F100-PW-220 Nozzle Optimization Problems

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1 Purpose

This document provides background on the formulation of optimization problems for the design of a F100-PW-220 nozzle with uncertain parameters and provides several example optimization problems.

2 Variables

Variables in the nozzle design problem include those related to geometry, supporting structure, material, inputs, and the environment.

2.1 Nozzle shape

The shape of the nozzle's interior wall is parameterized using a 3rd-order basis spline. In addition, the shape of the nozzle's wall thickness is parameterized using a piecewise-linear function. The nozzle is assumed to be radially symmetric.

Interior wall B-spline control points These control points govern the shape of the interior wall of the nozzle and therefore govern important parameters such as: nozzle length, inlet diameter, inlet to throat and outlet to throat area ratios, as well as the diameter $D(x)$. Certain limitations must be used in order to achieve an acceptable shape. First, the use of a 3rd order B-spline requires the first 2 and the last 2 control points to be duplicated. Next, for good control near the throat of the nozzle, it is preferable to limit the degrees of freedom of a small subset of control points near the throat. For example, see Figure **??**. As a result, the number of design variables corresponding to the shape of the nozzle will be less than the total possible number of degrees of freedom of the B-spline's control points, as there are some limitations in place. Finally, an appropriate set of linear constraints in the form $Ax \leq b$ should be used in order to achieve reasonable nozzle shapes.

Wall thickness piecewise linear control points These control points govern the shape of the thickness $t(x)$ of the nozzle wall. Note that:

$$R_{wall,outer} = R_{wall,inner} + t(x) \tag{1}$$

Each control point denotes the x-position and height (relative y-position) of the exterior nozzle wall from the interior nozzle wall. A linear interpolation is used between control points.

2.2 Supporting structure

The supporting structure consists of longerons and baffles which carry structural and thermal loads from the nozzle to the surrounding aircraft structure. The variables that can be adjusted will be written here shortly.

2.3 Material

The material properties of the nozzle greatly influence the heat transfer, stresses, and strength of the nozzle. A uniform density material is assumed.

Thermal conductivity k This parameter describes how resistant the material is to the transfer of heat.

Thermal expansion coefficient α This parameter describes how much the material expands when subjected to a temperature difference and is related to the thermal stresses in the material.

Elastic modulus E This parameter describes the stiffness of the material.

Max allowable stress σ_{max} This parameter is the maximum allowable amount of stress the material can be subjected to. For all intents and purposes, we choose it to be the yield stress at a fixed temperature.

Max allowable temperature T_{max} This parameter is the maximum allowable temperature the material can be subjected to.

2.4 Nozzle inlet

The boundary condition on the inlet of the nozzle is assumed to be uniform. The stagnation pressure and temperature of the inlet flow is governed by the upstream engine turbine and other components.

Inlet stagnation temperature T_{stag}

Inlet stagnation pressure P_{stag}

2.5 Environment

Heat transfer coefficient from external nozzle wall to ambient h_∞ A lot of complicated physics is lumped into this one parameter, but it does have a large effect on the heat transfer through the nozzle wall.

Freestream temperature T_∞

Freestream pressure P_∞

3 Quantities of Interest

3.1 Deterministic

Weight W

Thrust T

sfc sfc

Max stress σ_{max}

Max wall temperature $T_{w,max}$

Max wall temperature gradient $\Delta T_{w,max}$

 # Cycles to fatigue failure N_f

3.2 Stochastic

Mean, variance, or CDF of above deterministic quantities of interest.

3.3 Limitations

Maximum allowable temperature T_{allow}

Maximum allowable stress σ_{allow}

Required thrust T_{req}

Required minimum cycles to failure $N_{f,req}$

Maximum allowable sfc sfc_{allow}

4 Optimization Problems

4.1 Forms of objective functions

Deterministic:

$$W \tag{2}$$

Mean with variance penalty:

$$\mu(W) + \kappa\sigma(W) \tag{3}$$

Percentile requirement:

$$P[W < W_{req}] \tag{4}$$

Mean or variance:

$$\mu(W) \text{ or } \sigma^2(W) \tag{5}$$

CDF matching:

$$RI(W) = \int_\Omega |F_W(W) - \delta_W(RAO)| d\xi \tag{6}$$

4.2 Forms of constraint functions

Deterministic:

$$T > T_{req} \tag{7}$$

Moment specification:

$$\mu(T) - 3\sigma(T) > T_{req} \tag{8}$$

Percentile requirement:

$$P[T > T_{req}] > 0.99 \tag{9}$$

4.3 Design Variable Limits

$$\underline{\mathbf{x}} < \mathbf{x} < \overline{\mathbf{x}} \tag{10}$$

$$\underline{\xi} < f(\xi) < \overline{\xi} \tag{11}$$

4.4 Weight Focused

$$\begin{aligned} \underset{\mathbf{x},\xi}{\text{minimize}} \quad & f(W) \\ \text{subject to} \quad & f(T) > a \\ & f(T_w) < b \\ & f(\sigma_{max}) < c \end{aligned} \tag{12}$$

- Can be easily turned into a material selection and mixed programming problem by allowing multiple materials in the design space

4.5 Lifetime focused

$$
\begin{aligned}
\underset{\mathbf{x},\xi}{\text{maximize}} \quad & f(N_f) \\
\text{subject to} \quad & f(W) < a \\
& f(T) > b \\
& f(T_w) < c \\
& f(\sigma_{max}) < d
\end{aligned} \tag{13}
$$

- However if $N_f = f(T_w, \Delta T_w, \sigma_{max})$, then perhaps a simpler surrogate can be approximated by considering constraints with $f(T_w)$, ΔT_w, and σ_{max}.

4.6 Overall consumer cost

$$
\begin{aligned}
\underset{\mathbf{x},\xi}{\text{minimize}} \quad & \alpha f(W) + \beta f(sfc) + \gamma \frac{1}{f(N_f)} \\
\text{subject to} \quad & f(T) > a
\end{aligned} \tag{14}
$$

- Assumes $N_f = f(T_w, \sigma_{max})$