

ONLINE APPENDIX

“SURVEY EVIDENCE ON HABIT FORMATION: EXISTENCE, SPECIFICATION, AND IMPLICATION”

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This appendix contains the proof of the observational equivalence of linear and non-linear habit evolutions under general habit formation preferences, aggregation of the preference parameters, response distributions, proofs of the elicitation propositions, elicitation and estimate of time discount rate, and survey details not covered in the paper.

A Observational Equivalence of Linear and Nonlinear Habit Evolutions Under General Habit Formation Preferences

This section shows that the model with the linear habit evolution (Model L below) and the models with nonlinear habit evolutions (Model N below) are observationally equivalent (in the sense of Definition 1 below) by a monotonic transformation of the scale on which habit is measured.

- Model L : $\mathbb{E}_0 \int_0^\infty e^{-\rho t} u(C, H) dt$ s.t. $\dot{H} = \theta(C - H)$
- Model N : $\mathbb{E}_0 \int_0^\infty e^{-\rho t} v(C, \mathcal{H}) dt$ s.t. $\dot{\mathcal{H}} = f(C, \mathcal{H})$, where f can be a nonlinear function of C and \mathcal{H} .

Note that $H_t = h(C_0, H_0, t)$ if $C_t = \bar{C}$ for $t \geq 0$ where the subscripts index time. Similarly, $\mathcal{H}_t = k(C_0, \mathcal{H}_0, t)$ if $C_t = \bar{C}$ for $t \geq 0$. That is, if consumption does not change for $t \geq 0$, H_t and \mathcal{H}_t are functions of only time while C_0 , H_0 , and \mathcal{H}_0 are their parameters.

Definition 1. Two models are observationally equivalent if they lead to the same set of optimal choices.

Definition 2. Monotonicities of two functions are entangled with respect to a variable if 1) the two functions share this variable as an argument, and 2) ceteris paribus, when one function is monotonic in the variable, the other function is also monotonic in the variable.

Because H and \mathcal{H} are two measurements of one fundamental—habit, they change at the same time (though in potentially different ways) when habit changes and stop changing when habit stops changing. By Definition 2, their monotonicities are entangled with respect to time.¹

Proposition 10. *Model L and Model N are observationally equivalent if the monotonicities of H and \mathcal{H} are entangled with respect to time.*

Proof. Suppose that consumption changes at instant 0 and stays at that level afterward: $C_t = C_{t+\varepsilon} \neq C_{-\varepsilon} \forall t \geq 0$ and $\varepsilon > 0$. Without loss of generality, suppose also that habit reaches its new steady state at instant T . Because H and \mathcal{H} are entangled monotonically with respect to time, H and \mathcal{H} are monotonic from instant 0 to instant T and flat afterward (i.e., remain at constant levels), say at levels \bar{H} and $\bar{\mathcal{H}}$. That is, $H_t = a(t|C_0, H_0)$ and $\mathcal{H}_t = b(t|C_0, \mathcal{H}_0)$, where $a(\cdot)$ and $b(\cdot)$ are monotonic functions of t for $0 \leq t \leq T$ and flat for $t > T$.

Because

$$\begin{aligned}\mathcal{H}_t &= b(t|C_0, \mathcal{H}_0) \\ &= b(a^{-1}(a(t|C_0, H_0)|C_0, H_0)|C_0, \mathcal{H}_0) \\ &= b(a^{-1}(H_t|C_0, H_0)|C_0, \mathcal{H}_0)\end{aligned}$$

for $0 \leq t \leq T$ and

$$\mathcal{H}_t = \frac{\bar{\mathcal{H}}}{\bar{H}} H_t$$

for $t > T$, there always exists a bijective function G that maps H_t into \mathcal{H}_t :

$$\mathcal{H}_t = G(H_t) \equiv \begin{cases} b(a^{-1}(H_t|C_0, H_0)|C_0, \mathcal{H}_0) & 0 \leq t \leq T \\ \frac{\bar{\mathcal{H}}}{\bar{H}} H_t & t > T. \end{cases}$$

Because $v(C, \mathcal{H}) = v(C, G(H)) \equiv u(C, H)$, Model N gives the same utility as Model L for any consumption path that is constant for $t \geq 0$.

When the consumption path is not constant for $t \geq 0$, the utilities from the two models remain equal. To see this, start from the instant when consumption is changed for the last time and apply the above logic to get the same utility from the two models starting from that

¹That is, $\dot{H}_t \cdot \dot{\mathcal{H}}_t$ does not change its sign. It is possible that $\dot{H}_t \cdot \dot{\mathcal{H}}_t = 0$ in some time intervals, but $\dot{H}_t \cdot \dot{\mathcal{H}}_t$ does not change its sign around the intervals.

instant onward. Then go back to the instant when consumption is changed for the second-to-last time and apply the above logic. Same utility results again for the two models. Continue this process until the first instant of interest.

Because the utilities from the two models are the same, the consumption choices generated from these two models coincide. Proof by contradiction: suppose that the two models lead to different optimal consumption paths— $\{C_L^*\} \neq \{C_N^*\}$ for at least one instant, where

$$\{C_L^*\} = \arg \max_{\{C\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(C, H) dt \equiv \arg \max_{\{C_L\}} U(\{C_L\}, H_0)$$

and

$$\{C_N^*\} = \arg \max_{\{C\}} \mathbb{E}_0 \int_0^\infty e^{-\rho t} v(C, \mathcal{H}) dt \equiv \arg \max_{\{C_N\}} V(\{C_N\}, \mathcal{H}_0).$$

If $U(\{C_L^*\}, H_0) \neq V(\{C_N^*\}, \mathcal{H}_0)$, at least one of the two consumption paths is not maximizing lifetime utility, contradicting that they are optimal solutions to the respective models. If $U(\{C_L^*\}, H_0) = V(\{C_N^*\}, \mathcal{H}_0)$, the consumption path $\{C_L^*\}$ is also a solution to the Model N while $\{C_N^*\}$ also a solution to Model L . Therefore, $\{C_H^*\}$ and $\{C_{\mathcal{H}}^*\}$ are both solutions to the two models. In other words, the two models share the same set of solutions. Thus, by Definition 1, the two models are observationally equivalent. *Q.E.D.*

Because the monotonicities of H and \mathcal{H} are entangled with respect to time, by Proposition 10, Model L and Model N are observationally equivalent.

One can easily allow external habit formation and peer effect in the models; the proof of the equivalence result in these situations is straightforward because other people's spending is exogenous to the agent.

B Aggregation

What do the elicited preference parameters of individual respondents tell us about the preference parameters of a representative agent? This question is of particular interest because almost all current models with habit formation assume the representative agent. This section shows that, given its existence, the representative agent's preference parameters are averages of the individuals' preference parameters.

To aggregate individuals, their welfare needs to be comparable with that of each other (comparability), and the representative agent's welfare should represent the average of individuals' welfare (representativeness). To formalize the idea of comparability, I assume that

at the homogenous steady state of $\bar{C}_i = \bar{H}_i = \varphi \forall i$, spending an extra dollar² while holding habit constant, brings the same marginal utility to every individual: $u_{i,C}(\bar{C}_i, \bar{H}_i) = u_{j,C}(\bar{C}_j, \bar{H}_j) = \bar{u} \forall i, j$.

With the comparability of the individuals' utilities, the representativeness of the representative agent (R) implies that $Nu_R(C_R, H_R) = \sum_i u_i(C_i, H_i)$ when $C_R = C_i = A$ and $H_R = H_i = B \forall i$ and $\forall A, B$ in the domains of the utility functions, where N is the number of individuals in the economy. That is, when the heterogeneities in behaviors (consumption and habit) are homogenized, the representative agent is the average individual agent in terms of welfare. To see what this condition means, first note that the difference between a representative-agent model and a heterogeneous-agent model is that in the former, everyone in the economy is the same, while in the latter, each individual can be different. Imagine that everyone in the heterogeneous economy becomes the same (i.e., homogeneous in consumption, habit, and utility function, etc.), the representative agent model should behave exactly the same as the homogenized heterogeneous-agent model behaves, and hence the equality of $Nu_R(C_R, H_R) = \sum_i u_i(C_i, H_i)$ under $C_R = C_i$, $H_R = H_i$, and $u_R = u_i \forall i$. Now, allowing individuals to be heterogeneous along the dimension of utility function after the normalization of the comparability condition, the representativeness condition simply requires that the representative agent represents the individuals along the welfare dimension, after controlling for consumption and habit.

B.1 Aggregation of Habit Depreciation Rate

Even though habit depreciation rate (θ) and habit (H) are one-to-one mapped at each instant of time³ for any given consumption profile, there are infinitely many pairs of them that satisfy at the same time the representative agent's habit evolution ($\dot{H}_R = \theta_R(C_R - H_R)$), the individuals' habit evolutions ($\dot{H}_i = \theta_i(C_i - H_i)$), the comparability condition, and the representativeness condition. The intuition is that while habit depends on the habit depreciation rate, its steady-state level does not. In other words, a different value of θ leads to a different value of H at all the instants before the steady state, and this difference vanishes after the steady state is reached. The representativeness condition eliminates such indeterminacy and pins down a unique θ_R for individuals' given θ_i 's.

To find the mapping between the aggregate habit depreciation rate (θ_R) and the individuals' habit depreciation rates (θ_i 's), imagine that everyone starts at the homogenous

²An epsilon dollar, to be exact.

³Before a steady state is reached.

steady state and increases their consumption by the same iota amount. That is, starting from $C_i = C_j = C_R = H_i = H_j = H_R \forall i, j$, increase consumption by the iota amount $\Delta C_i = \Delta C_j = \Delta C_R \forall i, j$. The resulting changes to the utilities are

$$\begin{aligned}\Delta u_R (C_R, H_R) &= u_{R,C} (C_R, H_R) \Delta C_R + u_{R,H} (C_R, H_R) \Delta H_R \\ &= u_{R,C} (C_R, H_R) \Delta C_R + u_{R,H} (C_R, H_R) \theta_R \Delta C_R\end{aligned}$$

and

$$\begin{aligned}\Delta u_i (C_i, H_i) &= u_{i,C} (C_i, H_i) \Delta C_i + u_{i,H} (C_i, H_i) \Delta H_i \\ &= u_{i,C} (C_i, H_i) \Delta C_i + u_{i,H} (C_i, H_i) \theta_i \Delta C_i\end{aligned}$$

where $u_{i,X} \equiv \partial u_i / \partial X$.

By $N \Delta u_R (C_R, H_R) = \sum_i \Delta u_i (C_i, H_i)$, as implied by the representativeness condition,

$$\begin{aligned}u_{R,C} (C_R, H_R) \Delta C_R + u_{R,H} (C_R, H_R) \theta_R \Delta C_R \\ = \frac{1}{N} \sum_i [u_{i,C} (C_i, H_i) \Delta C_i + u_{i,H} (C_i, H_i) \theta_i \Delta C_i].\end{aligned}$$

Because $\Delta C_R = \Delta C_i$ and $u_{R,C} (C_R, H_R) = u_{i,C} (C_i, H_i) \forall i$ (see Section B.2),

$$\frac{u_{R,H} (C_R, H_R)}{u_{R,C} (C_R, H_R)} \theta_R = \frac{1}{N} \sum_i \frac{u_{i,H} (C_i, H_i)}{u_{i,C} (C_i, H_i)} \theta_i = \frac{1}{N} \sum_i \frac{u_{i,H} (C_i, H_i)}{u_{i,C} (C_i, H_i)} \cdot \frac{1}{N} \sum_i \theta_i$$

where the second equality holds because of the independence between slope of indifference curve and habit depreciation rate. With $u_{R,H} (C_R, H_R) = \frac{1}{N} \sum_i u_{i,H} (C_i, H_i)$ (see Section B.2) and $u_{R,C} (C_R, H_R) = u_{i,C} (C_i, H_i) \forall i$, it follows that

$$\theta_R = \frac{1}{N} \sum_i \theta_i.$$

That is, the representative agent's habit depreciation rate is the average of the individuals' habit depreciation rates.

B.2 Aggregation of Ratios of Utility Derivatives

First, I derive the relationships between utility derivatives of the representative agent and the heterogeneous agents at the homogenous steady state ($\bar{C}_R = \bar{H}_R = \bar{C}_i = \bar{H}_i \forall i$).

Because $Nu_R(C_R, H_R) = \sum_i u_i(C_i, H_i)$ for $C_R = C_i = A$ and $H_R = H_i = B \forall i$ and $\forall A, B$ in the domains of the utility functions, utility derivatives of the representative agent are the average of the utility derivatives of the individuals:

$$u_{R,X}(C_R, H_R) = \frac{1}{N} \sum_i u_{i,X}(C_i, H_i),$$

where X denotes the variable and order of differentiation of the utility derivatives (e.g., C, H, CC, CH, HH).

Next, I derive the relationships between ratios of utility derivatives of the representative agent and the heterogeneous agents at the steady state.

1. Under the normalization of $u_{R,C}(\bar{C}_R, \bar{H}_R) = u_{i,C}(\bar{C}_i, \bar{H}_i) = \bar{u} \forall i$,

$$\frac{1}{N} \sum_i u_{i,H} = -\frac{1}{N} \sum_i \left(-\frac{u_{i,H}}{u_{i,C}} \cdot u_{i,C} \right) = -\left[\frac{1}{N} \sum_i \left(-\frac{u_{i,H}}{u_{i,C}} \right) \right] \cdot u_{i,C} \equiv -\mu_{-\frac{u_{i,H}}{u_{i,C}}} \cdot \bar{u},$$

where $\mu_{-\frac{u_{i,H}}{u_{i,C}}} \equiv \frac{1}{N} \sum_i \left(-\frac{u_{i,H}}{u_{i,C}} \right)$. Similar notations are used hereafter.

2. Because the parameters are independent,

$$\begin{aligned} \frac{1}{N} \sum_i u_{i,HH} &= \frac{1}{N} \sum_i \left(\frac{Hu_{i,HH}}{u_{i,H}} \cdot u_{i,H} \cdot \frac{1}{H} \right) \\ &= \frac{1}{N} \sum_i \frac{Hu_{i,HH}}{u_{i,H}} \cdot \frac{1}{N} \sum_i u_{i,H} \cdot \frac{1}{H} \\ &= \mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \cdot \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \cdot \bar{u} \cdot \frac{1}{H}, \end{aligned}$$

$$\begin{aligned} \frac{1}{N} \sum_i u_{i,CH} &= \frac{1}{N} \sum_i \left(\frac{u_{i,CH}}{u_{i,HH}} \cdot u_{i,HH} \right) \\ &= \frac{1}{N} \sum_i \frac{u_{i,CH}}{u_{i,HH}} \cdot \frac{1}{N} \sum_i u_{i,HH} \end{aligned}$$

$$= \mu_{\frac{u_{i,CH}}{u_{i,HH}}} \cdot \mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \cdot \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \cdot \bar{u} \cdot \frac{1}{H},$$

$$\begin{aligned} \frac{1}{N} \sum_i u_{i,CC} &= \frac{1}{N} \sum_i \frac{u_{i,CC}}{u_{i,HH}} \cdot u_{i,HH} \\ &= \frac{1}{N} \sum_i \frac{u_{i,CC}}{u_{i,HH}} \cdot \frac{1}{N} \sum_i u_{i,HH} \\ &= \mu_{\frac{u_{i,CC}}{u_{i,HH}}} \cdot \mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \cdot \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \cdot \bar{u} \cdot \frac{1}{H}, \end{aligned}$$

and

$$\begin{aligned} \frac{1}{N} \sum_i u_{i,C_{\text{others}}} &= \frac{1}{N} \sum_i \frac{u_{i,C_{\text{others}}}}{u_{i,H}} \cdot u_{i,H} \\ &= \frac{1}{N} \sum_i \frac{u_{i,C_{\text{others}}}}{u_{i,H}} \cdot \frac{1}{N} \sum_i u_{i,H} \\ &= \mu_{\frac{u_{i,C_{\text{others}}}}{u_{i,H}}} \cdot \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \cdot \bar{u}. \end{aligned}$$

3. With these, the representative agent's parameters can be calculated:

$$-\frac{u_{R,H}}{u_{R,C}} = -\frac{\frac{1}{N} \sum_i u_{i,H}}{\frac{1}{N} \sum_i u_{i,C}} = -\frac{\frac{1}{N} \sum_i u_{i,H}}{\bar{u}} = \mu_{-\frac{u_{i,H}}{u_{i,C}}},$$

$$\frac{Hu_{R,HH}}{u_{R,H}} = \frac{H \frac{1}{N} \sum_i u_{i,HH}}{\frac{1}{N} \sum_i u_{i,H}} = \frac{H \mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \bar{u} \frac{1}{H}}{-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \bar{u}} = \mu_{\frac{Hu_{i,HH}}{u_{i,H}}},$$

$$\frac{u_{R,CH}}{u_{R,HH}} = \frac{\frac{1}{N} \sum_i u_{i,CH}}{\frac{1}{N} \sum_i u_{i,HH}} = \frac{\mu_{\frac{u_{i,CH}}{u_{i,HH}}} \mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \bar{u} \frac{1}{H}}{\mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \bar{u} \frac{1}{H}} = \mu_{\frac{u_{i,CH}}{u_{i,HH}}},$$

$$\frac{u_{R,CC}}{u_{R,HH}} = \frac{\frac{1}{N} \sum_i u_{i,CC}}{\frac{1}{N} \sum_i u_{i,HH}} = \frac{\mu_{\frac{u_{i,CC}}{u_{i,HH}}} \mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \bar{u} \frac{1}{H}}{\mu_{\frac{Hu_{i,HH}}{u_{i,H}}} \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \bar{u} \frac{1}{H}} = \mu_{\frac{u_{i,CC}}{u_{i,HH}}},$$

and

$$\frac{u_{R,C_{\text{others}}}}{u_{R,H}} = \frac{\frac{1}{N} \sum_i u_{i,C_{\text{others}}}}{\frac{1}{N} \sum_i u_{i,H}} = \frac{\mu_{\frac{u_{i,C_{\text{others}}}}{u_{i,C}}} \cdot \left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \cdot \bar{u}}{\left(-\mu_{-\frac{u_{i,H}}{u_{i,C}}} \right) \cdot \bar{u}} = \mu_{\frac{u_{i,C_{\text{others}}}}{u_{i,H}}}.$$

In summary, the representative agent's ratios of utility derivatives are averages of individuals' ratios of utility derivatives.

B.3 Aggregation of External Habit Mixture Coefficient

Imagine that everyone's consumption increases by the same iota amount so that the representative agent also increases her consumption by this same amount. The representativeness condition implies that the changes in utilities satisfy

$$\begin{aligned} & N [u_{R,C} (C_R, H_R) \Delta C_R + u_{R,H} (C_R, H_R) \Delta H_R] \\ &= \sum_i [u_{i,C} (C_i, H_i) \Delta C_R + u_{i,H} (C_i, H_i) \Delta H_i]. \end{aligned}$$

Using the comparability condition to get

$$N \frac{u_{R,H} (C_R, H_R)}{u_{R,C} (C_R, H_R)} \Delta H_R = \sum_i \frac{u_{i,H} (C_i, H_i)}{u_{i,C} (C_i, H_i)} \Delta H_i.$$

Because $\Delta H_R / \Delta H_i = \theta_R (1 - \omega_R) / [\theta_i (1 - \omega_i)]$,

$$\begin{aligned} \frac{u_{R,H} (C_R, H_R)}{u_{R,C} (C_R, H_R)} \theta_R (1 - \omega_R) &= \frac{1}{N} \sum_i \frac{u_{i,H} (C_i, H_i)}{u_{i,C} (C_i, H_i)} \theta_i (1 - \omega_i) \\ &= \left(\frac{1}{N} \sum_i \frac{u_{i,H} (C_i, H_i)}{u_{i,C} (C_i, H_i)} \right) \cdot \left(\frac{1}{N} \sum_i \theta_i \right) \cdot \left(\frac{1}{N} \sum_i (1 - \omega_i) \right) \end{aligned}$$

where the second equality holds because of the independence between the preference parameters. Finally, by

$$\frac{u_{R,H} (C_R, H_R)}{u_{R,C} (C_R, H_R)} = \frac{1}{N} \sum_i \frac{u_{i,H} (C_i, H_i)}{u_{i,C} (C_i, H_i)}$$

and $\theta_R = \frac{1}{N} \sum_i \theta_i$,

$$\omega_R = \frac{1}{N} \sum_i \omega_i.$$

In words, the representative agent's external habit mixture coefficient equals the average of individuals' external habit mixture coefficients.

C Response Distributions

Table A.1 summarizes the distributions of responses to survey questions.

D Proofs of Elicitation Propositions

Before proving the elicitation propositions, this section first proves three lemmas and then derives three quantities, which will be used repeatedly in the proofs of the propositions.

D.1 Lemmas

Lemma 1. For $a, b, c \in \mathbb{R}$, if $a(a+b) > 0$ and $0 \leq c \leq 1$, $a(a+cb) > 0$.

Proof. $a(a+b) > 0$ is equivalent to $a+b < 0$ if $a < 0$ and $a+b > 0$ if $a > 0$.

Suppose $a < 0$ and $a+b < 0$. Note that $a+cb = a+b+(c-1)b$. If $b \geq 0$, $(c-1)b \leq 0$ and thus $a+cb \leq a+b < 0$. If $b < 0$, by $a < 0$ and $c \geq 0$, $a+cb < 0$. Therefore, $a(a+cb) > 0$.

Suppose $a > 0$ and $a+b > 0$. If $b \leq 0$, $(c-1)b \geq 0$ and, therefore, $a+cb \geq a+b > 0$. If $b > 0$, by $a > 0$ and $c \geq 0$, $a+cb > 0$. In both cases, $a(a+cb) > 0$. *Q.E.D.*

In words, the lemma states that if a and $a+b$ share a sign and $0 \leq c \leq 1$, $a+cb$ share its sign with a and $a+b$.

Lemma 2. For $\Delta e, \Delta f, M \in \mathbb{R}^+$, if $M - \Delta e - \Delta f \geq 0$, $\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s} / M^n$ is decreasing in $n \in \mathbb{N}^+$.

Proof. $\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s} / M^n$ is decreasing in $n \in \mathbb{N}^+$ if $\forall n \in \mathbb{N}^+$,

$$\frac{\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s}}{M^n} - \frac{\sum_{s=0}^{n+1} (\Delta e)^s (\Delta f)^{n+1-s}}{M^{n+1}} > 0.$$

Now,

$$\begin{aligned}
& \frac{\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s}}{M^n} - \frac{\sum_{s=0}^{n+1} (\Delta e)^s (\Delta f)^{n+1-s}}{M^{n+1}} \\
&= \frac{\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s}}{M^n} - \frac{\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s} + \frac{(\Delta e)^{n+1}}{\Delta f} \Delta f}{M^{n+1}} \\
&= \frac{\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s}}{M^n} \left(1 - \frac{\Delta f}{M}\right) - \frac{(\Delta e)^{n+1}}{M^{n+1}} \\
&= \frac{1}{M^{n+1}} \left[\sum_{s=0}^n (\Delta e)^s (\Delta f)^{n-s} (M - \Delta f) - (\Delta e)^{n+1} \right] \\
&= \frac{(\Delta e)^n (M - \Delta f)}{M^{n+1}} \left[\sum_{s=0}^n \left(\frac{\Delta f}{\Delta e} \right)^{n-s} - \frac{\Delta e}{M - \Delta f} \right] \\
&= \frac{(\Delta e)^n (M - \Delta f)}{M^{n+1}} \left[\sum_{s=0}^{n-1} \left(\frac{\Delta f}{\Delta e} \right)^{n-s} + 1 - \frac{\Delta e}{M - \Delta f} \right] \\
&= \frac{(\Delta e)^n (M - \Delta f)}{M^{n+1}} \left[\sum_{s=0}^{n-1} \left(\frac{\Delta f}{\Delta e} \right)^{n-s} + \frac{M - \Delta e - \Delta f}{M - \Delta f} \right] \\
&\geq \frac{(\Delta e)^n (M - \Delta f)}{M^{n+1}} \sum_{s=0}^{n-1} \left(\frac{\Delta f}{\Delta e} \right)^{n-s} \\
&> 0
\end{aligned}$$

where the first inequality holds because $M - \Delta e - \Delta f \geq 0$ and $\Delta e > 0$. *Q.E.D.*

Lemma 3. For $\Delta e, \Delta f, M, \theta, t \in \mathbb{R}^+$, if $M - \Delta e - \Delta f \geq 0$, $u(C, H)$ is analytic with $u_H < 0 \forall H$, and $\partial^{i_k} u / \partial H^{i_k} \leq \Lambda_k \forall i_k \in I \equiv \{i \mid \partial^i u / \partial H^i > 0, i \in \mathbb{N}^+\}$ with $\partial^{i_1} u / \partial H^{i_1} < \Lambda_1$, where i_k is the k -th smallest element of I ,

$$\Lambda_k \equiv - \sum_{n=1}^{i_k-1} e^{(i_k-n)\theta t} \frac{i_k! \sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s}}{n! \sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s}} \left[\frac{\partial^n u}{\partial H^n} 1(n \notin I) + \Lambda_{K(n)} 1(n \in I) \right],$$

and $K(n) = k$ if $n = i_k$, then

$$\sum_{n=1}^{\infty} e^{-n\theta t} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) < 0.$$

Proof. By $u_H < 0 \forall H$, analyticity of $u(C, H)$, and $Me^{-\theta t} > 0$,

$$u(C, H + Me^{-\theta t}) - u(C, H) = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} (Me^{-\theta t})^n < 0. \quad (\text{A.1})$$

Because $\Delta e, \Delta f, M > 0$, and $M - \Delta e - \Delta f \geq 0$, $0 < \frac{\Delta e + \Delta f}{M} \leq 1$.

By $u_H < 0$, inequality (A.1), and Lemma 1,

$$u_H Me^{-\theta t} + \frac{\Delta e + \Delta f}{M} \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} (Me^{-\theta t})^n < 0. \quad (\text{A.2})$$

By Lemma 2,

$$0 \leq \frac{\frac{\sum_{s=0}^2 (\Delta e)^s (\Delta f)^{2-s}}{M^2}}{\frac{\Delta e + \Delta f}{M}} < 1.$$

If $u_{HH} \leq 0$, apply Lemma 1 to inequality (A.2) to get

$$\begin{aligned} & u_H Me^{-\theta t} + \frac{1}{2} u_{HH} (Me^{-\theta t})^2 \frac{\Delta e + \Delta f}{M} \\ & + \frac{\frac{\sum_{s=0}^2 (\Delta e)^s (\Delta f)^{2-s}}{M^2}}{\frac{\Delta e + \Delta f}{M}} \frac{\Delta e + \Delta f}{M} \sum_{n=3}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} (Me^{-\theta t})^n \\ & = \sum_{n=1}^2 e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\ & + \frac{\sum_{s=0}^2 (\Delta e)^s (\Delta f)^{2-s}}{M^2} \sum_{n=3}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} (Me^{-\theta t})^n \\ & < 0. \end{aligned} \quad (\text{A.3})$$

This process of successive applications of Lemmas 1 and 2 can continue until $\partial^{i_1} u / \partial H^{i_1} > 0$. Note that $i_1 \geq 2$ since $u_H < 0$.

In general, when $\partial^i u / \partial H^i > 0$ for $i \in I$, it is necessary to bound the $\partial^i u / \partial H^i$'s from above, so that

$$\sum_{n=1}^i e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} < 0,$$

enabling the continued applications of Lemmas 1 and 2. Next, I show that the bounds of

$\Lambda_{\{k\}}$ achieve this goal.⁴

Suppose, for some $i_k \in I$,

$$\sum_{n=1}^{z-1} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) < 0 \quad (\text{A.4})$$

$\forall z \leq i_k$.

By inequalities (A.1) and (A.4), Lemmas 1 and 2 can be applied to all $n \leq i_k$ in the fashion inequality (A.3) is derived from inequality (A.1), which gives

$$\begin{aligned} & \sum_{n=1}^{i_k-1} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\ & + \frac{\sum_{s=0}^{i_k-2} (\Delta e)^s (\Delta f)^{i_k-2-s}}{M^{i_k-2}} \sum_{i=i_k}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(M e^{-\theta t} \right)^n \\ & < 0. \end{aligned} \quad (\text{A.5})$$

Since

$$0 \leq \frac{\frac{\sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s}}{M^{i_k-1}}}{\frac{\sum_{s=0}^{i_k-2} (\Delta e)^s (\Delta f)^{i_k-2-s}}{M^{i_k-2}}} \leq 1,$$

as implied by Lemma 2, and inequality (A.4) with $z = i_k$, Lemma 1 can be applied to inequality (A.5) for getting

$$\begin{aligned} & \sum_{n=1}^{i_k-1} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\ & + \frac{\frac{\sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s}}{M^{i_k-1}}}{\frac{\sum_{s=0}^{i_k-2} (\Delta e)^s (\Delta f)^{i_k-2-s}}{M^{i_k-2}}} \sum_{i=i_k}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(M e^{-\theta t} \right)^n \\ & = \sum_{n=1}^{i_k} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\ & + \frac{\sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s}}{M^{i_k-1}} \sum_{n=i_k+1}^{\infty} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(M e^{-\theta t} \right)^n \end{aligned}$$

⁴As mentioned in the paper, the ubiquitous additive and multiplicative habits with power utility satisfy these bounds under common parameter values.

<0.

Now,

$$\begin{aligned}
& \sum_{n=1}^{i_k} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\
&= \sum_{n=1}^{i_k-1} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) + e^{-i_k \theta t} M \frac{1}{i_k!} \frac{\partial^{i_k} u}{\partial H^{i_k}} \left(\sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s} \right) \\
&\leq \sum_{n=1}^{i_k-1} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) + e^{-i_k \theta t} M \frac{1}{i_k!} \left(\sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s} \right) \\
&\quad \cdot \left(- \sum_{n=1}^{i_k-1} e^{(i_k-n)\theta t} \frac{i_k! \sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s}}{n! \sum_{s=0}^{i_k-1} (\Delta e)^s (\Delta f)^{i_k-1-s}} \left[\frac{\partial^n u}{\partial H^n} 1(n \notin I) + \Lambda_{K(n)} 1(n \in I) \right] \right) \\
&= \sum_{n=1}^{i_k-1} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\
&\quad - \sum_{n=1}^{i_k-1} e^{-n\theta t} M \frac{1}{n!} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \left[\frac{\partial^n u}{\partial H^n} 1(n \notin I) + \Lambda_{K(n)} 1(n \in I) \right] \\
&= \sum_{n=1}^{i_k-1} 1(n \in I) e^{-n\theta t} M \frac{1}{n!} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \left(\frac{\partial^n u}{\partial H^n} - \Lambda_{K(n)} \right) \\
&< 0
\end{aligned} \tag{A.6}$$

where the first inequality holds because $\partial^{i_k} u / \partial H^{i_k} \leq \Lambda_k$ and the last inequality because $\partial^i u / \partial H^i \leq \Lambda_{K(i)} \forall i \in I$ and $\partial^{i_1} u / \partial H^{i_1} < \Lambda_1$. That is, inequality (A.4) holds for $z = i_k + 1$.

Since $\partial^n u / \partial H^n < 0$ for $i_k < n < i_{k+1}$, inequality (A.4) also holds $\forall z \leq i_{k+1}$. In words, if inequality (A.4) holds $\forall z \leq i_k$, it also holds for $z \leq i_{k+1}$. Since it is trivially true that inequality (A.4) holds $\forall z \leq i_1$ (note $\partial^n u / \partial H^n < 0$ for $n < i_1$), inequality (A.4) holds $\forall z \leq i \forall i \in I$.

In particular, it holds for the largest element of I ($i_{|I|}$ ⁵). Replacing k by $|I|$ in the step

⁵ $|I|$ denotes the cardinality of set I .

of inequality (A.4) and following the subsequent derivation to inequality (A.6) leads to

$$\sum_{n=1}^{i_{|I|}} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) < 0. \quad (\text{A.7})$$

Because $\partial^n u / \partial H^n < 0 \ \forall n > i_{|I|}$,

$$\sum_{n=i_{|I|}+1}^{\infty} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) < 0. \quad (\text{A.8})$$

Finally,

$$\begin{aligned} & \sum_{n=1}^{\infty} e^{-n\theta t} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \\ &= \frac{1}{M} \left[\sum_{n=1}^{i_{|I|}} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \right. \\ & \quad \left. + \sum_{n=i_{|I|}+1}^{\infty} e^{-n\theta t} M \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} (\Delta e)^s (\Delta f)^{n-1-s} \right) \right] \\ &< 0, \end{aligned}$$

where the inequality follows from inequalities (A.7) and (A.8).

Q.E.D.

D.2 Three Quantities

This section presents the derivations of three quantities regarding the utility differences between spending paths, which will be used repeatedly in proving the propositions.

First, note that $\dot{H} = \theta (C - H)$ leads to

$$H(t) = e^{-\theta(t-t_0)} H(t_0) + \int_{t_0}^t e^{-\theta(t-s)} \theta C(s) ds.$$

Second, denote a steady state by $C(t) = H(t) = \bar{C} = \bar{H} \ \forall t$. Then

$$\begin{aligned} \Delta H(t) &\equiv H(t) - \bar{H} \\ &= e^{-\theta(t-t_0)} (H(t_0) - \bar{H}) + \int_{t_0}^t e^{-\theta(t-s)} \theta (C(s) - \bar{H}) ds \end{aligned}$$

$$\begin{aligned}
&= e^{-\theta(t-t_0)} (H(t_0) - \bar{H}) + \int_{t_0}^t e^{-\theta(t-s)} \theta (C(s) - \bar{C}) ds \\
&\equiv e^{-\theta(t-t_0)} \Delta H(t_0) + \int_{t_0}^t e^{-\theta(t-s)} \theta \Delta C(s) ds
\end{aligned}$$

where $\Delta C(t) \equiv C(t) - \bar{C}$.

The first quantity is for the class of spending paths with $\Delta C(t) = A \cdot 1(t \leq 0) + B \cdot 1(t > 0)$ (that is, $\Delta C(t) = A$ for $t \leq 0$ and $\Delta C(t) = B$ for $t > 0$). Under such spending paths, for $t \geq 0$,

$$\begin{aligned}
\Delta H(t) &= e^{-\theta t} \Delta H(-\infty) + \int_{-\infty}^0 e^{-\theta(t-s)} \theta A ds + \int_0^t e^{-\theta(t-s)} \theta B ds \\
&= e^{-\theta t} A + (1 - e^{-\theta t}) B.
\end{aligned} \tag{A.9}$$

For the second equality, note that $\Delta H(-\infty)$ is finite by the definition of habit.

The difference between lifetime utilities starting now ($t = 0$), $\int_0^\infty e^{-\rho t} u(C(t), H(t)) dt$, under this class of spending paths and under the steady state (\bar{C}, \bar{H}) is

$$\begin{aligned}
&\Psi(A, B) \\
&\equiv \int_0^\infty e^{-\rho t} u(C(t), H(t)) dt - \int_0^\infty e^{-\rho t} u(\bar{C}, \bar{H}) dt \\
&= \int_0^\infty e^{-\rho t} [u(C(t), H(t)) - u(\bar{C}, \bar{H})] dt \\
&= \int_0^\infty e^{-\rho t} [u(\bar{C} + \Delta C(t), \bar{H} + \Delta H(t)) - u(\bar{C}, \bar{H})] dt \\
&= \int_0^\infty e^{-\rho t} \left[u\left(\bar{C} + B, \bar{H} + e^{-\theta t} A + (1 - e^{-\theta t}) B\right) - u(\bar{C}, \bar{H}) \right] dt \\
&= \int_0^\infty e^{-\rho t} \left\{ u_C B + \frac{1}{2} u_{CC} B^2 + u_{CH} B \left(e^{-\theta t} A + (1 - e^{-\theta t}) B \right) + \dots \right. \\
&\quad \left. + \left[u_H \left(e^{-\theta t} A + (1 - e^{-\theta t}) B \right) + \frac{1}{2} u_{HH} \left(e^{-\theta t} A + (1 - e^{-\theta t}) B \right)^2 + \dots \right] \right\} dt
\end{aligned}$$

where the last equality holds because u is analytic, and all the utility derivatives are evaluated at the steady state.

For example, under the second-order approximation,

$$\Psi(A, B)$$

$$= \frac{1}{\rho} \left[u_C B + u_H \frac{\rho A + \theta B}{\rho + \theta} + \frac{1}{2} \left(u_{CC} B^2 + 2u_{CH} \frac{\rho AB + \theta B^2}{\rho + \theta} + u_{HH} \frac{\rho(\rho + \theta) A^2 + 2\rho\theta AB + 2\theta^2 B^2}{(\rho + \theta)(\rho + 2\theta)} \right) \right]$$

The second quantity is related to the class of spending paths with

$$\Delta C(t) = A \cdot 1(-1 < t \leq 0) + B \cdot 1(0 < t \leq 1) + 0 \cdot 1(t \leq -1 \text{ or } t > 1).$$

It follows that, for $t \geq 0$,

$$\begin{aligned} \Delta H(t) &= e^{-\theta\infty} \Delta H(-\infty) + \int_{-\infty}^{-1} e^{-\theta(t-s)} \theta 0 ds + \int_{-1}^0 e^{-\theta(t-s)} \theta A ds \\ &\quad + \int_0^{\min\{1,t\}} e^{-\theta(t-s)} \theta B ds + \int_{\min\{1,t\}}^t e^{-\theta(t-s)} \theta 0 ds \\ &= e^{-\theta t} (1 - e^{-\theta}) A + (e^{-\theta(t-\min\{1,t\})} - e^{-\theta t}) B \\ &= e^{-\theta t} \left[(1 - e^{-\theta}) A + (e^{\theta \min\{1,t\}} - 1) B \right]. \end{aligned}$$

The difference between lifetime utilities starting now, $\int_0^\infty e^{-\rho t} u(C(t), H(t)) dt$, under this class of spending paths and under the steady state (\bar{C}, \bar{H}) is

$$\begin{aligned} &\Upsilon(A, B) \\ &\equiv \int_0^\infty e^{-\rho t} u(C(t), H(t)) dt - \int_0^\infty e^{-\rho t} u(\bar{C}, \bar{H}) dt \\ &= \int_0^\infty e^{-\rho t} [u(C(t), H(t)) - u(\bar{C}, \bar{H})] dt \\ &= \int_0^\infty e^{-\rho t} \left[u(\bar{C} + B \cdot 1(0 < t \leq 1), \bar{H} + e^{-\theta t} ((1 - e^{-\theta}) A + (e^{\theta \min\{1,t\}} - 1) B)) \right. \\ &\quad \left. - u(\bar{C}, \bar{H}) \right] dt \\ &= \int_0^1 e^{-\rho t} \left[u(\bar{C} + B, \bar{H} + e^{-\theta t} ((1 - e^{-\theta}) A + (e^{\theta t} - 1) B)) - u(\bar{C}, \bar{H}) \right] dt \\ &\quad + \int_1^\infty e^{-\rho t} \left[u(\bar{C}, \bar{H} + e^{-\theta t} (1 - e^{-\theta}) (A + B e^\theta)) - u(\bar{C}, \bar{H}) \right] dt \\ &= \int_0^1 e^{-\rho t} \left\{ u_C B + \frac{1}{2} u_{CC} B^2 + u_{CH} B e^{-\theta t} ((1 - e^{-\theta}) A + (e^{\theta t} - 1) B) + \dots \right. \end{aligned}$$

$$\begin{aligned}
& + \left[u_H e^{-\theta t} \left((1 - e^{-\theta}) A + (e^{\theta t} - 1) B \right) + \frac{1}{2} u_{HH} \left(e^{-\theta t} \left((1 - e^{-\theta}) A + (e^{\theta t} - 1) B \right) \right)^2 \right. \\
& \left. + \dots \right] dt + \int_1^\infty e^{-\rho t} \left[u_H e^{-\theta t} (1 - e^{-\theta}) (A + B e^\theta) \right. \\
& \left. + \frac{1}{2} u_{HH} \left(e^{-\theta t} (1 - e^{-\theta}) (A + B e^\theta) \right)^2 + \dots \right] dt.
\end{aligned}$$

When others' spending varies, it is necessary to consider peer effect and external habit: $u(C, C_{\text{others}}, H)$ with $\dot{H} = \theta((1 - \omega)C + \omega C_{\text{others}} - H)$. Under this habit evolution,

$$\Delta H(t) = e^{-\theta(t-t_0)} H(t_0) + \int_{t_0}^t e^{-\theta(t-s)} \theta [(1 - \omega) \Delta C(s) + \omega \Delta C_{\text{others}}(s)] ds,$$

where $\Delta C_{\text{others}}(t) \equiv C_{\text{others}}(t) - \bar{C}_{\text{others}}$.

For the class of spending paths with $\Delta C(t) = A \cdot 1(t \leq 0) + B \cdot 1(t > 0)$ and $\Delta C_{\text{others}}(t) = D \cdot 1(t \leq 0) + E \cdot 1(t > 0)$, similar to equation (A.9),

$$\Delta H(t) = e^{-\theta t} ((1 - \omega) A + \omega D) + (1 - e^{-\theta t}) ((1 - \omega) B + \omega E).$$

The difference between lifetime utilities starting now, $\int_0^\infty e^{-\rho t} u(C(t), C_{\text{others}}(t), H(t)) dt$, under this class of spending paths and under the steady state $(\bar{C}, \bar{C}_{\text{others}}, \bar{H})$, the third quantity, is

$$\begin{aligned}
& \Phi(A, B, D, E) \\
& \equiv \int_0^\infty e^{-\rho t} u(C(t), C_{\text{others}}(t), H(t)) dt - \int_0^\infty e^{-\rho t} u(\bar{C}, \bar{C}_{\text{others}}, \bar{H}) dt \\
& = \int_0^\infty e^{-\rho t} [u(C(t), C_{\text{others}}(t), H(t)) - u(\bar{C}, \bar{C}_{\text{others}}, \bar{H})] dt \\
& = \int_0^\infty e^{-\rho t} \left[u\left(\bar{C} + B, \bar{C}_{\text{others}} + E, \right. \right. \\
& \quad \left. \left. \bar{H} + e^{-\theta t} ((1 - \omega) A + \omega D) + (1 - e^{-\theta t}) ((1 - \omega) B + \omega E) \right) - u(\bar{C}, \bar{C}_{\text{others}}, \bar{H}) \right] dt \\
& = \int_0^\infty e^{-\rho t} \left\{ u_C B + u_{C_{\text{others}}} E + u_H \left[e^{-\theta t} ((1 - \omega) A + \omega D) \right. \right. \\
& \quad \left. \left. + (1 - e^{-\theta t}) ((1 - \omega) B + \omega E) \right] + \frac{1}{2} u_{CC} B^2 + \frac{1}{2} u_{C_{\text{others}} C_{\text{others}}} E^2 \right. \\
& \quad \left. + \frac{1}{2} u_{HH} \left[e^{-\theta t} ((1 - \omega) A + \omega D) + (1 - e^{-\theta t}) ((1 - \omega) B + \omega E) \right]^2 \right\} dt
\end{aligned}$$

$$\begin{aligned}
& + u_{CC_{\text{others}}} BE + u_{CH} B \left[e^{-\theta t} ((1 - \omega) A + \omega D) + (1 - e^{-\theta t}) ((1 - \omega) B + \omega E) \right] \\
& + u_{C_{\text{others}}H} E \left[e^{-\theta t} ((1 - \omega) A + \omega D) + (1 - e^{-\theta t}) ((1 - \omega) B + \omega E) \right] + \dots \Big\} dt.
\end{aligned}$$

D.3 Proof of Proposition 1

Proof. That θ is habit depreciation rate implies $\theta \in \mathbb{R}^+$. Taking $M = 5000$ gives $M - \Delta C_{U1} - (1 - e^{-\theta}) \Delta C_{U2} > 0$ in all the questions for habit depreciation rate.⁶

A respondent preferring Universe One for a better future experience (U) in a habit depreciation rate question implies

$$\begin{aligned}
& U (\text{Universe One}) - U (\text{Universe Two}) \\
& = [U (\text{Universe One}) - U (\text{Baseline})] - [U (\text{Universe Two}) - U (\text{Baseline})] \\
& = \Psi (\Delta C_{U1}, 0) - \Upsilon (\Delta C_{U2}, 0) \\
& = \int_0^\infty e^{-\rho t} \left[u_H e^{-\theta t} \Delta C_{U1} + \frac{1}{2} u_{HH} (e^{-\theta t} \Delta C_{U1})^2 + \dots \right] dt \\
& \quad - \int_0^\infty e^{-\rho t} \left[u_H e^{-\theta t} (1 - e^{-\theta}) \Delta C_{U2} + \frac{1}{2} u_{HH} (e^{-\theta t} (1 - e^{-\theta}) \Delta C_{U2})^2 + \dots \right] dt \\
& = \int_0^\infty e^{-\rho t} \left\{ u_H e^{-\theta t} [\Delta C_{U1} - (1 - e^{-\theta}) \Delta C_{U2}] \right. \\
& \quad \left. + \frac{1}{2} u_{HH} e^{-2\theta t} [(\Delta C_{U1})^2 - ((1 - e^{-\theta}) \Delta C_{U2})^2] + \dots \right\} dt \\
& = [\Delta C_{U1} - (1 - e^{-\theta}) \Delta C_{U2}] \int_0^\infty e^{-\rho t} \left\{ u_H e^{-\theta t} \right. \\
& \quad \left. + \frac{1}{2} u_{HH} e^{-2\theta t} [\Delta C_{U1} + (1 - e^{-\theta}) \Delta C_{U2}] + \dots \right\} dt \\
& = [\Delta C_{U1} - (1 - e^{-\theta}) \Delta C_{U2}] \\
& \quad \cdot \int_0^\infty e^{-\rho t} \sum_{n=1}^\infty e^{-n\theta t} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left[\sum_{s=0}^{n-1} (\Delta C_{U1})^s ((1 - e^{-\theta}) \Delta C_{U2})^{n-1-s} \right] dt \\
& > 0.
\end{aligned}$$

⁶See Figure 1b of the paper for the initial monthly spending graphs and Table A.2 for all the values of ΔC_{U1} and ΔC_{U2} for this survey module.

The inequality, by Lemma 3, implies $\Delta C_{U1} - (1 - e^{-\theta}) \Delta C_{U2} < 0$ or equivalently

$$\theta > -\ln \left(1 - \frac{\Delta C_{U1}}{\Delta C_{U2}} \right).$$

Q.E.D.

It is worth noting that when others' spending does not vary, the elicitation propositions in here and the following, which are based on $\dot{H} = \theta (C - H)$, give exactly the same thresholds for the preference parameters of interest as under $\dot{H} = \theta ((1 - \omega) C + \omega C_{\text{others}} - H)$, and therefore lead to precisely the same estimates.

D.4 Proof of Proposition 2

Proof.

$$\frac{u_{CH}}{u_{HH}} \frac{u_H}{u_C} = \frac{-\alpha v''}{\alpha^2 v''} \frac{-\alpha v'}{v'} = 1,$$

and

$$\frac{u_{CH}}{u_{CC}} \frac{u_C}{u_H} = \frac{-\alpha v''}{v''} \frac{v'}{-\alpha v'} = 1.$$

Q.E.D.

D.5 Proof of Proposition 3

Proof.

$$\frac{H u_H u_{CH}}{u_C (u_H + H u_{HH})} = \frac{H \left(-\alpha \frac{C}{H^{\alpha+1}} v' \right) \left(-\alpha \frac{1}{H^{\alpha+1}} v' - \alpha \frac{C}{H^{2\alpha+1}} v'' \right)}{\frac{1}{H^\alpha} v' \left\{ -\alpha \frac{C}{H^{\alpha+1}} v' + H \alpha \frac{C}{H^2} \left[(\alpha + 1) \frac{1}{H^\alpha} v' + \alpha \frac{C}{H^{2\alpha}} v'' \right] \right\}} = 1,$$

and

$$\frac{C u_C u_{CH}}{u_H (u_C + C u_{CC})} = \frac{\frac{C}{H^\alpha} v' \left(-\alpha \frac{1}{H^{\alpha+1}} v' - \alpha \frac{C}{H^{2\alpha+1}} v'' \right)}{\left(-\alpha \frac{C}{H^{\alpha+1}} v' \right) \left(\frac{1}{H^\alpha} v' + \frac{C}{H^{2\alpha}} v'' \right)} = 1.$$

Q.E.D.

D.6 Proof of Proposition 4

Proof. A respondent preferring Universe One for a better future experience (U) in a slope of indifference curve question⁷ implies

$$\begin{aligned}
& U(\text{Universe One}) - U(\text{Universe Two}) \\
&= [U(\text{Universe One}) - U(\text{Baseline})] - [U(\text{Universe Two}) - U(\text{Baseline})] \\
&= \Psi(\Delta C_{\text{past}}, \Delta C_{\text{future}}) - \Psi(-\Delta C_{\text{past}}, -\Delta C_{\text{future}}) \\
&= \frac{1}{\rho} \left\{ u_C \Delta C_{\text{future}} + u_H \frac{\rho \Delta C_{\text{past}} + \theta \Delta C_{\text{future}}}{\rho + \theta} \right. \\
&\quad + \frac{1}{2} \left[u_{CC} (\Delta C_{\text{future}})^2 + 2u_{CH} \frac{\rho \Delta C_{\text{past}} \Delta C_{\text{future}} + \theta (\Delta C_{\text{future}})^2}{\rho + \theta} \right. \\
&\quad \left. \left. + u_{HH} \frac{\rho(\rho + \theta)(\Delta C_{\text{past}})^2 + 2\rho\theta \Delta C_{\text{past}} \Delta C_{\text{future}} + 2\theta^2 (\Delta C_{\text{future}})^2}{(\rho + \theta)(\rho + 2\theta)} \right] \right\} \\
&\quad - \frac{1}{\rho} \left\{ u_C (-\Delta C_{\text{future}}) + u_H \frac{\rho(-\Delta C_{\text{past}}) + \theta(-\Delta C_{\text{future}})}{\rho + \theta} \right. \\
&\quad + \frac{1}{2} \left[u_{CC} (\Delta C_{\text{future}})^2 + 2u_{CH} \frac{\rho \Delta C_{\text{past}} \Delta C_{\text{future}} + \theta (\Delta C_{\text{future}})^2}{\rho + \theta} \right. \\
&\quad \left. \left. + u_{HH} \frac{\rho(\rho + \theta)(\Delta C_{\text{past}})^2 + 2\rho\theta \Delta C_{\text{past}} \Delta C_{\text{future}} + 2\theta^2 (\Delta C_{\text{future}})^2}{(\rho + \theta)(\rho + 2\theta)} \right] \right\} \\
&= \frac{2}{\rho} \left(u_C \Delta C_{\text{future}} + u_H \frac{\rho \Delta C_{\text{past}} + \theta \Delta C_{\text{future}}}{\rho + \theta} \right) \\
&> 0,
\end{aligned}$$

where the third equality holds under the second-order approximation.

The inequality, by $u_C > 0$ and $\rho > 0$,⁸ implies

$$-\frac{u_H}{u_C} < \frac{(\rho + \theta) \Delta C_{\text{future}}}{\rho \Delta C_{\text{past}} + \theta \Delta C_{\text{future}}}.$$

Q.E.D.

⁷See Figure A.11 of the supplemental material for the initial monthly spending graphs and Table A.2 for all the values of ΔC_{past} and ΔC_{future} for this survey module.

⁸The sign of ρ is elicited in the time discount rate question.

D.7 Proof of Proposition 5

Proof. A respondent preferring Universe One for a better future experience (U) in a $\frac{Hu_{HH}}{u_H}$ question⁹ implies

$$\begin{aligned}
& U \text{ (Universe One)} - U \text{ (Universe Two)} \\
&= U \text{ (Universe One)} - U \text{ (Baseline)} \\
&= \frac{1}{2} \Psi(-\Delta C_1, 0) + \frac{1}{2} \Psi(\Delta C_2, 0) \\
&= \frac{1}{2} \left[u_H \frac{-\Delta C_1}{\rho + \theta} + \frac{1}{2} u_{HH} \frac{(-\Delta C_1)^2}{\rho + 2\theta} \right] + \frac{1}{2} \left[u_H \frac{\Delta C_2}{\rho + \theta} + \frac{1}{2} u_{HH} \frac{(\Delta C_2)^2}{\rho + 2\theta} \right] \\
&= \frac{1}{2} \left[u_H \frac{\Delta C_2 - \Delta C_1}{\rho + \theta} + \frac{1}{2} u_{HH} \frac{(\Delta C_1)^2 + (\Delta C_2)^2}{\rho + 2\theta} \right] \\
&> 0,
\end{aligned}$$

where the third equality holds under the second-order approximation.

The inequality, by $u_H < 0$ ¹⁰ and $H > 0$, implies

$$\frac{Hu_{HH}}{u_H} < \frac{2(\rho + 2\theta)}{\rho + \theta} \frac{\Delta C_1 / \Delta C_2 - 1}{(\Delta C_1 / \Delta C_2)^2 + 1} \frac{H}{\Delta C_2}.$$

Q.E.D.

D.8 Proof of Proposition 6

Proof. A respondent preferring Universe One for a better future experience (U) in a $\frac{u_{CH}}{u_{HH}}$ question¹¹ implies

$$\begin{aligned}
& U \text{ (Universe One)} - U \text{ (Universe Two)} \\
&= [U \text{ (Universe One)} - U \text{ (Baseline)}] - [U \text{ (Universe Two)} - U \text{ (Baseline)}] \\
&= \frac{1}{2} \Psi(\Delta C_{\text{past}}, \Delta C_{\text{future}}) + \frac{1}{2} \Psi(-\Delta C_{\text{past}}, -\Delta C_{\text{future}}) - \frac{1}{2} \Psi(0, \Delta C_{\text{future}}) - \frac{1}{2} \Psi(0, -\Delta C_{\text{future}})
\end{aligned}$$

⁹See Figure A.12 of the supplemental material for the initial monthly spending graphs and Table A.2 for all the values of ΔC_1 and ΔC_2 for this survey module.

¹⁰This sign is elicited in the existence of (internal) habit formation question.

¹¹See Figure A.13 of the supplemental material for the initial monthly spending graphs and Table A.2 for all the values of ΔC_{past} and ΔC_{future} for this survey module.

$$\begin{aligned}
&= \frac{1}{2} \frac{1}{\rho} \left\{ \left[u_C \Delta C_{\text{future}} + u_H \frac{\rho \Delta C_{\text{past}} + \theta \Delta C_{\text{future}}}{\rho + \theta} + \frac{1}{2} \left(u_{CC} (\Delta C_{\text{future}})^2 \right. \right. \right. \\
&\quad \left. \left. + 2u_{CH} \frac{\rho \Delta C_{\text{past}} \Delta C_{\text{future}} + \theta (\Delta C_{\text{future}})^2}{\rho + \theta} \right. \right. \\
&\quad \left. \left. + u_{HH} \frac{\rho (\rho + \theta) (\Delta C_{\text{past}})^2 + 2\rho\theta \Delta C_{\text{past}} \Delta C_{\text{future}} + 2\theta^2 (\Delta C_{\text{future}})^2}{(\rho + \theta) (\rho + 2\theta)} \right) \right] \\
&\quad + \left[-u_C \Delta C_{\text{future}} - u_H \frac{\rho \Delta C_{\text{past}} + \theta \Delta C_{\text{future}}}{\rho + \theta} + \frac{1}{2} \left(u_{CC} (\Delta C_{\text{future}})^2 \right. \right. \\
&\quad \left. \left. + 2u_{CH} \frac{\rho \Delta C_{\text{past}} \Delta C_{\text{future}} + \theta (\Delta C_{\text{future}})^2}{\rho + \theta} \right. \right. \\
&\quad \left. \left. + u_{HH} \frac{\rho (\rho + \theta) (\Delta C_{\text{past}})^2 + 2\rho\theta \Delta C_{\text{past}} \Delta C_{\text{future}} + 2\theta^2 (\Delta C_{\text{future}})^2}{(\rho + \theta) (\rho + 2\theta)} \right) \right] \\
&\quad - \left[\left(u_C + \frac{u_H \theta}{\rho + \theta} \right) \Delta C_{\text{future}} + \frac{1}{2} \left(u_{CC} + \frac{2u_{CH}\theta}{\rho + \theta} + \frac{2u_{HH}\theta^2}{(\rho + \theta) (\rho + 2\theta)} \right) (\Delta C_{\text{future}})^2 \right] \\
&\quad - \left[- \left(u_C + \frac{u_H \theta}{\rho + \theta} \right) \Delta C_{\text{future}} + \frac{1}{2} \left(u_{CC} + \frac{2u_{CH}\theta}{\rho + \theta} + \frac{2u_{HH}\theta^2}{(\rho + \theta) (\rho + 2\theta)} \right) (\Delta C_{\text{future}})^2 \right] \Big\} \\
&= \frac{1}{2} \frac{1}{\rho} \left[\left(u_{CC} (\Delta C_{\text{future}})^2 + 2u_{CH} \frac{\rho \Delta C_{\text{past}} \Delta C_{\text{future}} + \theta (\Delta C_{\text{future}})^2}{\rho + \theta} \right. \right. \\
&\quad \left. \left. + u_{HH} \frac{\rho (\rho + \theta) (\Delta C_{\text{past}})^2 + 2\rho\theta \Delta C_{\text{past}} \Delta C_{\text{future}} + 2\theta^2 (\Delta C_{\text{future}})^2}{(\rho + \theta) (\rho + 2\theta)} \right) \right. \\
&\quad \left. - \left(u_{CC} + \frac{2u_{CH}\theta}{\rho + \theta} + \frac{2u_{HH}\theta^2}{(\rho + \theta) (\rho + 2\theta)} \right) (\Delta C_{\text{future}})^2 \right] \\
&= u_{CH} \frac{\Delta C_{\text{past}} \Delta C_{\text{future}}}{\rho + \theta} + u_{HH} \frac{(\rho + \theta) (\Delta C_{\text{past}})^2 + 2\theta \Delta C_{\text{past}} \Delta C_{\text{future}}}{2(\rho + \theta) (\rho + 2\theta)} \\
&> 0,
\end{aligned}$$

where the third equality holds under the third-order approximation.¹²

The inequality, by $u_{HH} < 0$ and $\rho > 0$,¹³ implies

$$\frac{u_{CH}}{u_{HH}} < - \frac{(\rho + \theta) \Delta C_{\text{past}} + 2\theta \Delta C_{\text{future}}}{2(\rho + 2\theta) \Delta C_{\text{future}}}.$$

¹²The third-order terms cancel each other and are omitted for space consideration.

¹³The signs are elicited in the Hu_{HH}/u_H and time discount rate questions.

Q.E.D.

D.9 Proof of Proposition 7

Proof. A respondent preferring Universe One for a better future experience (U) in a $\frac{u_{CC}}{u_{HH}}$ question¹⁴ implies

$$\begin{aligned}
& U(\text{Universe One}) - U(\text{Universe Two}) \\
&= [U(\text{Universe One}) - U(\text{Baseline})] - [U(\text{Universe Two}) - U(\text{Baseline})] \\
&= \frac{1}{2} \Psi(0, \Delta C_{\text{future}}) + \frac{1}{2} \Psi(0, -\Delta C_{\text{future}}) - \frac{1}{2} \Psi(\Delta C_{\text{past}}, 0) - \frac{1}{2} \Psi(-\Delta C_{\text{past}}, 0) \\
&= \frac{1}{2} \frac{1}{\rho} \left\{ \left[\left(u_C + \frac{u_H \theta}{\rho + \theta} \right) \Delta C_{\text{future}} + \frac{1}{2} \left(u_{CC} + \frac{2u_{CH}\theta}{\rho + \theta} + \frac{2u_{HH}\theta^2}{(\rho + \theta)(\rho + 2\theta)} \right) (\Delta C_{\text{future}})^2 \right] \right. \\
&\quad + \left[- \left(u_C + \frac{u_H \theta}{\rho + \theta} \right) \Delta C_{\text{future}} + \frac{1}{2} \left(u_{CC} + \frac{2u_{CH}\theta}{\rho + \theta} + \frac{2u_{HH}\theta^2}{(\rho + \theta)(\rho + 2\theta)} \right) (\Delta C_{\text{future}})^2 \right] \\
&\quad \left. - \left[u_H \frac{\rho \Delta C_{\text{past}}}{\rho + \theta} + \frac{1}{2} u_{HH} \frac{\rho (\Delta C_{\text{past}})^2}{\rho + 2\theta} \right] - \left[-u_H \frac{\rho \Delta C_{\text{past}}}{\rho + \theta} + \frac{1}{2} u_{HH} \frac{\rho (\Delta C_{\text{past}})^2}{\rho + 2\theta} \right] \right\} \\
&= \frac{1}{2} \frac{1}{\rho} \left(u_{CC} + 2u_{CH} \frac{\theta}{\rho + \theta} + u_{HH} \frac{2\theta^2}{(\rho + \theta)(\rho + 2\theta)} \right) (\Delta C_{\text{future}})^2 - \frac{1}{2} \frac{1}{\rho} u_{HH} \frac{\rho (\Delta C_{\text{past}})^2}{\rho + 2\theta} \\
&> 0,
\end{aligned}$$

where the second equality holds under the third-order approximation.¹⁵

The inequality, by $u_{HH} < 0$ and $\rho > 0$,¹⁶ implies

$$\frac{u_{CC}}{u_{HH}} < \frac{\rho}{\rho + 2\theta} \left(\frac{\Delta C_{\text{past}}}{\Delta C_{\text{future}}} \right)^2 - \frac{2\theta}{\rho + \theta} \frac{u_{CH}}{u_{HH}} - \frac{2\theta^2}{(\rho + \theta)(\rho + 2\theta)}.$$

Q.E.D.

¹⁴See Figure A.14 of the supplemental material for the initial monthly spending graphs and Table A.2 for all the values of ΔC_{past} and ΔC_{future} for this survey module.

¹⁵The third-order terms cancel each other and are omitted for space consideration.

¹⁶The signs are elicited in the Hu_{HH}/u_H and time discount rate questions.

D.10 Proof of Proposition 8

Proof. Taking $M = 5000$ gives $M - (1 - \omega) \Delta C - \omega \Delta C_{\text{others}} > 0$ in all the questions for external habit formation.¹⁷

A respondent preferring Universe One for a better future experience (U) in an external habit formation question implies

$$\begin{aligned}
& U(\text{Universe One}) - U(\text{Universe Two}) \\
&= [U(\text{Universe One}) - U(\text{Baseline})] - [U(\text{Universe Two}) - U(\text{Baseline})] \\
&= \Phi(\Delta C, 0, 0, 0) - \Phi(0, 0, \Delta C_{\text{others}}, 0) \\
&= \int_0^\infty e^{-\rho t} \left[u_H e^{-\theta t} (1 - \omega) \Delta C + \frac{1}{2} u_{HH} \left(e^{-\theta t} (1 - \omega) \Delta C \right)^2 + \dots \right] dt \\
&\quad - \int_0^\infty e^{-\rho t} \left[u_H e^{-\theta t} \omega \Delta C_{\text{others}} + \frac{1}{2} u_{HH} \left(e^{-\theta t} \omega \Delta C_{\text{others}} \right)^2 + \dots \right] dt \\
&= [(1 - \omega) \Delta C - \omega \Delta C_{\text{others}}] \int_0^\infty e^{-\rho t} \left[u_H e^{-\theta t} \right. \\
&\quad \left. + \frac{1}{2} u_{HH} \left(e^{-\theta t} \right)^2 [(1 - \omega) \Delta C + \omega \Delta C_{\text{others}}] + \dots \right] dt \\
&= [(1 - \omega) \Delta C - \omega \Delta C_{\text{others}}] \\
&\quad \cdot \int_0^\infty e^{-\rho t} \sum_{n=1}^\infty e^{-n\theta t} \frac{1}{n!} \frac{\partial^n u}{\partial H^n} \left(\sum_{s=0}^{n-1} ((1 - \omega) \Delta C)^s (\omega \Delta C_{\text{others}})^{n-1-s} \right) dt \\
&> 0.
\end{aligned}$$

The inequality, by Lemma 3, implies $(1 - \omega) \Delta C - \omega \Delta C_{\text{others}} < 0$ or equivalently

$$\omega > \frac{\Delta C}{\Delta C + \Delta C_{\text{others}}}.$$

Q.E.D.

¹⁷See Figure A.15 of the supplemental material for the initial monthly spending graphs and Table A.2 for all the values of ΔC and ΔC_{others} for this survey module.

D.11 Proof of Proposition 9

Proof. A respondent preferring Universe One for a better future experience (U) in a $\frac{u_{C_{\text{others}}}}{u_H}$ question¹⁸ implies

$$\begin{aligned}
& U (\text{Universe One}) - U (\text{Universe Two}) \\
&= [U (\text{Universe One}) - U (\text{Baseline})] - [U (\text{Universe Two}) - U (\text{Baseline})] \\
&= \Phi (0, 0, 0, \Delta C_{\text{others}}^{U1}) - \Phi (0, 0, \Delta C_{\text{others}}^{U2}, 0) \\
&= \int_0^\infty e^{-\rho t} \left[u_{C_{\text{others}}} \Delta C_{\text{others}}^{U1} + u_H (1 - e^{-\theta t}) \omega \Delta C_{\text{others}}^{U1} \right] dt - \int_0^\infty e^{-\rho t} u_H e^{-\theta t} \omega \Delta C_{\text{others}}^{U2} dt \\
&= \frac{1}{\rho} u_{C_{\text{others}}} \Delta C_{\text{others}}^{U1} + u_H \frac{\omega}{\rho (\rho + \theta)} (\theta \Delta C_{\text{others}}^{U1} - \rho \Delta C_{\text{others}}^{U2}) \\
&> 0,
\end{aligned}$$

where the third equality holds under the first-order approximation.

The last inequality, by $u_H < 0$ and $\rho > 0$,¹⁹ implies

$$\frac{u_{C_{\text{others}}}}{u_H} < \frac{\omega}{\rho + \theta} \left(\rho \frac{\Delta C_{\text{others}}^{U2}}{\Delta C_{\text{others}}^{U1}} - \theta \right).$$

Q.E.D.

E Elicitation and Estimate of Time Discount Rate

To elicit the time discount rate, I increase spending in the next year and the year after next year. The resulting survey question has monthly spending graphs as in Figure A.1.

Proposition 11. $\rho > -\ln \frac{\Delta C_{U1}}{\Delta C_{U2}}$ if the respondent chooses Universe One over Universe Two for a better future experience in a time discount rate question.

Proof. A respondent preferring Universe One over Universe Two for a better future experience (U) in the time discount rate question implies

$$U (\text{Universe One}) - U (\text{Universe Two})$$

¹⁸See Figure A.16 of the supplemental material for the initial monthly spending graphs and Table A.2 for all the values of $\Delta C_{\text{others}}^{U1}$ and $\Delta C_{\text{others}}^{U2}$ for this survey module.

¹⁹The signs are elicited in the existence of (internal) habit formation and time discount rate questions.

$$\begin{aligned}
&= [U(\text{Universe One}) - U(\text{Baseline})] - [U(\text{Universe Two}) - U(\text{Baseline})] \\
&= \gamma(0, \Delta C_{U1}) - e^{-\rho} \gamma(0, \Delta C_{U2}) \\
&= \Delta C_{U1} \frac{\gamma(0, \Delta C_{U1})}{\Delta C_{U1}} - e^{-\rho} \Delta C_{U2} \frac{\gamma(0, \Delta C_{U2})}{\Delta C_{U2}} \\
&> (\Delta C_{U1} - e^{-\rho} \Delta C_{U2}) \frac{\gamma(0, \Delta C_{U1})}{\Delta C_{U1}} \\
&> 0,
\end{aligned}$$

where the first inequality follows from diminishing marginal utility: $\frac{\gamma(0, \Delta C_{U1})}{\Delta C_{U1}} > \frac{\gamma(0, \Delta C_{U2})}{\Delta C_{U2}} > 0$ for $\Delta C_{U2} > \Delta C_{U1} > 0$.

The last inequality implies $\Delta C_{U1} - e^{-\rho} \Delta C_{U2} > 0$ or equivalently

$$\rho > -\ln \frac{\Delta C_{U1}}{\Delta C_{U2}}.$$

Q.E.D.

Estimation pins down a value of 0.13 for the time discount rate with the 95% HPDI of [0.03, 0.23]. See Table A.2 of the online appendix for all the values of ΔC_{U1} and ΔC_{U2} for this survey module.

F Additional Survey Details

The survey starts with a consent form detailing the purpose, procedure, potential benefits, payment for participation, confidentiality, withdrawal procedure, and investigator contact information. Consent was obtained from all the respondents.

F.1 Instructions

After consent is obtained, the survey continues with instructions on the basic hypothetical situation discussed in Section 2.2 of the paper. Two practice questions follow to test the respondents' understanding of the hypothetical situation:

With no inflation and prices of everything staying the same, if you can buy 3 bananas with one dollar in the last year, how many bananas can you buy with one dollar in the next year?

- 5
- 3
- 1
- No idea

If you rent the durable goods you consume, select any of the following that you own (that is, not rent):

- Residence
- Car
- Furniture
- I do not own any of the above
- No idea

If a respondent makes a mistake in the practice questions, they need to go over the instructions again and redo the practice questions. A maximum of three attempts of the practice questions is allowed.

After the practice questions, the survey continues with instructions on reading the monthly spending graphs:

In this survey, you'll compare your experience in several universes that are identical except that your monthly spending differs.

- Monthly spending refers to the total amount of money you spend, rather than earn, in each month.
- You will be asked to find out in which universe you will have (had) a better experience given how much you spend (spent).
- 'Better' means more satisfying.
- You can afford the monthly spending specified in the questions.

The difference of your monthly spending between the universes is detailed in monthly spending graphs, like the one below.

[Figure A.2a]

Now let's learn to read a monthly spending graph.

The first element of a monthly spending graph is the timeline, with past on the left, now in the middle, future on the right. A thick vertical line representing now separates the past from the future.

[Figure A.2b]

To fix the idea, the 'Past' means as far back in the past as you can remember and the 'Future' as far in the future as you can imagine. If easier, think of the 'Past' as the past 30 years and the 'Future' as the next 30 years.

The second element of a monthly spending graph is the bars above the timeline.

- The height of the bars represents the level of monthly spending (again, not income) in time frames covered by the bars.
- The exact level of monthly spending is labeled on top of the corresponding bar. The words 'per month' are saved for space consideration from now on, but you should always remember that the numbers are per month spending.
- The bars are colored differently to help you distinguish different time frames.

For example, if the following monthly spending graph describes your monthly spending,

[Figure A.2c]

you spent/spend

- \$5,500 per month in the 'past' until '1 year ago';
- \$7,500 per month from '1 year ago' until 'now' (or in the 'past year');
- \$6,500 per month from 'now' to '1 year from now' (or in the 'next year');
- \$5,000 per month from '1 year from now' onward.

To highlight the difference of monthly spending, the time frames as in the above example are sometimes collapsed into two or three time frames. For instance, if in Universe One your monthly spending graph is

[Figure A.3a]

while in Universe Two your monthly spending is

[Figure A.3b]

then the difference and the similarity of your monthly spending in the two universes are that

- in Universe Two you spent \$1,000 more per month in the ‘past’ than you did in Universe One where you spent \$5,000 per month in the ‘past’; and that
- in both universes, you will spend \$5,000 per month from ‘now’ onward.

Then, the respondents received a set of practice questions testing their understanding of the instructions.

Imagine that your monthly spending is detailed in the following monthly spending graph

[Figure A.3c]

How much will you spend per month in the next year?

- \$5,000
- \$6,000
- \$6,500
- \$8,000

How much did you spend per month from ‘as far back as you can remember in the past’ until ‘1 year ago’?

- \$5,000
- \$6,000
- \$6,500
- \$8,000

Imagine that your monthly spending in Universe One and Universe Two are detailed in the following monthly spending graphs.

[The graphs are in the choices below. You can directly click on the graph to give your answer.]

In which universe did you spend more per month in the ‘past year’?

- Universe One

[Figure A.2c]

- Universe Two

[Figure A.3c]

In which universe will you spend more per month from ‘1 year from now’ onward?

- Universe One

[Figure A.2c]

- Universe Two

[Figure A.3c]

In the graphs of last question, how much more did you spend in Universe One than in Universe Two from ‘as far back in the past as you can remember’ until ‘1 year ago’?

- \$0
- \$500
- \$1,000
- \$5,500

A final set of instructions inoculates the respondents against the seeming repetitiveness of follow-up questions and encourages effort with attention checks and a lottery reward. An opportunity to review previous instructions was also presented.

We designed the survey to learn as much as possible from your answers. To increase the power of the study result, each question is normally followed by additional questions that vary slightly from previous questions. Although the survey may look repetitive, please pay careful attention and answer each question the best you can.

Implicit and explicit attention checks are integrated into this survey. Responses show signs of inattentiveness will be rejected. A small lottery (\$1) will be randomly paid as a bonus to workers who show excellence in the responses.

The chance to get this lottery reward is 1 in 100. Your normal HIT payment won't be affected by this lottery.

If you would like to view these instructions again before beginning the survey, please check the following box.

☐ View Instructions Again

F.2 Core Modules

The survey has nine core modules, each corresponding to one or two preference parameters of interest.

The flow of each core module is

1. Ask the respondent to choose the universe that brings them a better past experience (e.g., Figure A.4);
2. Ask the respondent to choose the universe that brings them a better future experience (e.g., Figure 1b in the paper);
3. Present the respondent with one to two follow-up questions of monthly spending graphs that vary slightly from the initial question (cf. Figure 2b in the paper).

The main elements of the survey questions in each core module are the monthly spending graphs. To save space, I present below only the initial monthly spending graphs from each module. Table A.2 of the online Appendix summarizes the changes between follow-up questions and initial questions in terms of the spending amounts necessary to elicit the preference parameters in Propositions 1, 4 to 9, and 11.

- Existence of internal habit formation: [Figure 3 in the paper]
- Habit depreciation speed: [Figure 1b in the paper]
- Time discount rate: [Figure A.1]
- Slope of indifference curve: [Figure A.5]
- $\frac{Hu_{HH}}{u_H}$: [Figure A.6]
- $\frac{u_{CH}}{u_{HH}}$: [Figure A.7]

- $\frac{u_{CC}}{u_{HH}}$: [Figure A.8]
- External habit formation and composition of habit: [Figure A.9]
- Relative strength of habit formation and peer effect: [Figure A.10]

F.3 End-of-Survey Quiz on the Basic Hypothetical Situation

At the end of the survey, I check the respondents' understanding of the basic hypothetical situation again using the following questions, which serve as an implicit attention check.

Under the hypothetical situation of this survey, if you can buy 3 bananas with one dollar in the last year, how many bananas can you buy with one dollar in the next year?

- 5
- 3
- 1
- No idea

Under the hypothetical situation of this survey, select any of the following that you own (that is, not rent):

- Residence
- Car
- Furniture
- I do not own any of the above
- No idea

Under the hypothetical situation of this survey, do things you want change over time?

- Yes
- Maybe
- No

Under the hypothetical situation of this survey, do things not mentioned in the questions change?

- Yes
- Maybe
- No

Under the hypothetical situation of this survey, how much do people not mentioned in questions always spend per month?

- \$4,000
- \$5,000
- \$6,500
- \$8,000
- No idea

TABLE A.1: RESPONSE DISTRIBUTIONS (PERCENTAGE)

Question	Wave	Response					
<i>Panel A. Parameters Identifiable to Scale</i>		U1U1U1	U1U1U2	U1U2	U2U1	U2U2U1	U2U2U2
Habit depreciation rate	1	28	9	17	11	6	28
	2	29	10	14	11	6	30
External habit mixture coefficient	1	17	5	9	14	9	46
	2	24	5	16	6	4	46
$-u_H/u_C$	1	33	4	7	7	12	38
	2	32	4	6	2	20	36
Hu_{HH}/u_H	1	14	5	8	11	3	59
	2	24	2	6	5	1	61
u_{CH}/u_{HH}	1	7	4	12	9	28	40
	2	9	3	10	9	34	35
u_{CC}/u_{HH}	1	23	30	11	10	5	21
	2	24	19	8	10	9	30
$u_{C_{\text{others}}}/u_H$	1	26	18	10	8	3	36
	2	24	19	8	6	3	40
<i>Panel B. Parameter Identifiable to Sign</i>		U1	U2	U3	U4	U5	
Existence of internal habit formation	1	56	4	10	2	29	
	2	60	1	6	1	30	

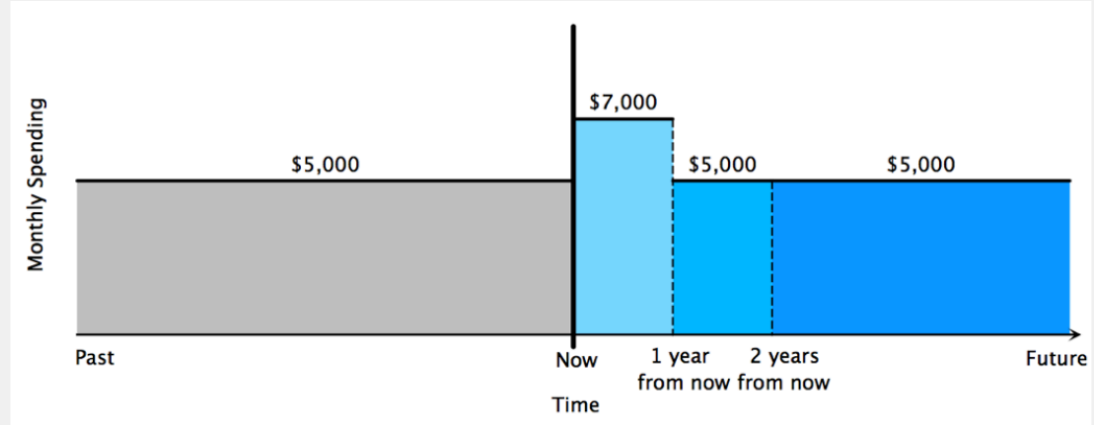
Notes: UX stands for Universe X. U1U1U2 denotes the response sequence of first choosing U1, then U1 again in the first follow-up question, and finally U2 in the second follow-up question. Similar notations are used to denote other response sequences.

TABLE A.2: QUANTITIES IN MONTHLY SPENDING GRAPHS

		If choosing U2 in initial and 1st follow-up questions (2nd follow-up question)	If choosing U2 in initial question (1st follow-up question)	Initial question	If choosing U1 in initial question (1st follow-up question)	If choosing U1 in initial and 1st follow-up questions (2nd follow-up question)
Habit depreciation	ΔC_{U1}	400	1200	2000	2000	2000
speed	ΔC_{U2}	4000	4000	4000	2800	2200
External habit	ΔC	4500	1200	500	500	500
mixture coefficient	ΔC_{others}	500	500	500	1200	4500
$\frac{u_H}{u_C}$	ΔC_{past}	2000	2000	2000	2000	2000
	ΔC_{future}	20	80	200	400	1000
$\frac{Hu_{HH}}{u_H}$	ΔC_1	540	600	650	700	800
	ΔC_2	500	500	500	500	500
$\frac{u_{CH}}{u_{HH}}$	ΔC_{past}	2000	1600	1000	600	100
	ΔC_{future}	200	200	200	200	200
$\frac{u_{CC}}{u_{HH}}$	ΔC_{past}	500	1500	2200	3000	3500
	ΔC_{future}	500	500	500	500	500
$\frac{u_{C_{\text{others}}}}{u_H}$	$\Delta C_{\text{others}}^{U1}$	3000	600	300	150	100
	$\Delta C_{\text{others}}^{U2}$	3000	3000	3000	3000	3000

Notes: U1 and U2 denote Universe One and Universe Two of the monthly spending graphs, respectively. Choosing U1 in the initial question and then U2 in the 1st follow-up question, or choosing U2 in the initial question and then U1 in the 1st follow-up question, ends a module at the end of the 1st follow-up question (cf. Figure 2b in the paper). All amounts are in U.S. dollars.

Universe One



Universe Two

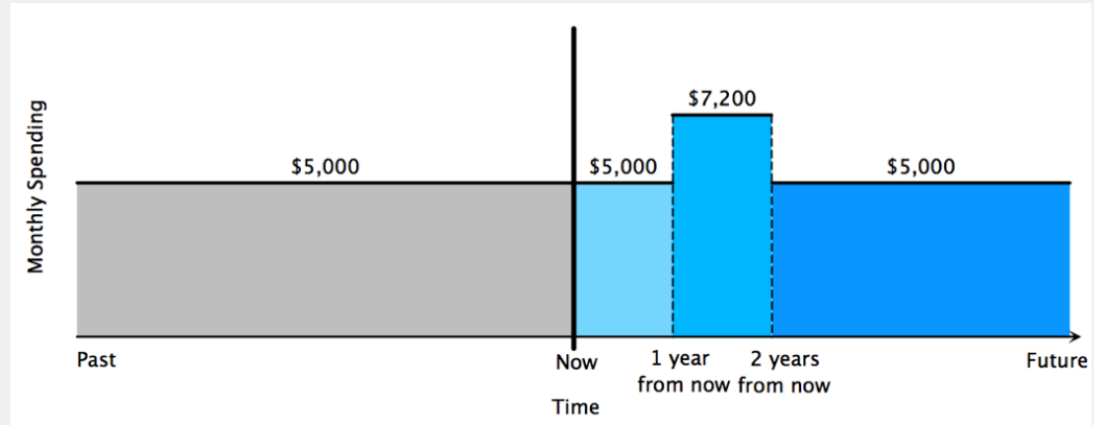
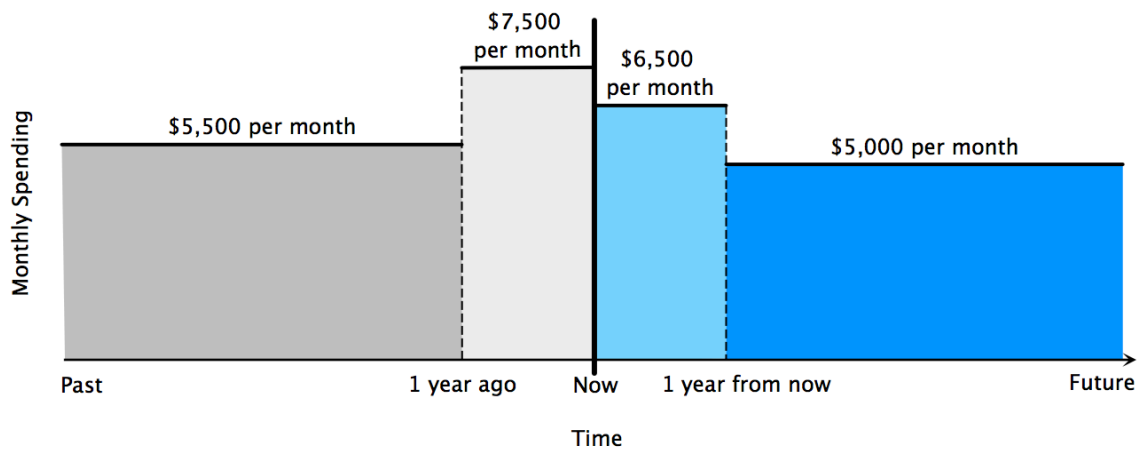
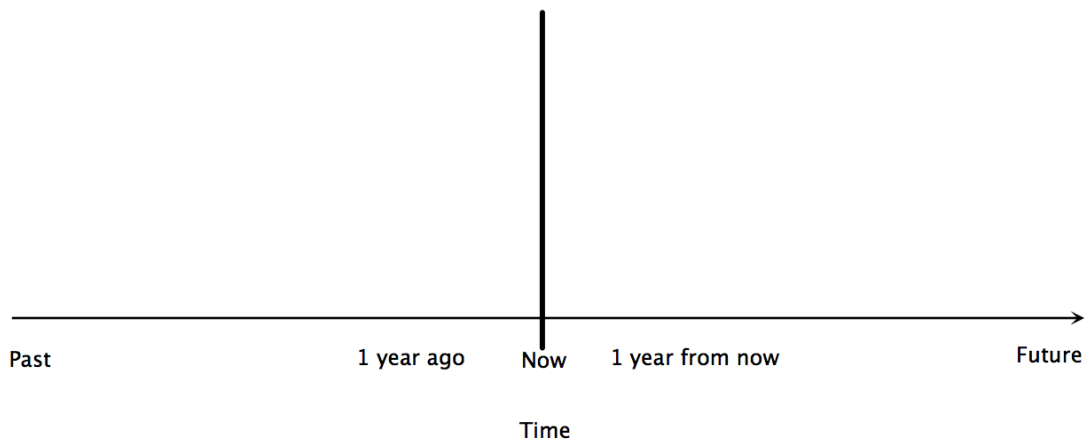


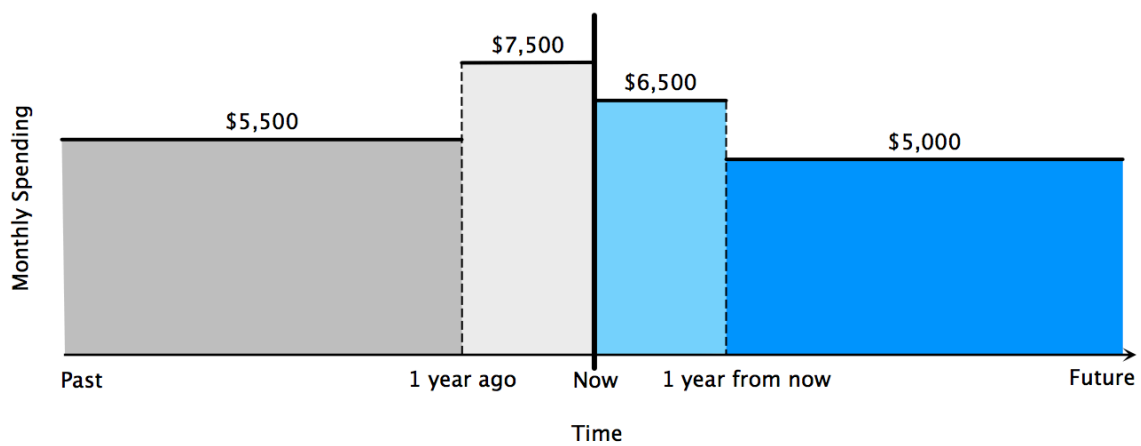
Figure A.1: Monthly spending graphs of a survey question for time discount rate.



(a) More detailed monthly spending graph.



(b) Timeline.



(c)

Figure A.2: Instruction—monthly spending graphs and timeline.

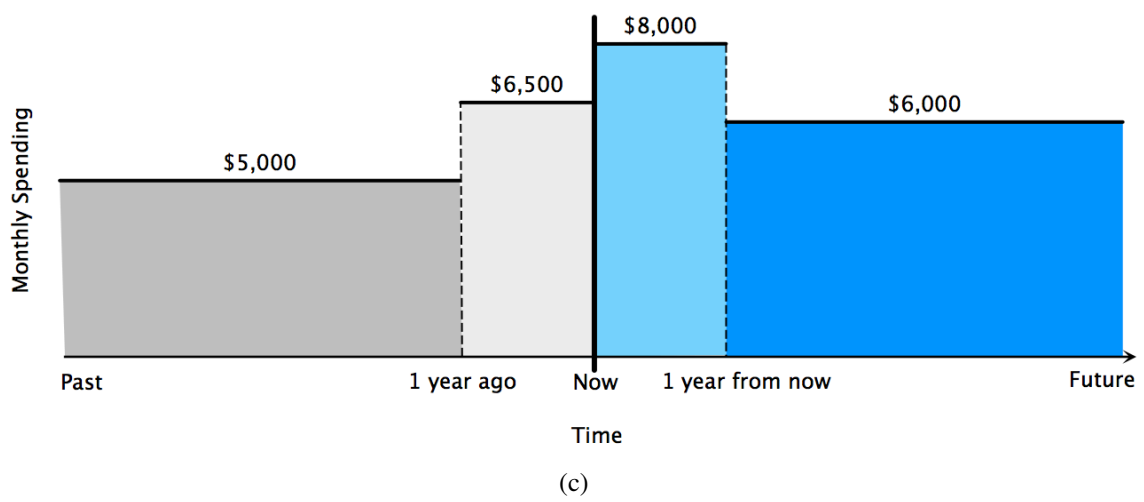
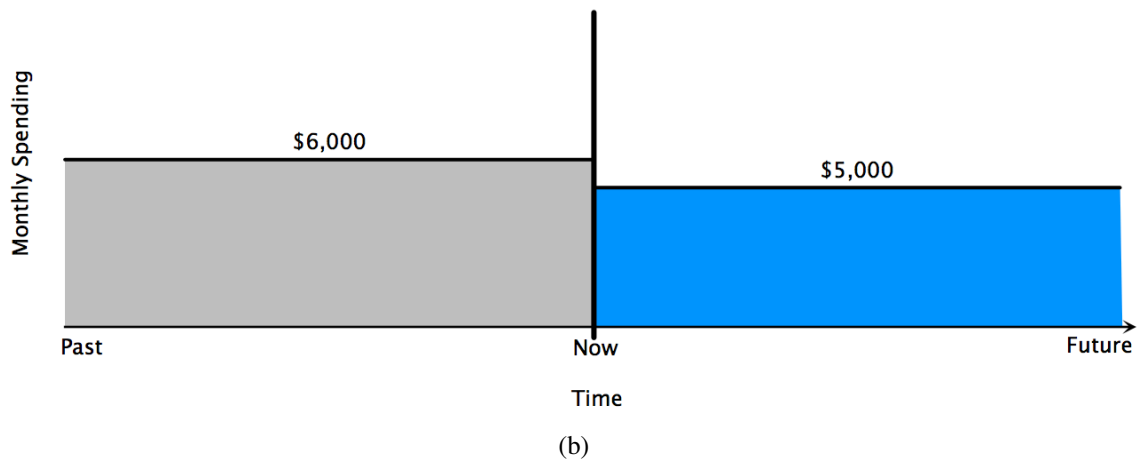
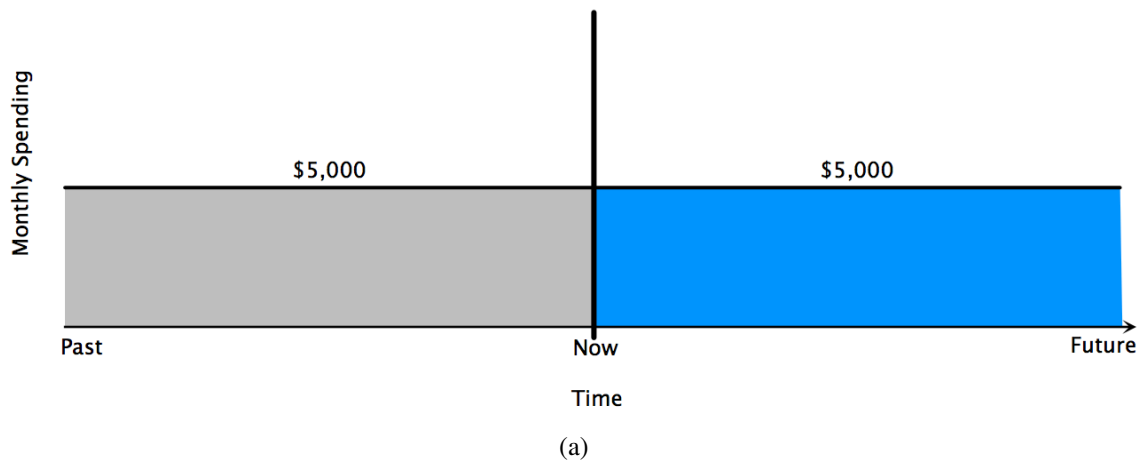


Figure A.3: Instruction—monthly spending graphs.

- Imagine that Universes One and Two are identical except your monthly spending in the 'past'.
- Remember that past experience is how you felt about the 'past' until 'now'.

Which universe gave you a better **past** experience?

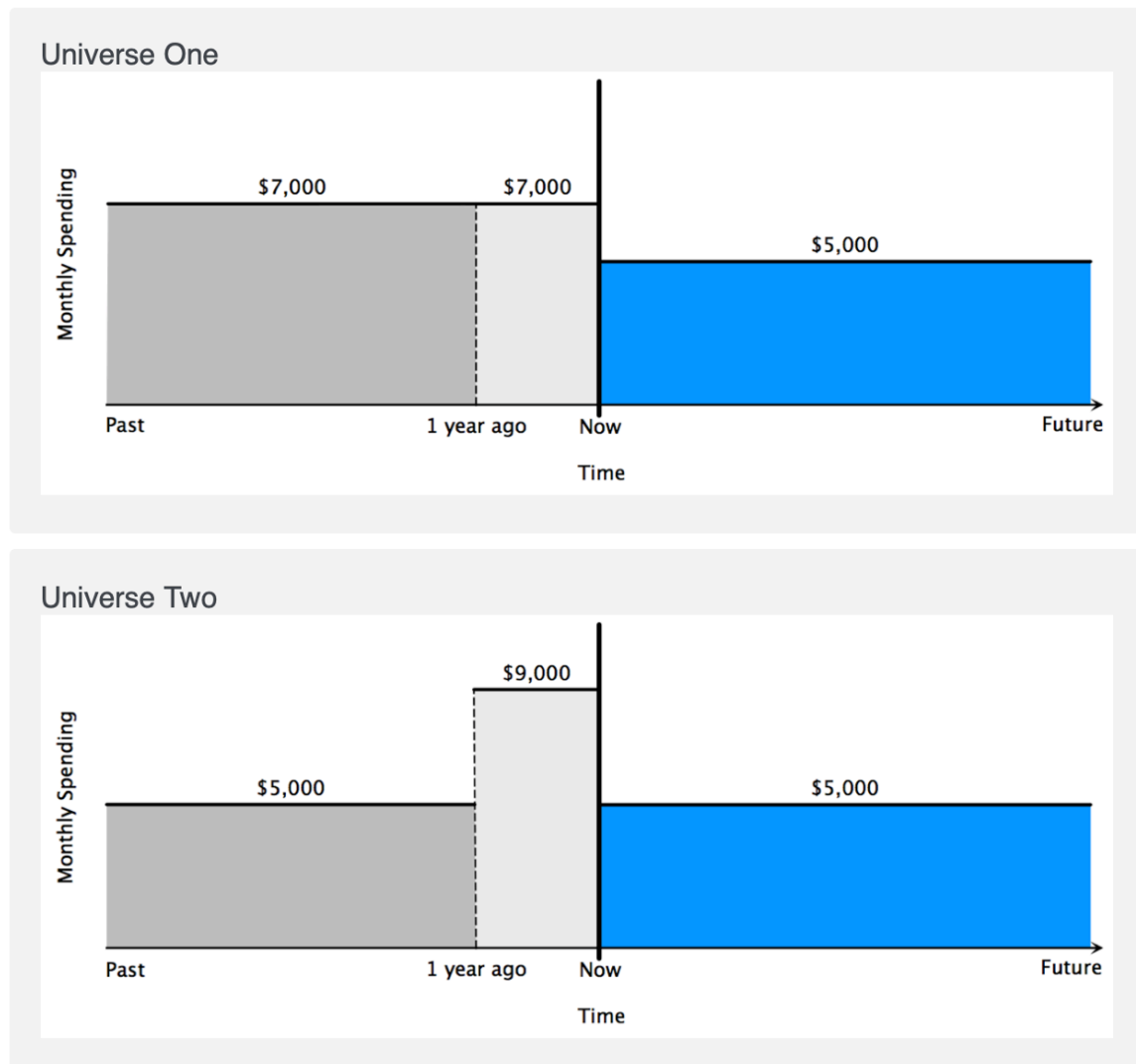


Figure A.4: A typical survey question asking about past experience.

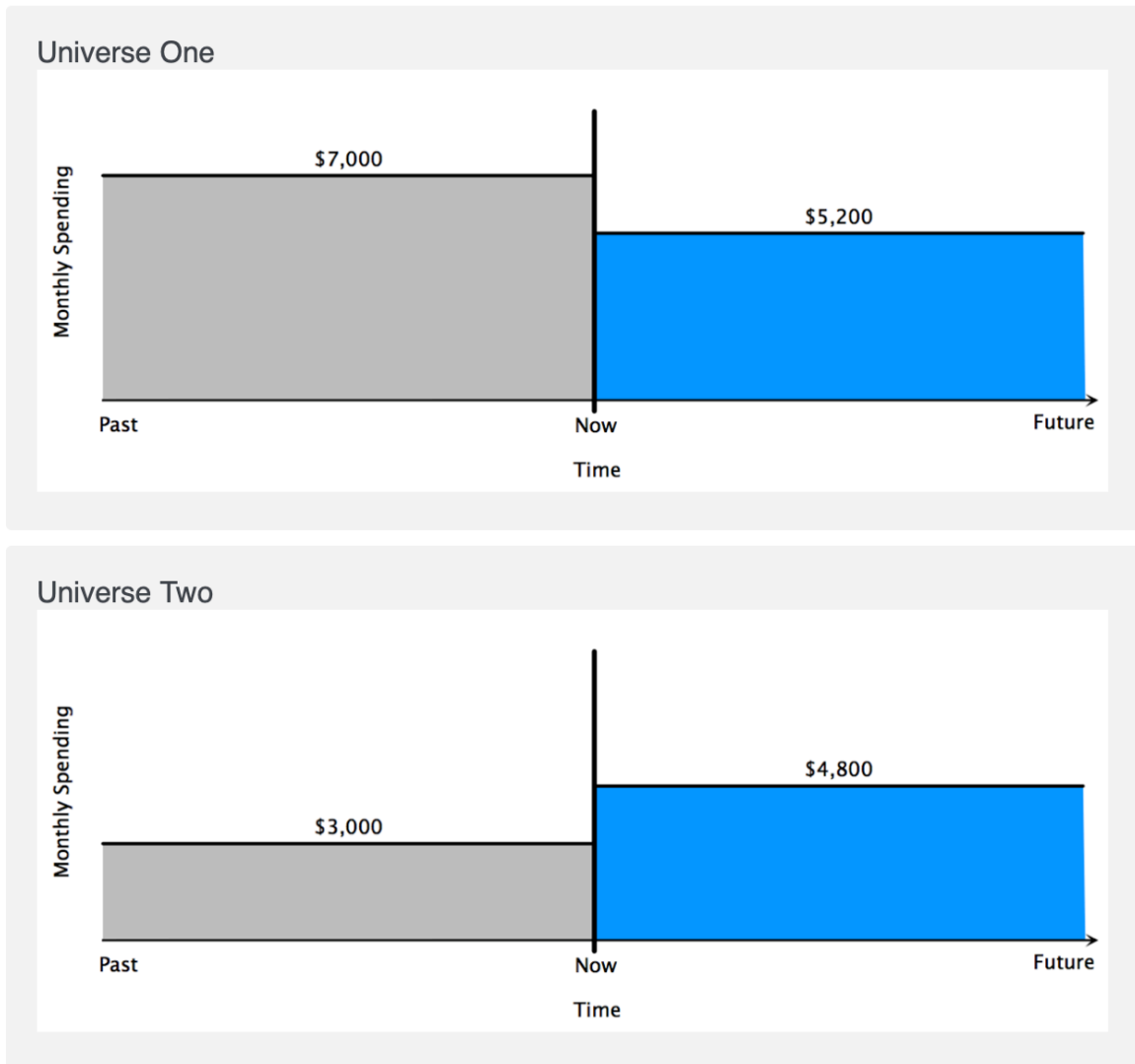


Figure A.5: Monthly spending graphs of a survey question for slope of indifference curve.

Universe One: 50-50 chance that your monthly spending is either Monthly Spending 1 or Monthly Spending 2 below



Universe Two: for sure that your monthly spending is

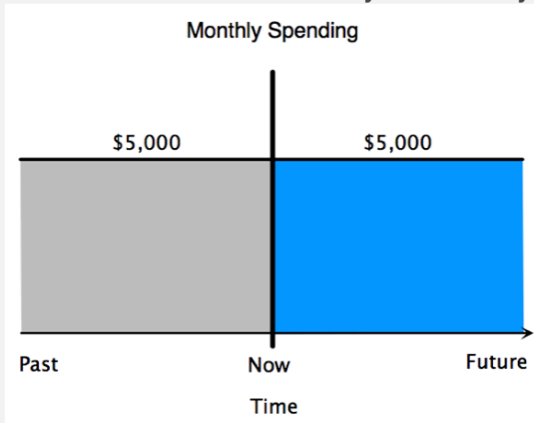
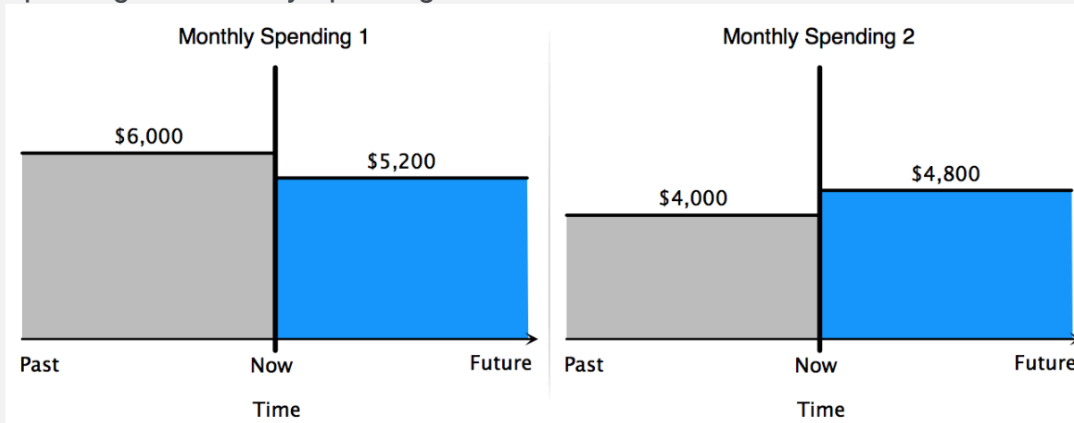


Figure A.6: Monthly spending graphs of a survey question for $\frac{Hu_{HH}}{u_H}$.

Universe One: 50-50 chance that your monthly spending is either Monthly Spending 1 or Monthly Spending 2 below



Universe Two: 50-50 chance that your monthly spending is either Monthly Spending 1 or Monthly Spending 2 below

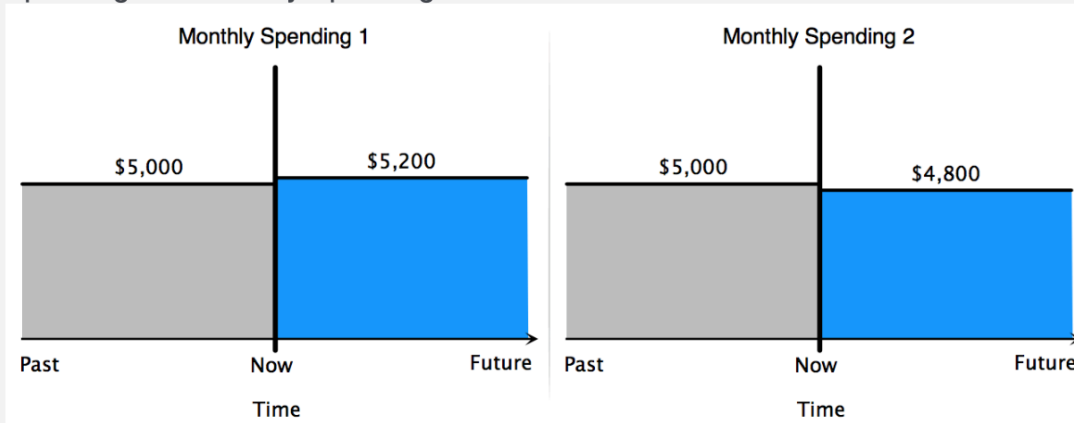
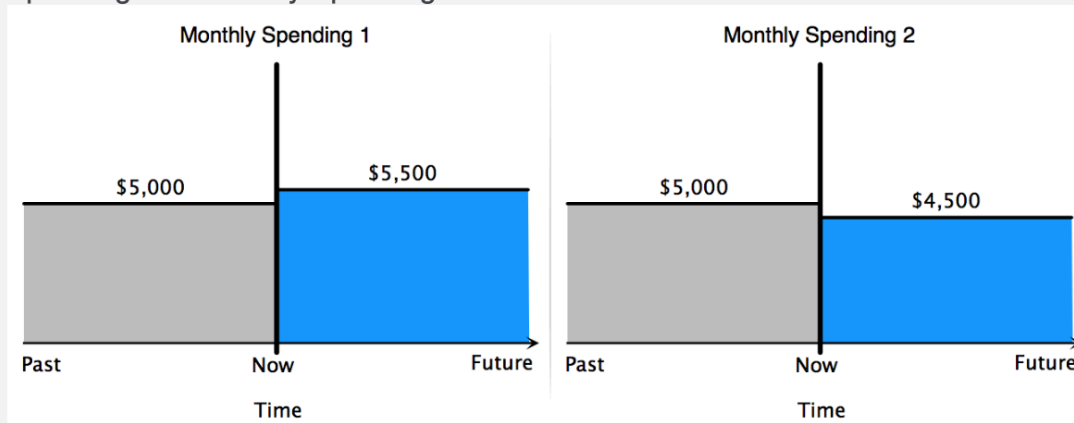


Figure A.7: Monthly spending graphs of a survey question for $\frac{u_{CH}}{u_{HH}}$.

Universe One: 50-50 chance that your monthly spending is either Monthly Spending 1 or Monthly Spending 2 below



Universe Two: 50-50 chance that your monthly spending is either Monthly Spending 1 or Monthly Spending 2 below

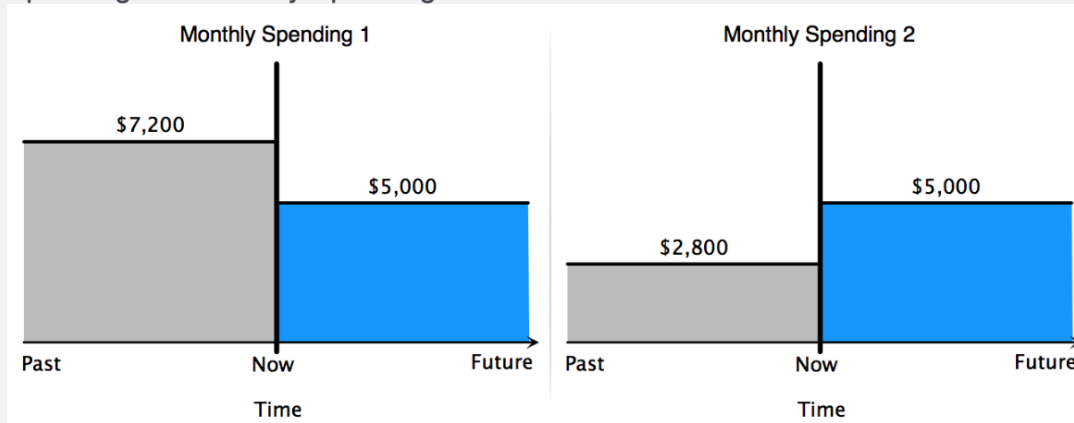
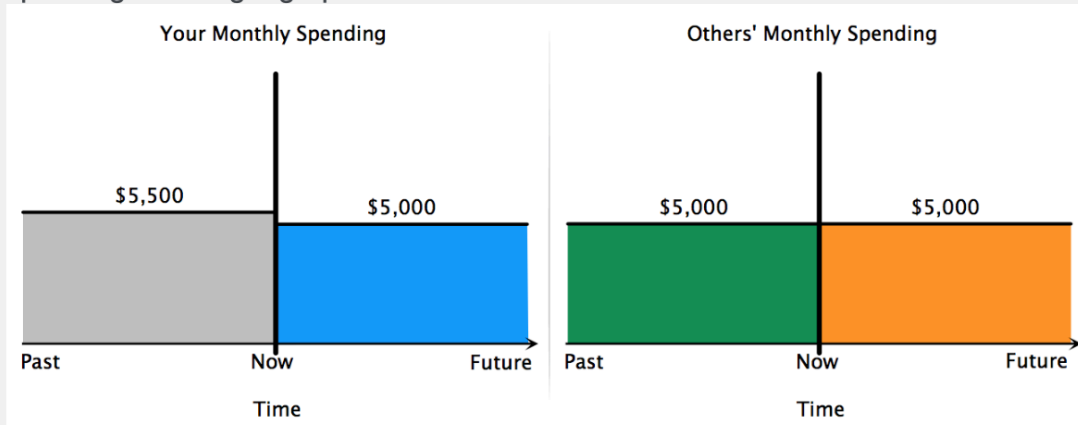


Figure A.8: Monthly spending graphs of a survey question for $\frac{u_{CC}}{u_{HH}}$.

Universe One: your monthly spending graph is the left graph and others' monthly spending is the right graph below



Universe Two: your monthly spending graph is the left graph and others' monthly spending is the right graph below

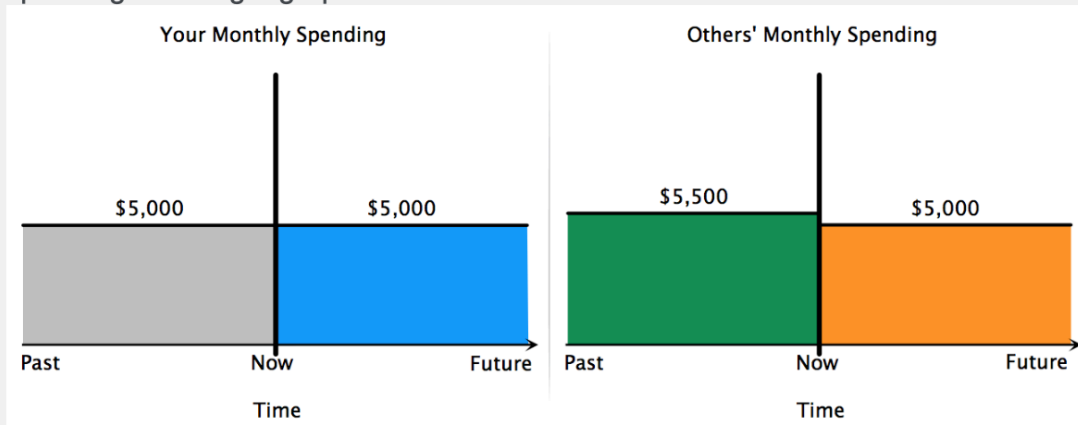
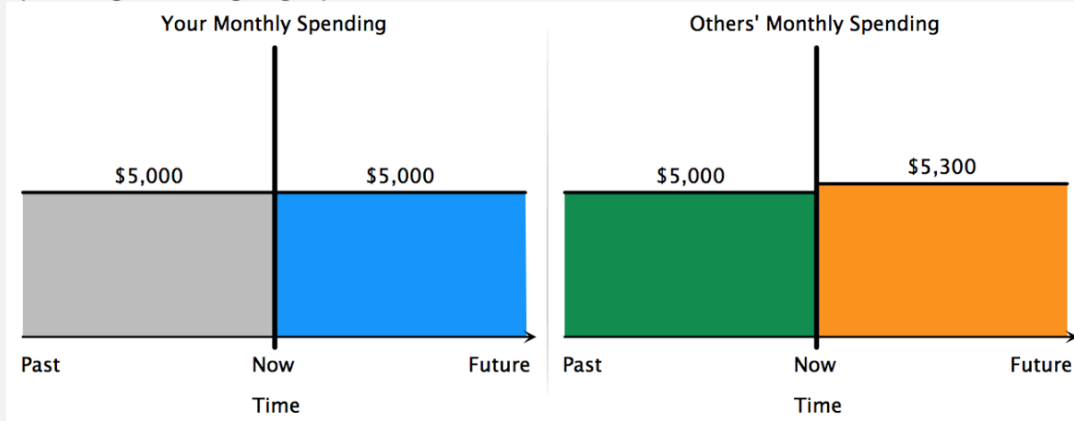


Figure A.9: Monthly spending graphs of a survey question for external habit formation.

Universe One: your monthly spending graph is the left graph and others' monthly spending is the right graph below



Universe Two: your monthly spending graph is the left graph and others' monthly spending is the right graph below

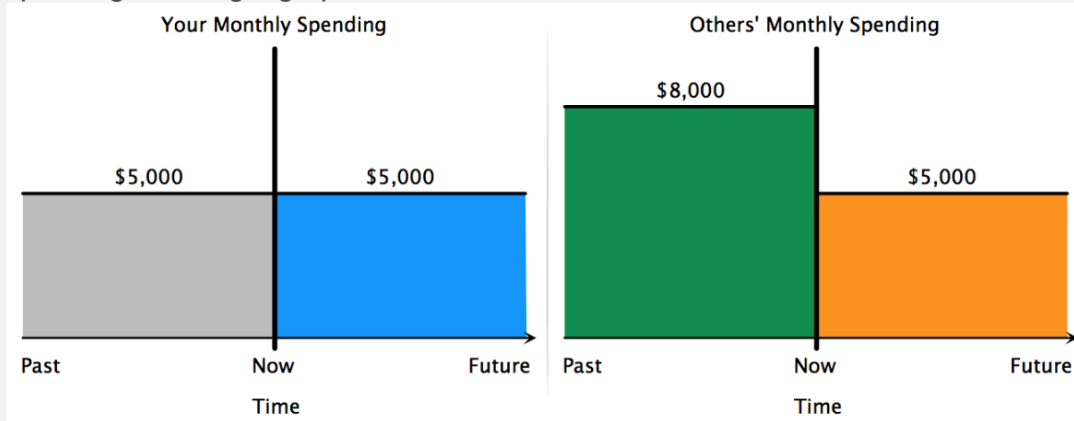


Figure A.10: Monthly spending graphs of a survey question for $\frac{u_{C_{\text{others}}}}{u_H}$.