

## **Evidence #11:**

The Ref. 64 in Version 2 (first released online to ChemRxiv on May 27, 2022) or Ref. 65 in Version 3 (accepted online on July 29, 2022), which were both notes, did not appear in Version 1 (first released online to ChemRxiv on May 20, 2022).

Huo and his coworkers added the note to illustrate that they were never aware of the kernel form of eq (27) of Version 2 or Version 3 in pervious articles, while they did not mention this point in Version 1.

Table 1: Comparisons of Version 1 and Version 2

| Version 1 (first released online to ChemRxiv on May 20, 2022)   | Version 2 (first released online to ChemRxiv on May 27, 2022)   |
|---|---|
| where $\hbar \mathbf{\Omega}$ is equivalent to the Bloch vector and its components' detailed expressions can be found in Eqs. B2-B4. In terms of the spin coherent states, we can also express the kernel as $\hat{w}_{\rm s}(\mathbf{\Omega}) = \frac{1-r_{\rm s}}{N} \hat{\mathcal{I}} + r_{\rm s}  \mathbf{\Omega}\rangle \langle \mathbf{\Omega} . \tag{26}$ The kernel defines an identity | eters $\{\theta_n, \varphi_n\}$ of the manifold. Using the spin coherent states, we can also express the S-W kernel in Eq. 26 as $\hat{w}_s(\Omega) = \frac{1-r_s}{N}\hat{\mathcal{I}} + r_s \Omega\rangle\langle\Omega $ , (27) which can be easily verified by using $ \Omega\rangle\langle\Omega $ expressed with generators in Eq. 24. Note that when $r_s = 1$ (so |
| No corresponding part   | <sup>64</sup> To the best of our knowledge, we do not see this expression in the previous literature. We also made an incorrect statement in our previous work in Ref. 35 by suggesting that for $s\neq Q$ , there is no simple relation between $\hat{w}_s$ and $ \Omega\rangle\langle\Omega $ , which is not true because of Eq. 27.                                  |

Table 2: Comparisons of Version 1 and Version 3

| Table 2. Comparisons of version 1 and version 2  |  |
|--|--|
| Version 1 (first released online to ChemRxiv on May 20, 2022)  | Version 3 (accepted online on July 29, 2022)   |
| where $\hbar\Omega$ is equivalent to the Bloch vector and its components' detailed expressions can be found in Eqs. B2-B4. In terms of the spin coherent states, we can also express the kernel as | rameters $\{\theta_n, \varphi_n\}$ of the manifold. Using the spin coherent states, we can also express the S-W kernel in Eq. 26 as  |
| $\hat{w}_{s}(\mathbf{\Omega}) = \frac{1 - r_{s}}{N} \hat{\mathcal{I}} + r_{s}  \mathbf{\Omega}\rangle \langle \mathbf{\Omega} . $ (26)   | $\hat{w}_{s}(\mathbf{\Omega}) = \frac{1 - r_{s}}{N} \hat{\mathcal{I}} + r_{s}  \mathbf{\Omega}\rangle\langle\mathbf{\Omega} , \qquad (27)$   |
| The kernel defines an identity   | which can be easily verified by using $ \Omega\rangle\langle\Omega $ expressed with generators in Eq. 24. Using the MMST-type vari-  |
| No corresponding part  | Fo the best of our knowledge, we do not see this expression in the previous literature. We also made an incorrect statement in our previous work in Ref. 36 by suggesting that for $s\neq Q$ , there is no simple relation between $\hat{w}_s$ and $ \Omega\Gamma.E.\ Wigner,\ Phys.Rev.\ 40,\ 749(1932).$ |

In fact, the kernel formalism of eq (27) had already been proposed in J. Phys. A:

Math. Theor. 45, 015302 (2012) as stated in Appendix 3 of Wiley Interdiscip. Rev. Comput. Mol. Sci. e1619 (2022) [submitted on February 5, 2022, released on arXiv on May 8, 2022 and officially published on May 13, 2022]:

Table 3: Comparisons of Version 2 and Liu's WCMS article

## Version 2 (first released online to ChemRxiv on May 27, 2022)

Wiley Interdiscip. Rev. Comput. Mol. Sci. e1619 (2022) (submitted on February 5, 2022, released on arXiv on May 8, 2022 and officially published on May 13, 2022)

eters  $\{\theta_n, \varphi_n\}$  of the manifold. Using the spin coherent states, we can also express the S-W kernel in Eq. 26 as<sup>64</sup>

$$\hat{w}_{\rm s}(\mathbf{\Omega}) = \frac{1 - r_{\rm s}}{N} \hat{\mathcal{I}} + r_{\rm s} |\mathbf{\Omega}\rangle\langle\mathbf{\Omega}|,\tag{27}$$

which can be easily verified by using  $|\Omega\rangle\langle\Omega|$  expressed with generators in Eq. 24. Note that when  $r_s = 1$  (so

The mapping kernel of the SU(F)/U(F-1) Stratonovich phase space of ref <sup>11</sup> is

$$\hat{K}_{\text{ele}}^{\text{SU}(F)}(\boldsymbol{\theta}, \boldsymbol{\varphi}; s) = (1+F)^{(1+s)/2} \mid \boldsymbol{\theta}, \boldsymbol{\varphi} \rangle \langle \boldsymbol{\theta}, \boldsymbol{\varphi} \mid + \frac{\hat{\mathbf{I}}}{F} (1-(1+F)^{(1+s)/2}), \quad s \in \mathbb{R}$$
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Ref. 11 [*J. Phys. A: Math. Theor.* 45, 015302 (2012)] cited in Appendix 3 of *Wiley Interdiscip. Rev. Comput. Mol. Sci.* e1619 (2022) had clearly presented a kernel formalism as eqs (3.13) and (4.7):

For example, if we define  $F_{N,M}^{+1}(\theta, \phi) = |(\theta, \phi)_N^M\rangle\langle(\theta, \phi)_N^M|$  and  $f_{N,M,\rho}^{+1}(\theta, \phi) = Q(\theta, \phi)$  then we can see that (3.9) and (3.10) are satisfied by (2.13) and (3.2). Lastly, to recover the density

$$F_{N,1}^{+1}(\boldsymbol{\theta}, \boldsymbol{\phi}) = \frac{\left| (\boldsymbol{\theta}, \boldsymbol{\phi})_{N}^{1} \right\rangle \left\langle (\boldsymbol{\theta}, \boldsymbol{\phi})_{N}^{1} \right|}{= \frac{1}{N} \mathbb{1}_{N} + \frac{1}{2} \sum_{k=1}^{N^{2}-1} \left\langle (\boldsymbol{\theta}, \boldsymbol{\phi})_{N}^{1} \middle| \Lambda_{N,1}(k) \middle| (\boldsymbol{\theta}, \boldsymbol{\phi})_{N}^{1} \middle| \Lambda_{N,1}(k).$$
(3.13)

$$F_{N,1}^{s'}(\boldsymbol{\theta}, \boldsymbol{\phi}) = \frac{\Omega(s')}{\Omega(s)} \tilde{F}_{N,1}^{s}(\boldsymbol{\theta}, \boldsymbol{\phi}) + \frac{1}{N} \mathbb{1}_{N},$$

$$= \frac{\Omega(s')}{\Omega(s)} \left( F_{N,1}^{s}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \frac{1}{N} \mathbb{1}_{N} \right) + \frac{1}{N} \mathbb{1}_{N},$$

$$= \frac{\Omega(s')}{\Omega(s)} F_{N,1}^{s}(\boldsymbol{\theta}, \boldsymbol{\phi}) + \left( 1 - \frac{\Omega(s')}{\Omega(s)} \right) \frac{1}{N} \mathbb{1}_{N}.$$

$$(4.7)$$

,,

It is a *serious plagiarism* that Huo and his coworkers stated that "to the best of our knowledge, we do not see this expression in the previous literature", while they cited our work nowhere but still cited *J. Phys. A: Math. Theor.* 45, 015302 (2012) in Version 1 (see Ref. 43), Version 2 (see Ref. 50) and Version 3 (see Ref. 53). This "expression", which is the mapping kernel formalism, had been **already in literature**, in both *J. Phys. A: Math. Theor.* 45, 015302 (2012) and in our *Wiley Interdiscip. Rev. Comput. Mol. Sci.* e1619 (2022). This evidence also indicates that they had studied our work *Wiley Interdiscip. Rev. Comput. Mol. Sci.* e1619 (2022) [submitted on **February 5, 2022**, released on **arXiv on May 8**, 2022 and officially published on **May 13, 2022**] before they released their Version 2 (first released online to ChemRxiv on **May 27, 2022**) and

Version 3 (accepted online on **July 29, 2022**), and pretended others' scientific contributions to be their first ideas. Huo and his coworkers should explain why they added this note in Version 2.