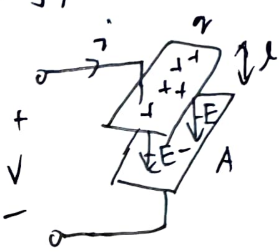
Zhuang Jianming

A0214561M Group 4B Week 5 Studio 2 10/9/2020

Up to this point, time has not been important in our study of electronic circuits. All analysis and design have been static where circuit responses at a given time only depend on circuit input at that time. This assumes circuits respond to input changes infinitely fast, but this is not the case in reality. To explain the temporal behavior of circuit responses, 2 new elements, capacitors and Inductors are needed

Capacitors

An idealized 2-terminal linear capacitor has each terminal connected to a conducting plate. Plates are parallel, separated by gap l and area of overlap is A. The gap is filled with an insulating linear dielectric with permittivity ε

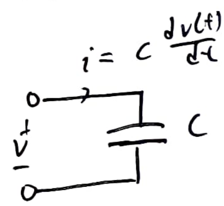


As current enters positive terminal, it transports charge q to the plate

An identical current exits the negative terminal and transports equal charge q off

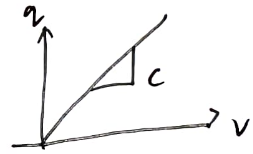
charge q on positive plate and its image charge $-q$ produce an electric field within dielectric E

$i = C \frac{dv(t)}{dt}$



From maxwell's equations : $E(t) = \frac{q(t)}{\varepsilon A}$ (1)

By integrating E-field from positive to negative plate : $\Delta V = \int E \cdot ds \Rightarrow v(t) = E(t) \cdot l$ (2)



Combining (1) and (2) $\Rightarrow q(t) = \frac{\varepsilon A}{l} v(t)$

we define capacitance $C(t) = \frac{\varepsilon A}{l} \Rightarrow q(t) = C(t)\, v(t)$

$$\boxed{C(t) = \frac{q(t)}{v(t)}}$$

[Farads]

$1F = \frac{1C}{1V}$

Resistor : algebraic r/n between branch V and I

Capacitor : algebraic r/n between branch V and stored charge

$$i(t) = \frac{d\,q(t)}{dt} = \frac{d\,(C(t)\,v(t))}{dt}$$

we assume capacitors are linear & time invariant

$$= C \frac{dv(t)}{dt}$$

At steady state, when V is constant, $i(t) = 0 \Rightarrow$ capacitors behave as open circuit at steady state

$$i(t) = C\frac{dv(t)}{dt}$$

If voltage changes instantaneously, $\frac{dv(t)}{dt} = \infty$

which is impossible since it requires $i(t) = \infty$

Hence, capacitor voltage cannot change instantaneously, but continuously

Energy storage

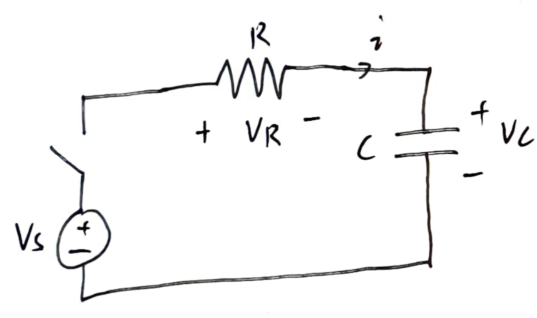
voltage is a measure of energy transfer / work done per unit charge

WD in moving charge dq from one plate to another $\Rightarrow dw = v\,dq$

$$W = \int_0^q v\,dq = \int_0^q \frac{q}{C}\,dq = \frac{q^2}{2C} = \boxed{\frac{1}{2}CV^2}$$

First Order Transients in linear networks

An RC circuit contains resistors and capacitors.



From KVL : $V_S = V_R + V_C$

By current-voltage rln of capacitor : $i = C\frac{dV_C}{dt}$

By element laws : $V_R = iR$

Hence $V_S = RC\frac{dV_C}{dt} + V_C$ (first order DE)

To solve first order DE of the form

$\dot{x} + p(t)\,x = q(t)$, we multiply both sides by an integrating factor $e^{\int p(t)\,dt}$

$$\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V_S}{RC}$$

multiply by integrating factor $e^{\int \frac{1}{RC}dt}$

$$\underbrace{\frac{dV_C}{dt}e^{\int \frac{1}{RC}dt} + \frac{V_C}{RC}e^{\int \frac{1}{RC}dt}}_{\frac{d}{dt}\left(V_C\, e^{\int \frac{1}{RC}dt}\right)} = \frac{V_S}{RC}e^{\int \frac{1}{RC}dt}$$

$$\frac{d}{dt}\left(V_c e^{\int \frac{1}{RC} dt}\right) = \frac{V_s}{RC} e^{\int \frac{1}{RC} dt}$$

integrate both sides.

$$V_c e^{\frac{t}{RC}} = \int \frac{V_s}{RC} e^{\frac{t}{RC}} \, dt$$

$$V_c = e^{-\frac{t}{RC}} \left[\frac{V_s}{RC} \frac{e^{\frac{t}{RC}}}{\frac{1}{RC}} + c \right]$$

$$V_c = V_s + c e^{-\frac{t}{RC}}$$

To solve for arbitrary constant c, we can plug in initial conditions, $t=0$

let $V_c(0)$ be initial conditions when $t=0$

$$V_c(0) = V_s + c \cancel{e^{-\frac{0}{RC}}} \to 1$$

$\boxed{\text{solve for arbitrary constant } c}$

$$c = V_c(0) - V_s$$

$$V_c(t) = V_s + (V_c(0) - V_s) e^{-\frac{t}{RC}}$$

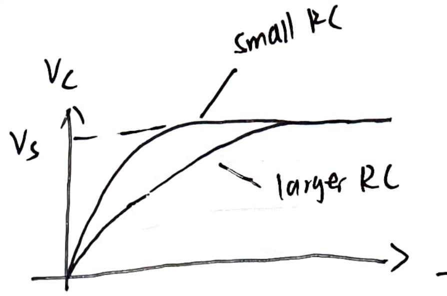
$$= V_c(0) e^{-\frac{t}{RC}} + V_s\left[1 - e^{-\frac{t}{RC}}\right]$$

If $V_c(0) = 0 \Rightarrow c = -V_s \Rightarrow V_c(t) = V_s\left[1 - e^{-\frac{t}{RC}}\right]$

$\boxed{RC}$ has dimension of $\boxed{\text{time}}$ and is also known as the $\boxed{\text{time constant}}$

As $t \to \infty$, $e^{-\frac{t}{RC}} \to 0$

$$\boxed{V_c(\infty) = V_s}$$



graph of $V_c(t) = V_s\left[1 - e^{-\frac{t}{RC}}\right]$ when $V_c(0) = 0$

RC discharge transient

when the RC is discharging, voltage source $= 0$ when switch is open

$$V_s = \frac{dV_C}{dt} + \frac{V_C}{RC}$$

$$0 = \frac{dV_C}{dt} + \frac{V_C}{RC} \quad \text{(homogeneous DE)}$$

can solve by separating variables,

$$\int \frac{1}{V_C}\, dV_C = \int -\frac{1}{RC}\, dt$$

$$\ln|V_C| = -\frac{t}{RC} + C$$

$$V_C = A e^{-\frac{t}{RC}} \quad \text{where } A = e^{C}$$

let $V_C(0)$ be initial conditions when $t=0$

$$V_C(0) = A \cancel{e^{-\frac{t}{RC}}} \rightarrow 1 \quad \Rightarrow \quad A = V_C(0)$$

$$V_C(t) = V_C(0)\, e^{-\frac{t}{RC}} \quad \text{where } RC \text{ is the same time constant}$$

If capacitor was fully charged, $V_C(0) = V_C(\infty) = V_S \Rightarrow A = V_S$

Properties of decaying exponentials

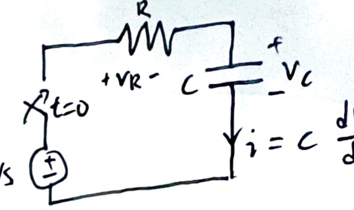
$$x = A e^{-\frac{t}{\tau}}$$

initial slope $\left.\frac{dx}{dt}\right|_{t=0} = -\frac{A}{\tau}$ $\Rightarrow$ initial slope projected onto time axis cuts time axis at $t = \tau$, regardless of A

when $t = \tau \Rightarrow x = \frac{A}{e}$ (function reaches $\frac{1}{e}$ of its initial value ≈ 0.368)

when $t > 5\tau \Rightarrow e^{-5} = 0.0067$ (can assume function is essentially 0)

Activity 1



Key equation

When capacitors are being charged / discharged through a series RC circuit, its transient behaviour of voltage changes exponentially with time

$$V_c(t) = V_c(0)e^{-\frac{t}{\tau}} + V_c(\infty)\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{where} \quad \tau = RC$$

Charging

when $V_c(0) = 0$ (voltage source not connected and capacitor is fully discharged)

and $V_c(\infty) = V_s$ (see rigorous proof in notes pg 3 when solving first order DE by integrating factor)

$$V_c(t) = V_s\left(1 - e^{-\frac{t}{\tau}}\right), \quad \tau = RC \quad \text{(time constant)}$$

Discharging

when $V_c(\infty) = 0$ (capacitor fully discharged)

and $V_c(0) = V_s$ (capacitor fully charged)

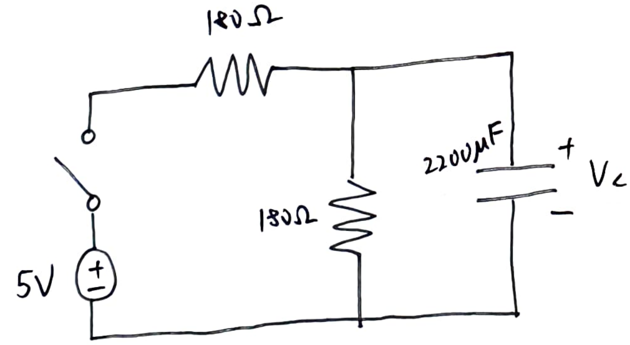
$$V_c(t) = V_s e^{-\frac{t}{\tau}}, \quad \tau = RC$$

(see rigorous proof in notes pg 4 by separating variables)

1. a) Electrolytic capacitors have polarity. I will ensure that the polarities are correct before turning on the power supply.

   b) I will wear protective goggles when the circuits are powered

3.



when $t = 0^-$

capacitor is in steady state and is fully discharged. Hence it acts like an open circuit and does not produce current. As the voltage source is not connected too, no current runs in the circuit.

$$\underline{V_c(0^-) = 0V}$$

when $t = 0^+$

$\underline{V_c(0^+) = 0V}$ (as voltage cannot change instantaneously) through analysis of $i(t) = C\frac{dv(t)}{dt}$

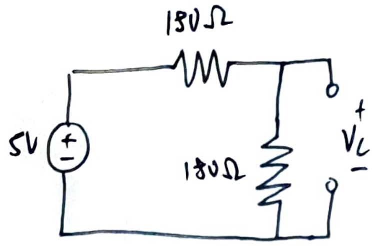
proven in notes pg2

when $t = \infty$

capacitor is in steady state and behaves like an open circuit

$$e^{-\frac{t}{RC}} \to 0$$

$$V_c(t) = V_c(\infty)$$



By voltage divider principle

$$V_c = \frac{180\Omega}{180\Omega + 180\Omega} \times 5V$$

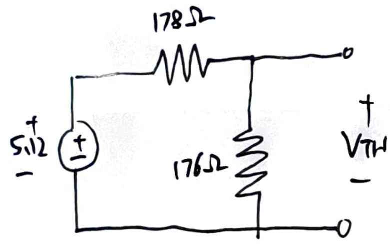
$$= \underline{2.5V}$$

4. Derive the Thevenin equivalent circuit that the capacitor sees for $t \geq 0$

$V_{open} = 5.12V$

$R_1 = 178\ \Omega$

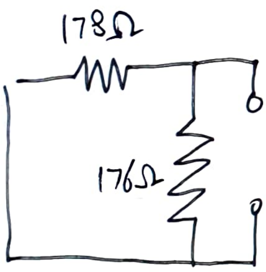
$R_2 = 176\ \Omega$



$$V_{TH} = \frac{176\Omega}{(176+178)\Omega} \times 5.12V$$

$$= \underline{2.55V}$$

R_{TH} can be found by setting all internal voltage sources to short, $v=0$



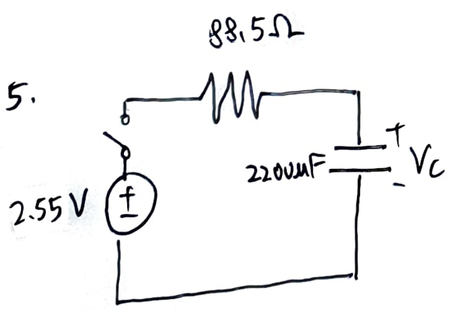
R_{TH} = equivalent resistance seen across terminals

$$= 176\Omega \,//\, 178\Omega$$

$$= \frac{176 \times 178}{176 + 178}$$

$$= \underline{88.497\Omega}$$

5.



6. for $t \geq 0$

from 3)

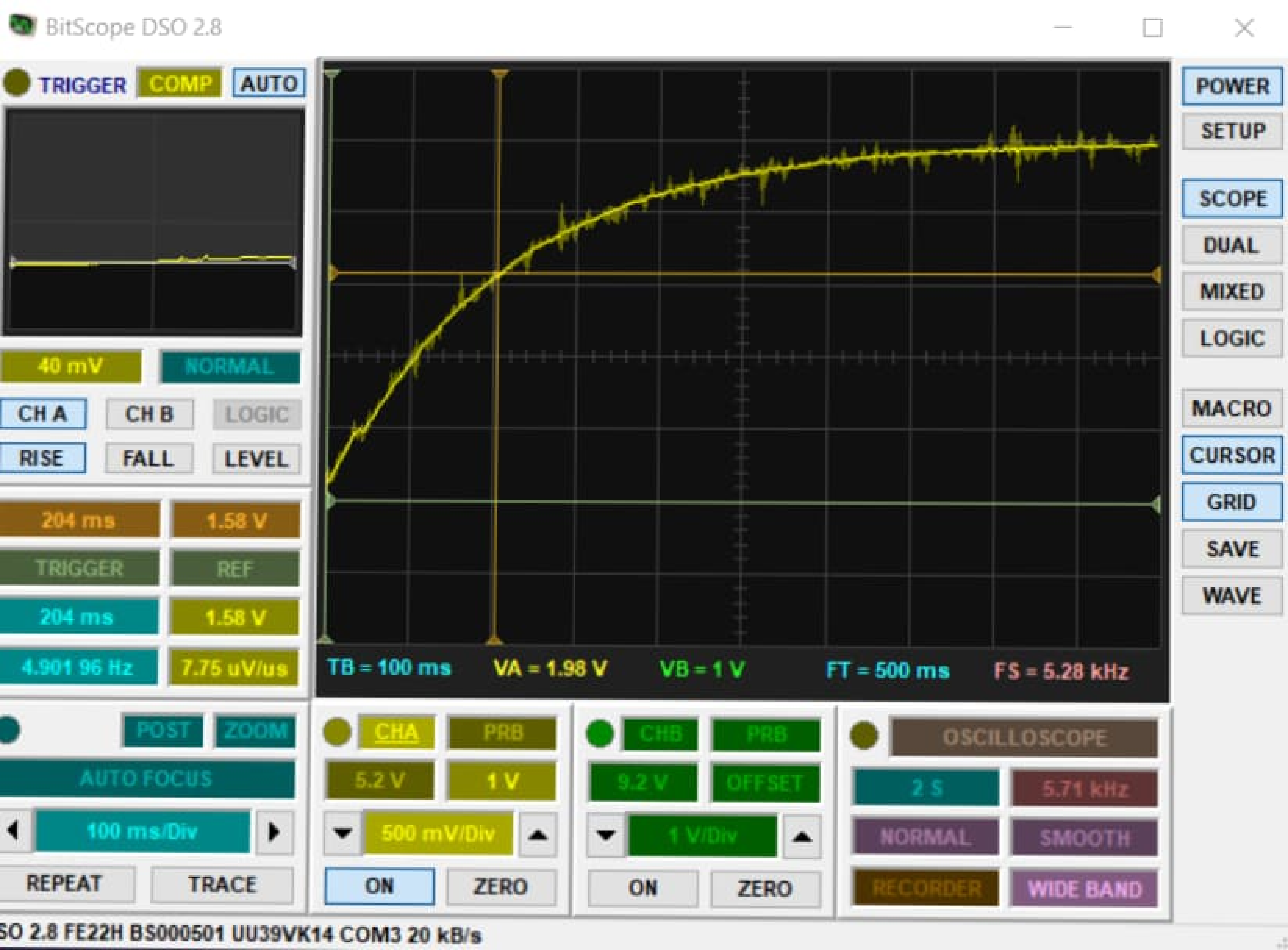
$$V_c(t) = \overset{0}{\cancel{v_c(0)}}\, e^{-\frac{t}{\tau}} + \overset{V_{TH}}{\cancel{v_c(\infty)}}\,(1 - e^{-\frac{t}{\tau}}), \quad \tau = RC$$

$$= 2.55(1 - e^{-\frac{t}{\tau}})$$

$\tau = 88.5 \times 2200 \times 10^{-6}$ s

$= \underline{0.195\text{ s}}$

~ insert screenshot of waveform ~


BitScope DSO 2.8
TRIGGER
COMP
AUTO
40 mV
NORMAL
CH A
CH B
LOGIC
RISE
FALL
LEVEL
204 ms
1.58 V
TRIGGER
REF
204 ms
1.58 V
4.901 96 Hz
7.75 uV/us
POST
ZOOM
AUTO FOCUS
100 ms/Div
REPEAT
TRACE
TB = 100 ms
VA = 1.98 V
VB = 1 V
FT = 500 ms
FS = 5.28 kHz
CHA
PRB
5.2 V
1 V
500 mV/Div
ON
ZERO
CHB
PRB
9.2 V
OFFSET
1 V/Div
ON
ZERO
OSCILLOSCOPE
2 S
5.71 kHz
NORMAL
SMOOTH
RECORDER
WIDE BAND
POWER
SETUP
SCOPE
DUAL
MIXED
LOGIC
MACRO
CURSOR
GRID
SAVE
WAVE
SO 2.8 FE22H BS000501 UU39VK14 COM3 20 kB/s

13. $V_C(\infty) \approx 2.5V$

63.2% of $V_C(\infty) = 1.58V$

~ set both horizontal and vertical cursor line to coincide with 63.2% of $V_C(\infty)$ ~

time measured = 204ms = $\underline{0.204s}$

14. <u>Analysis and Discussion</u>

i) $0.632 \times 2.55 = 2.55(1 - e^{-\frac{t}{\tau}})$ $\qquad \tau = 0.195s$ (from step 6)

$1 - e^{-\frac{t}{\tau}} = 0.632$

$e^{-\frac{t}{\tau}} = 0.368$

$-\frac{t}{\tau} = \ln 0.368$

$t = -\tau \ln 0.368$

$= \underline{0.19494} \approx \tau$

Actually, from analysis of decaying exponentials.

$x = e^{-\frac{t}{\tau}}$

when $\boxed{t = \tau}$, $x = \frac{1}{e} \approx 0.368$

$(1 - \frac{1}{e}) \approx 0.632$

Theoretically, $t = \tau$ for $V_C(t)$ to reach 63.2% of $V_C(\infty)$ //

My experimental value of 0.204s agrees strongly with theoretical value of 0.195s //

ii) <u>Sources of error</u>

① The actual value of resistance and capacitance may be different than the values used in the theoretical analysis due to tolerance of elements. Hence $\boxed{\text{time taken to reach 63.2\% of } V_C(\infty) = \tau = RC}$ will be slightly different

② There could be error from reading off the graph generated by the Bitscope

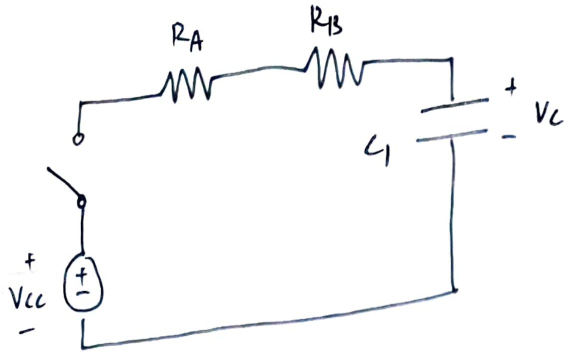
Ⓐ $V_C(\infty)$ read off the plateau of the waveform might not be accurate. Hence setting the cursor line to 63.2% of $V_C(\infty)$ would give smaller value of t if $V_C(\infty)$ read off is lower than actual $V_C(\infty)$ and vice versa

Ⓑ There could be error from dragging the vertical cursor line to coincide with the horizontal line and the graph

③ If step 11 is repeated to capture the waveform, the capacitor might not be fully discharged, Hence $V_C(0) \neq 0$, t would be smaller

Activity 2

1. charging voltage of capacitor



when $t=0$

capacitor starts charging when $V_C(t) = \frac{1}{3}V_{cc}$

$V_C(0) = \frac{1}{3}V_{cc}$

when $t=\infty$

If capacitor is allowed to charge indefinitely

$V_C(\infty) = V_{cc}$ (capacitor behaves as an open circuit)

$$V_C(t) = \frac{1}{3}V_{cc}e^{-\frac{t}{\tau}} + V_{cc}\left(1-e^{-\frac{t}{\tau}}\right), \tau = (R_A+R_B)C_1$$

when $V_C(t) = \frac{1}{3}V_{cc}$:

$$\frac{1}{3}V_{cc} = V_{cc}\left(1-\frac{2}{3}e^{-\frac{t}{\tau}}\right)$$

$$\frac{2}{3}e^{-\frac{t}{\tau}} = 1-\frac{1}{3}$$

$$-\frac{t}{\tau} = \ln 1$$

$$t = 0 \quad ; \quad t_{start} = 0 \qquad (1)$$

when $V_C(t) = \frac{2}{3}V_{cc}$:

$$\frac{2}{3}V_{cc} = V_{cc}\left(1-\frac{2}{3}e^{-\frac{t}{\tau}}\right)$$

$$\frac{2}{3}e^{-\frac{t}{\tau}} = \frac{1}{3}$$

$$-\frac{t}{\tau} = \ln\frac{1}{2}$$

$$t = -\tau\ln\frac{1}{2} \quad , \quad t_{end} = \tau\ln 2 \quad (2)$$

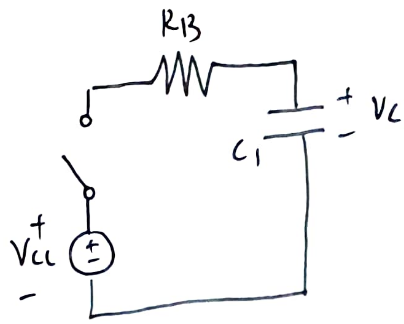
From (1) and (2)

$$t_1 = t_{end} - t_{start}$$

$$= \tau\ln 2$$

$$= \ln 2\,(R_A+R_B)C_1$$

2. Discharge voltage of capacitor



<u>when $t = 0$</u>

capacitor discharges when $V_C(t) = \frac{2}{3} V_{CC}$

$V_C(0) = \frac{2}{3} V_{CC}$

<u>when $t = \infty$</u>

If capacitor is allowed to discharge indefinitely

$V_C(\infty) = 0$ (switch open)

$$V_C(t) = \frac{2}{3} V_{CC} e^{-\frac{t}{\tau}}, \quad \tau = R_B C_1$$

when $V_C(t) = \frac{2}{3} V_{CC}$:

$$\frac{2}{3} V_{CC} = \frac{2}{3} V_{CC} e^{-\frac{t}{\tau}}$$

$$e^{-\frac{t}{\tau}} = 1$$

$$t = 0, \quad t_{start} = 0 \qquad (1)$$

when $V_C(t) = \frac{1}{3} V_{CC}$:

$$\frac{1}{3} V_{CC} = \frac{2}{3} V_{CC} e^{-\frac{t}{\tau}}$$

$$e^{-\frac{t}{\tau}} = \frac{1}{2}$$

$$t = -\tau \ln\frac{1}{2}, \quad t_{end} = \tau \ln 2 \qquad (2)$$

<u>From (1) and (2)</u>

$$t_2 = t_{end} - t_{start}$$
$$= \tau \ln 2$$
$$= \ln 2 \, (R_B) C_1$$

3. period T of square wave $= t_1 + t_2$

$$= \ln 2 \, (R_A + R_B) C_1 + \ln 2 \, (R_B) C_1$$
$$= \ln 2 \, (R_A + 2R_B) C_1$$

$$\text{frequency} = \frac{1}{T} = \frac{1}{\ln 2 \, (R_A + 2R_B) C_1}$$

$$\frac{1}{\ln 2} \approx 1.44$$

$$\approx \boxed{\frac{1.44}{(R_A + 2R_B) C_1}}$$

4. $R_A = 10k\Omega$

$R_B = 100k\Omega$

$C_1 = 100nF$

$$t_1 = \ln 2\,(10\times10^3 + 100\times10^3)\,100\times10^{-9}\ s$$
$$= 0.00762465$$
$$= \boxed{7.6246ms}$$

$$t_2 = \ln 2\,(100\times10^3)\,100\times10^{-9}\ s$$
$$= \boxed{6.9315ms}$$

$$f = \frac{1}{t_1+t_2} = \frac{1}{(6.9315+7.6246)\times10^{-3}} = \boxed{68.699\ Hz}$$

~ insert screenshots for t_1, t_2 and f ~

* included both 2ms/DIV and 5ms/DIV timebase

8. drag both vertical orange and green cursor lines to output's falling and rising edge respectively

$\boxed{t_1 = 8.18ms}$

9. $\boxed{t_2 = 6.92ms}$

$T = t_1 + t_2 = 15.1ms$

$f = \frac{1}{T} = \boxed{66.225Hz}$

10. Analysis and Discussion

My experimental measurements of t_1, t_2 and f agree strongly with my theoretical values (see highlighted boxes)

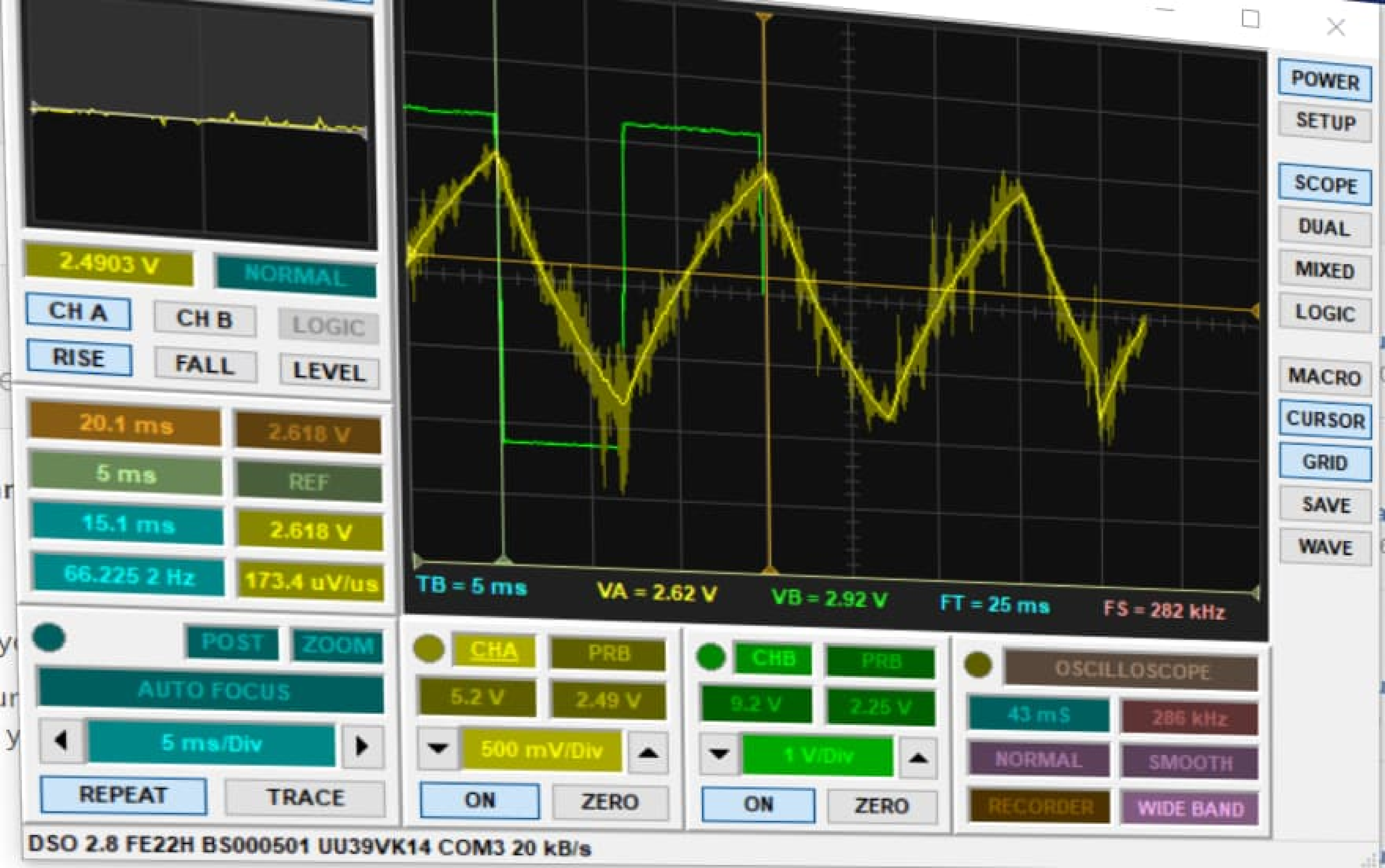
Sources of error

* resistors and capacitors have tolerance ± %

① The actual measured values of resistance and capacitance, R_A, R_B, C_1 most likely differ from $10k\Omega$, $100k\Omega$ and $100nF$ used in the theoretical analysis. Hence t_1, t_2 and f calculated will differ from measured values

② There could be error from reading off the graph generated by the Bitscope such as dragging the vertical cursor lines to where the "falling edge" and "rising edge" occurs and ends. This would result in different measured values in the cyan box than calculated values


TRIGGER
COMP
AUTO
2.4903 V
NORMAL
CH A
CH B
LOGIC
RISE
FALL
LEVEL
5 ms
2.618 V
11.92 ms
REF
-6.92 ms
2.618 V
-144.509 Hz
-378.3 uV/us
POST
ZOOM
AUTO FOCUS
2 ms/Div
REPEAT
TRACE
TB = 2 ms
VA = 2.62 V
VB = 2.93 V
FT = 10 ms
FS = 264 kHz
CHA
PRB
5.2 V
2.49 V
500 mV/Div
ON
ZERO
CHB
PRB
9.2 V
2.24 V
1 V/Div
ON
ZERO
OSCILLOSCOPE
40 mS
286 kHz
NORMAL
SMOOTH
RECORDER
WIDE BAND
POWER
SETUP
SCOPE
DUAL
MIXED
LOGIC
MACRO
CURSOR
GRID
SAVE
WAVE
SO 2.8 FE22H BS000501 UU39VK14 COM3 21 kB/s


2.4903 V
NORMAL
CH A
CH B
LOGIC
RISE
FALL
LEVEL
20.1 ms
2.618 V
5 ms
REF
15.1 ms
2.618 V
66.225 2 Hz
173.4 uV/us
POST
ZOOM
AUTO FOCUS
5 ms/Div
REPEAT
TRACE
TB = 5 ms
VA = 2.62 V
VB = 2.92 V
FT = 25 ms
FS = 282 kHz
CHA
PRB
5.2 V
2.49 V
500 mV/Div
ON
ZERO
CHB
PRB
9.2 V
2.25 V
1 V/Div
ON
ZERO
OSCILLOSCOPE
43 mS
286 kHz
NORMAL
SMOOTH
RECORDER
WIDE BAND
POWER
SETUP
SCOPE
DUAL
MIXED
LOGIC
MACRO
CURSOR
GRID
SAVE
WAVE
DSO 2.8 FE22H BS000501 UU39VK14 COM3 20 kB/s


TRIGGER
COMP
AUTO
2.4903 V
NORMAL
CH A
CH B
LOGIC
RISE
FALL
LEVEL
20.1 ms
2.618 V
11.92 ms
REF
8.18 ms
2.618 V
122.249 Hz
320 uV/us
POST
ZOOM
AUTO FOCUS
5 ms/Div
REPEAT
TRACE
TB = 5 ms
VA = 2.62 V
VB = 2.92 V
FT = 25 ms
FS = 282 kHz
CHA
PRB
5.2 V
2.49 V
500 mV/Div
ON
ZERO
CHB
PRB
9.2 V
2.25 V
1 V/Div
ON
ZERO
OSCILLOSCOPE
43 mS
286 kHz
NORMAL
SMOOTH
RECORDER
WIDE BAND
POWER
SETUP
SCOPE
DUAL
MIXED
LOGIC
MACRO
CURSOR
GRID
SAVE
WAVE