CG1111: Engineering Principles and Practice I

Summary of Key Points for Week 6 to 11

NUS
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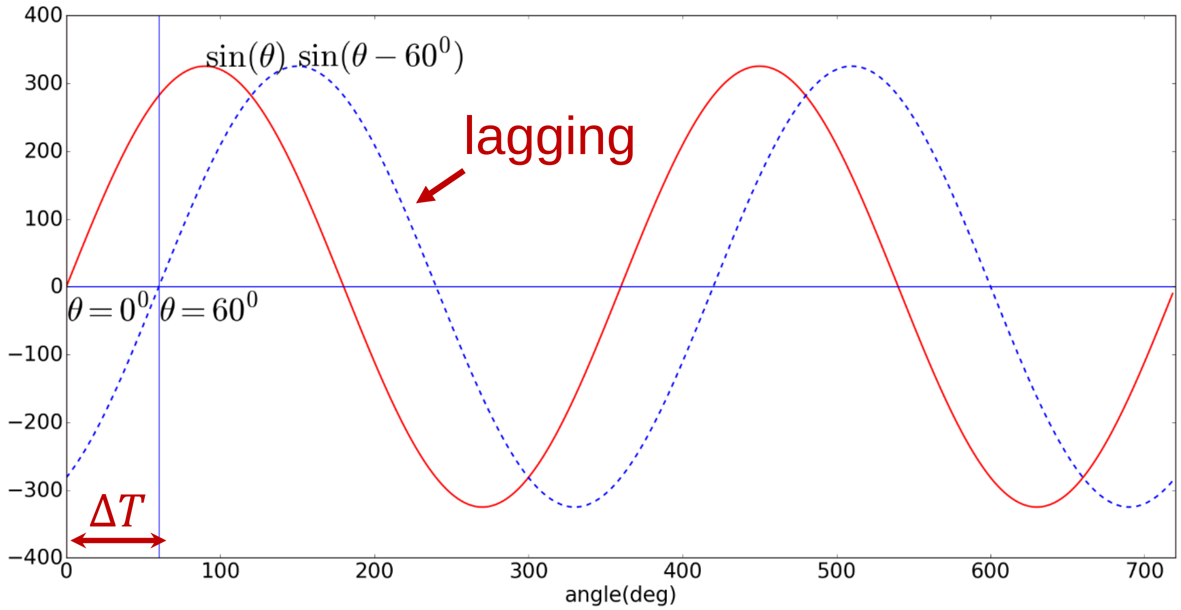
Principles of AC Circuits

Sinusoidal waveform



$$v(t) = V_m \cos(\omega t \pm \emptyset)$$

$$\emptyset = \frac{\Delta T}{T} \times 360°$$

'+' if leading
'–' if lagging

V_m: Amplitude (or peak)
ω: Angular frequency in rad/s
$\emptyset$: Phase angle

$$\omega = 2\pi f$$
$$T = \frac{1}{f}$$

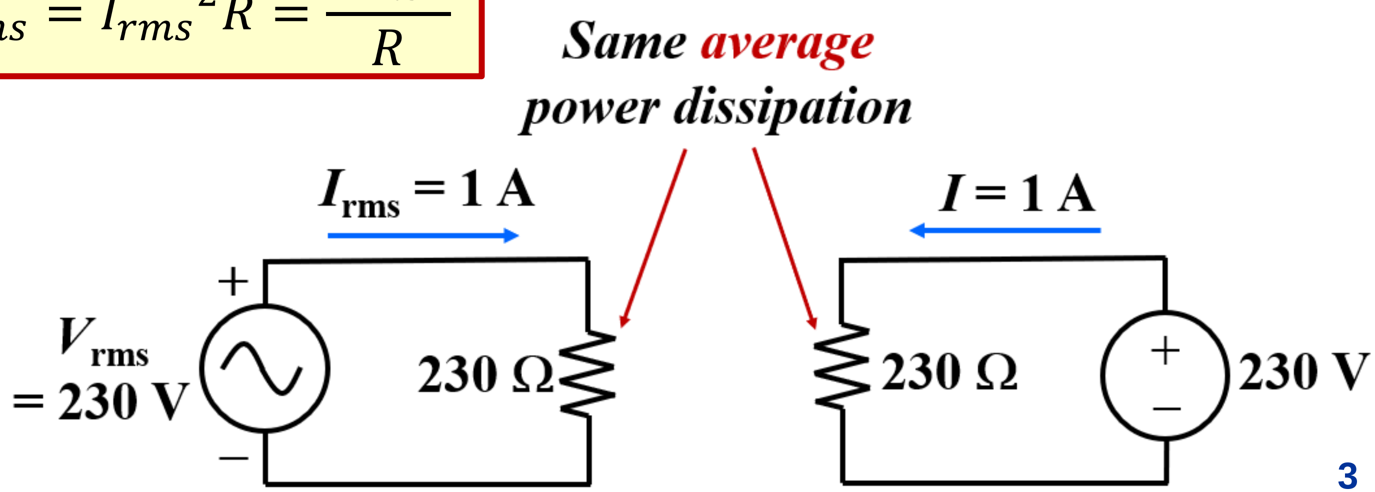
Root-Mean-Square (RMS)

- Significance of rms value:
  - They are the equivalent values of the DC voltage & current that would have the **same average power dissipation** in a **resistive load**
  - So that you can apply the same formula as DC!
  - Average power dissipation of resistive load in AC:

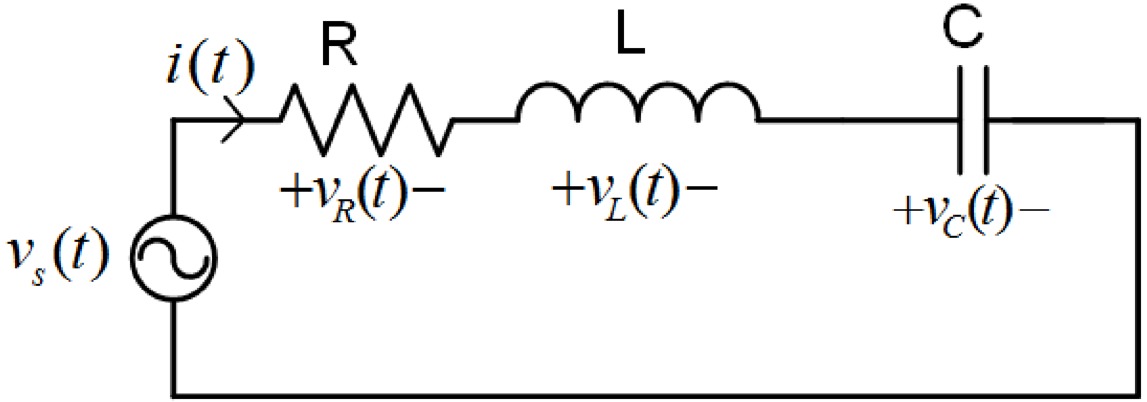
$$\boxed{P = V_{rms} \times I_{rms} = I_{rms}{}^2 R = \frac{V_{rms}{}^2}{R}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$



AC Circuit Analysis in Time Domain?

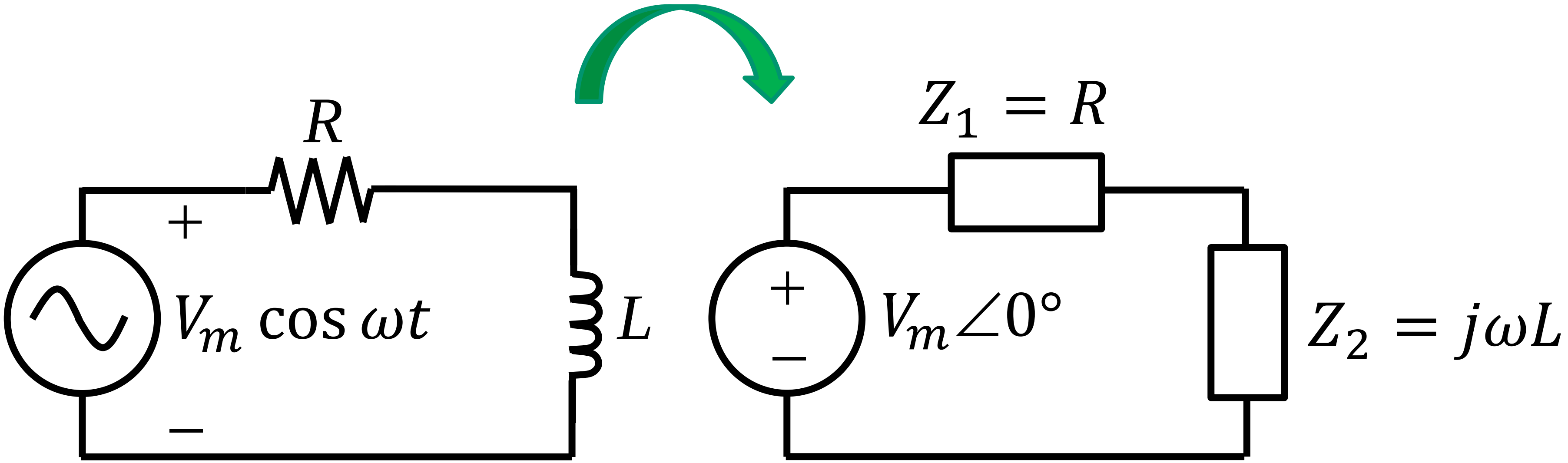


- In AC circuits, inductors/capacitors result in differential equations:

$$V_m \cos(\omega t) = v_R(t) + v_L(t) + v_C(t) = iR + L\frac{di}{dt} + \frac{\int i\, dt}{C}$$

$\rightarrow$ difficult to solve in time-domain!

AC Circuit Analysis



Time Domain

Complex Domain

Can then solve using DC circuit analysis techniques:

- KVL, KCL, Ohm's Law, Potential divider principle, current division principle, Thevenin equivalent, NVA, series equivalent impedance, parallel equivalent impedance, etc.

AC Circuit Analysis with Phasors & Impedances

We must work with KVL & KCL in Phasor form

Steps:

1. Replace voltage sources with their phasors (all must have same frequency)
2. Replace R, L, C elements with their impedances
3. Analyse circuit using DC circuit analysis techniques (work within complex domain)
4. Convert final results back to time-domain

Phasors

- Sinusoidal voltage:

$$V_m \cos(\omega t + \theta)$$

- Phasor:

$$V_m \angle \theta$$

Note:
Another common practice is to represent phasors using the RMS value instead of the magnitude. In that case, the phasor will be written as $\frac{V_m}{\sqrt{2}} \angle \theta$.

Note:

- Phasor is just a definition. It leads to mathematical convenience, but has no physical significance

Impedances

- For resistance: $\boxed{R}$

- For inductor: $\boxed{j\omega L = \omega L \angle 90°}$

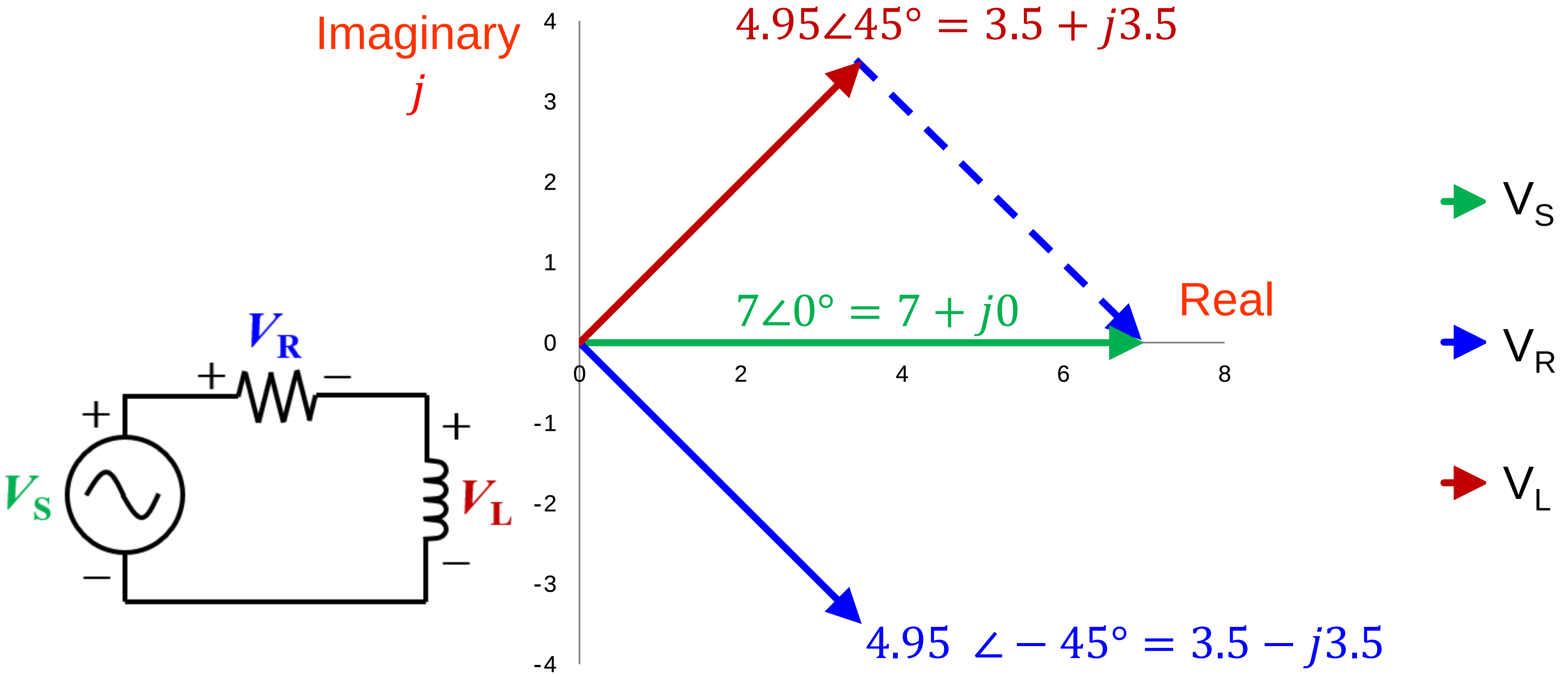
- For capacitor: $\boxed{\dfrac{1}{j\omega C} = \dfrac{-j}{\omega C} = \dfrac{1}{\omega C} \angle -90°}$

Illustration: KVL Applies for Phasors

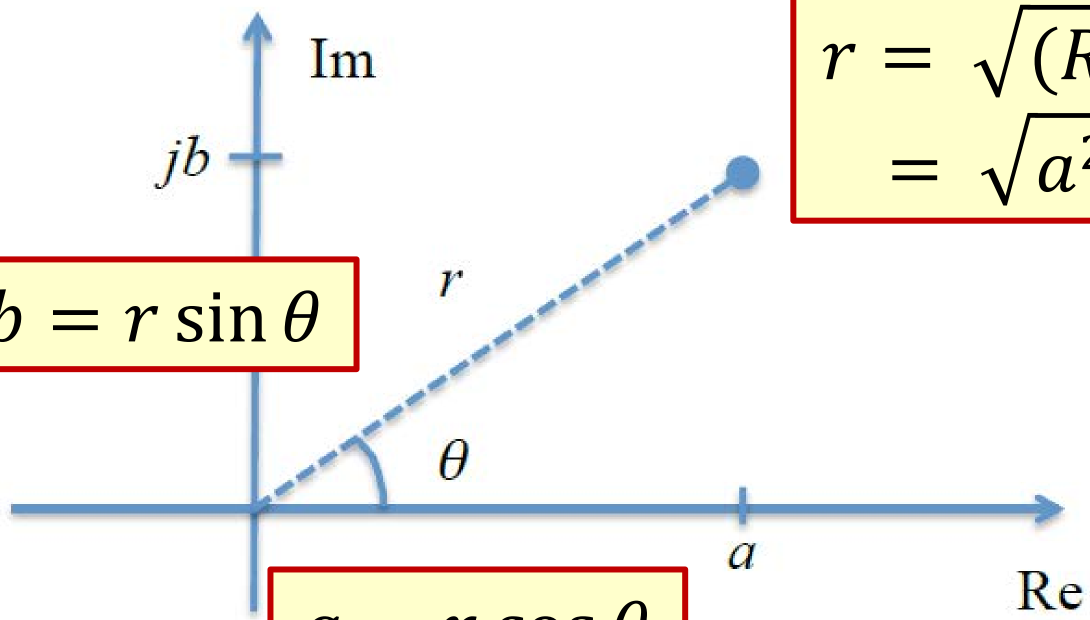
Voltage	Magnitude	Phase Angle	Real	Imaginary
V_S	7	0°	7	0
V_R	4.95	−45°	3.5	−3.5
V_L	4.95	45°	3.5	3.5

In AC circuits with R, L, C elements, because of the phase differences, we can only apply KVL in complex domain:

$$V_S = V_R + V_L$$



Relationship Among Magnitude, Phase, Real & Imaginary Parts



$$r = \sqrt{(Re)^2 + (Im)^2} = \sqrt{a^2 + b^2}$$

$$b = r \sin\theta$$

$$a = r \cos\theta$$

$$\theta = \tan^{-1}\left(\frac{Im}{Re}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

Phasor Division & Multiplication

- Division:

$$\boxed{\frac{A\angle\theta_1}{B\angle\theta_2} = \frac{A}{B}\angle(\theta_1 - \theta_2)}$$

- Multiplication:

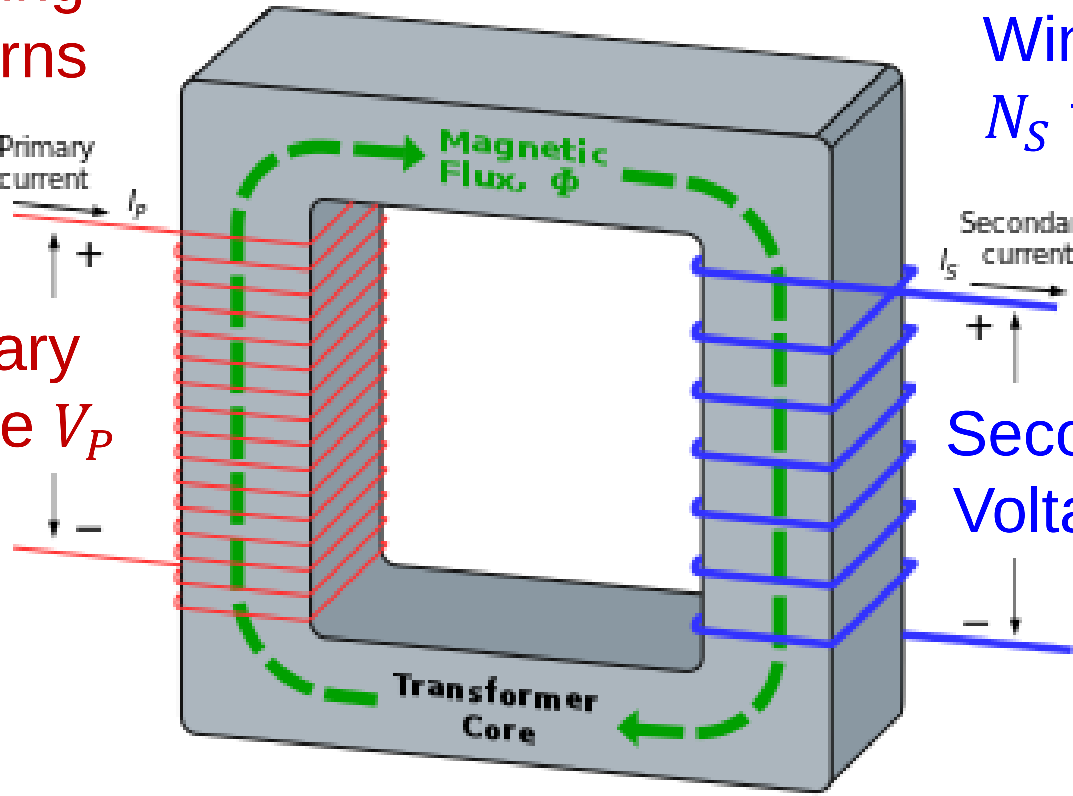
$$\boxed{A\angle\theta_1 \times B\angle\theta_2 = AB\angle(\theta_1 + \theta_2)}$$

Step-Up/Down Transformer

Primary Winding N_P turns

Primary Current I_P

Primary Voltage V_P



Secondary Winding N_S turns

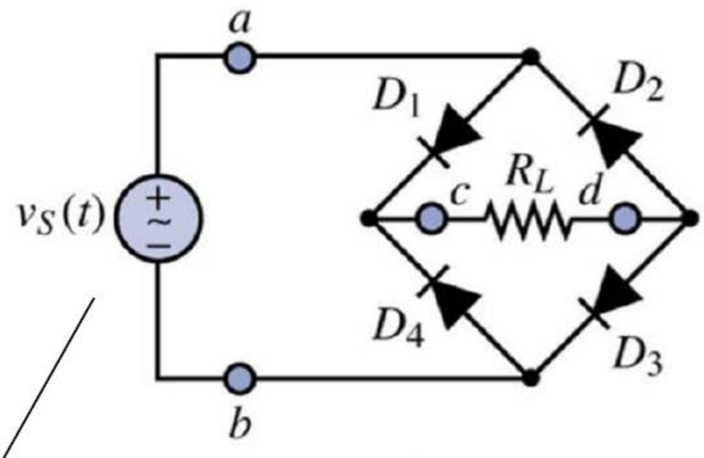
Secondary Current I_S

Secondary Voltage V_S

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

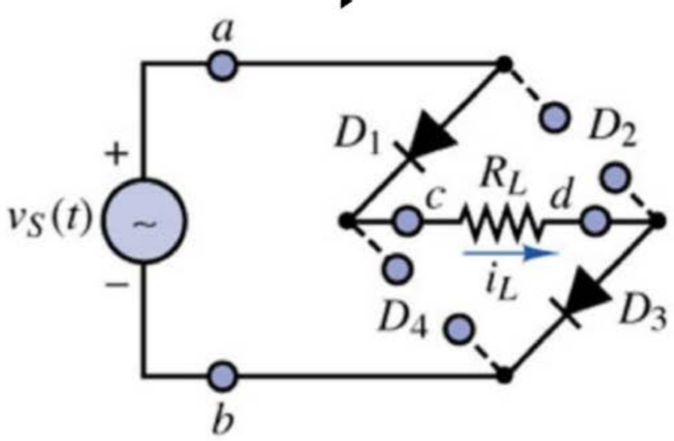
$$\frac{I_S}{I_P} = \frac{N_P}{N_S}$$

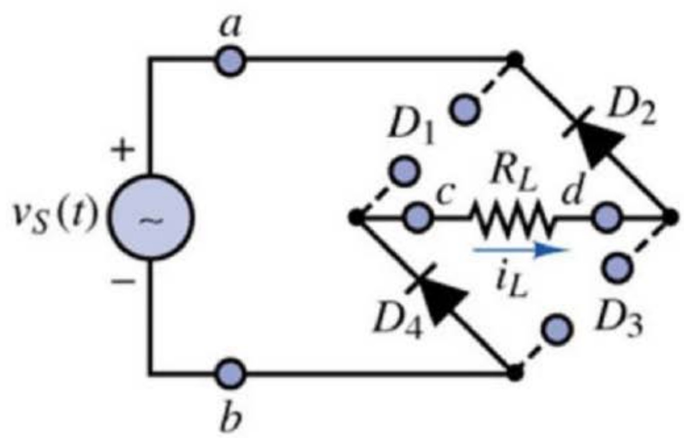
Why Rectifier?

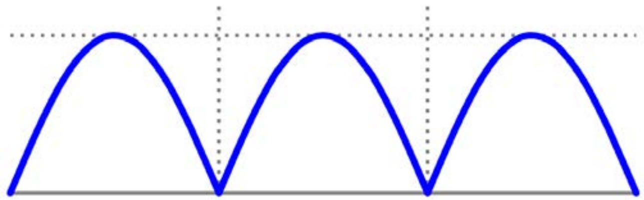


Positive
half-cycle

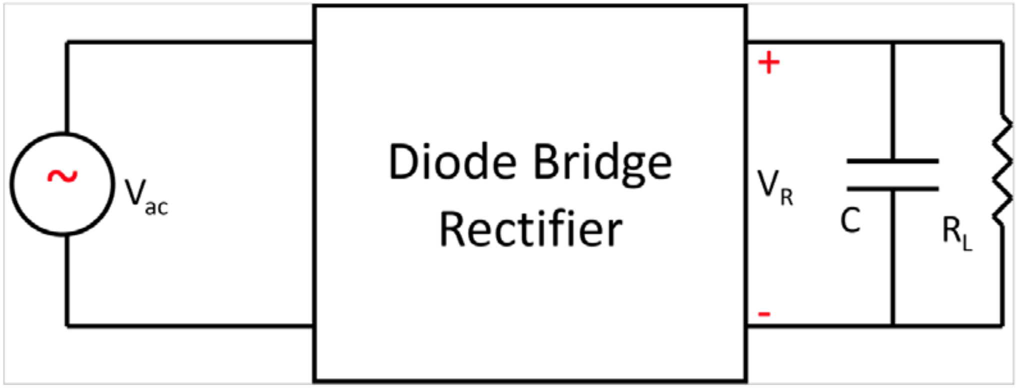
Negative
half-cycle

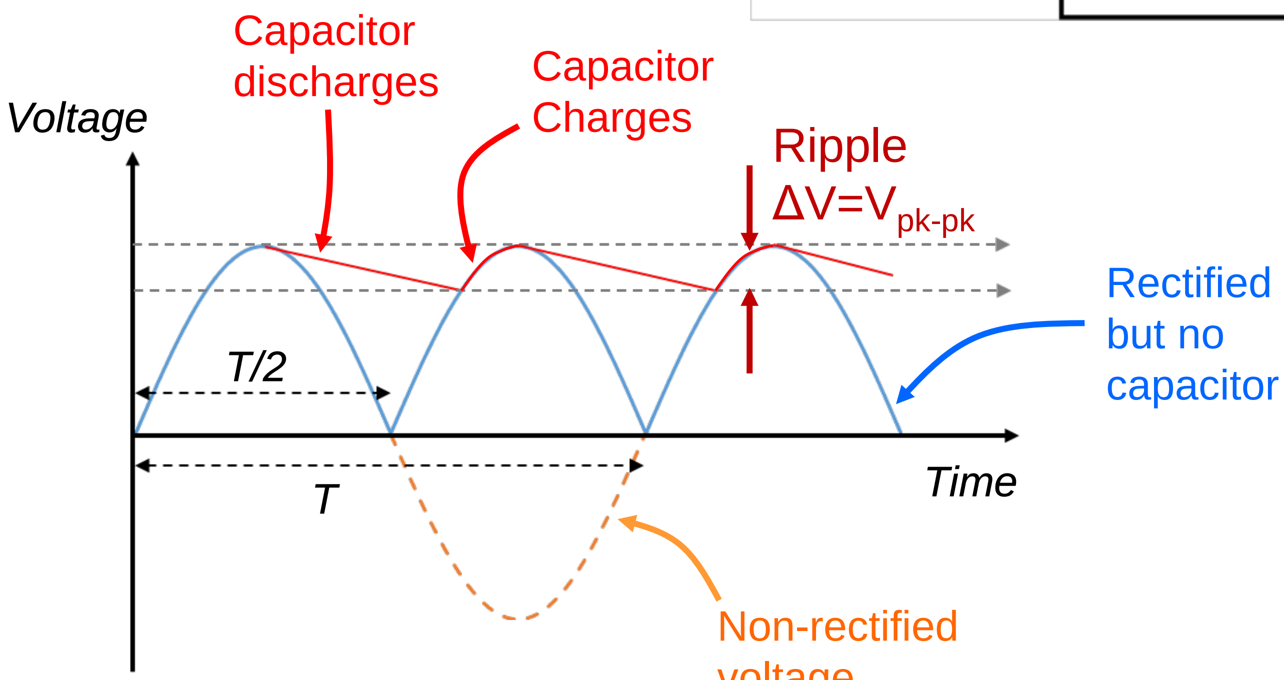




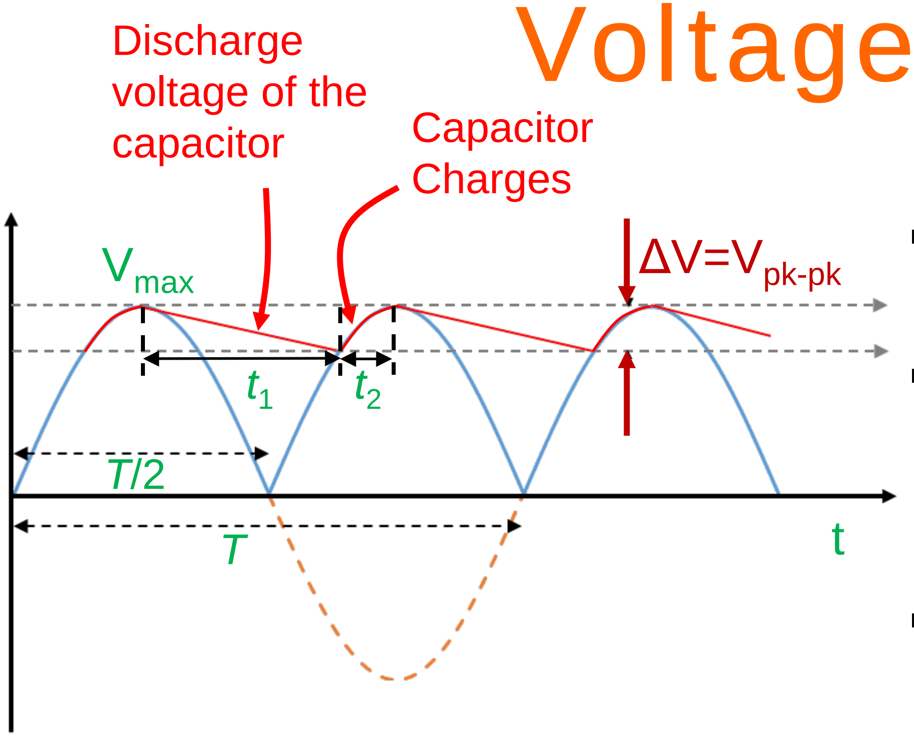


Why Filter Capacitor?





Voltage Ripple



- Let V_{Load}: average load voltage
- Average load current is given by
$$I_L = \frac{V_{Load}}{R_L}$$
- Since it is a full-wave diode bridge rectifier, the pattern repeats every $T/2$

- Let ΔV be the peak-to-peak ripple voltage
- $\Delta V = \frac{\Delta Q}{C}$ (capacitance's definition)
- For small ΔV, $t_1 \approx T/2$. Since $i(t) = \frac{dQ}{dt}$, average current $I_L \approx \frac{\Delta Q}{T/2}$.
- Hence $$\boxed{\Delta V \approx \frac{I_L * T/2}{C} = \frac{V_{Load}}{R_L} * \frac{1}{2f_S} * \frac{1}{C}}$$

Characteristics of DC Motors

$$\boxed{T_{\text{shaft}} = K_t\, I_m \;\; [\mathrm{N.m}]}$$

$$\boxed{E_b = K_e\, \omega \;\; [\mathrm{V}]}$$

For PMDC motor: $\boxed{K_t = K_e}$

Note: $\boxed{\omega = 2\pi \times \frac{RPM}{60} \;\; [\mathrm{rad/s}]}$

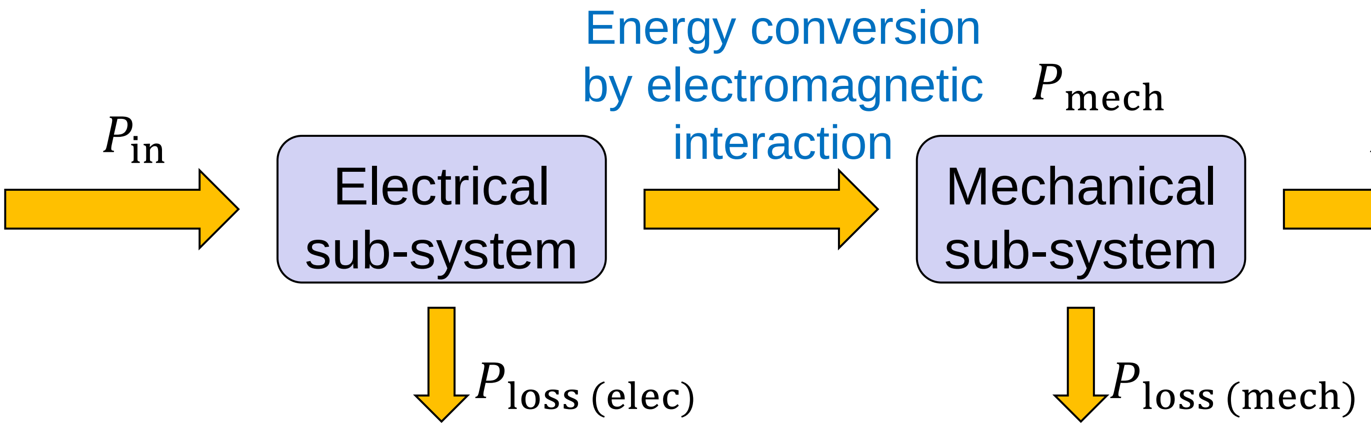
Power Conversion in Motors

- Mechanical power at motor shaft:

$$\boxed{P_{\text{mech}} = T_{\text{shaft}}\,\omega \quad [\text{W}]}$$

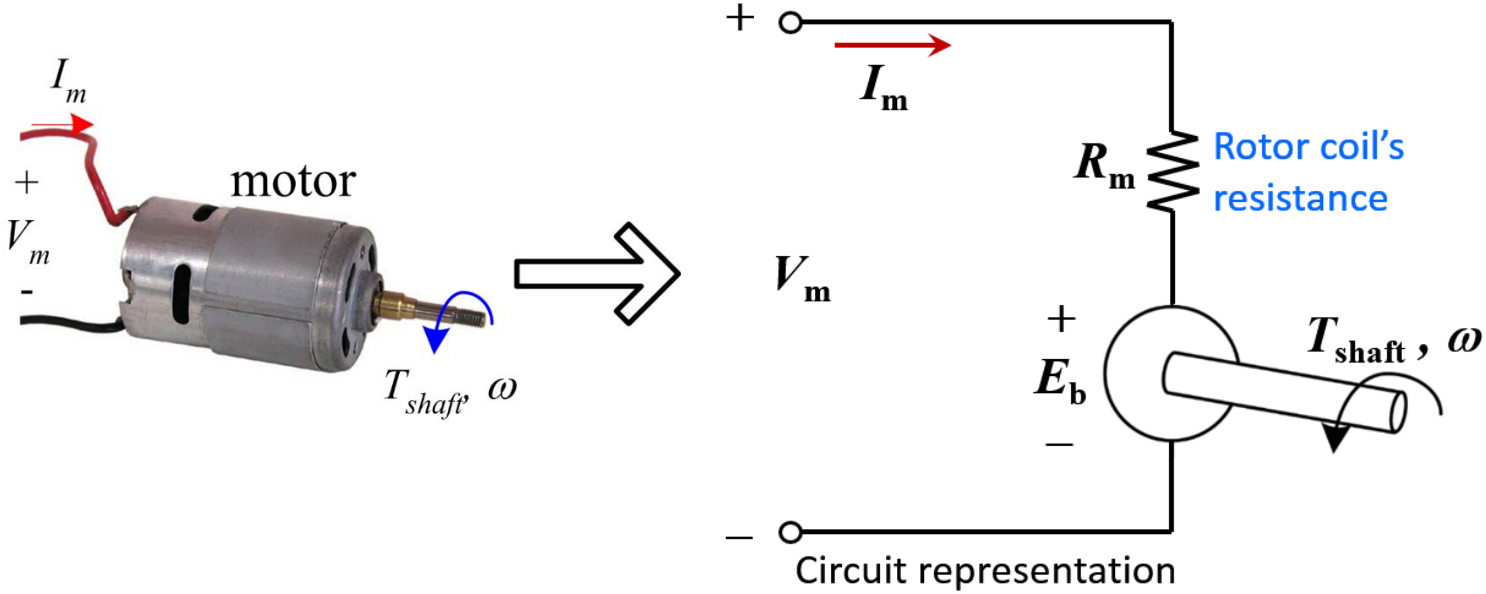
- Electrical power supplied to motor:

$$\boxed{P_{\text{in}} = V_m\, I_m \quad [\text{W}]}$$



$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}}}$$

Circuit Representation: PMDC Motor



- From the circuit: $\boxed{I_m = \frac{V_m - E_b}{R_m}}$
- Since $E_b = K_e\,\omega$, we have: $\boxed{I_m = \frac{V_m}{R_m} - \frac{K_e\,\omega}{R_m}}$

Basic Properties of PMDC Motor

Rearranging: $$\boxed{\omega = \frac{V_m}{K_e} - \frac{R_m\, I_m}{K_e}}$$

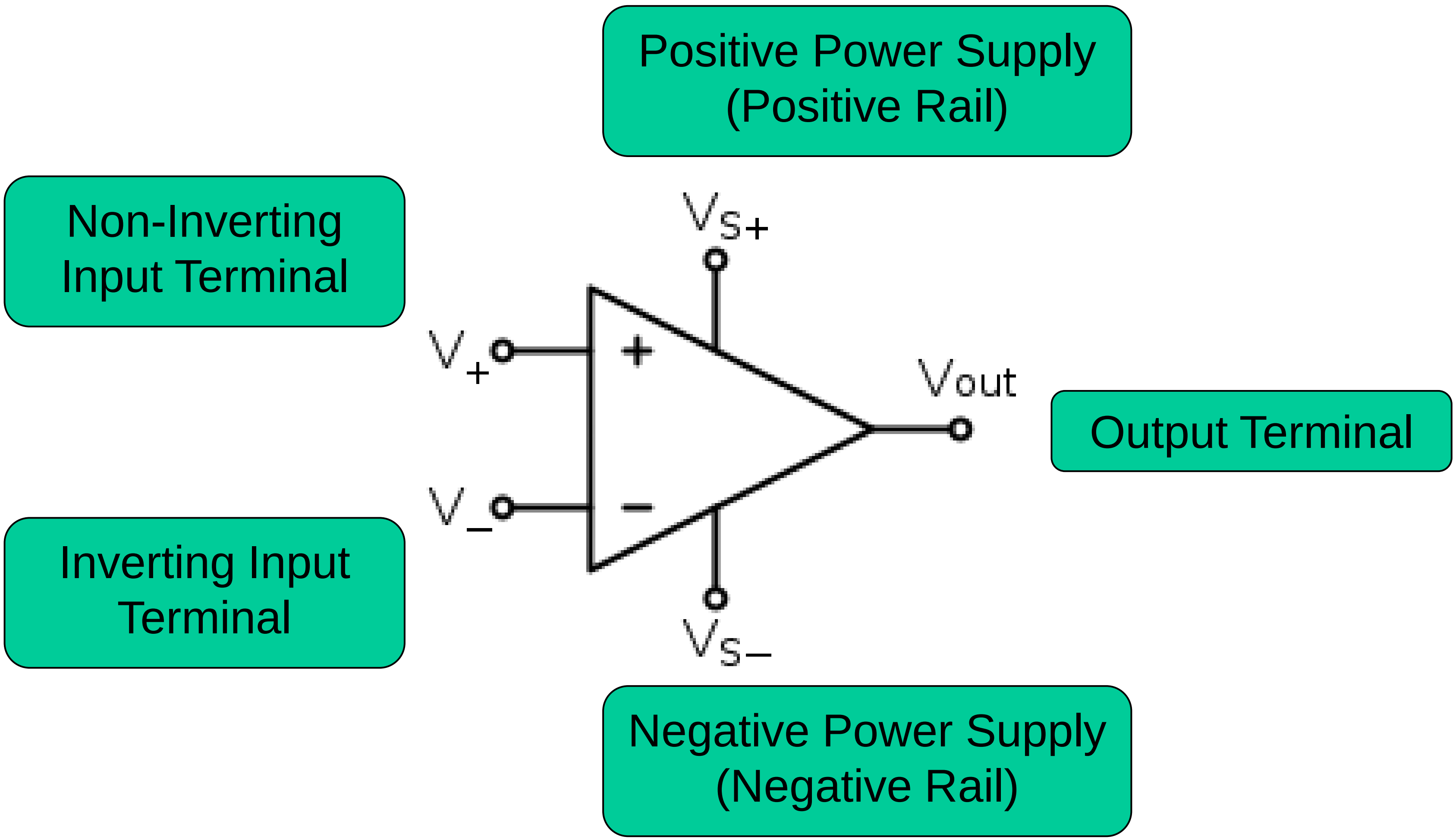
- For a fixed load (i.e., fixed T_{shaft}, which implies fixed I_m since $T_{\text{shaft}} = K_t\, I_m$):

> Shaft speed ω can be increased by increasing motor voltage V_m

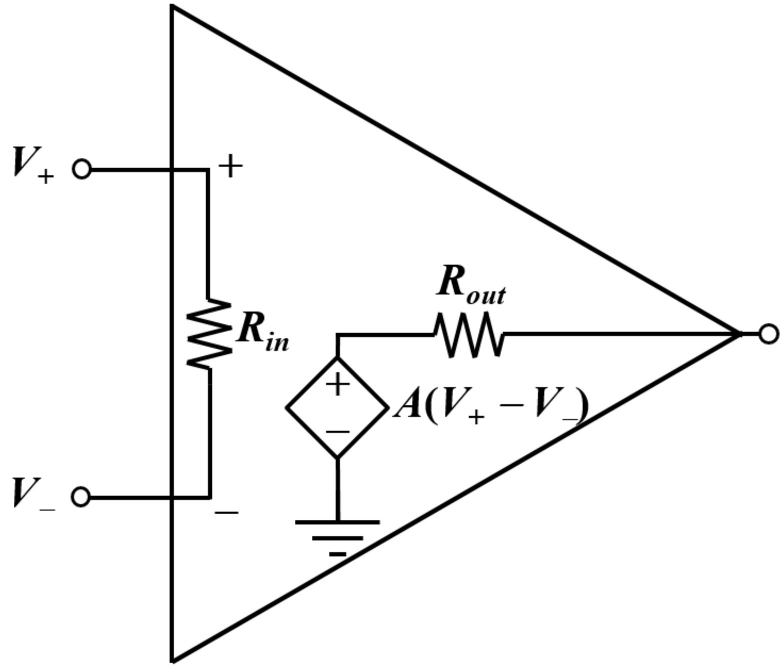
- For a fixed voltage, if T_{shaft} increases, I_m increases, and hence ω decreases:

> Shaft speed ω decreases with increasing load T_{shaft}

Op-Amp Terminals



Op-Amp Equivalent Circuit



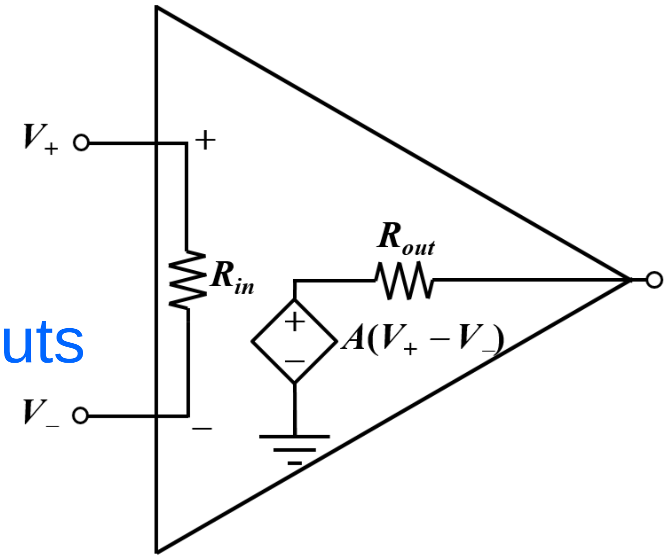
- *A* is the open-loop voltage gain
  - –It is very large, approaching infinity
- R_{in} is the input impedance & R_{out} is the output impedance

Typical Op Amp Parameters

Parameter	Variable	Typical Ranges	Ideal Values
Voltage Gain	A	10^5 to 10^8	∞
Input Impedance	R_{in}	10^5 to 10^8 Ω	∞ Ω
Output Impedance	R_{out}	10 to 100 Ω	0 Ω
Supply Voltage	V_S $-V_S$	5 to 30 V –30 to 0 V	N/A N/A

Op-amp Golden Rules

- Rule 1: In a closed loop, the output attempts to do whatever is necessary to make the voltage difference between the inputs zero
  - –The voltage gain of a real op-amp is so high that a fraction of a mV difference between the V_+ & V_- inputs will swing the output to saturation

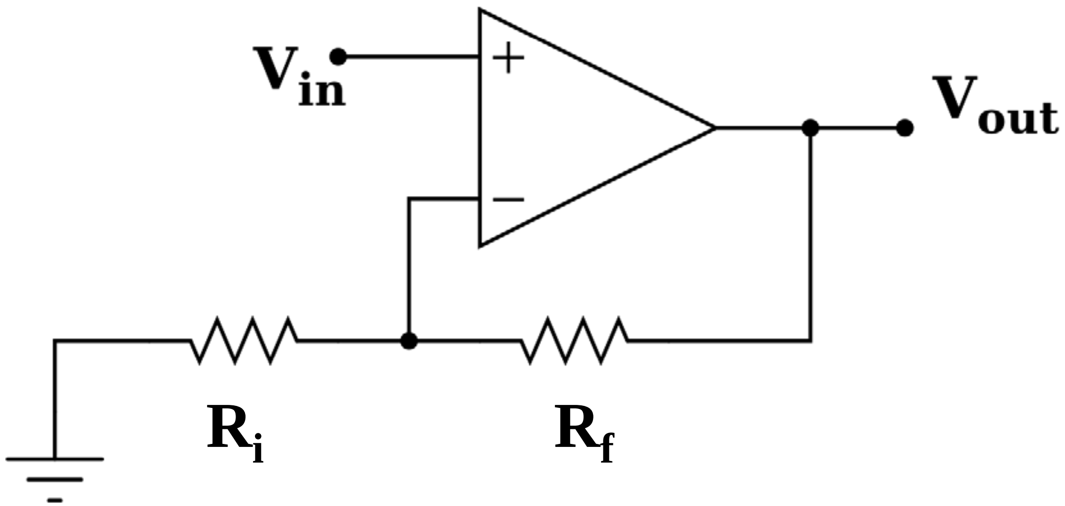


- Rule 2: The inputs draw no current
  - –The ideal op-amp has infinite input impedance (R_{in}). Thus, the current drawn at the two terminals is zero

Non-Inverting Amplifier

- For an ideal op-amp, the non-inverting amplifier gain is given simply by

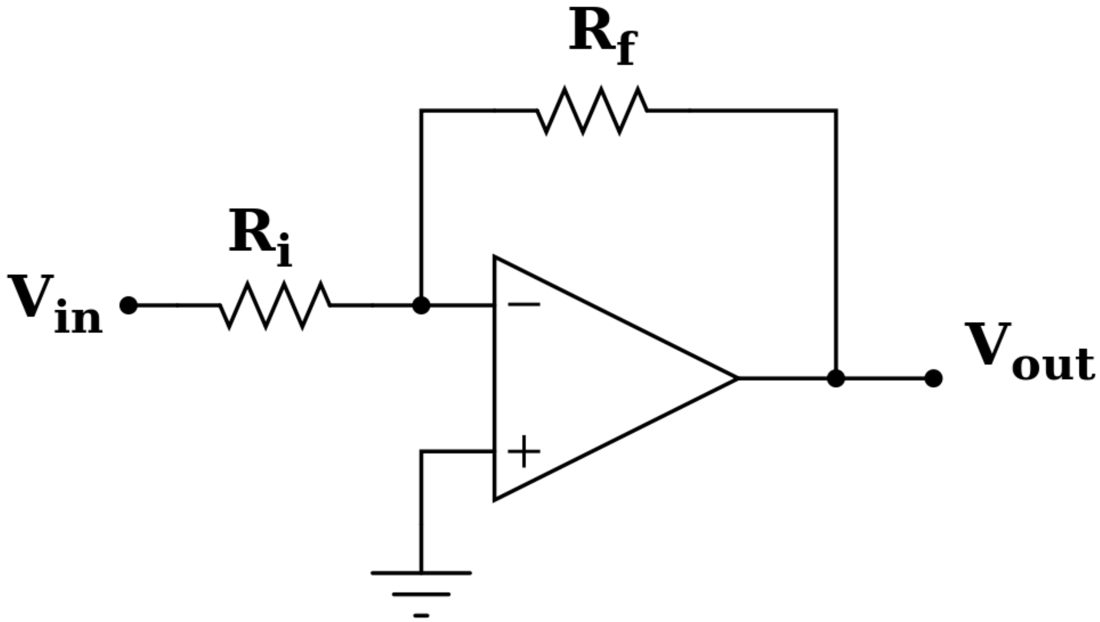
$$\boxed{\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}} = 1 + \frac{R_{\mathrm{f}}}{R_{\mathrm{i}}}}$$



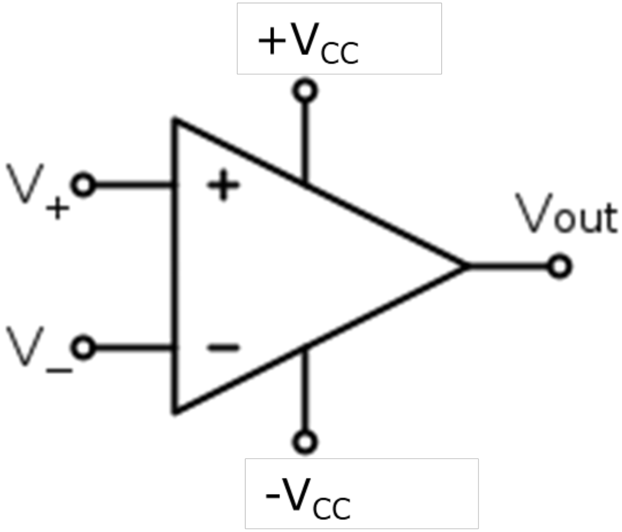
Inverting Amplifier

- For an ideal op-amp, the inverting amplifier gain is given simply by

$$\boxed{\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_{\text{f}}}{R_{\text{i}}}}$$



Comparator

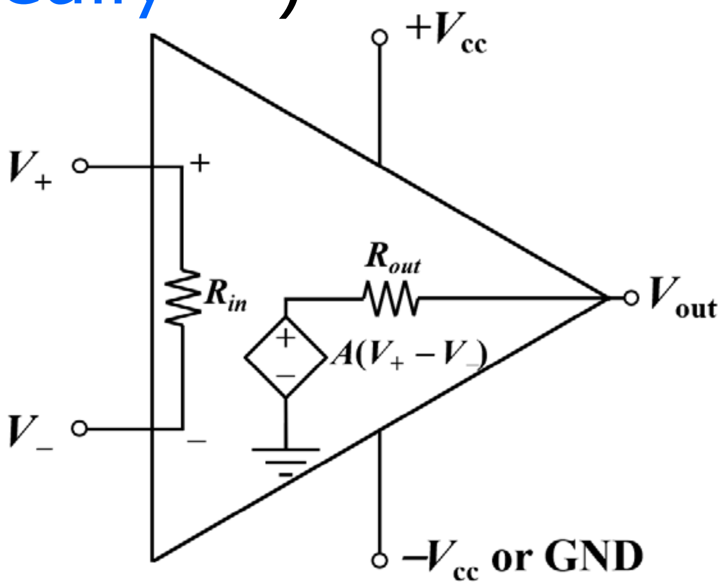


- The comparator is an electronic decision-making circuit that makes use of an op-amp's very high gain in its open-loop state (i.e., there is no feedback resistor)

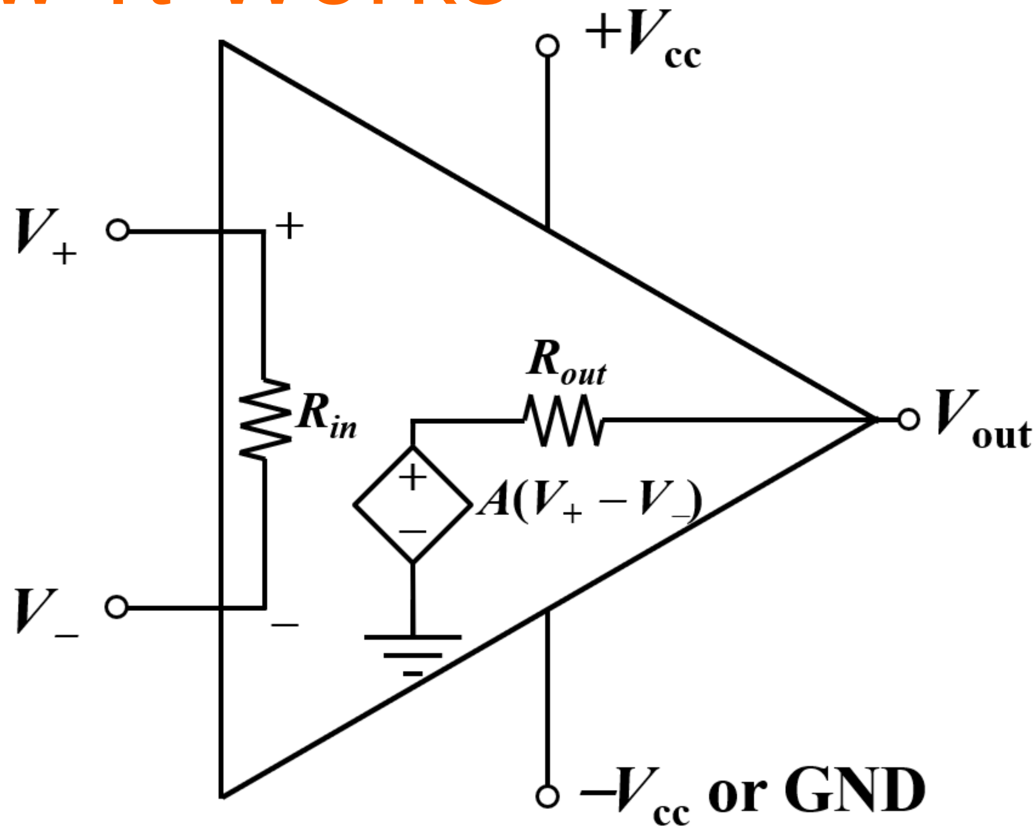
Op-Amp as a Comparator – How It Works

Recall that for op-amp:

- The difference between the two inputs is amplified as '$A(V_+ - V_-)$' at the output
- The open-loop voltage gain ('A') of the op-amp is very high (ideally ∞)
- Even if there is a very small difference between the inputs, the high 'A' will pull the output to "saturation"



Op-Amp as a Comparator – How It Works



- If $V_+ > V_-$:

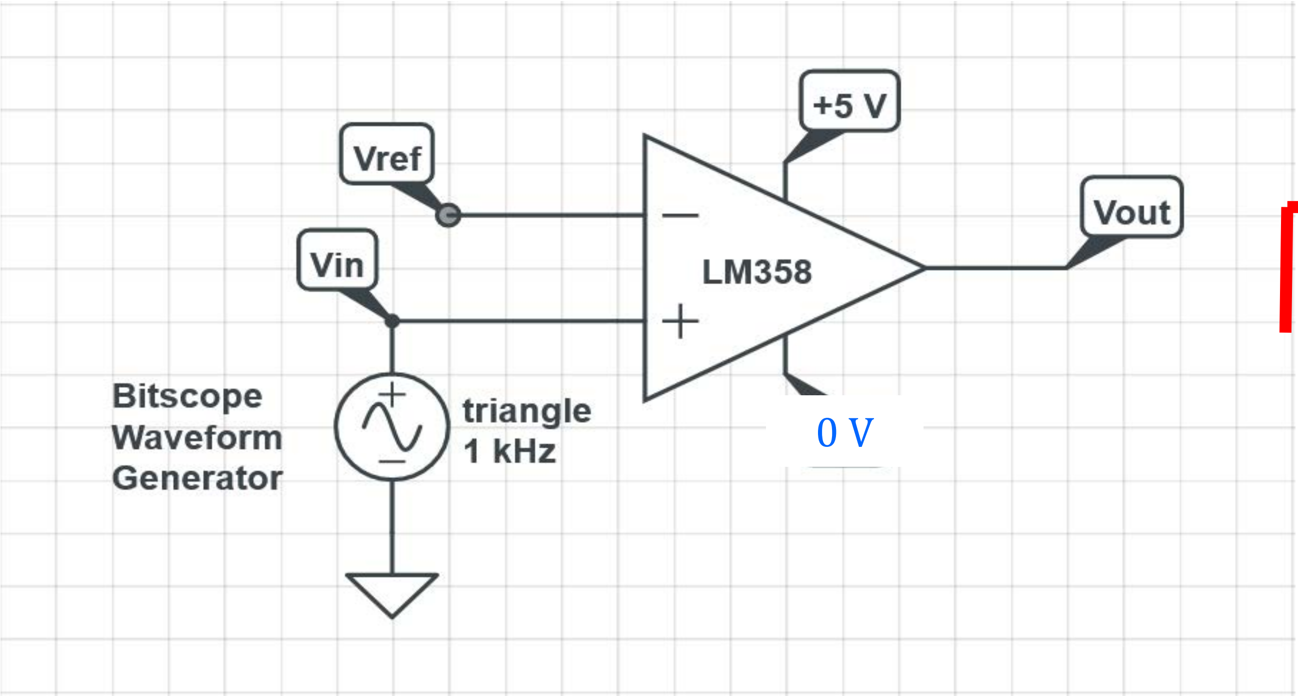
$$V_{\text{out}} = +V_{\text{sat}}$$

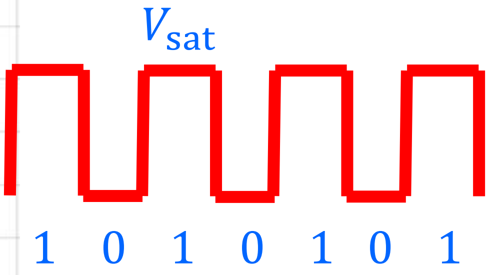
- If $V_- > V_+$:

$$V_{\text{out}} = \begin{cases} -V_{\text{sat}} & \text{if dual power supply} \\ 0 & \text{if single power supply} \end{cases}$$

Common Application of Comparator

- The comparator is ideal for converting analog signals to digital signals at certain threshold values





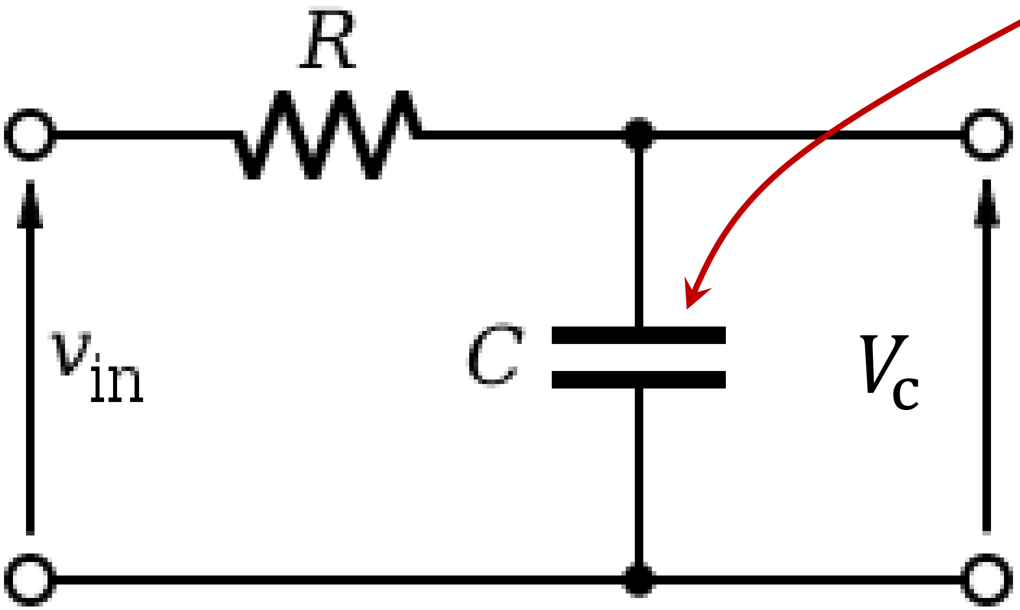
Filter

- A filter is a device or process that removes some unwanted components or features from a signal

- Examples:
  - Removing the noise from measured ECG signal using a filter to help a doctor understand the heart better

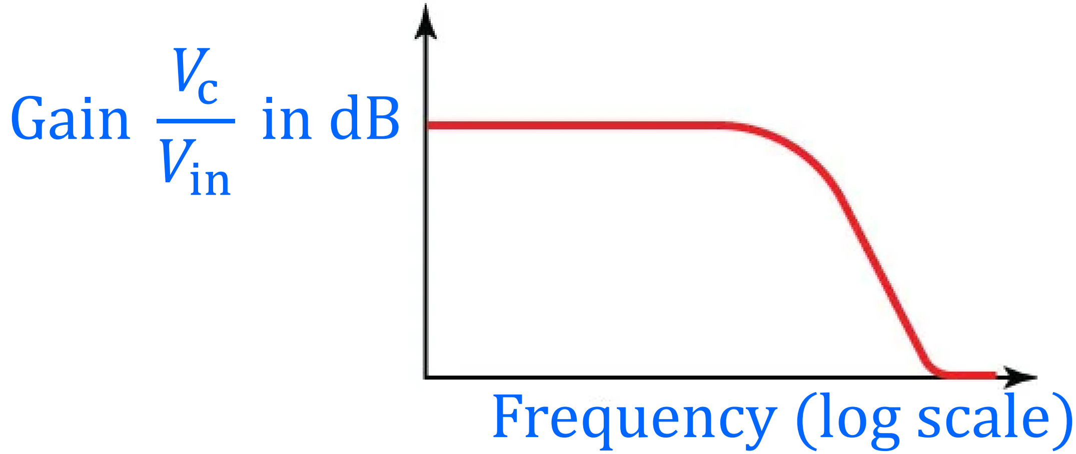
  - Removing some frequencies or frequency bands from an audio signal

Passive Low-Pass Filter



Voltage divider:

The capacitor's impedance decreases as the frequency increases, hence V_c decreases



Power Gain in decibels (dB)

- The Voltage Amplification (A_v) or Gain of a voltage amplifier/filter is given by:

$$\boxed{A_v = \frac{V_{\text{out}}}{V_{\text{in}}}}$$

- The voltage gain is commonly expressed in terms of the resulting power gain in dB:

$$\begin{aligned} \text{Power Gain (dB)} &= 10\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2 \text{ dB} \\ &= 20\log_{10}\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| \text{ dB} \end{aligned}$$

Frequency Response

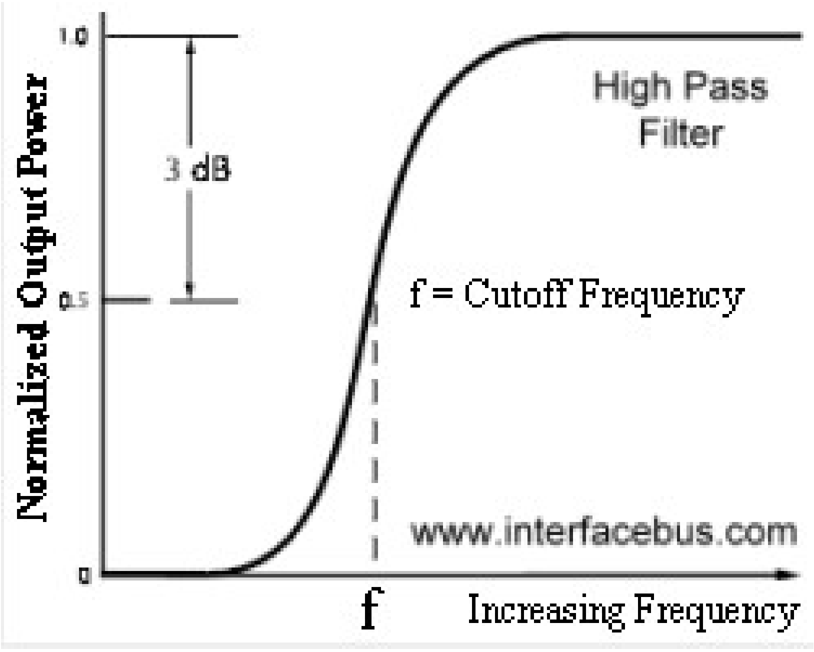
- It is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system
  - Frequency in logarithmic Scale: horizontal x-axis

  - Power Gain in decibels (dB): vertical y-axis
    To describe a change in output power over the whole frequency range

$$\boxed{\text{Power Gain in dB }(f) = 20\log_{10}|A_v(f)|}$$

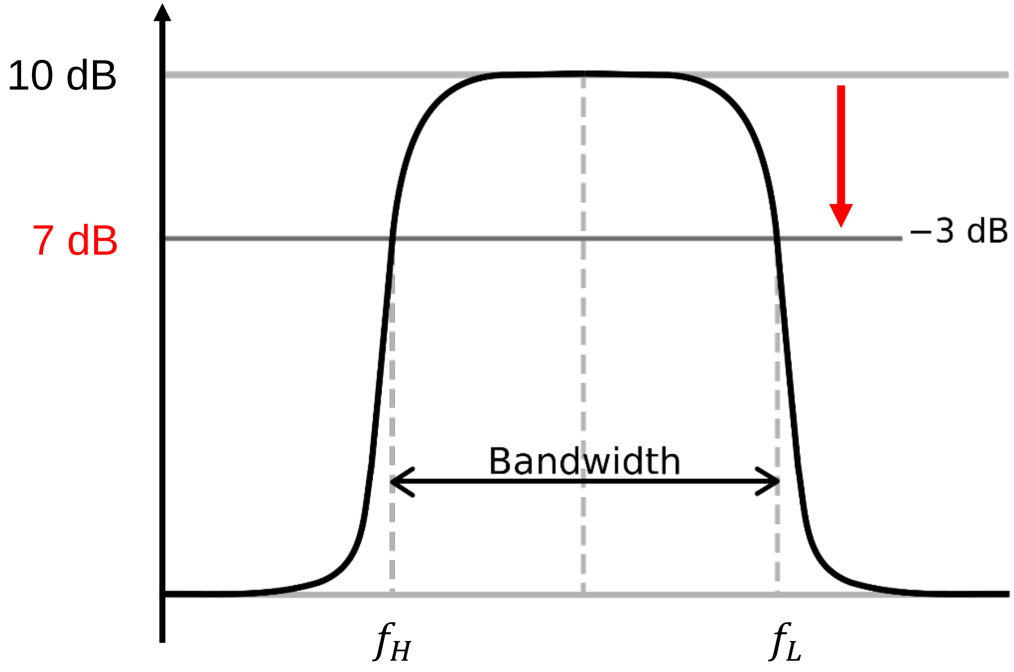
Cut-off Frequency

- In filters, the cut-off frequency characterizes a boundary between a passband and a stopband
- The cut-off frequency is taken as the frequency at which the output of the circuit is −3 dB (corresponding to half the power) of the nominal passband value



Cut-off Frequency: –3 dB Point (i.e., Half-power Point)

- Graphical approach:
  - Find the passband gain from the magnitude vs frequency plot
  - Subtract 3 dB from the passband gain and draw a line on the plot
  - The points where this line cuts the plot corresponds to the cut-off frequency(s)



Cut-off Frequency: –3 dB Point (i.e., Half-power Point)

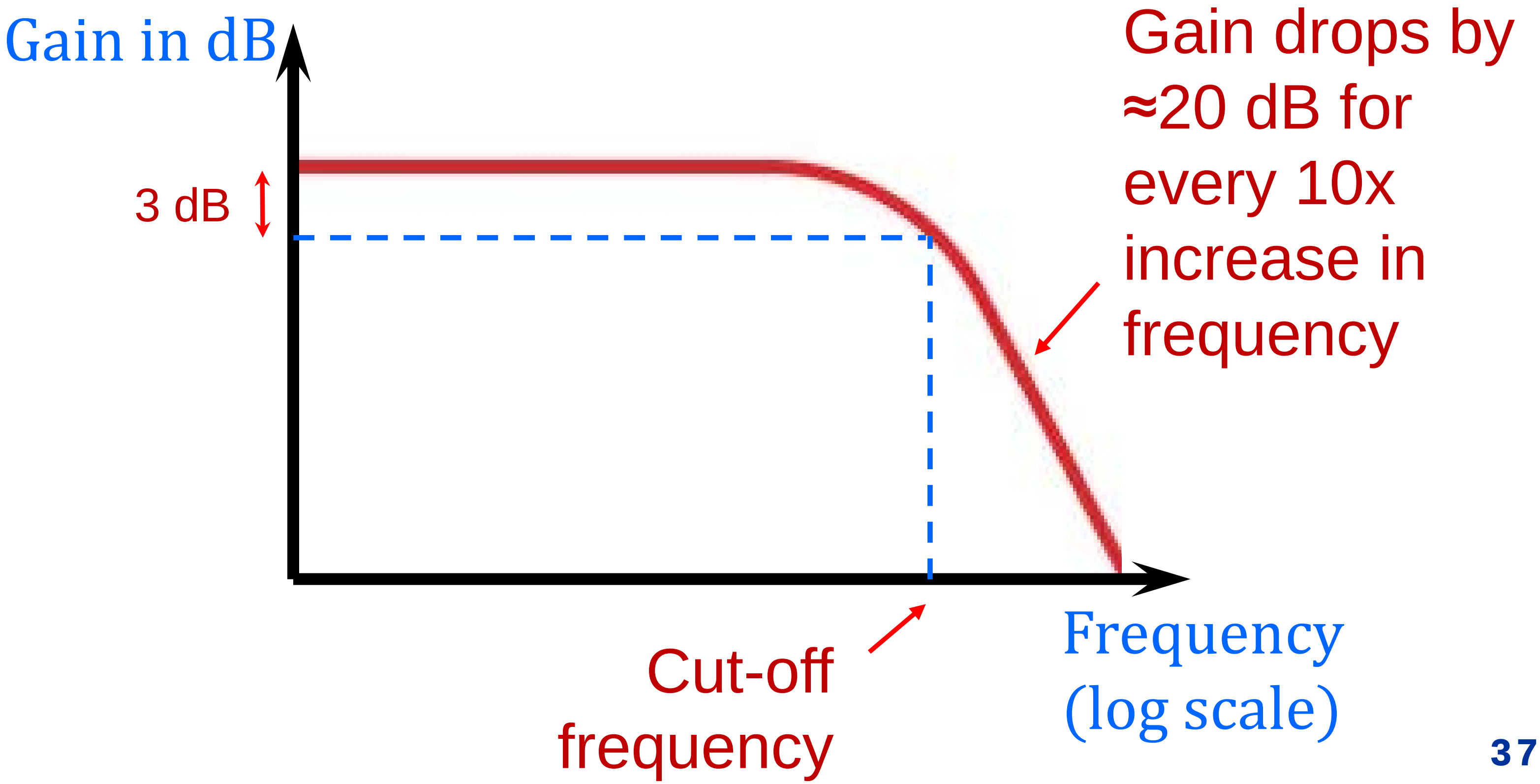
- Quantitative approach (for first-order filters):

$$\boxed{f_H = \frac{1}{2\pi R_H C_H}, \quad f_L = \frac{1}{2\pi R_L C_L}}$$

First-order Low-Pass Filter

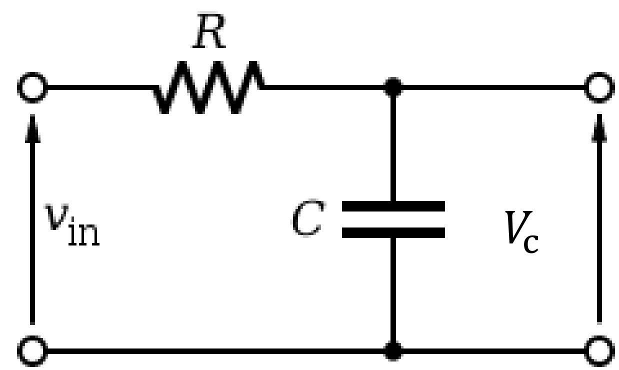
- Slope after cut-off frequency

  ≈ –20 dB/decade



Passive Low-Pass Filter

- We can use the voltage divider rule to find the voltage gain



$$\rightarrow\ V_c = \left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R}\right] V_{\text{in}}$$

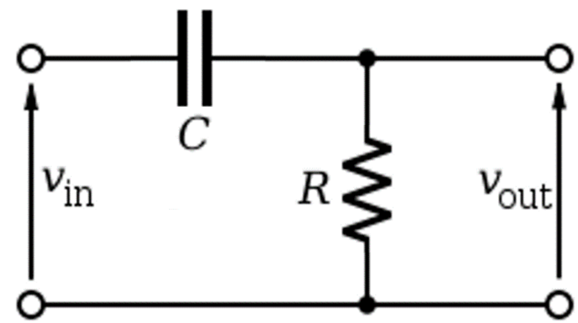
$$\rightarrow\ \frac{V_c}{V_{\text{in}}} = \left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R}\right] = \left[\frac{1}{1+jR\omega C}\right] = \frac{1\angle 0^\circ}{\sqrt{1^2+(R\omega C)^2}\angle\theta^\circ}$$

$$\rightarrow\ \left|\frac{V_c}{V_{\text{in}}}\right| = \left[\frac{1}{\sqrt{1+(R\omega C)^2}}\right]$$

$$\text{Gain in dB} = \underbrace{20\log_{10} 1}_{\text{Passband gain (= 0 dB)}} \underbrace{- 20\log_{10}\sqrt{1+(\omega CR)^2}}_{\text{Change in gain with }\omega}$$

Passive High-Pass Filter

- Using voltage divider rule to find the voltage gain:

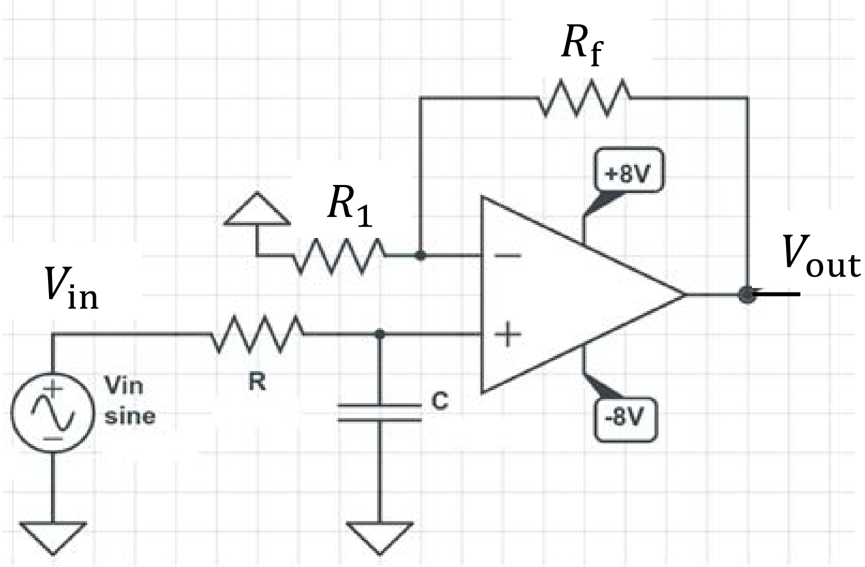


$$\rightarrow V_{\text{out}} = \left[\frac{R}{R+\frac{1}{j\omega C}}\right] V_{\text{in}}$$

$$\rightarrow \frac{V_{\text{out}}}{V_{\text{in}}} = \left[\frac{R}{R+\frac{1}{j\omega C}}\right] = \left[\frac{1}{1-\frac{j}{\omega CR}}\right] = \frac{1\angle 0°}{\sqrt{1^2+\left(\frac{1}{\omega CR}\right)^2}\angle\theta°}$$

$$\rightarrow \underbrace{\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right|}_{\text{Magnitude}} = \left[\frac{1}{\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}\right] = \underbrace{20\log_{10}1}_{\substack{\text{Passband gain} \\ (=\ 0\ \text{dB})}} \underbrace{- 20\log_{10}\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}}_{\substack{\text{Change in gain with } \omega \\ (-3\ \text{dB occurs at } f_H)}}\ \text{dB}$$

Active Low-Pass Filter



$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_+} \times \frac{V_+}{V_{in}} = \left(1 + \frac{R_f}{R_1}\right) \frac{V_+}{V_{in}}$$

Hence, $$\left|\frac{V_{out}}{V_{in}}\right| = \left(1 + \frac{R_f}{R_1}\right) \frac{1}{\sqrt{1 + (R\omega C)^2}}$$

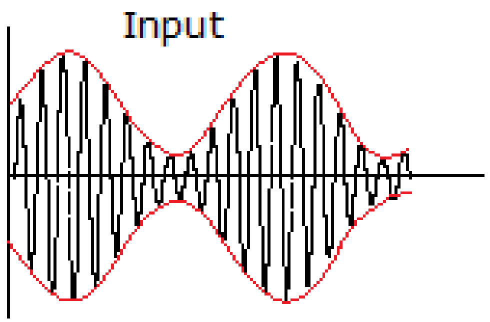
The Need for Amplifying Signals

- Voltage output from sensors may be in the order of mV, e.g., microphone signals

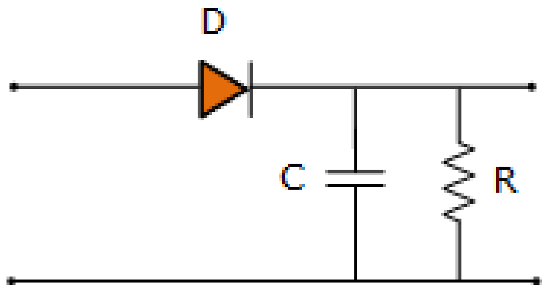
- The sensor voltage output would need to be scaled before A-to-D conversion for more accurate measurements (e.g., using Arduino Uno)

Envelope Detector

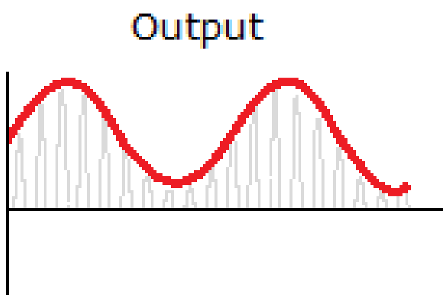
- An envelope detector is an electronic circuit that takes a high-frequency signal as input (**sound**) and provides an output which is the envelope of the original signal
- The capacitor in the circuit stores up charge on the rising edge, and releases it slowly (through the load) when signal falls



(Sound Signal)

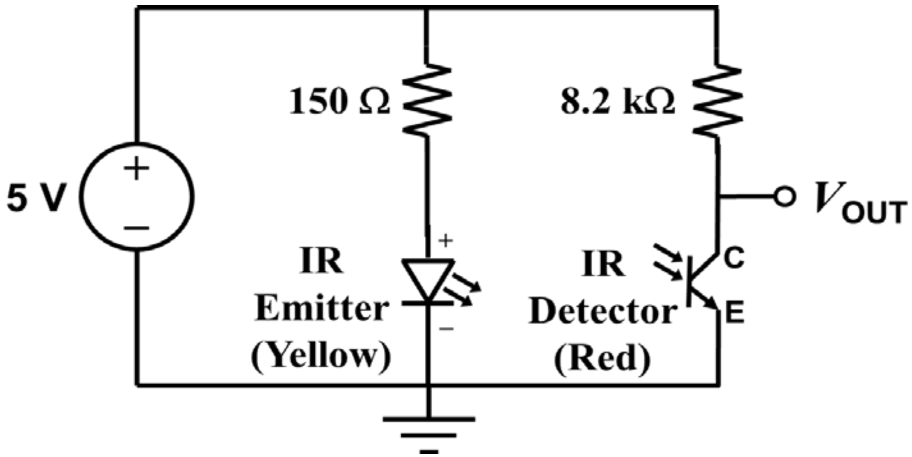


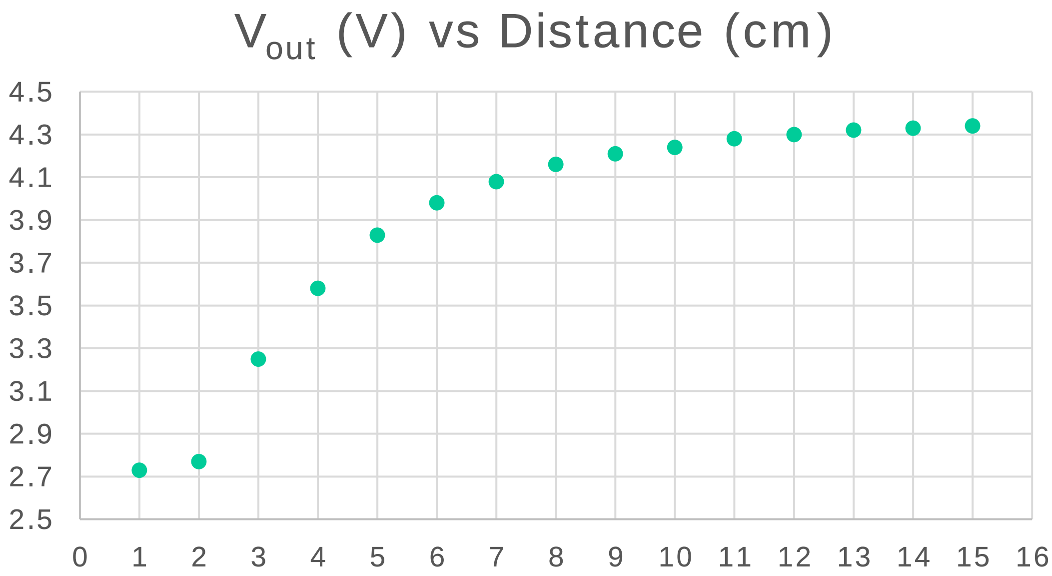
Envelope Detector



(Envelope of the Input)

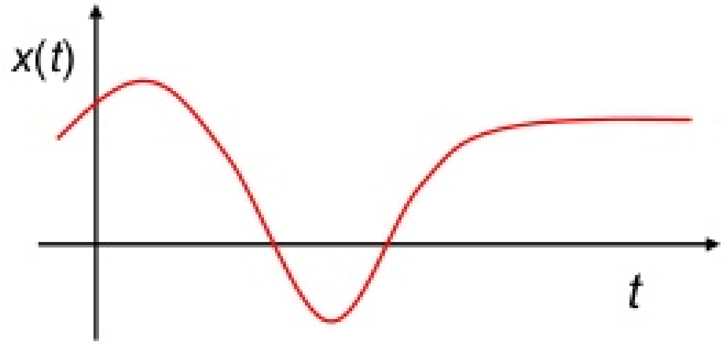
IR Proximity Sensor



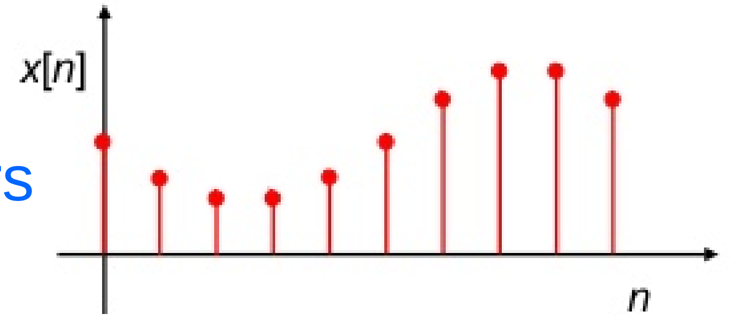


Classification of Signals

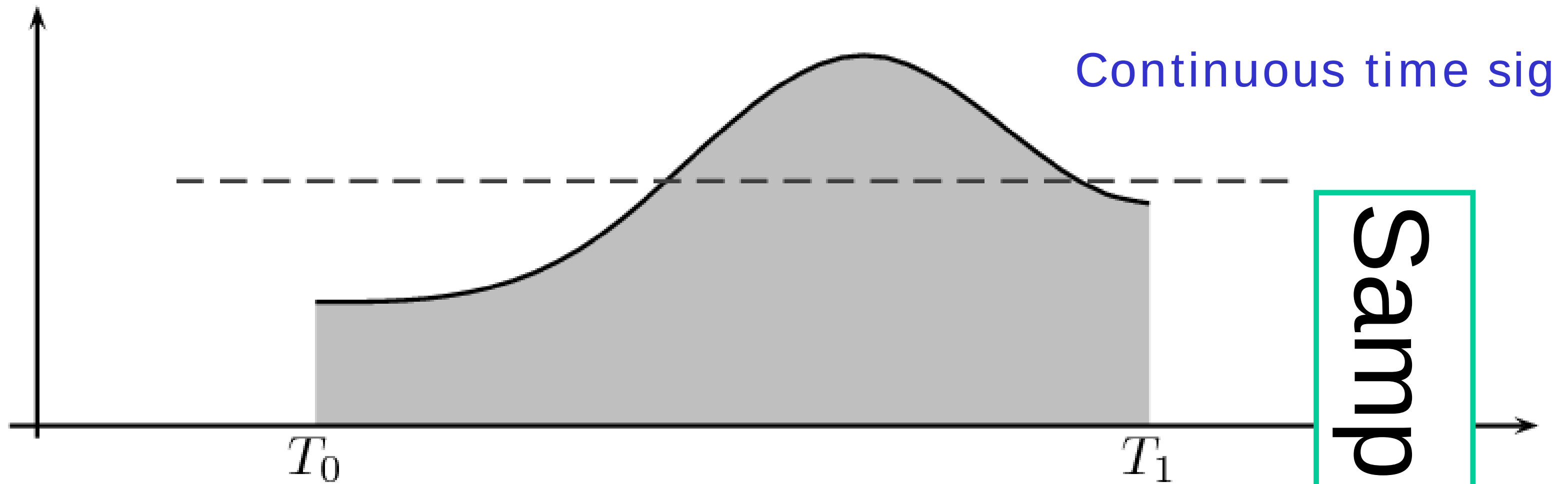
- Continuous signals
  - Independent variable is a continuous variable
  - Examples
    - ✓ Sine wave from a function generator
    - ✓ Speech signal received from a microphone

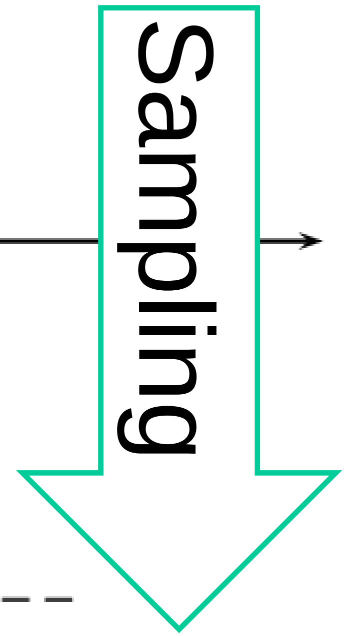


- Discrete signals
  - Independent variable takes on discrete values, e.g., integers
  - Examples
    - ✓ Weekly stock market index
    - ✓ Speech signal stored on a digital computer

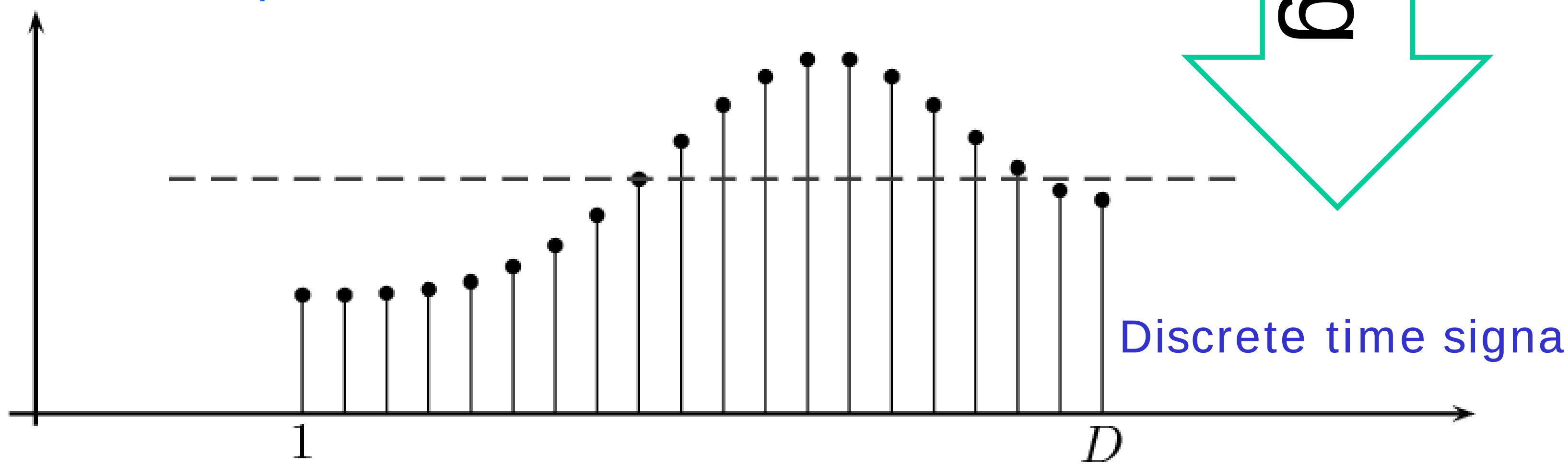


Continuous vs. Discrete Time Signals





The sampling rate or frequency is the number of times a signal is read per second



Sampling Theorem

- How frequently should the signal be sampled to ensure that information contained in the signal is preserved?

$f_s \geq 2 \times$ max frequency component in the signal

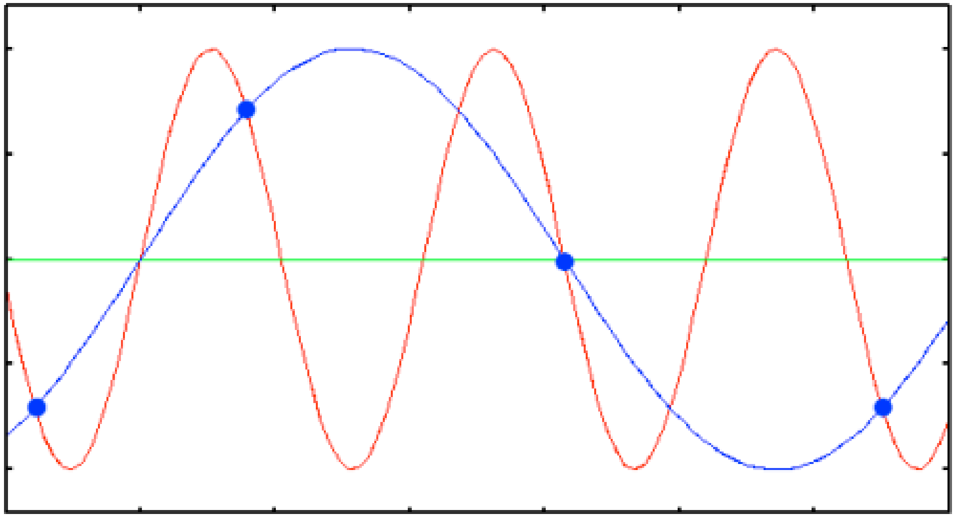
Nyquist Rate & Nyquist Frequency

- The minimum sampling rate that is required to well represent a continuous time signal with highest frequency component f is given by $2 \times f$ and this is known as the Nyquist rate

- For a given sampling rate f_s, perfect reconstruction is possible for a continuous signal whose highest frequency component is $f_s/2$, also known as the Nyquist frequency
  - For example, audio CDs use a sampling rate of 44.1 kHz. Therefore, the Nyquist frequency is 22.05 kHz

Aliasing

- When the sampling rate is lower than the Nyquist rate, the signal reconstructed from samples (using DAC) is <u>different</u> from the original continuous signal
- This effect is known as aliasing



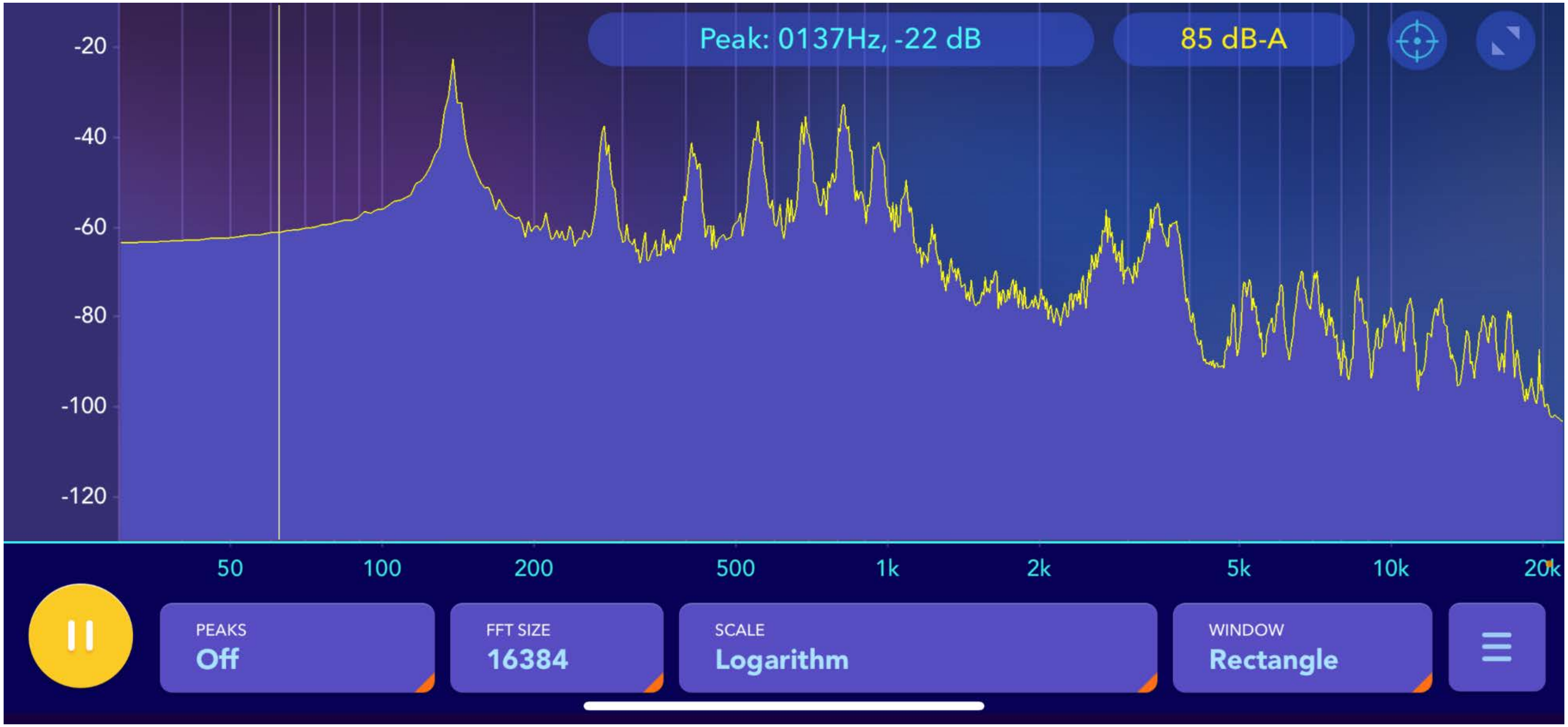
The red waveform represents the original continuous time signal

The blue dots represent samples obtained from the continuous signal with sampling rate < Nyquist rate

Spectral Analysis

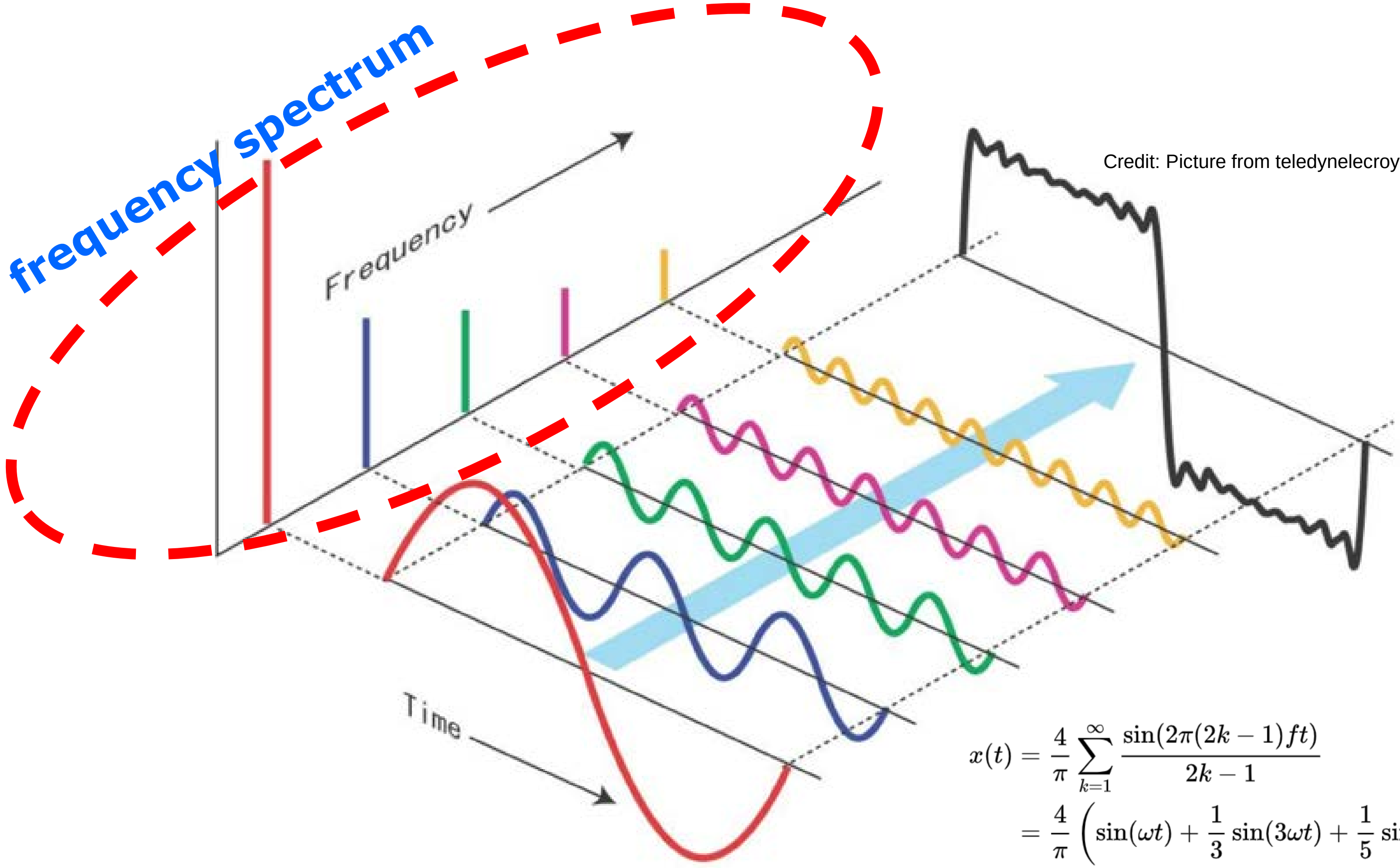
- Any function of time can be described as a sum of sine waves, each with different amplitudes and frequencies
- **Spectral analysis** investigates the distribution of a signal's frequency components
- The plot of a signal's frequency components and their corresponding magnitudes is called "**frequency spectrum**"

Example of Frequency Spectrum



The frequency spectrum of a particular audio tone

Oscilloscope Can Help Us Perform Spectral Analysis

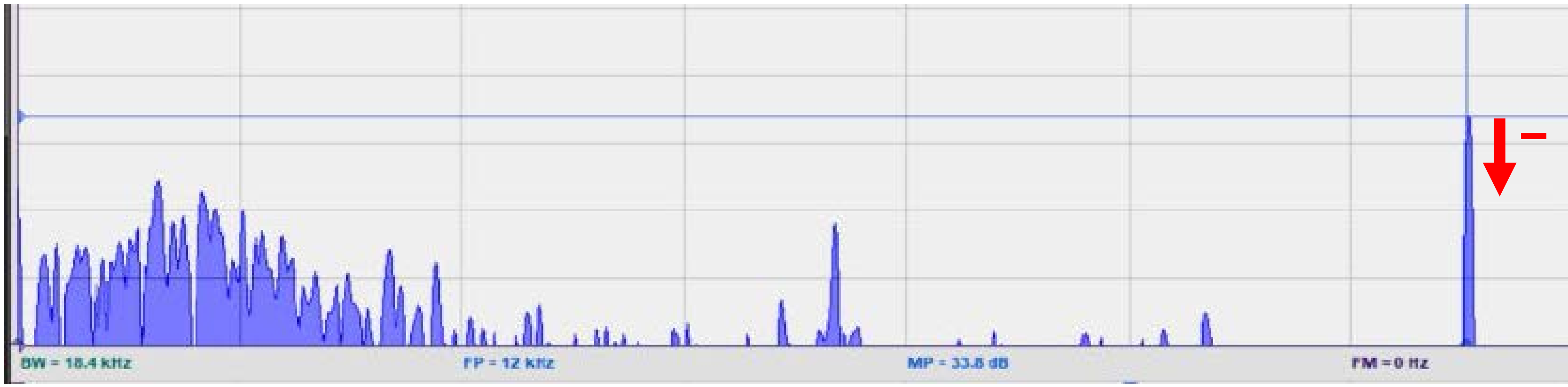


Credit: Picture from teledynelecroy.com

$$x(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi(2k-1)ft)}{2k-1}$$
$$= \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \dots\right)$$

- A square wave (triangular waveform too) can be decomposed into an infinite sum of sinusoidal waves

Filters Can Help Suppress (Attenuate) Undesirable Frequencies



– 10 dB