CG1111: Engineering Principles and Practice I

DC Transient Behaviour of Capacitors

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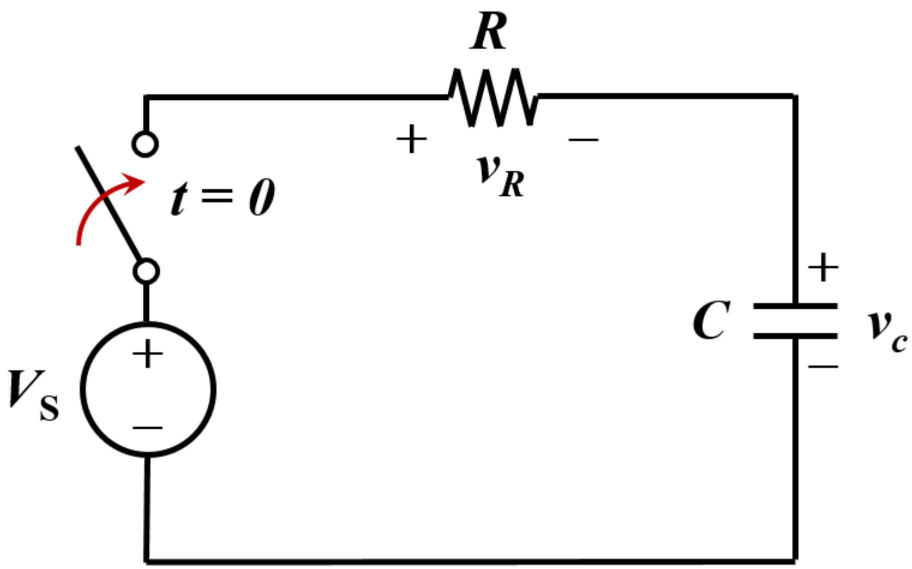
DC Transients

What are "Transients"?

- The time-varying voltages and currents resulting from the adding or removing of a power source to circuits containing energy storage elements (e.g., capacitors/ inductors)

RC Circuit with a DC Source



- From KVL:

$$V_S = v_R + v_c$$

- Recall:

$$i = C\frac{dv_c}{dt} \text{ and } v_R = iR$$

- Hence,

$$V_S = RC\frac{dv_c}{dt} + v_c$$

First-order differential equation

First-order Differential Equation's Solution

- First-order differential equation:

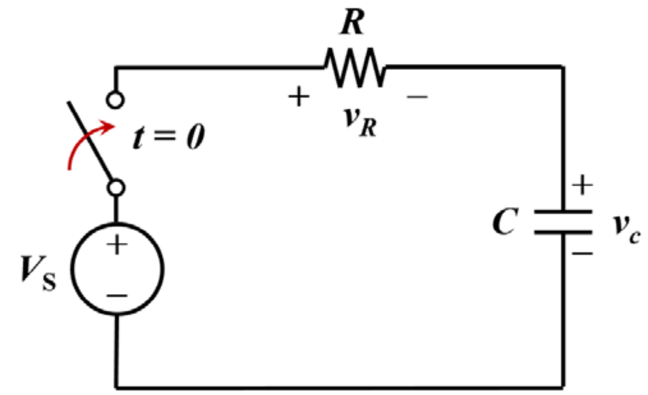
$$\tau \frac{dx}{dt} + x = K, \text{where } \tau \ \& \ K \text{ are constants}$$

- Solution has the following form:

$$x(t) = x(0)e^{-\frac{t}{\tau}} + K[1 - e^{-\frac{t}{\tau}}]$$

- τ is called the "time constant"

Capacitor's Charging Voltage in a Series RC Circuit



- From Slide 3:

$$V_S = RC\frac{dv_c}{dt} + v_c$$

- Compare with 1st order differential equation from Slide 4:

$$K = \tau\frac{dx}{dt} + x$$

- Hence, the solution is

Note that V_S is actually $v_c(\infty)$

$$\boxed{v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + V_S[1 - e^{-\frac{t}{\tau}}],\ \tau = RC}$$

- If $v_c(0) = 0$, then $\boxed{v_c(t) = V_S[1 - e^{-\frac{t}{\tau}}],\ \tau = RC}$

Capacitor's Discharging Voltage in a Series RC Circuit

- Following the same procedure, we can derive a capacitor's voltage when it is discharging through a resistor connected in series:

$$\boxed{v_c(t) = v_c(0)e^{-\frac{t}{\tau}}, \text{ where } \tau = RC}$$

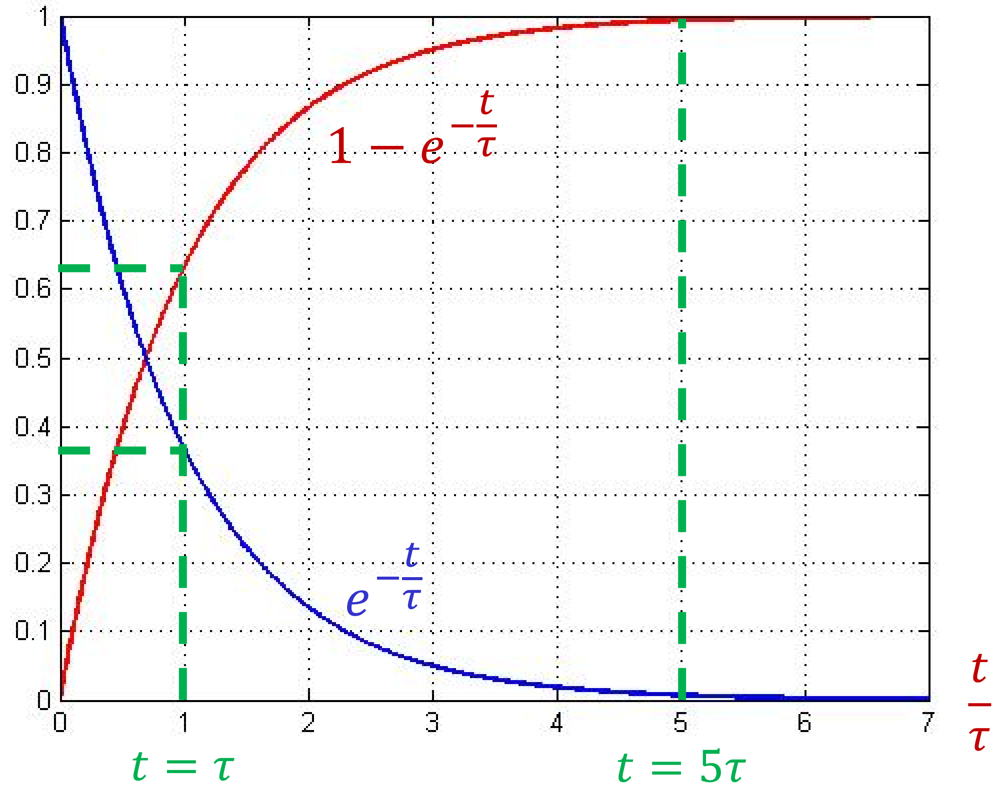
- Hence, a more general form of a capacitor's transient voltage in a series RC circuit can be expressed as

$$\boxed{v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + v_c(\infty)[1 - e^{-\frac{t}{\tau}}],\ \tau = RC}$$

Shape of First-order Transient

Charging: $v_c(t) = V_S[1 - e^{-\frac{t}{\tau}}]$ Discharging: $v_c(t) = v_c(0)e^{-\frac{t}{\tau}}$

t	$e^{-\frac{t}{\tau}}$	$1 - e^{-\frac{t}{\tau}}$
τ	0.368	0.632
2τ	0.135	0.865
3τ	0.050	0.950
4τ	0.018	0.982
5τ	0.007	0.993



Steps for Solving $v_c(t)$ Transient in General RC Circuits

- Use DC steady-state analysis of the circuit before the transient starts to find the initial values:

$$v_c(0^-) = v_c(0^+)$$

- And also the final steady-state values:

$$v_c(\infty)$$

- Use Thevenin equivalent to reduce the circuit to the standard series RC circuit
- Use the general form of the solution:

$$\boxed{v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + v_c(\infty)[1 - e^{-\frac{t}{\tau}}],\ \tau = RC}$$

THANK YOU