marginal	( )
( )	Poor	( )

3. Recommendation:
( ) Publish without change (please give reasons in report).
( ) Publish after authors have considered the optional revisions mentioned in the report.
( ) Publish after authors have made the revisions mentioned in the report. (I do not need to see the manuscript again.)
( ) Revisions are necessary. Return to me on resubmittal.
( ) Revisions are necessary. On resubmittal, send to:

(Use as much space as needed to list names of alternative reviewers.)
( ) Manuscript is more appropriate for another
( ) journal (specify):
or ( ) section (specify):
( ) Do not publish; see report.
( ) Other; see report.

****
begin_report (use as much space as is needed):

end_report
****