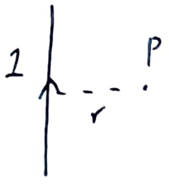
Principles of Transformers

A magnetic field is a region of space where a magnetic pole experiences a force.
Strength of a magnetic field can be expressed by a quantity called magnetic flux density (unit Tesla)

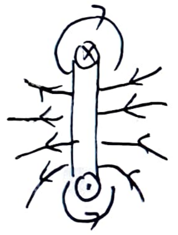
Magnetic fields are produced and experienced by moving charges.

Direction of magnetic field can be determined using Maxwell's right hand rule

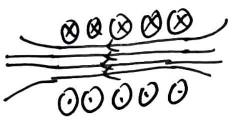


Magnetic flux density B at point P $= \frac{\mu_0 I}{2\pi r}$
and is directed into the paper

Solenoid



per individual coil
by RHR



turns of wire on solenoid closely spaced

net field inside is uniform

Magnetic flux density of an infinitely long solenoid is

$$B = \mu_0 n I$$

Magnetic flux

Magnetic flux is the product of magnetic flux density and the area normal to flux
≈ amount of magnetic field lines passing through the area

$$\phi = BA\cos\theta = \vec{B} \cdot \vec{A}$$

Electromagnetic Induction

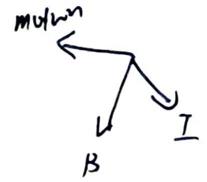
induced current flows in a direction to oppose the change that produced induced current
Faraday's law of electromagnetic induction states that the induced emf is proportional to rate of change of magnetic flux linkage $N\phi$

$$E = -\frac{d(N\phi)}{dt}$$

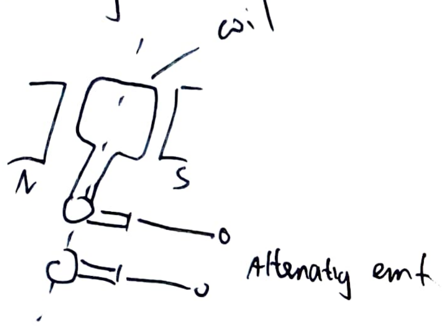
If source is DC, magnetic flux is constant, hence no induced emf

If source is AC, there is varying magnetic flux, hence emf is generated

direction of emf determined by Fleming's RHR



Alternating current is generated by electromagnetic induction



A flat coil of N turns and area A is placed in magnetic field of flux density B

magnetic flux of each turn $= BA\cos\theta$

total linkage $= NBA\cos\theta$

If the coil is made to turn with angular velocity ω, $\theta = \omega t$

By Faraday's Law,

$$E = -\frac{d(N\phi)}{dt}$$

$$= -NBA\frac{d\cos\omega t}{dt}$$

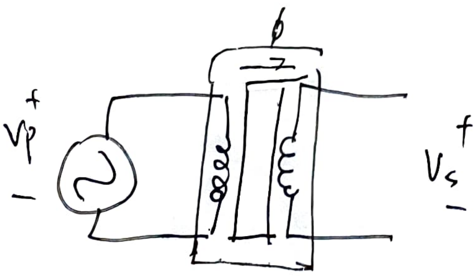
$$= NBA\omega\sin\omega t$$

Transformer

a device that uses mutual electromagnetic induction to step up or down an alternating voltage

consists of 2 coils, primary and secondary wound on an iron core

when AC flows through primary coil, it sets up a varying magnetic field in iron core and links primary coil to secondary coil



$$V_p = N_p\frac{d\phi_p}{dt} = N_pBA\omega\sin\omega t$$

$$V_s = N_s\frac{d\phi_s}{dt} = N_sBA\omega\sin\omega t$$

If transformer is ideal and no flux leakage, $\phi_p = \phi_s$

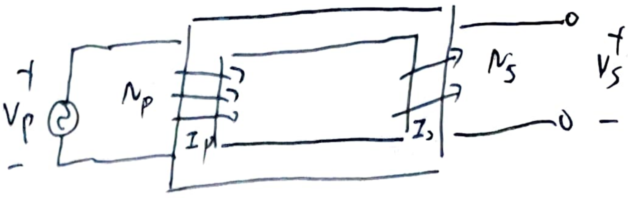
$$\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

Ampere's Law

Magnetic flux along path equals the net current enclosed by the path

$$\oint B \cdot dl = N \mu_0 I$$

$$\frac{B \, l_{path}}{\mu_0} = I_p N_p - I_s N_s$$



For an ideal transformer, magnetic permeability is $\infty \Rightarrow \frac{Bl}{\mu_0} = 0$

$$I_p N_p = I_s N_s \Rightarrow \boxed{\frac{I_p}{I_s} = \frac{N_s}{N_p}}$$

Energy is conserved for ideal transformer

$$\text{input} = V_p I_p = \left(\frac{N_p}{N_s} V_s\right)\left(\frac{N_s}{N_p} I_s\right) = V_s I_s$$

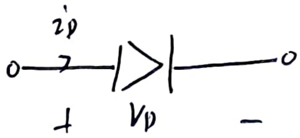
For complex analysis, are conjugates, $V_p I_p^*$

Applications

high voltage transmission is more efficient due to low power loss in cables I^2R

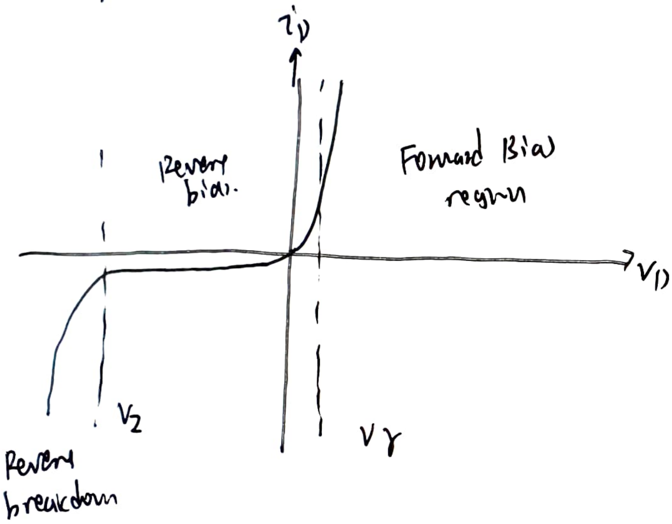
Diodes and Bridge Rectifiers

PN junction diode



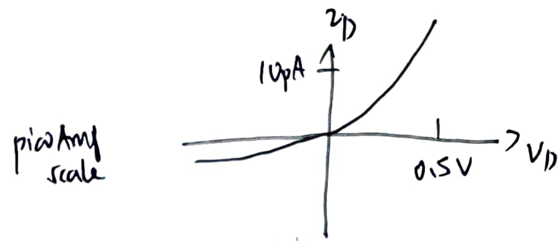
A diode is a polarized device that allows current to flow in only 1 direction

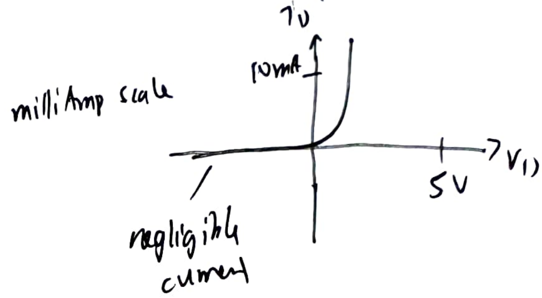
current is exponentially related to voltage across its terminals



Analytical expression for relation between V_D and i_D of a diode

$$i_D = I_S\left(e^{\frac{V_D}{V_{TH}}} - 1\right)$$





Forward Bias region

positive voltage is applied across the diode

when voltage exceeds threshold V_γ, diode starts to conduct current, which grows exponentially with increased voltage

low resistance in this region

Assumed state $\approx V_D = 0$ (short circuit)

Diode is in ON state

Reverse Bias Region

negligible current flows through diode

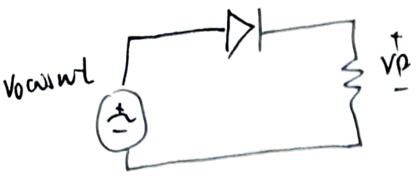
high resistance

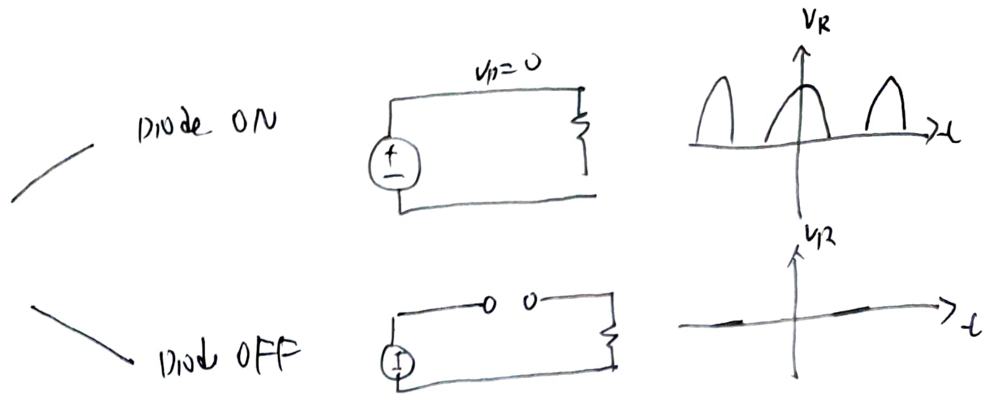
Diode is OFF state

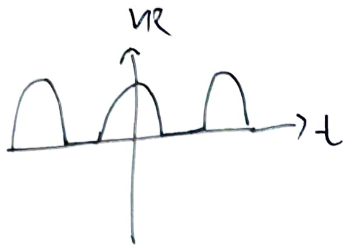
Assumed state $\approx i_D = 0$ (open circuit)

Beyond a certain limit, the diode breaks down, known as breakdown voltage

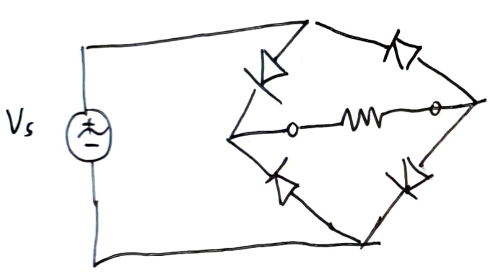
Half wave rectifier

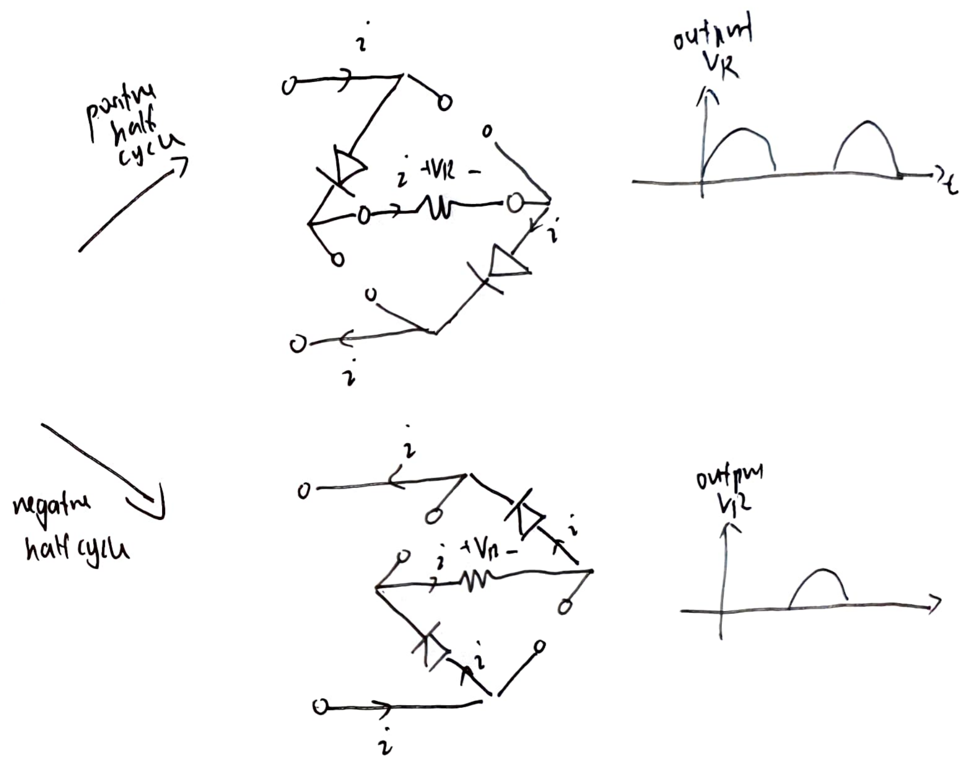


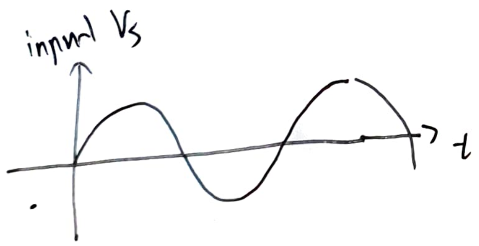


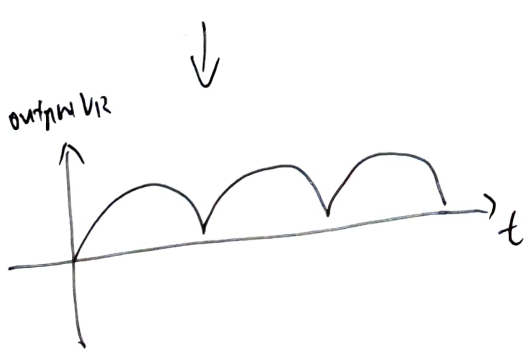


Diode Bridge rectifier





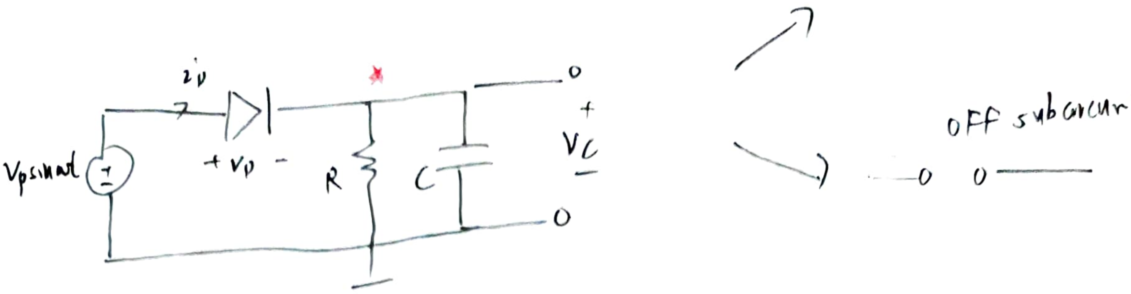




full rectified DC

Nonlinear analysis with RL and RC

ON subcircuit



$$V_D = \cancelto{V_i}{V_p \sin\omega t} - V_c$$

$$i_D = I_s\left(e^{q\overbrace{(V_i - V_c)}^{V_D}\overbrace{/kT}^{V_T^{-1}}} - 1\right)$$

At node * $\quad i_D = \dfrac{V_c}{R} + i_c$

$$= \frac{V_c}{R} + C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = -\frac{V_c}{RC} + \frac{I_s}{C}\left[e^{q(V_i - V_c)/kT} - 1\right]$$

Analyse by piecewise method

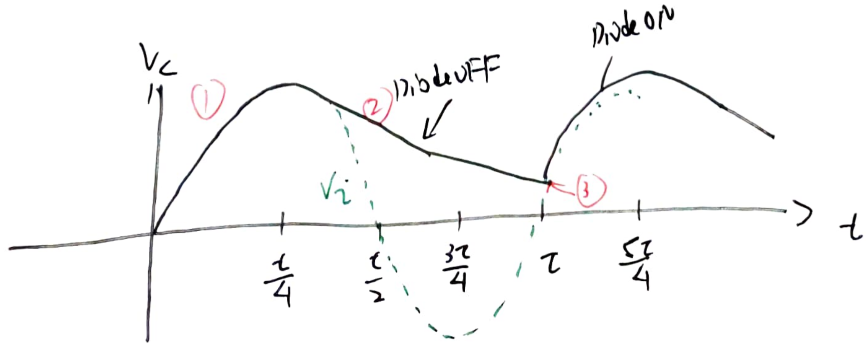
For diode ON $\quad V_i = V_c \quad (V_D = 0)$

For diode OFF, capacitor is disconnected from drive $\quad (i_D = 0)$

$$V_c = V_c(0)\, e^{-\frac{t}{RC}}$$

In ON state $\quad i_D > 0 \Rightarrow \left[\dfrac{V_c}{R} + C\dfrac{dV_c}{dt}\right] > 0$

In OFF state, to keep V_D negative $\quad V_i < V_c$



solution for sine wave V_i on output V_C [illegible]

(1) In the first quarter cycle, AC source is charging capacitor, $i_D > 0$ diode is ON

After peak, capacitor starts to discharge, $i_D = 0$, diode OFF

(2) simple exponential discharge follows $V_C = V_C(0)e^{-\frac{t}{RC}}$

(3) At some point, input voltage V_i rises to surpass capacitor voltage

$V_i > V_C$ which violates $V_i < V_C$ for OFF state condition

diode is ON again

* It is desirable to have RC time constant much longer than sine wave period

Thus, diode switches to OFF state ($V_i < V_C$) only a few degrees beyond peak

$V_C(0)$ is roughly peak sine wave input

V_C discharge $\approx V_p e^{-\frac{(t-\frac{\tau}{4})}{RC}}$

For long time constant RC, discharge also becomes approximately linear

$$V_C \approx V_p\left(1 - \frac{t-\frac{\tau}{4}}{RC}\right)$$ (maclaurin of e^x)

ON state when $V_i = V_C \Rightarrow V_p \sin \omega t = V_p\left(1 - \frac{t-\frac{\tau}{4}}{RC}\right)$

peak to peak ripple $\approx V_p \frac{\tau}{RC}$

If RC time constant is chosen to be 10X period peak to peak ripple $\approx 10\% V_p$

DC voltage is ave value of V_C $= \frac{1}{\tau}\int_0^{\tau} V_p\left(1 - \frac{t}{RC}\right) dt$

$= V_p\left(1 - \frac{\tau}{2RC}\right)$

$RC = 10\tau$

$DC \approx 0.95 V_p$

Week 7 Studio

Activity 1

1. The observed waveform is sinusoidal
9V is the root mean square voltage.

Actual measured voltage = 9.60V rms

3.

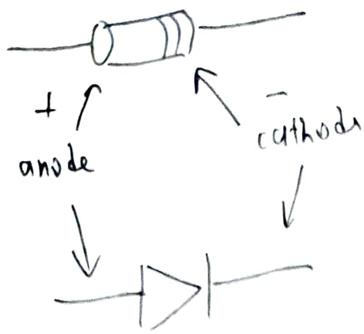
	Vmax	primary	secondary	calculated maximum voltage (V)	Measured Maximum voltage (V)	step up or step down?
e.g	20V	0-12V	0-18V	20.3	20.8	step up
	13.5V	0-9V	0-9V	13.5	13.8	no change
	27V	0-9V	0-18V	27	25.2	step up
	6.5V	0-18V	0-9V	6.75	7.4	step down
	10V	0-12V	0-9V	10.1	10.6	step down

4. There is variation between measured voltage and calculated voltage

voltage regulator LM7805 → 5V regulator

any input
5V output

Activity 2



Vavg across 1kΩ

$$\Delta V \approx \frac{V_{load}}{R_L} * \frac{1}{2f_s} * \frac{1}{C}$$

1kΩ frequency

3. without capacitance

rms of rectified voltage = 6.00 V

V_{max} = 9.20 V

e.g. 10µF $\frac{6.8}{1000} \times \frac{1}{2 \times 100.2} \times \frac{1}{10 \times 10^{-6}}$

$\approx 3.4V$

voltage ripple primer

Capacitance	Vavg	Vrms	Vmax	Vmin	voltage ripple ΔV
no C	5.40 V	6.00 V	9.20 V	0.00 V	—
10 µF	6.80 V	7.00 V	9.20 V	4.80 V	3.4 V
22 µF	7.80 V	7.80 V	9.20 V	6.40 V	1.78 V
100 µF	8.40 V	8.40 V	9.20 V	8.20 V	0.42 V
2200 µF	8.60 V	8.60 V	8.80 V	8.60 V	0.02 V

10. The larger the capacitance, the lower the voltage ripple and the better the output voltage can be used as a DC source.