CG1111 Engineering Principles and Practice I

DC Circuit Principles IV – Node Voltage Analysis

(Week 5, Studio 1)

Time	Duration (mins)	Activity
0:00	15	Briefing on Activity #1
0:15	70	Activity #1: Practising node voltage analysis on a bridge network, and verifying your calculations through experiments
1:25	10	Discussion on Activity #1's results, and briefing on Activity #2
1:35	55	Activity #2: Practising node voltage analysis on a circuit with **two** voltage sources, and verifying your calculations through experiments
2:30	5	Final discussions and wrap-up

Introduction:

- In a previous studio, we have seen the Wheatstone bridge, which is a member of the family of bridge circuits. In this studio, we shall examine a slightly different bridge circuit as shown in Figure 1, which allows current to flow across the bridge. Such topologies are useful because the current may flow in either direction, depending on the relative values of all the resistances. This feature could allow, say, a motor connected across the bridge, to spin either clockwise or counter-clockwise.
- As there are no resistors in the circuit that are in series or parallel, we cannot use any shortcut to simplify the circuit to help our circuit analysis. Hence, it is a perfect candidate where the ***node voltage analysis (NVA) method*** comes in handy.
- In Activity #1, you will practise analysing the circuit using the NVA method for the circuit shown in Figure 1. From the node voltages you have calculated, you shall solve for the voltages and currents across all the five resistors. You will then perform experiments to measure these values to verify your calculations.
- In Activity #2, you will repeat the procedure using a slight variant of the circuit. In this variant, you cannot write the KCL equation the same way as before for any of the nodes. You will have to think about how you can redefine the concept of a "node" to obtain the KCL equation you need.

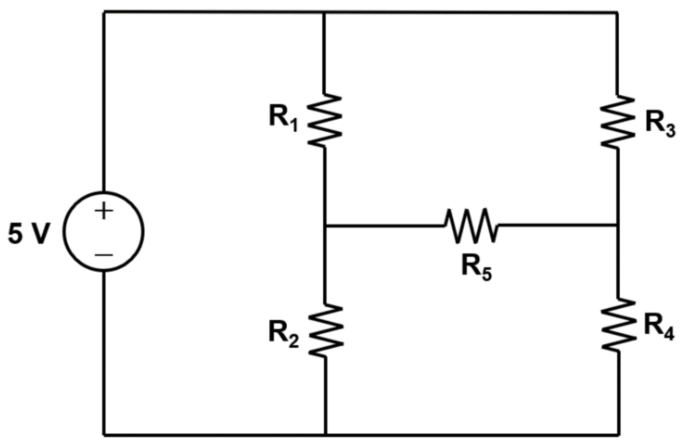

Figure 1: A bridge circuit.

Objectives:

- Practise analysing a circuit using the node voltage analysis method
- Practise forming the simultaneous equations from KCL equations, and solving them
- Able to apply Ohm's Law and KCL
- Become more skilful in interpreting ***voltage polarity*** and ***current direction***

Materials:

- Breadboard and connecting wires
- USB breakout cable + your own USB charger
- Digital multimeter
- Arduino Uno (to provide second DC voltage source)
- 5% tolerance resistors {560 Ω x 4, 330 Ω x 1}

Activity #1: Practising node voltage analysis on a bridge network, and verifying your calculations through experiments (70 mins)

Procedure:

1. Construct the circuit shown in Figure 1 using the resistor values shown in Table I. Before you insert each resistor into the circuit, take some time to measure its actual resistance value using the multimeter, and note it down using the format shown in Table I. As there are four resistors having the same nominal value of 560 Ω, you should pay attention to which resistor is used as R_1, R_2, R_3, and R_5.

Table I

Resistor	Nominal resistance	Measured resistance
R_1	560 Ω	
R_2	560 Ω	
R_3	560 Ω	
R_4	330 Ω	
R_5	560 Ω	

2. Next, you shall practise applying the NVA method to solve for all the node voltages. Keep in mind that the **final objective** is to **solve** for all the **voltages** and **currents** of each of the five resistors. As the first step, proceed to pick a node in the circuit shown in Figure 1 as a **reference node**. This node will be taken to have a node voltage of 0 V, and **all other node voltages are relative to this reference node's voltage**.

   Hint: It is more convenient if you pick one end of a voltage source to be the reference node (usually the negative terminal).

3. Identify the unknown node voltages in the circuit. If you have picked one terminal of the voltage source to be the reference node, then you can proceed to write down the node voltage of the voltage source's other terminal without defining it as an additional unknown. How many unknown node voltages do you have for this circuit?

4. If you have ***N* unknown node voltages**, you need to have ***N* linearly independent equations** to solve for these unknowns. To obtain the *N* equations, you shall apply KCL at *N* nodes.

   Hint: Pick the nodes that are adjacent to resistor branches only, so that you can obtain an expression for each branch current using Ohm's Law. If you pick a node that is adjacent to a voltage source, you cannot write down the KCL equation because you do not know what current is passing through that voltage source for now.

5. Rearrange your equations **in the following form** by consolidating the coefficients (you should learn to **visually extract them** from the KCL equations) for each unknown node voltage V_i:

$$a_1V_a + b_1V_b = c_1$$
$$a_2V_a + b_2V_b = c_2$$

   The coefficients a_i, b_i, and c_i are in terms of the resistances R_1, R_2, R_3, R_4, and R_5. Do not substitute their numerical values into the equation when you write them down, else they get very messy and are hard to check. If any coefficient a_i, b_i, or c_i appears as the sum of multiple fractions, leave them as they are. A common mistake is to waste time combining them into a single fraction of variables (e.g., using cross-multiplication).

6. Now, calculate the coefficients in each equation using your calculator by substituting the resistances R_1, R_2, R_3, R_4, and R_5 with their numerical values. You may also want to use the measured voltage of your USB breakout cable instead of the nominal value of 5 V. Solve the *N* simultaneous equations you have derived. If your calculator has the built-in feature to solve simultaneous equations, you should learn how to use it. There are also some online resources that can help you solve them (e.g., http://www.webmath.com/solver2.html).

7. Now that you have obtained all the node voltages in the circuit, you shall use them to calculate the voltage across each of the five resistors, and the current flowing through them. Draw/copy the circuit from Figure 1 in your learning journal. **Then label the voltages and currents for the resistors clearly with the correct voltage polarity and current's direction**. Tabulate your answers using the template shown in Table II.

Table II

Resistor	Measured resistance (Ω)	Calculated voltage (V)	Calculated current (mA)	Measured voltage (V)	Measured current (mA)
R_1					
R_2					
R_3					
R_4					
R_5					

8. Power up your circuit, and measure the voltages and currents through each of the resistors. Tabulate your readings in Table II. Be aware of how you connect the red and black clips of the multimeter when you perform the measurements, because voltage has polarity, and current has direction!

   Note:
   When measuring current, it is convenient to think of the ammeter as a two-part wire consisting of a red tip and a black tip. Put this "two-part wire" **in series** with any resistor for which you are trying to measure its current. The ammeter's reading measures the current flowing from the red tip to the black tip.

9. Compare your measured readings with the calculated values in Table II.

Activity #2: Practising node voltage analysis on a circuit with two voltage sources, and verifying your calculations through experiments (55 mins)

In this part of the activity, you will repeat the procedure using a slight variant of the circuit, as shown in Figure 2 below. Notice that the circuit now consists of two voltage sources.

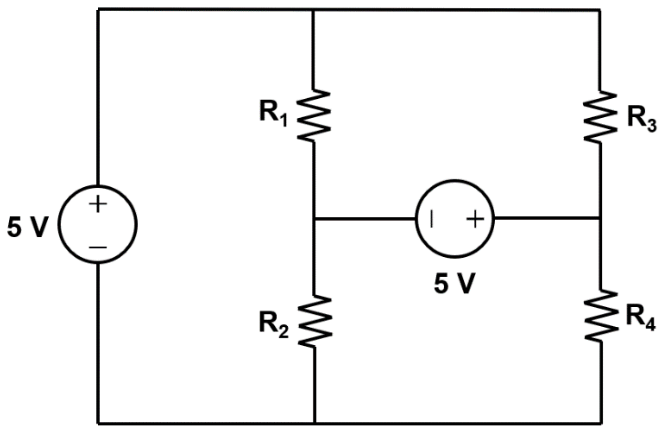

Figure 2: A modified bridge circuit with two voltage sources.

Procedure:

1. In this activity, you shall obtain the second 5 V voltage source from your Arduino Uno. Connect your Arduino Uno (using the USB cable that came with it) to another USB power adapter that is **isolated** from the one powering your USB breakout cable. This can be another USB power bank/adapter, or even your laptop's USB port.

   Note:
   It is **very important** that the second power source is **isolated from the first**. If they are from the same device (e.g., two USB ports on the same power bank or laptop), their grounds (i.e., negative terminals) are connected internally and you will get wrong results!

2. Notice that the Arduino Uno has a pin that is labelled as "5V". This serves as the "+" terminal of your second 5 V source. There are also two ground pins beside it that are labelled as "GND". Either one of this can be used as the "–" terminal of your second 5 V source.

3. Modify your previous circuit by removing resistor R_5, and replacing it with the second 5 V voltage source provided by Arduino Uno. You may also want to use your multimeter to measure the actual voltage provided by this second voltage source for more accurate experimental results. Pay attention to the polarity of the voltage source when you connect it to the circuit.

4. Pick a node in the circuit as the reference node. You can pick the same node as Activity #1.

5. Identify the unknown node voltage(s) in the circuit. Recall from the e-lecture that if one terminal of a voltage source is labelled to have an unknown node voltage, then its other terminal's node voltage can be expressed in terms of this unknown without the need to define an additional unknown. How many unknowns do you have for this circuit?

6. Similar to Activity #1, you need N equations to solve for N unknowns. For this circuit, you would notice that **there is no node that is adjacent to resistors only** (because one of the adjacent branches always contains a voltage source). Since we do not know what current is passing through either of the voltage sources at this moment, we cannot write the KCL equation in terms of just node voltages and resistances for any of these nodes. Now, think about how you can **redefine the concept of a "node"** so that you can conveniently obtain the KCL equations in terms of just node voltages and resistances without introducing additional variables.

7. Solve the equation(s) to obtain all the node voltages in the circuit. Use them to calculate the voltage across each of the four resistors, and the current flowing through them. Draw the circuit from Figure 2 in your learning journal. **Then label the voltages and currents for the resistors clearly with the correct voltage polarity and current's direction**. Also calculate the current flowing through the 5 V voltage source provided by Arduino Uno, and **label its direction clearly in your figure**. Tabulate your answers using the template in Table III.

Table III

Element	Measured resistance (Ω)	Calculated voltage (V)	Calculated current (mA)	Measured voltage (V)	Measured current (mA)
R_1					
R_2					
R_3					
R_4					
5 V source (Arduino)	---	---		---	

8. Power up your circuit, and measure the voltages and currents through each of the resistors. Also measure the current flowing through the 5 V voltage source provided by Arduino Uno. Tabulate your readings in Table III. Once again, be aware of how you connect the red and black clips of the multimeter when you perform the measurements.

9. Compare your measured readings with the calculated values in Table III.

10. Is the 5 V voltage source from Arduino Uno supplying power to the circuit, or consuming power from the circuit? How do you tell?

END OF STUDIO SESSION

CHALLENGE YOURSELF (Optional)

In this extra activity, perform NVA for the circuit shown in Figure 3. Solve for all the node voltages.

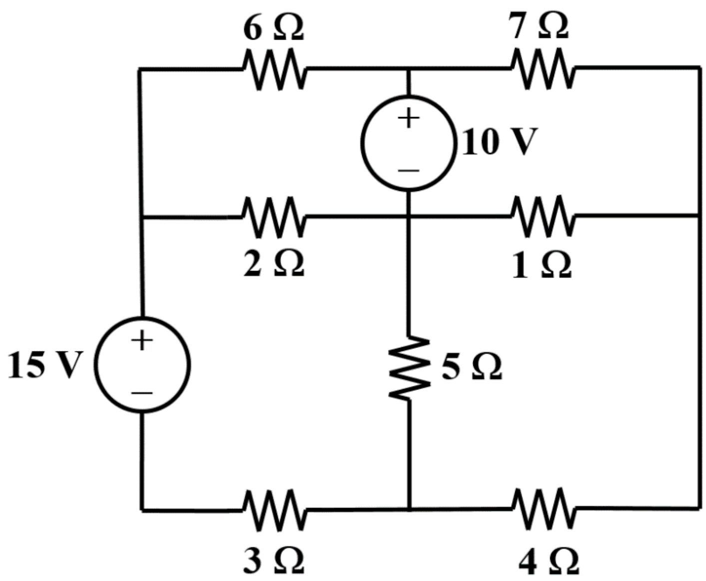

Figure 3: A more complicated circuit for practising NVA.

You may find this tool useful: http://math.bd.psu.edu/~jpp4/finitemath/3x3solver.html