CG1111 Week 3 Studio 1 Grp 4b

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Electric charge

physical property of matter that causes it to experience a force when placed in an electromagnetic field

charge of an electron $q_e = -1.602 \times 10^{-19}$ C [C] coulomb

movement of electrons in conductors $\rightarrow$ electric current

Electric Current

time rate of flow of electrical charges through an element

$I = \frac{dq}{dt}$ [A] $1 \text{ Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ second}}$

positive current (conventional current) in the direction of flow of positive charges
opposite flow of electrons in conductors

Voltage

difference in electric potential between 2 points.

work needed per unit charge to move a test charge between the 2 points.

[V] $1 \text{ Volt} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}}$

'+' terminal has higher potential than '−' terminal

Electric Power

voltage × current = rate of energy transfer / power

[W] $1 \text{ Watt} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}} \times \frac{1 \text{ Coulomb}}{1 \text{ second}} = \frac{1 \text{ Joule}}{1 \text{ second}}$

Resistance

over a <u>limited</u> range of voltage and current, the voltage measured across the terminals of a resistor is linearly proportional to current flowing through it

$V = IR$ (Ohm's law) [Ω]

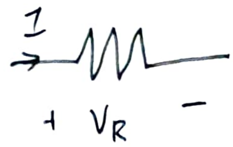
constant of proportionality known as resistance

resistivity (Ωm)

$R = \frac{\rho l}{A}$ — length (m), area (m^2)

dependent on geometry and material of element

Practical Circuit Elements



resistors are used to control behavior in other parts of circuit

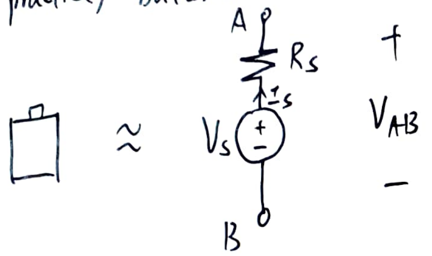
e.g. limit current flowing through an LED to control brightness

voltage drop across resistor $V = RI$

power loss $P = VI = I^2R$

voltage sources supply constant voltage

in practice, batteries have internal resistance



$\approx$ simple case of a thevenin circuit ?

V_{TH} R_{TH}

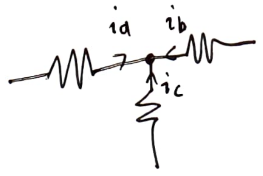
Kirchoff's laws

Maxwell's Equations are simplified into 2 algebraic relationships — KVL, KCL — under certain constraints. (lumped matter discipline)

Kirchoff's Current Law : The current flowing into any node must equal the current flowing out

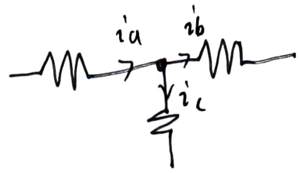
$$\sum_{k=1}^{N} i_k = 0$$

direction matters.



$i_a + i_b + i_c = 0$

vs

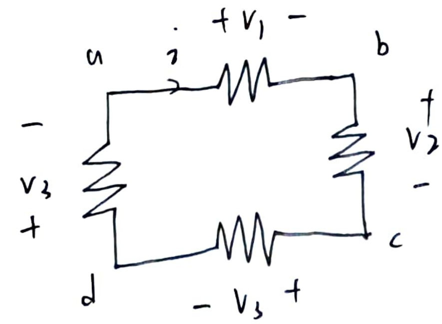


$i_a = i_b + i_c$ or $i_a - i_b - i_c = 0$

If KCL does not hold at a node, electric charge must accumulate at that point which is inconsistent with lumped matter discipline that $\boxed{\frac{\partial q}{\partial t} = 0}$

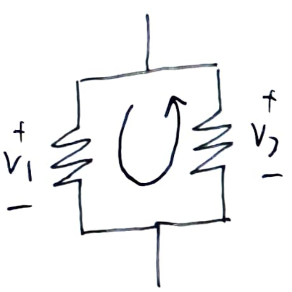
* KCL is also a statement of the conservation of charge

Kirchoff's Voltage Law : The sum of branch voltage around any closed loop in a circuit is zero



$$v_1 + v_2 + v_3 + v_4 = 0$$

* polarity is important
need to be consistent with assignment of polarity to each voltage
positive voltage when going from positive to negative terminal



$$v_1 + (-v_2) = 0$$

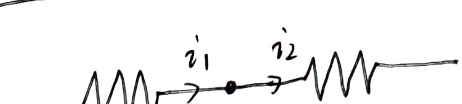
$v_1 = v_2$ (parallel connected elements have same branch voltage)

If KVL does not hold around a loop, magnetic flux linkage will accumulate through the loop which is inconsistent with lumped matter discipline that $\boxed{\frac{d\phi_B}{dt} = 0}$

* intuitive justification of KVL

voltage between a pair of nodes is their potential difference
potential difference between any 2 points is the sum of potential difference along any path (independent of path)
for a loop, start and end point are the same, 0 potential difference
from the definition of voltage, KVL is a statement of conservation of energy

Applications of KVL and KCL



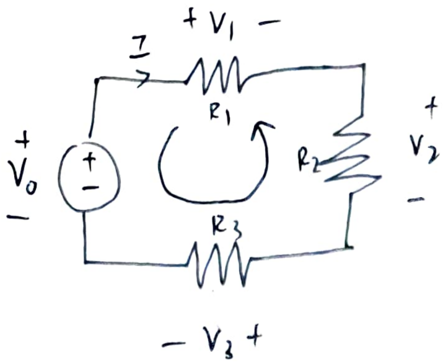
$$i_1 - i_2 = 0$$
$$i_1 = i_2$$

simplification of KCL shows 2 series connected resistors / elements have the same current flowing through them as no net current can flow into a node

with multiple applications of KCL, this can be extended to longer strings of series connected elements
* common branch current passes through series connected elements.

Application to resistances in series

from KCL, a common branch current passes through series connected element.



from KVL, $V_0 - V_3 - V_2 - V_1 = 0$

$$V_0 = V_1 + V_2 + V_3$$

from ohm's law $V_0 = IR_1 + IR_2 + IR_3$

$$= I(R_1 + R_2 + R_3)$$

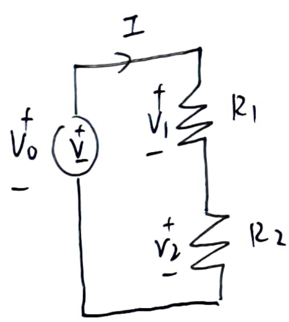
equivalent to

$$R_{eff} = R_1 + R_2 + R_3$$

equivalent resistance of resistance in series is equal to the sum of their individual resistance

resistance in series $\longrightarrow$ $\uparrow$ effective resistance

Resistance in series causes voltage division



element laws

$V_0 = V$

$V_1 = R_1 I$

$V_2 = R_2 I$

KVL

$V_0 - V_2 - V_1 = 0$

$V_0 = V_1 + V_2$

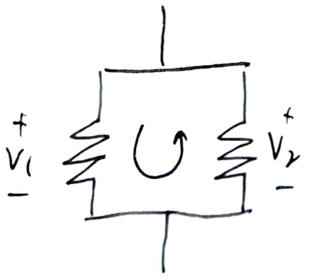
$V = I(R_1 + R_2)$

$$I = \frac{1}{R_1 + R_2} V$$

$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$

In a series circuit, voltage across each resistance is a fraction of the total voltage equal to the ratio of concerned resistance to total resistance



simplification of KVL shows voltage across 2 parallel connected elements have the same voltage across them

$V_1 - V_2 = 0$

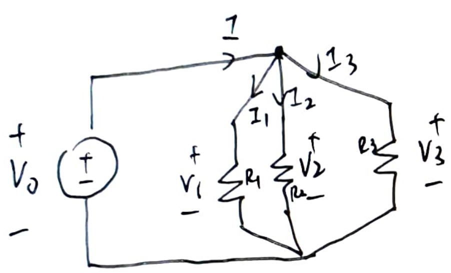
$V_1 = V_2$

with multiple applications of KVL, this can be extended to a longer string of parallel-connected elements.

* common voltage across all parallel-connected elements

Applications to resistances in parallel

from KVL, a common branch voltage runs across all parallel-connected elements



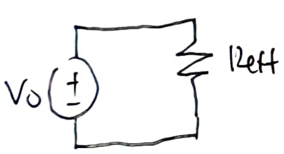
from KCL, $I = I_1 + I_2 + I_3$

$V_0 = V_1 = V_2 = V_3$

from ohm's law, $I = \frac{V_0}{R_1} + \frac{V_0}{R_2} + \frac{V_0}{R_3}$

$= V_0\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

equivalent to

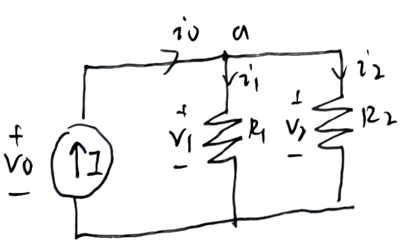


$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eff} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

resistances in parallel → ↓ effective resistance

Resistance in parallel causes current division



element laws

$i_0 = I$

$V_1 = R_1 i_1$

$V_2 = R_2 i_2$

KCL at node a

$i_0 = i_1 + i_2$

KVL

$v_0 = V_1 = V_2$

$V_1 = V_2 \Rightarrow R_1 i_1 = R_2 i_2$

$i_1 = \frac{R_2 i_2}{R_1}$

$I = i_2\left(\frac{R_2}{R_1} + 1\right) \rightarrow \frac{R_1 + R_2}{R_1}$

$i_2 = \frac{R_1}{R_1 + R_2} I \qquad i_1 = \frac{R_2}{R_1 + R_2} I$

current flowing into each resistor is a fraction of total current equal to the ratio of the other resistance to sum of both resistances

Circuit Analysis

B branches $\Rightarrow$ 2B branch variables

B current $i_1\ i_2 \cdots i_B$

B voltage $V_1\ V_2 \cdots V_b$

to solve for each unknown, 2B independent equations required

① B from element laws & sources

$$V_i = I_i R_i$$

$$V_j = V$$

$$I_k = I$$

② N-1 from KCL

If a circuit has N nodes, they form N-1 independent nodes

③ B-N+1 from KVL

If a circuit has B branches and N nodes, there are B-N+1 independent loops.

$$B + N - 1 + B - N + 1 = 2B$$

Usually, use more intuitive methods

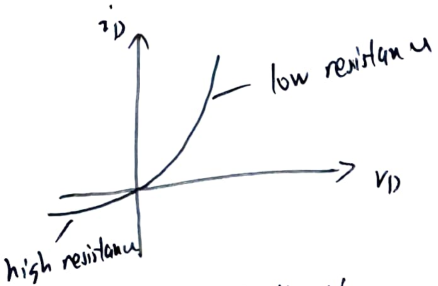
- Thevenin's theorem
- Superposition
- Node Analysis
- voltage divider, current divider

Activity 1

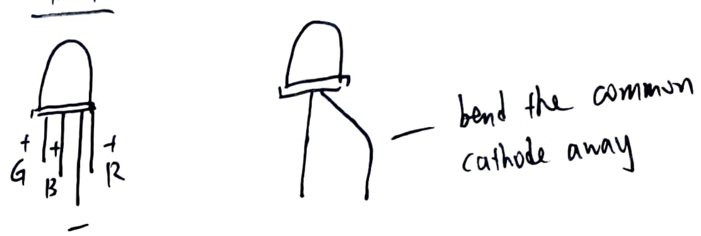
LED lamp contains 3 colour LEDs, Red, Green, Blue (RGB) — 3 primary colours of light

A diode is a polarized device that allows current to flow in only 1 direction

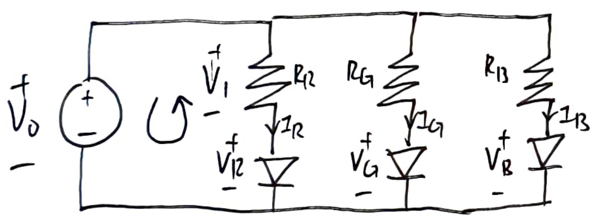
Diodes are 2 terminal, non-linear resistors whose current is exponentially related to voltage across its terminals



i-v characteristics of a diode

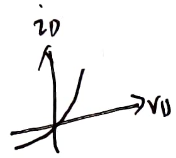


Derive equation that expresses I_R in terms of V_R, R_R and DC supply voltage



polarity of V_1, voltage across R_R due to I_R entering R_R at positive terminal and leaving through negative terminal

* polarity matters. Keep it consistent



By ohm's law, $V_1 = I_R R_R$

By KVL,

$$V_0 - V_R - V_1 = 0$$

$$V_1 = V_0 - V_R$$

$$I_R R_R = V_0 - V_R$$

$$\boxed{I_R = \frac{V_0 - V_R}{R_R}}$$

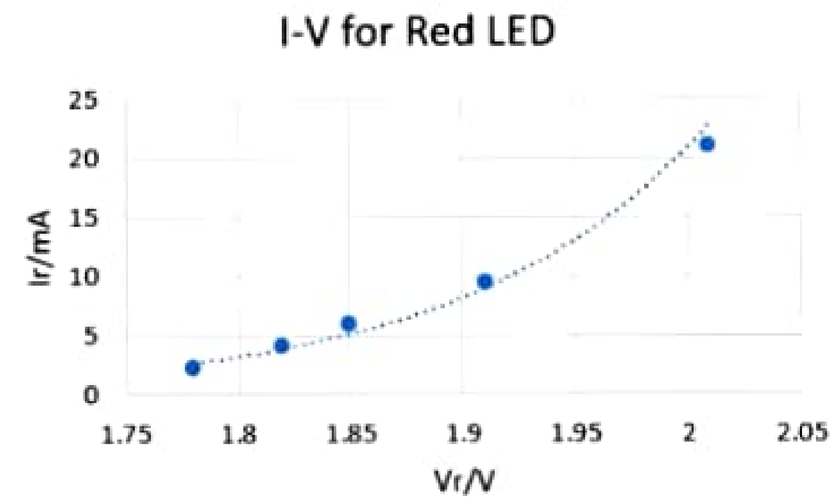
As the diode is a non-linear resistor, we cannot apply ohm's law for LEDs

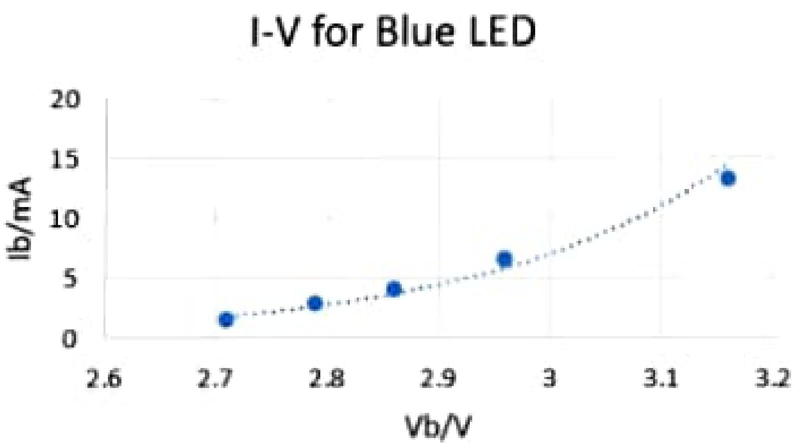
$V_0 = 5.11\,V$

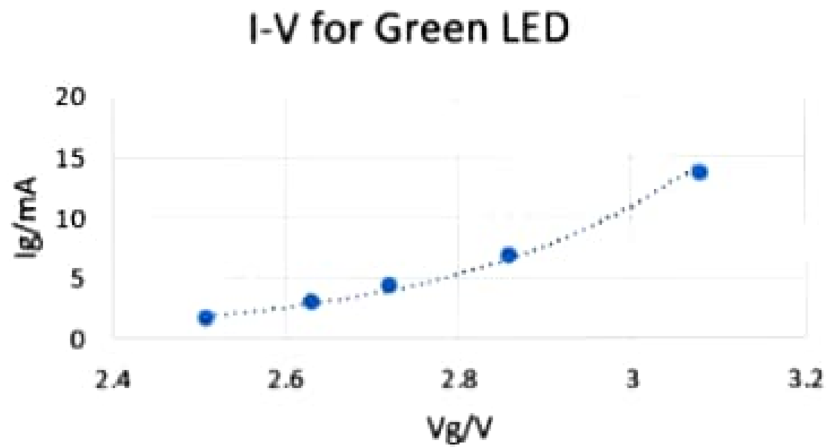
similarly for I_G and I_B

refer to excel sheet for measurements and calculations

Nominal Res/Ω		150	330	560	820	1500
Measured Res/Ω		148	326	546	804	1474
Red LED	Vr/V	2.01	1.91	1.85	1.82	1.78
Red LED	Ir/mA	20.94	9.48	5.97	4.09	2.22
Green LED	Vg/V	3.08	2.86	2.72	2.63	2.51
Green LED	Ig/mA	13.72	6.9	4.38	3.08	1.76
Blue LED	Vb/V	3.16	2.96	2.86	2.79	2.71
Blue LED	Ib/mA	13.18	6.6	4.12	2.89	1.63





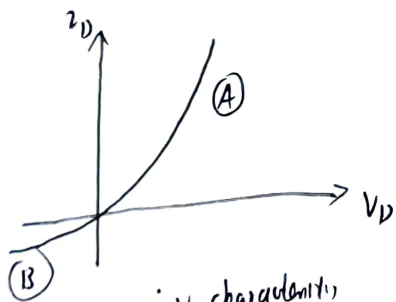


Observations and Analysis

The graphs are not linear as diodes are non linear resistors that allow current to flow easily in 1 direction but restricts current flow in the opposite direction

low resistance (A)

high resistance (B)



i-V characteristics of a diode

Comparing my plots with the RGB lamp datasheet, the values are not identical but follow quite closely, especially for the lower ranges of voltage

e.g my RED LED plot has current around 3.5 mA at 1.8 V while the I_R from the datasheet is around 3 mA for the same voltage.

This should be due to the exponential nature of the i-V graph of diodes where at higher voltage ranges, a small increase in voltage can lead to a huge difference in current

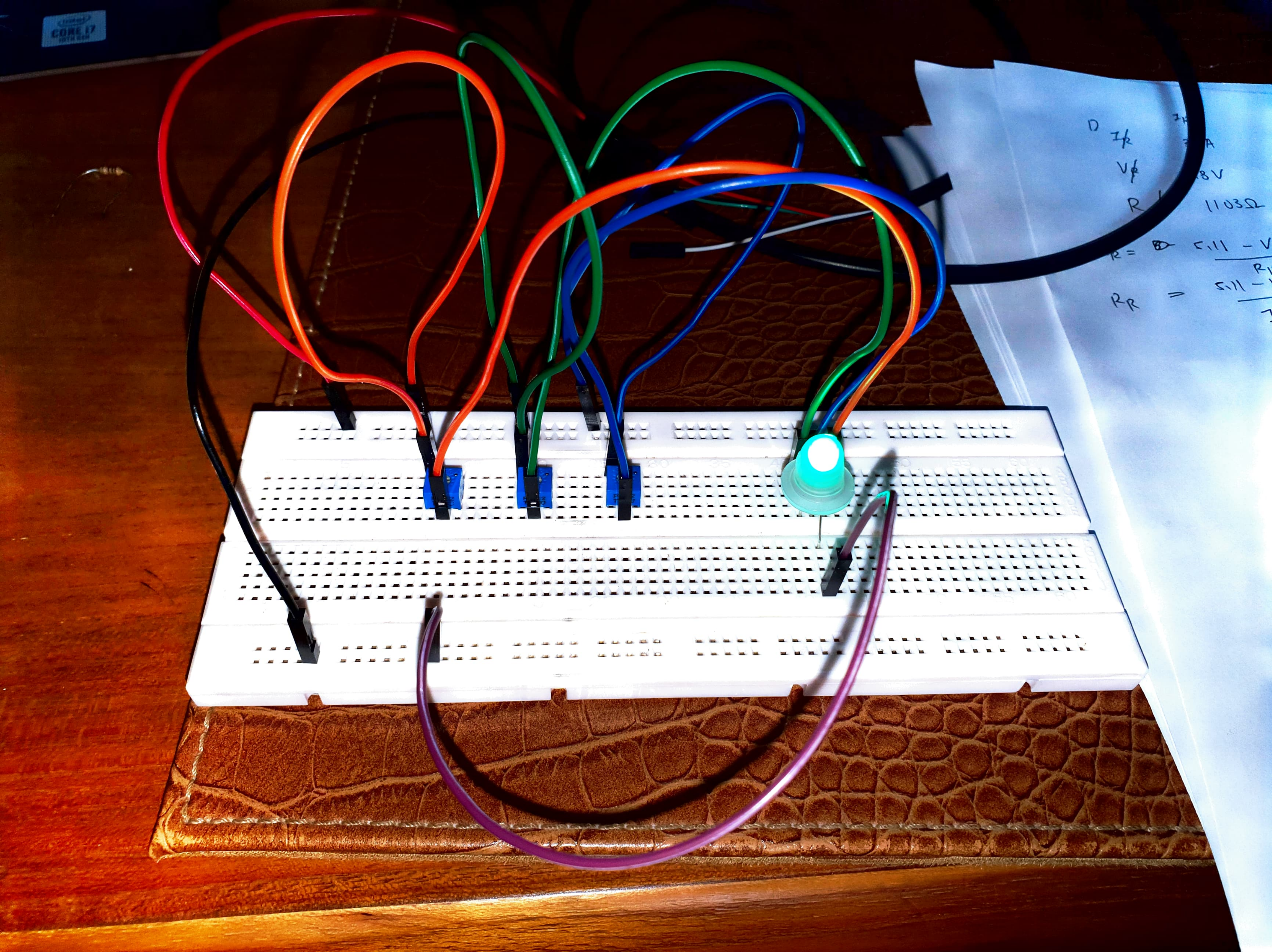
Overall, both follow an exponential trend and minute differences can be accounted for due to manufacturing differences.

Adding more data points would help extend the plot if more resistances are used which cover the full range of the voltage values for each LED in the datasheet. This might make the graph values more accurate to the exponential trend of the LED and thus in turn closer to the values in the datasheet.

However, the range of resistance used to plot the I-V graphs do cover most of the working range corresponding to the bold lines of each LED I-V characteristic in the datasheet

e.g $150\,\Omega$ to $1500\,\Omega$ gives me a V_R range from 1.78 V to 2.01 V while the range of the curve in the Red LED datasheet goes from 1.65 V to ~2 V

Adding more data points within the range would not change the 'polynomial' line fitting too much but might give more accurate values as there would be less extrapolation between points.


CORE i7
1103Ω

Activity 2

Mystery colour $\boxed{D}$ — let me guess!

	Red	Green	Blue
Current I (mA)	3	13.5	1.5
Extrapolated V (V)	1.8	3.0	2.6
calculated R (Ω)	1103	156	1673

voltage across each diode is extrapolated / read off plot for respective LEDs from previous activity

$$I_R = \frac{5.11 - V_R}{R_R} \Rightarrow R_R = \frac{5.11 - V_R}{I_R}$$

similarly for green & blue resistance.

~ insert picture of circuit ~

efficiency of biasing each of the R, G, B LED

$$\eta_R = \frac{P_{RLED}}{P_{RLED} + P_{Rloss}} \times 100\% = \frac{V_R I_R}{V_R I_R + I_R^2 R_R} \times 100\% = \frac{0.0054}{0.0054 + 0.00993} \times 100\% = \boxed{35.2\%}$$

$$\eta_G = \frac{V_G I_G}{V_G I_G + I_G^2 R_G} \times 100\% = \frac{0.0405}{0.0405 + 0.0284} \times 100\% = \boxed{58.8\%}$$

$$\eta_B = \frac{V_B I_B}{V_B I_B + I_B^2 R_B} \times 100\% = \frac{0.0039}{0.0039 + 0.00376} \times 100\% = \boxed{50.9\%}$$

Using current-limiting resistors for biasing LEDs is quite inefficient as a huge fraction of the total power is dissipated through the resistor

Pulse width Modulation (PWM) may be a better approach to improve efficiency and reducing average power delivered by an electrical signal.