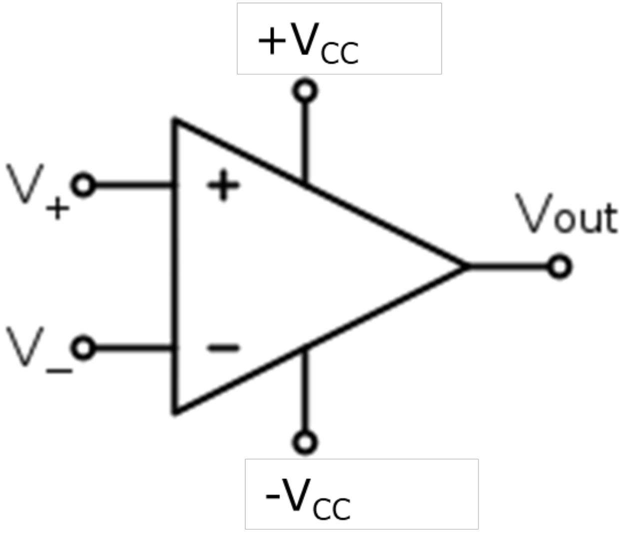
CG1111: Engineering Principles and Practice I

Debrief and Tutorial for Week 9

National University of Singapore

Comparator

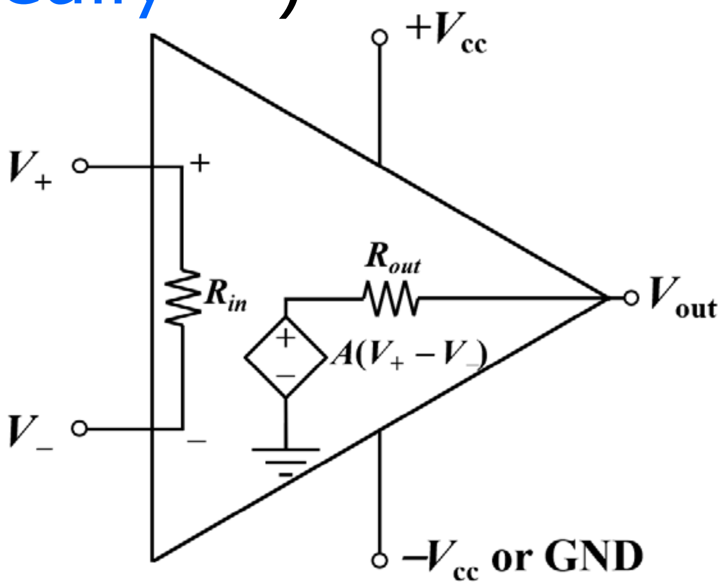


- The comparator is an electronic decision-making circuit that makes use of an op-amp's very high gain in its open-loop state (i.e., there is no feedback resistor)

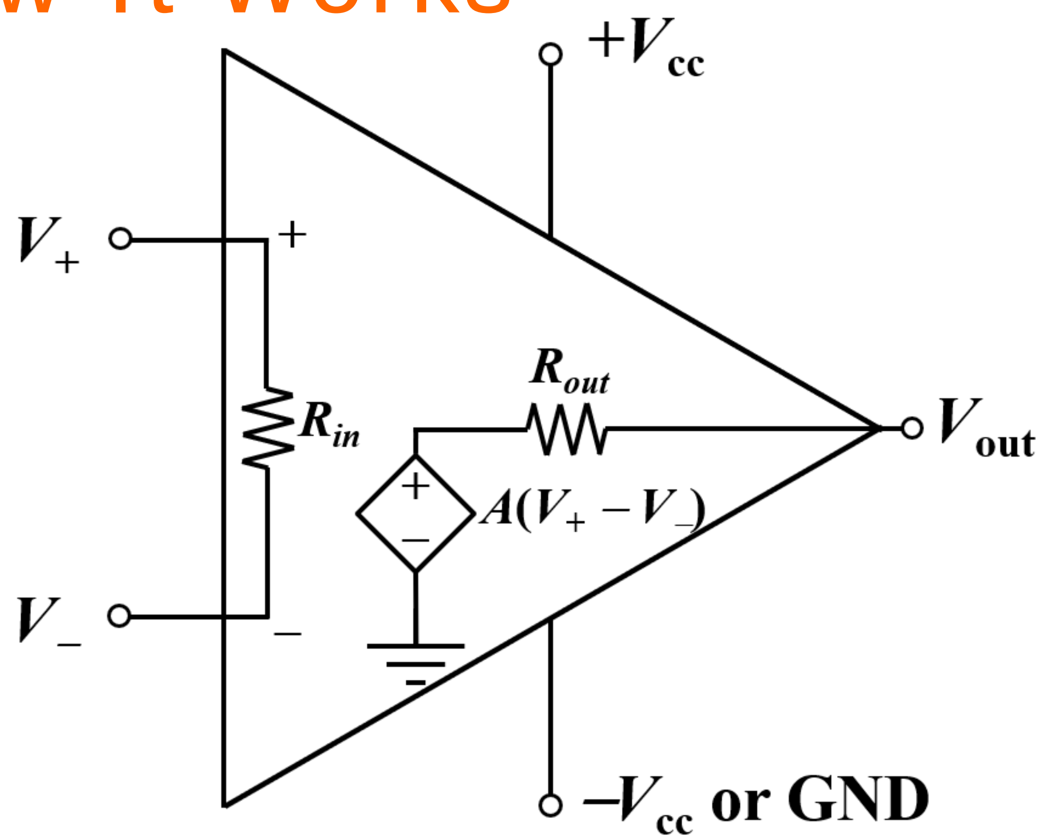
Op-Amp as a Comparator – How It Works

Recall that for op-amp:

- The difference between the two inputs is amplified as '$A(V_+ - V_-)$' at the output
- The open-loop voltage gain ('A') of the op-amp is very high (ideally ∞)
- Even if there is a very small difference between the inputs, the high 'A' will pull the output to "saturation"



Op-Amp as a Comparator – How It Works



- If $V_+ > V_-$:

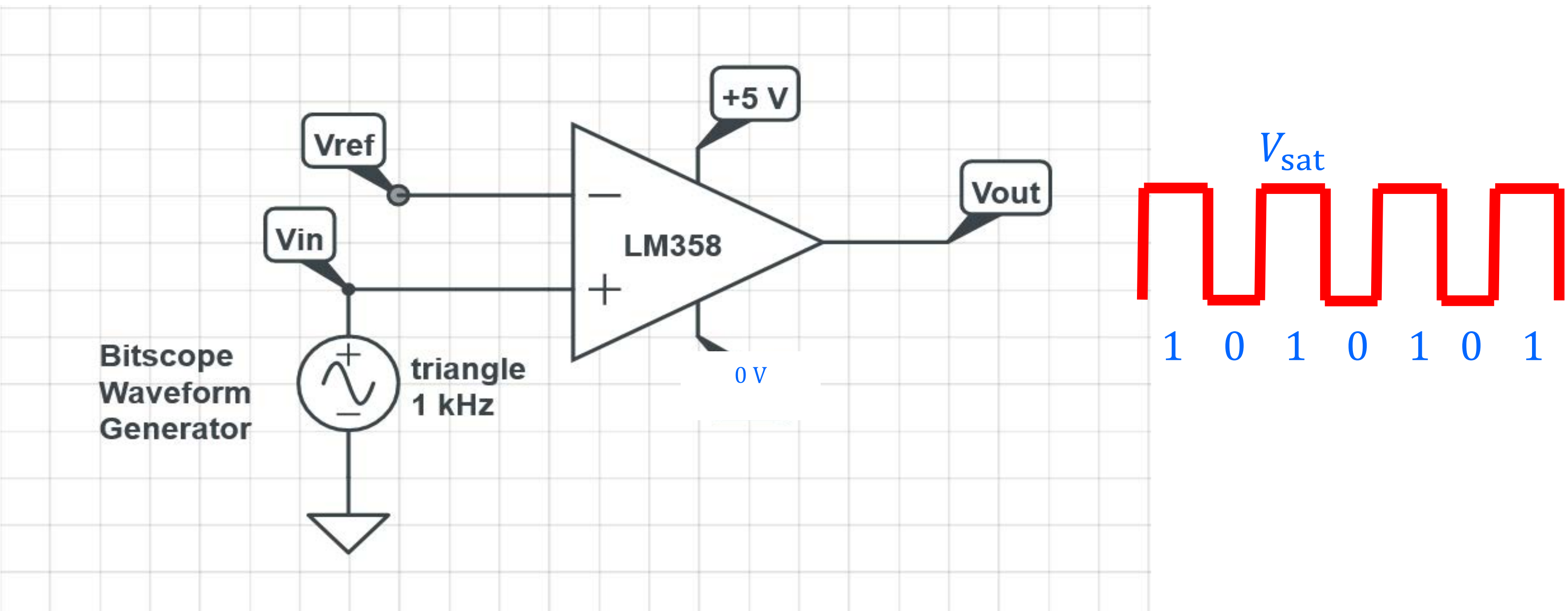
$$V_{\text{out}} = +V_{\text{sat}}$$

- If $V_- > V_+$:

$$V_{\text{out}} = \begin{cases} -V_{\text{sat}} & \text{if dual power supply} \\ 0 & \text{if single power supply} \end{cases}$$

Common Application of Comparator

- The comparator is ideal for converting analog signals to digital signals at certain threshold values



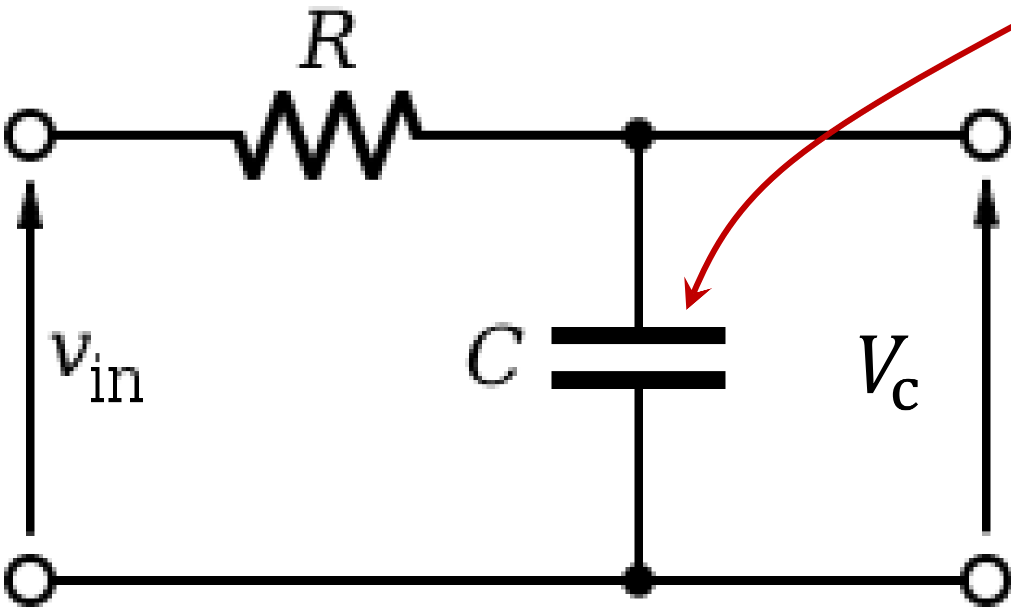
Filter

- A filter is a device or process that removes some unwanted components or features from a signal

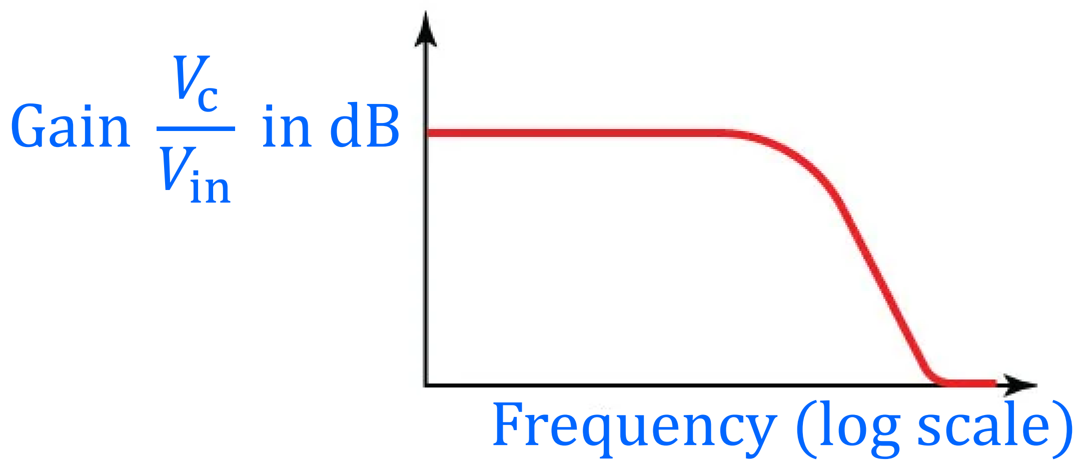
- Examples:
  - –Removing the noise from measured ECG signal using a filter to help a doctor understand the heart better

  - –Removing some frequencies or frequency bands from an audio signal

Passive Low-Pass Filter



Voltage divider:
The capacitor's impedance decreases as the frequency increases, hence V_c decreases



Power Gain in decibels (dB)

- The Voltage Amplification (A_v) or Gain of a voltage amplifier/filter is given by:

$$\boxed{A_v = \frac{V_{\text{out}}}{V_{\text{in}}}}$$

- The voltage gain is commonly expressed in terms of the resulting power gain in dB:

$$\boxed{\begin{aligned} \text{Power Gain (dB)} &= 10\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2 \text{ dB} \\ &= 20\log_{10}\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| \text{ dB} \end{aligned}}$$

Frequency Response

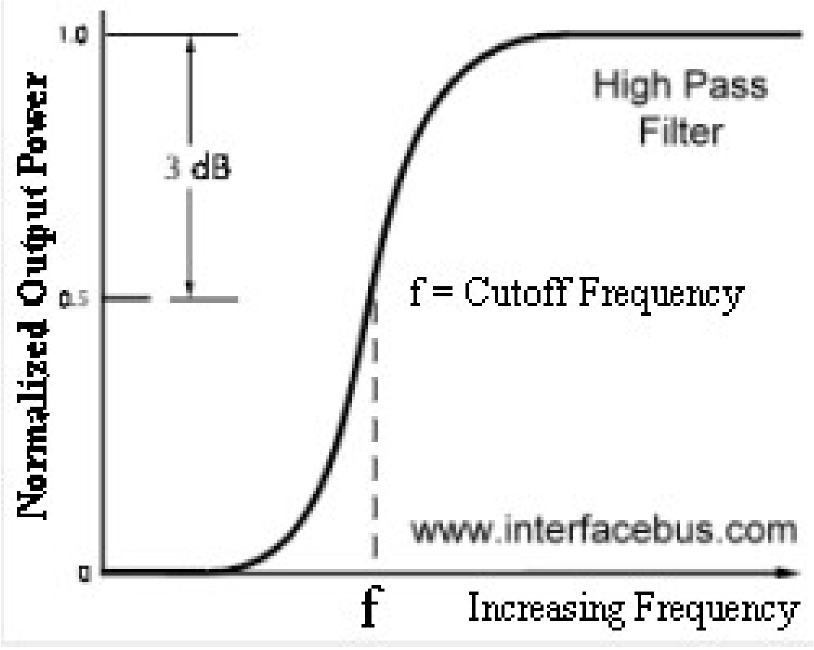
- It is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system
  - –Frequency in logarithmic Scale: horizontal x-axis

  - –Power Gain in decibels (dB): vertical y-axis
    To describe a change in output power over the whole frequency range

$$\text{Power Gain in dB } (f) = 20 \log_{10} |A_v(f)|$$

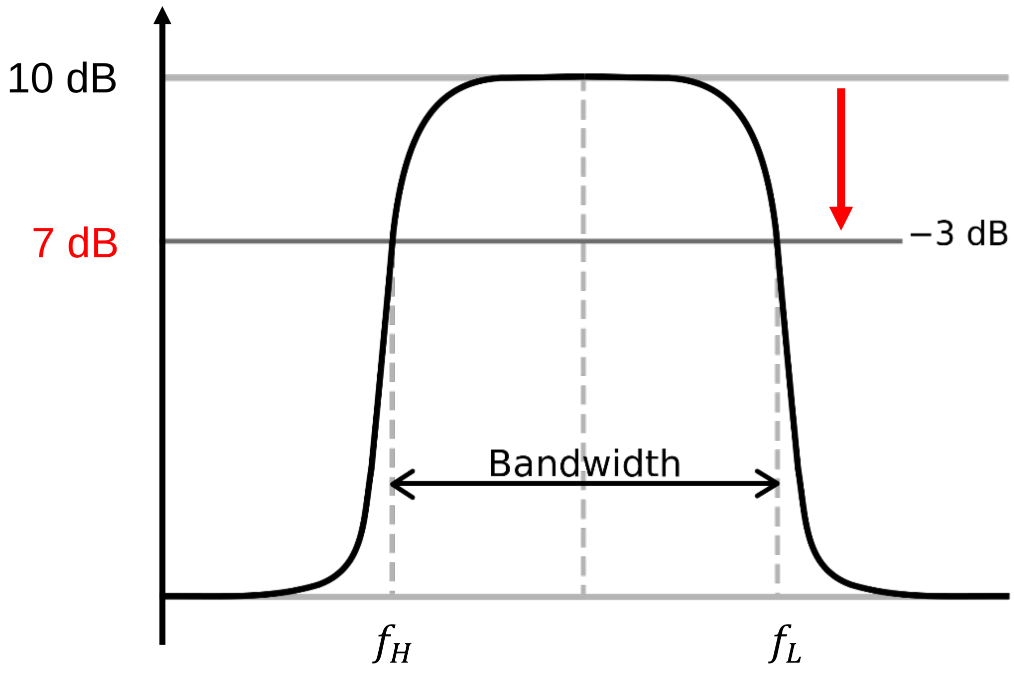
Cut-off Frequency

- In filters, the cut-off frequency characterizes a boundary between a passband and a stopband
- The cut-off frequency is taken as the frequency at which the output of the circuit is −3 dB (corresponding to half the power) of the nominal passband value



Cut-off Frequency: –3 dB Point (i.e., Half-power Point)

- Graphical approach:
  - Find the passband gain from the magnitude vs frequency plot
  - Subtract 3 dB from the passband gain and draw a line on the plot
  - The points where this line cuts the plot corresponds to the cut-off frequency(s)



Cut-off Frequency: –3 dB Point (i.e., Half-power Point)

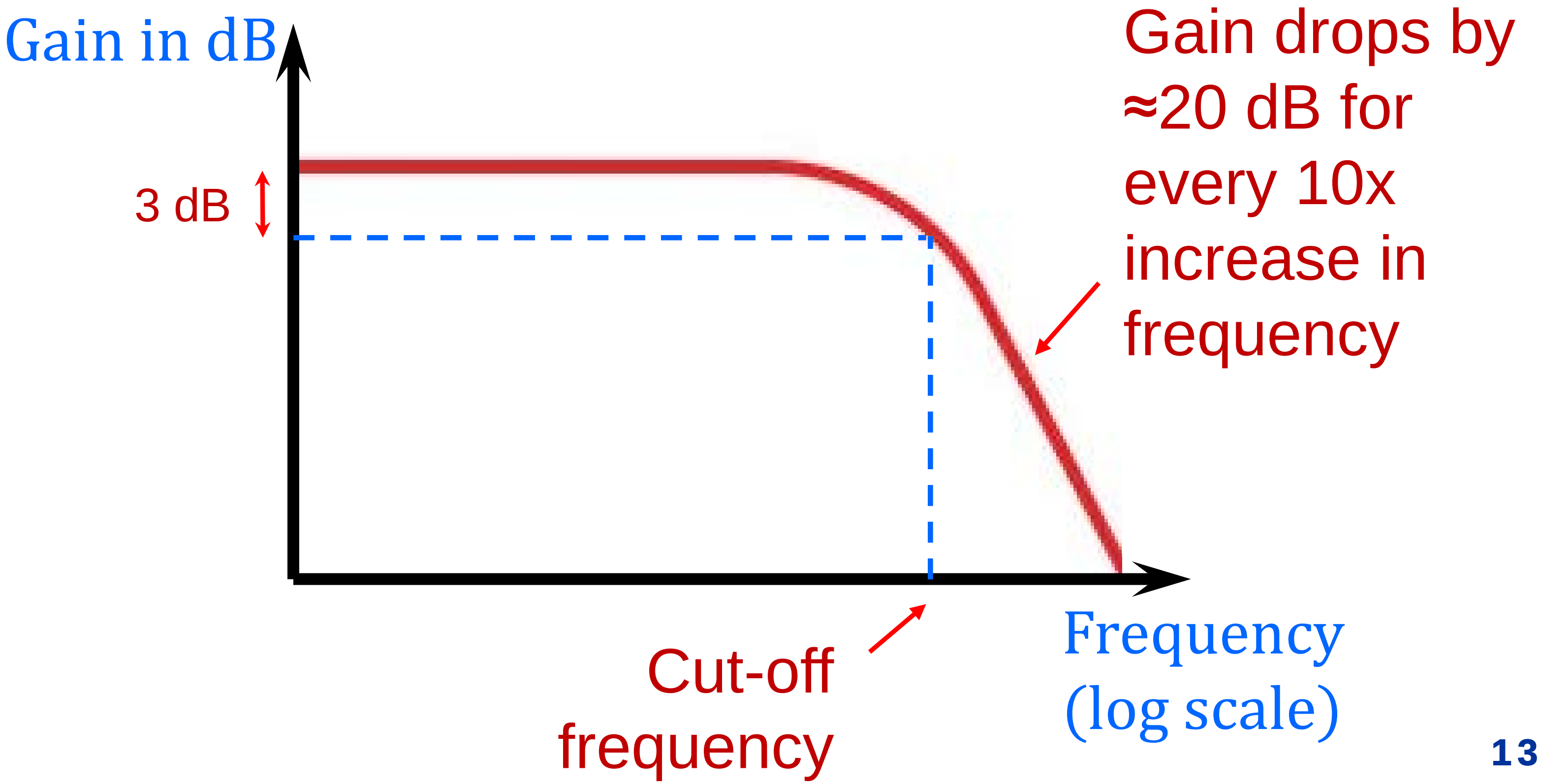
- Quantitative approach (for first-order filters):

$$f_H = \frac{1}{2\pi R_H C_H}, \quad f_L = \frac{1}{2\pi R_L C_L}$$

First-order Low-Pass Filter

- Slope after cut-off frequency

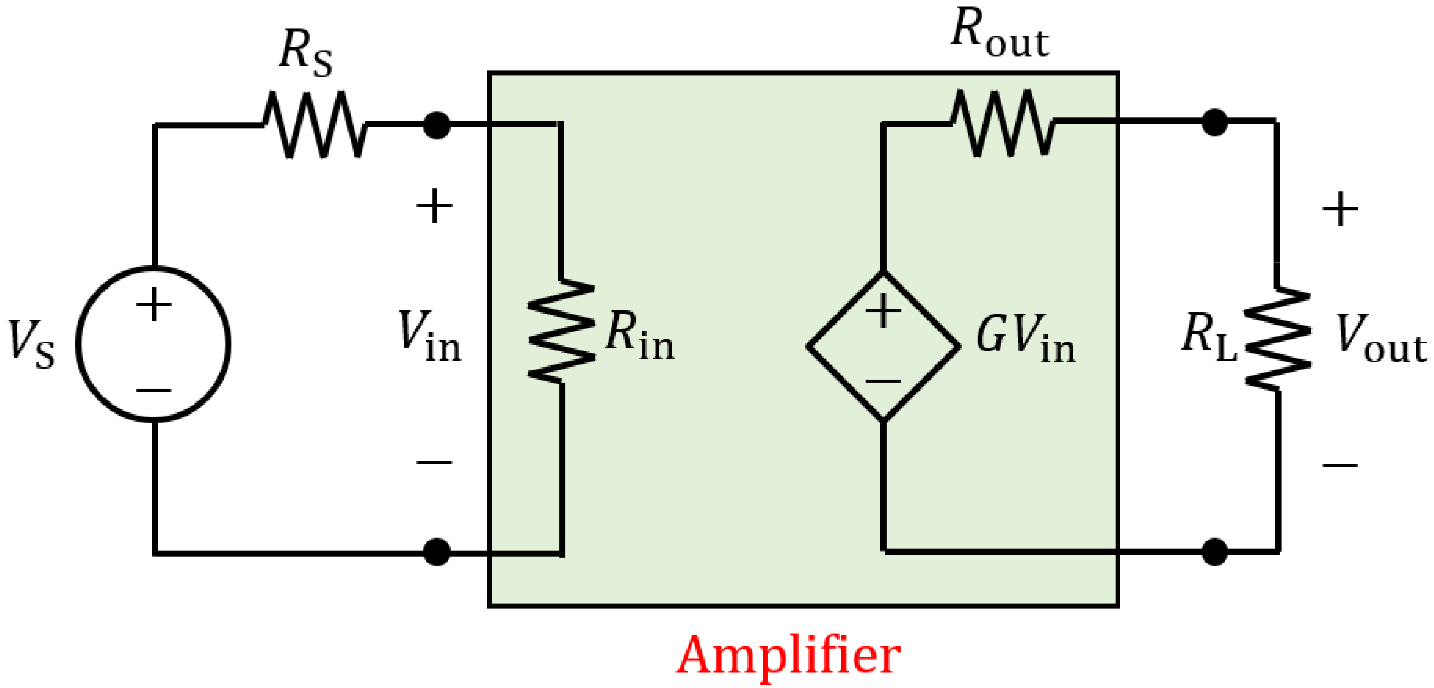
  ≈ -20 dB/decade



Question 1

Show that if $R_{\mathrm{in}} = R_{\mathrm{L}}$, then the power gain in dB for an amplifier circuit is given by

$$\boxed{\text{Power gain (dB)} = 20\log_{10}\left|\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}}\right| \text{ dB}}$$



Solution to Q1

- Power at amplifier's input: $P_{\text{in}} = \dfrac{V_{\text{in}}^2}{R_{\text{in}}}$
- Power delivered to load: $P_{\text{out}} = \dfrac{V_{\text{out}}^2}{R_{\text{L}}}$
- Power Gain in dB

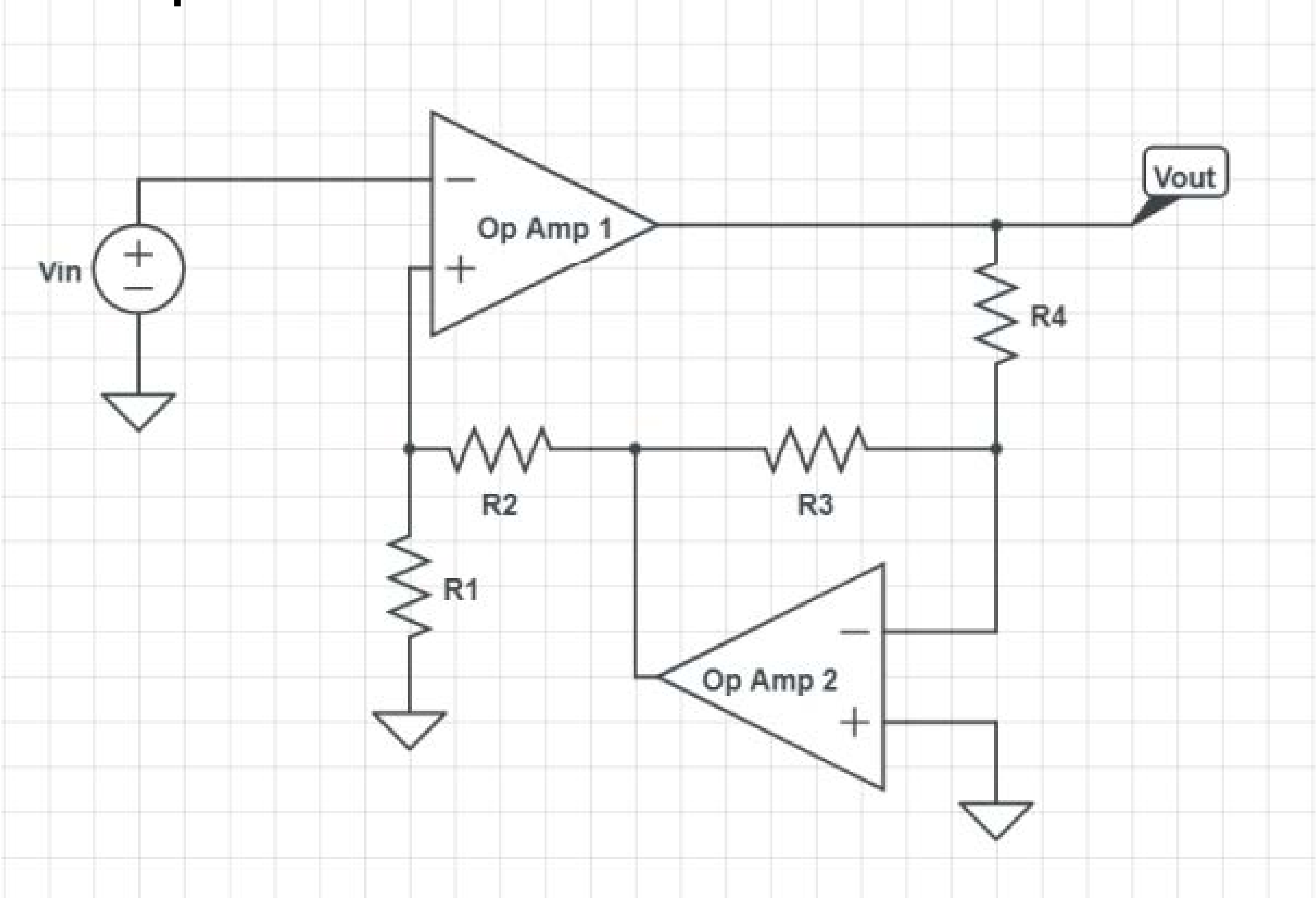
$$= 10 \log_{10}\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)$$

$$= 10 \log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2 + \overbrace{10 \log_{10}\left(\frac{R_{\text{in}}}{R_{\text{L}}}\right)}^{\text{Equals 0 if } R_{\text{in}} = R_{\text{L}}}$$

$$= 20 \log_{10}\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right|$$

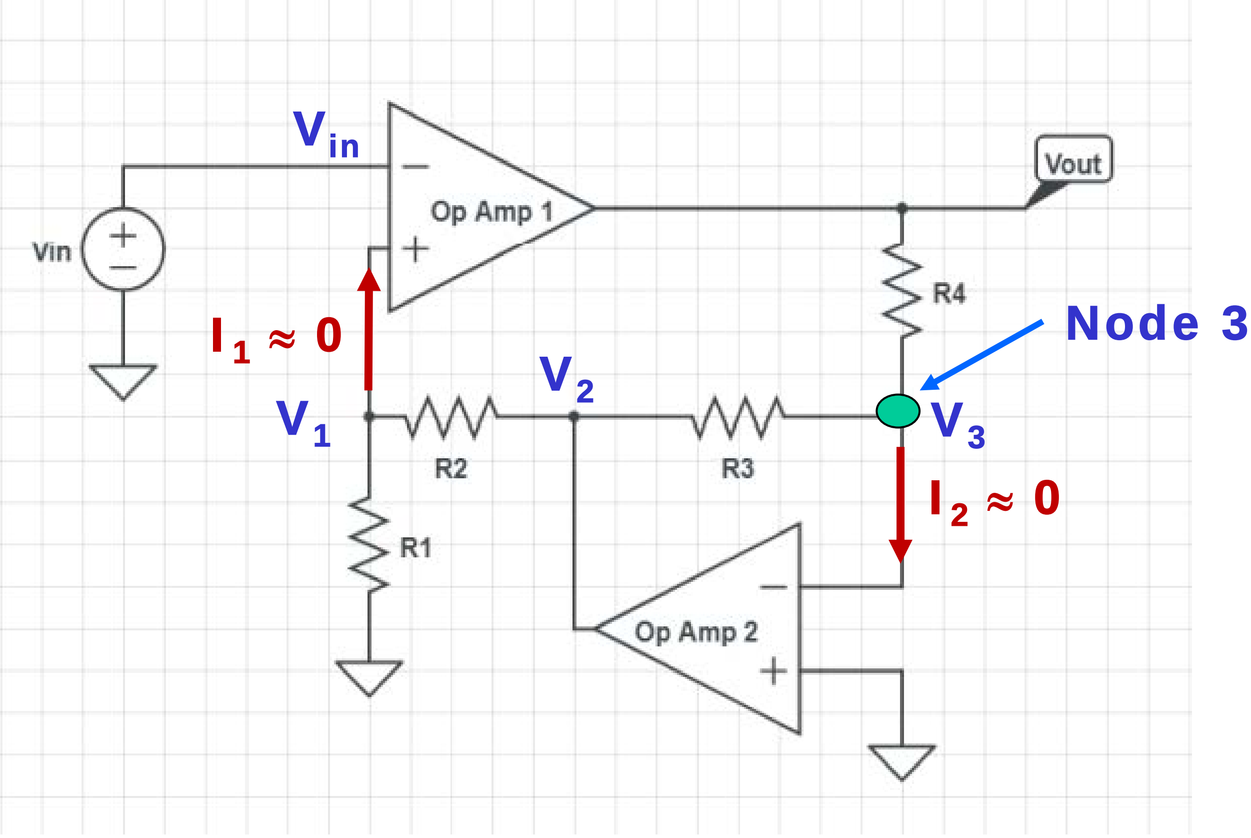
Question 2

- Calculate the voltage gain (V_{out}/V_{in}) of Op Amp 1



Solution to Q2

- Using op amp Golden rules,
  - For Op amp 1, $V_+ = V_- = V_{in}$. Hence, $V_1 = V_{in}$
  - For Op amp 2, $V_- = V_+ = 0$. Hence, $V_3 = 0$



Solution to Q2

- Current through R_1 (downwards)

$$I_{R1} = \frac{V_{\text{in}}}{R_1}$$

- Current through R_2 (right to left)

$$I_{R2} = I_{R1} = \frac{V_{\text{in}}}{R_1}$$

$V_2 = V_1 + (I_{R2} \times R_2)$

$= V_{\text{in}} + \frac{V_{\text{in}}}{R_1} \times R_2 = (1 + \frac{R_2}{R_1}) \times V_{\text{in}}$

$\rightarrow$ $V_2 = (1 + \frac{R_2}{R_1}) \times V_{\text{in}}$ ------>Equation 1

Solution to Q2

Applying KCL at Node 3,

$$\frac{V_2 - 0}{R_3} + \frac{V_{\text{out}} - 0}{R_4} = 0$$

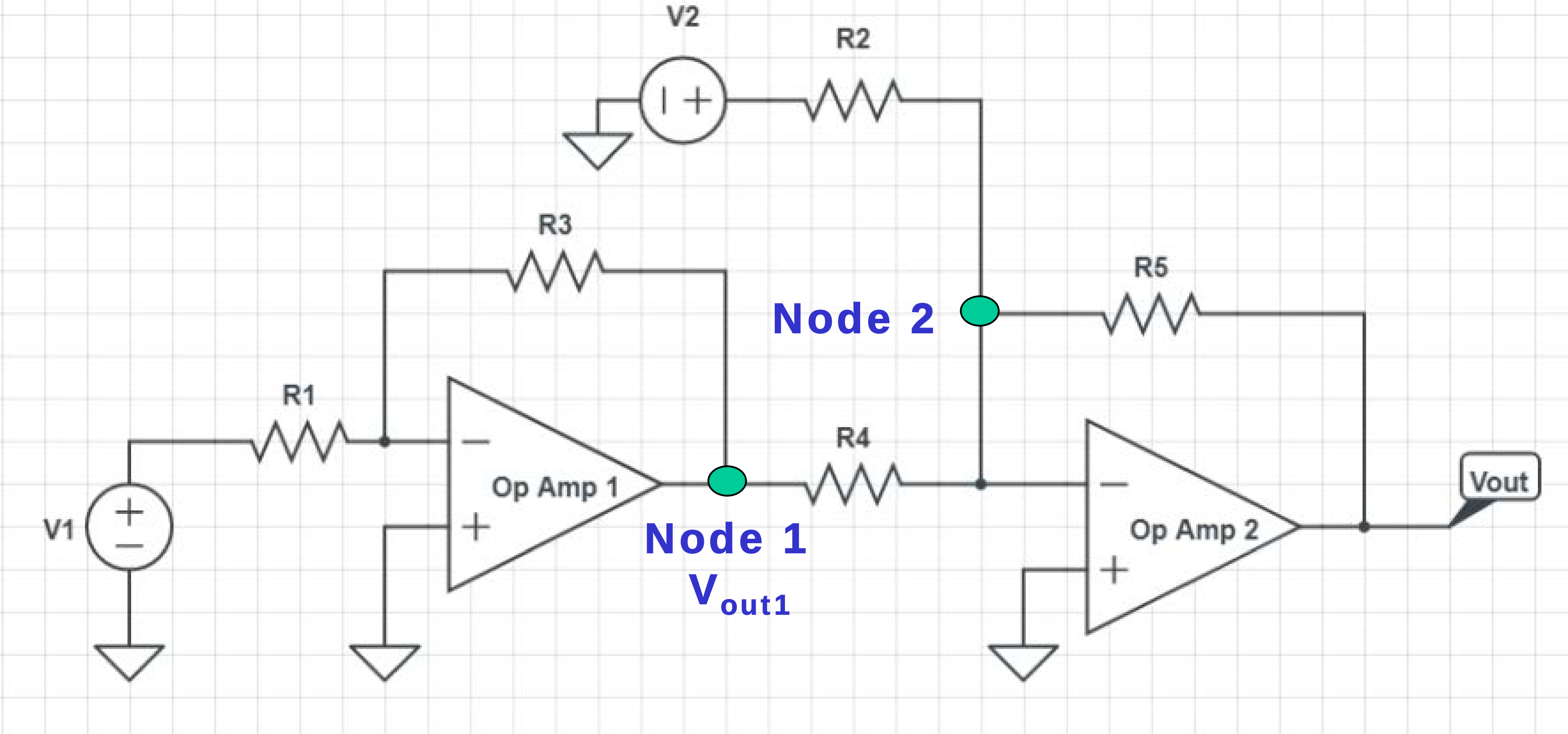
Applying expression for V_2 from Equation 1,

$$(1+\frac{R_2}{R_1}) \times \frac{V_{\text{in}}}{R_3} = \frac{-V_{\text{out}}}{R_4}$$

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\,(1+\frac{R_2}{R_1}) \times \frac{R_4}{R_3}$$

Question 3

- Derive the expression relating V_{out} and the two inputs, V_1 and V_2
- Design R values such that $V_{out} \propto (V_1 - V_2)$



Solution to Q3

- For both op amps, $V_{-} = V_{+} = 0$ (op amp golden rules)

$$\frac{V_1 - 0}{R_1} + \frac{V_{\text{out1}} - 0}{R_3} = 0 \rightarrow \quad V_{\text{out1}} = -\frac{R_3}{R_1} V_1$$

Applying KCL at Node 2,

$$\frac{V_2 - 0}{R_2} + \frac{V_{\text{out1}} - 0}{R_4} + \frac{V_{\text{out}} - 0}{R_5} = 0$$

$$V_{\text{out}} = \left(\frac{R_5}{R_4} * \frac{R_3}{R_1} * V_1\right) - \left(\frac{R_5}{R_2} * V_2\right)$$

Solution to Q3

If $\dfrac{R_5 R_3}{R_4 R_1} = \dfrac{R_5}{R_2}$, which gives $\dfrac{R_3}{R_4 R_1} = \dfrac{1}{R_2}$, then

$$V_{\text{out}} = \frac{R_5}{R_2}\left(V_1 - V_2\right)$$

$$= K\left(V_1 - V_2\right)$$

Question 4

- Audio song frequencies: 100-3000 Hz
- Corrupted with 10 kHz noise

- Design a low-pass filter to suppress the 10 kHz noise by 20 dB relative to the passband gain

- What is the cut-off frequency of the low-pass filter?

A Note About –20 dB in Power

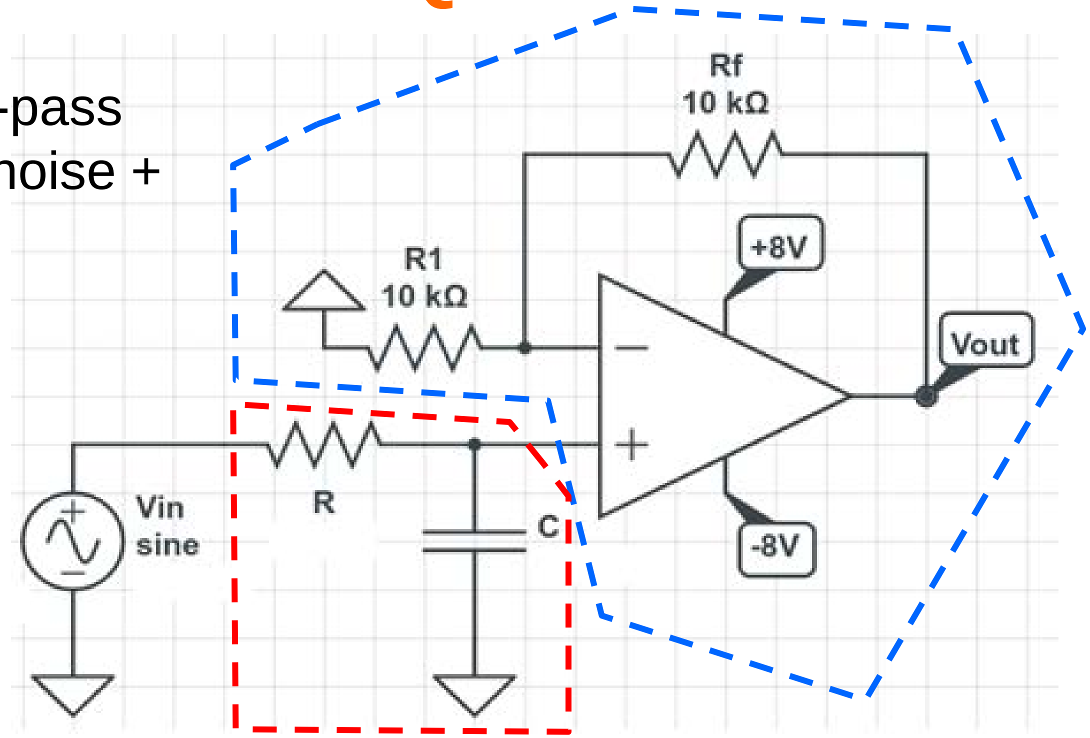
- Suppressing the noise by 20 dB is equivalent to reducing its power to just 1% compared to no filtering
- Also equivalent to reducing its voltage to just 10% compared to no filtering

- $10\ \log_{10}\left(\frac{P_{\text{noise(filtered)}}}{P_{\text{noise(no filter)}}}\right) = 10\ \log_{10}(0.01) = -20\ \text{dB}$

- $20\ \log_{10}\left(\frac{V_{\text{noise(filtered)}}}{V_{\text{noise(no filter)}}}\right) = 20\ \log_{10}(0.1) = -20\ \text{dB}$

Solution to Q4

Suppose we take an <u>active</u> low-pass filtering approach (i.e., filtering noise + <u>amplifying</u> entire signal)



Blue:
Active gain (= 2) due to non-inverting amplifier

Red:
Passive filter's gain (< 1) due to RC potential divider (ω-dependent)

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{V_{\text{out}}}{V_+} \times \frac{V_+}{V_{\text{in}}} = \left(1 + \frac{R_f}{R_1}\right)\frac{V_+}{V_{\text{in}}} = 2\left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right] = \frac{2}{1 + j\omega CR}$$

Hence, gain's magnitude w.r.t. ω = $\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{2}{\sqrt{1+(\omega CR)^2}}$

Solution to Q4

$$\text{Gain in dB} = \underbrace{20\log_{10} 2}_{\text{Passband gain}} \underbrace{- 20\log_{10}\sqrt{1+(\omega CR)^2}}_{\text{Change in gain with } \omega}$$

A gain reduction of 20 dB at f = 10 kHz means:

$$-20\log_{10}\sqrt{1+(\omega CR)^2}\Big|_{f\,=\,10\text{ kHz}} = -20\text{ dB}$$

Hence, $\sqrt{1+(\omega CR)^2} = 10$ when f = 10 kHz

Our low-pass filter needs to have:

$$RC = \frac{\sqrt{10^2-1}}{2\pi\times 10000} = 1.584\times 10^{-4}\text{ s}$$

Solution to Q4

- Cutoff frequency is the frequency at which the gain decreases by 3 dB from passband gain

- A gain reduction of 3 dB at $f = f_c$ means:

$$-20\log_{10}\sqrt{1+(\omega CR)^2}\bigg|_{f=f_c} = -3\ \text{dB}$$

- Hence, $\sqrt{1+(\omega CR)^2} = 10^{3/20} = \sqrt{2}$ when $f = f_c$

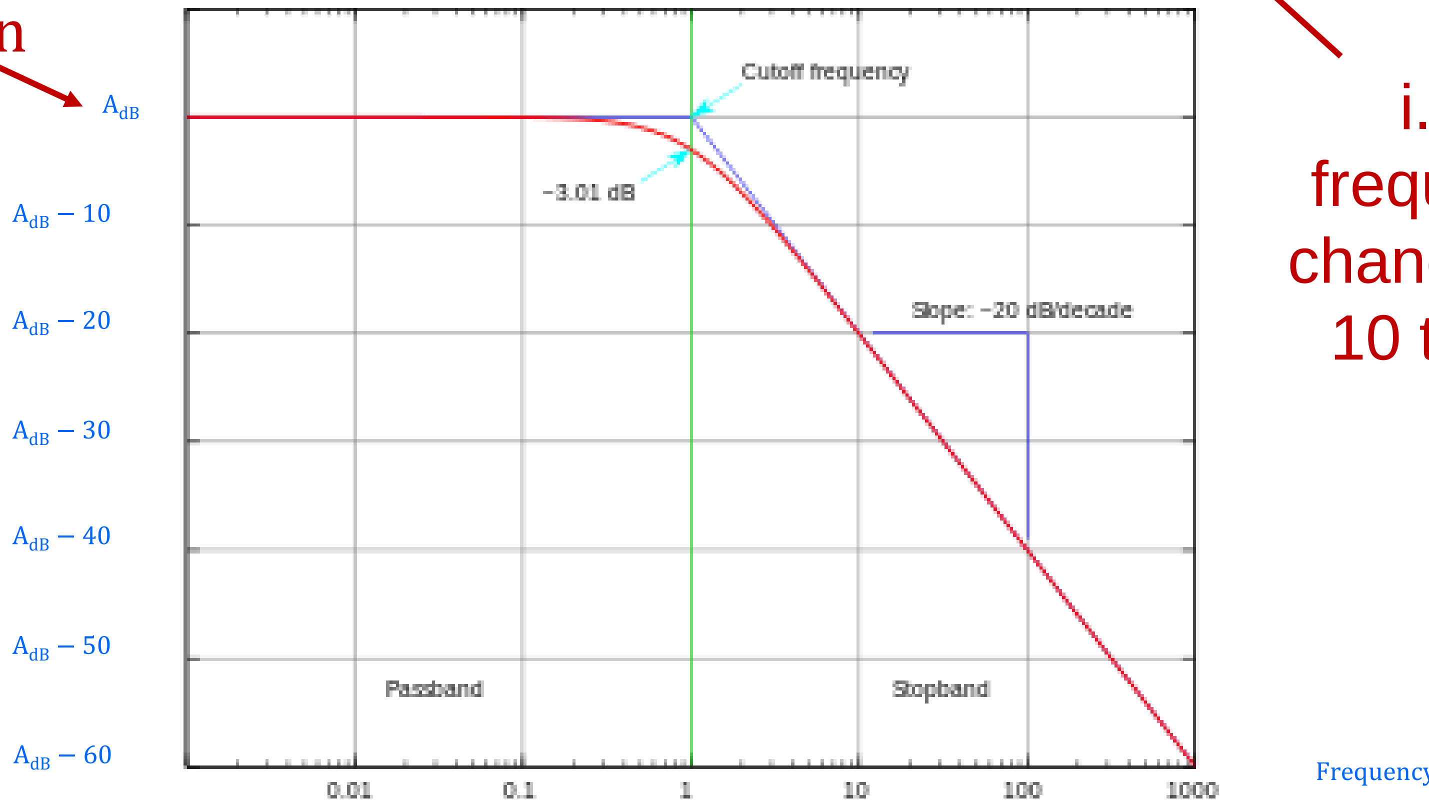
$\omega CR = 1$ when $f = f_c$

$$\boxed{f_c = \frac{1}{2\pi RC}} = \frac{1}{2\pi \times 1.584 \times 10^{-4}} \approx 1\ \text{kHz}$$

Graphical Visualization for Q4

- First-order low-pass filter: –20 dB/decade
- Each horizontal box is “1 decade”

DC gain



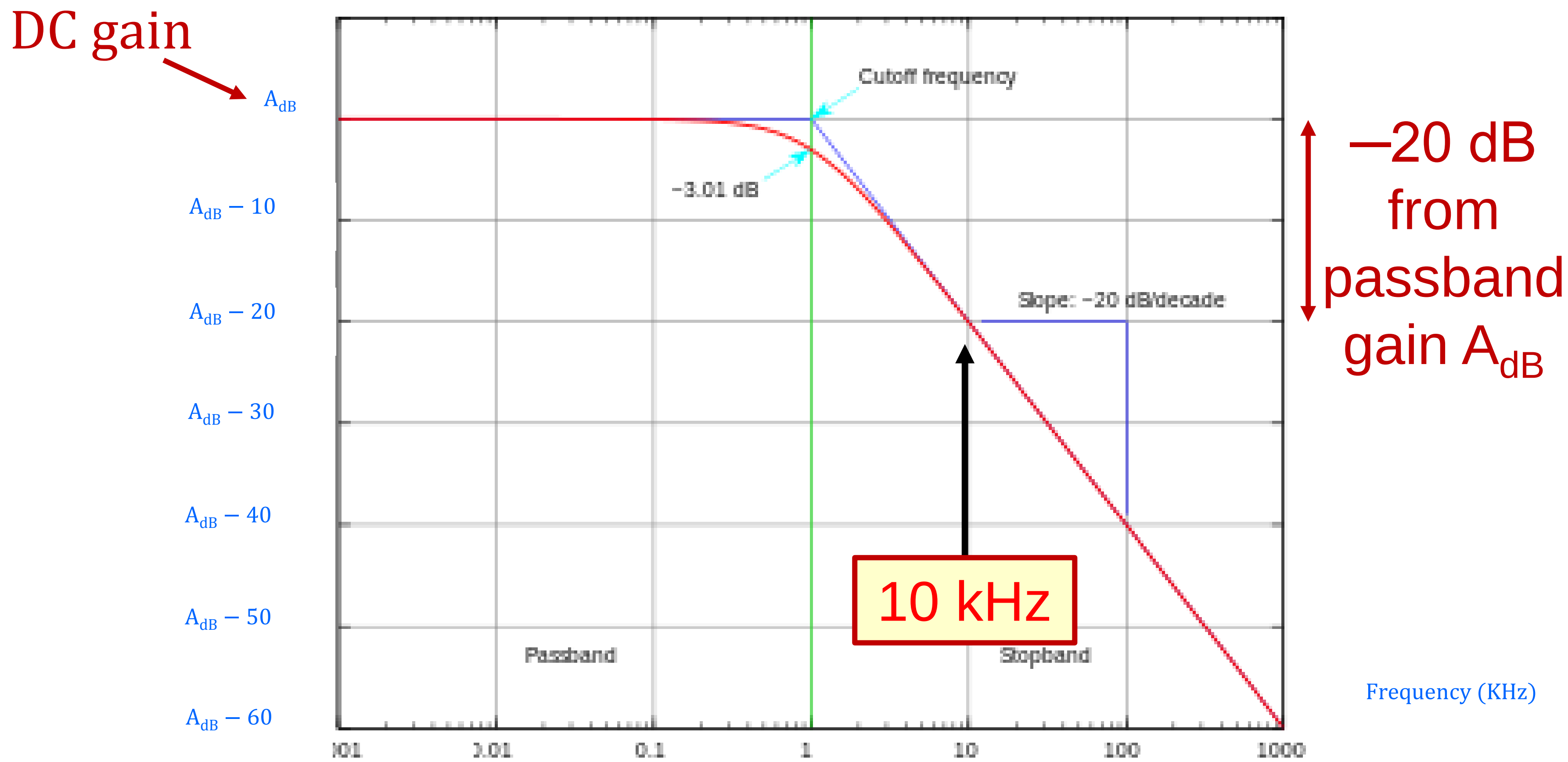
i.e., frequency changes by 10 times

Why –20 dB/decade?

f	$-20\log_{10}\sqrt{1+(\omega CR)^2}$	
f_c	≈ –3 dB	
10 x f_c	≈ –20 dB	–17
100 x f_c	≈ –40 dB	–20
1000 x f_c	≈ –60 dB	–20

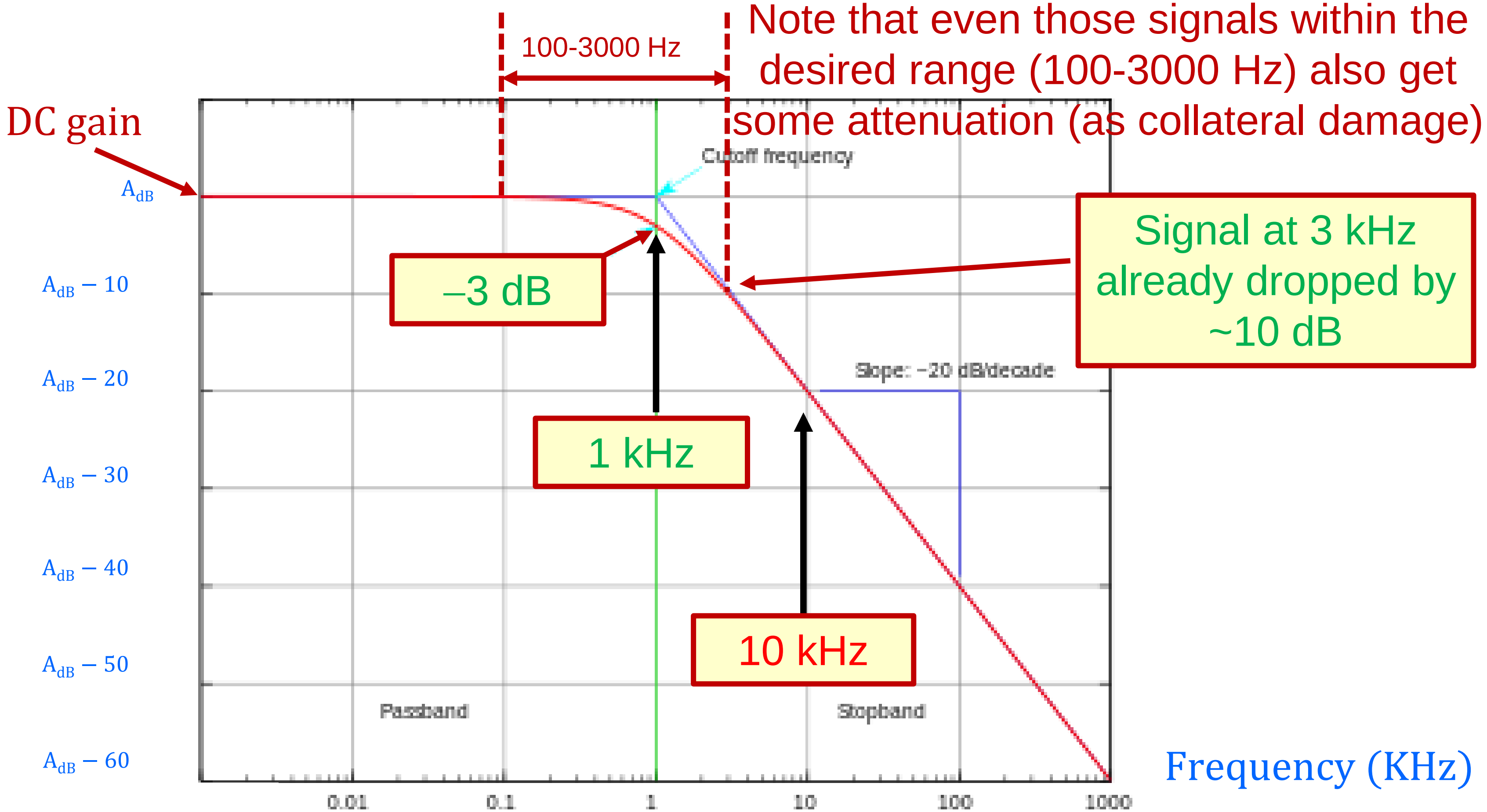
Graphical Visualization for Q4

- We need to reduce the gain at 10 kHz by 20 dB (always relative to passband gain)



Graphical Visualization for Q4

- If 10 kHz noise has to be reduced by 20 dB, we need to have the cutoff frequency at 1 kHz



Extra Points to Note for Q4

For your curiosity only:

As can be seen, with a first-order filter, we also lose some audio signals that we desire. How do we improve this?

We can use higher-order filters! This allows us to have sharper attenuation slope, so that our desired passband is not attenuated too much!

- 2^{nd} order: 40 dB/decade
  - http://www.electronics-tutorials.ws/filter/second-order-filters.html
- 3^{rd} order: 60 dB/decade
  - http://www.circuitstoday.com/higher-order-filters