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Group 4B

Week 4 studio 2

3/9/2020

## Node Analysis

Branch voltage is the potential difference across the element in a branch.
Node voltage is the potential difference between the given node and another reference node / ground node

Since node voltages are defined with respect to a common reference, choice of reference ground is important

↳ maximum number of circuit elements connected to it
↳ connects to maximum number of voltage sources
↳ ground node that leads to more intuitive operation in circuit

## Node Method

① Select a reference / ground node. Define its potential to be 0V.

② Label potentials of remaining nodes wrt to ground node.
Nodes connected to ground through independent / dependent voltage sources should be labelled with the voltage of the source
Remaining unknowns labelled $e_1, e_2 \cdots e_n$

③ For each unknown node voltage, write KCL for that node
Use KVL and element laws to replace current with voltage & element parameters.
1 eqn for each unknown node voltage

④ Solve simultaneous equations

⑤ Back solve for branch voltage and currents.

* general comments on equations produced by node method
If circuit is made of linear elements ⟹ source term enters equations as sums, not products

Gives rise to intuition for superposition for linear networks

## Superposition

Node analysis results in expressions $e_i = \dfrac{V_1 \cdot f_1(R) + V_2 \cdot f_2(R) + \cdots I_1 f_\alpha(R) + I_2 f_\beta(R) + \cdots}{g(R)}$

where RHS consists of each source term multiplied by its resistive / conductive factor

in a linear circuit there are no products of source terms.

In Matrix form $[R][e] = [S][s]$

$[R]$: resistance / conductance matrix

$[e] = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$ col vector of unknown voltages

$[S]$: source matrix

$[s] = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ I_1 \end{bmatrix}$ col vector of sources

$[R]$ is symmetric

Due to linearity, each source term remains unchanged if all other sources are set to 0

$f(ax_1 + bx_2 + cx_3 + \cdots) = af(x_1) + bf(x_2) + \cdots$

$V = 0$ (short) $\quad I = 0$ (open)

## Superposition method

① For each independent source, form a subcircuit with all other independent sources set to 0.

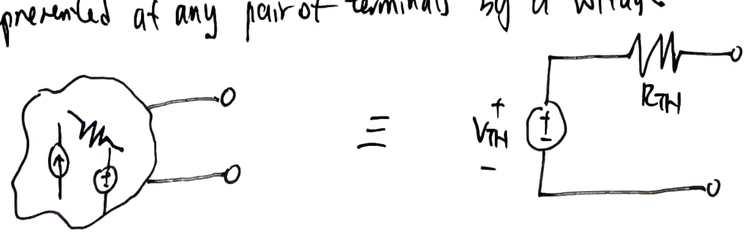
② Find response of each independent source acting alone

③ Total response can be found by summing response to each independent source acting alone

## Thevenin's Theorem

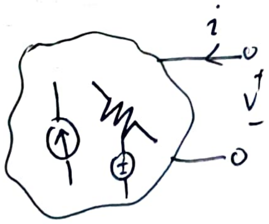
simple extension of the concept of superposition allows us to suppress details in circuits to focus only on parts of networks we are interested in.

If the system is linear, any collection of voltage sources, current sources, resistors can be represented at any pair of terminals by a voltage source and resistor



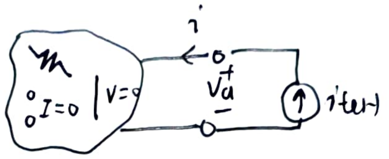

* Thevenin equivalent circuit

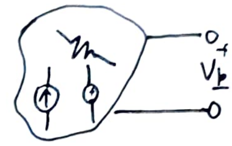
A general linear network containing sources and resistors can be shown as an amorphous box, with the terminals of interest emerging on the right.




To find relationship between $v$ & $i$ at terminal of interest, apply excitation $i_{test}$ and measure the response

we can choose to apply a test current source for simplicity.

By superposition, response = 

+ 

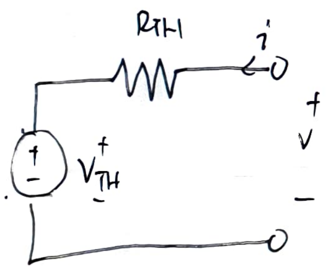
$i_{test} = 0$

all internal sources set to 0 + $i_{test} = 0$

$V_a = i_{test} R_{TH}$ * Thevenin equivalent resistance

where $R_{TH}$ is the net resistance measured between the 2 terminals when all internal independent sources are set to 0

response $= V_a + V_b$
$= i_{test} R_{TH} + V_{TH}$




$V_b = V_{TH}$

where $V_b$ is just the open circuit voltage at the terminals when no current is flowing

Thevenin equivalent circuit

Thevenin equivalent circuit for any linear network at a given pair of terminals consists of a voltage source $V_{TH}$ in series with a resistor $R_{TH}$

① $V_{TH}$ found by measuring / calculating open circuit voltage at terminal pair
② $R_{TH}$ found by measuring / calculating resistance of network as seen from terminal pair with all independent sources internal to network set to zero.

Norton equivalent is similar to Thevenin except we apply test voltage instead of test current.

to find $i_N$ and $R_N = R_{TH}$

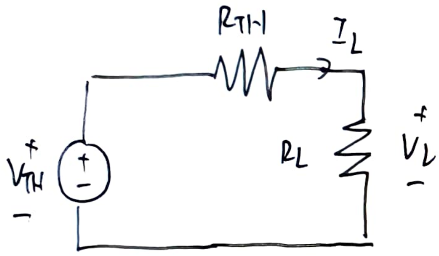
# Measurements for Thevenin parameters

2 independent measurements are required to determine parameters for Thevenin model

① open circuit voltage at terminal pair | $i_{term} = 0$
$V_{TH}$

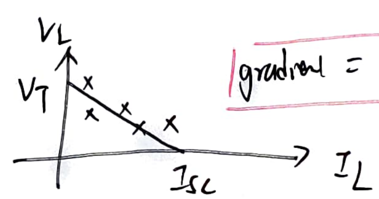
② $R_{TH}$ using Ohm's law if we know the short circuit current $\qquad R_{TH} = \dfrac{V_{TH}}{I_{SC}}$

However, there might be a risk that $I_{SC}$ is large,
it is safer to connect circuit to multiple resistive loads and solve for $R_{TH}$
using a graphical approach.




By KVL: $V_{TH} - V_L - I_L R_{TH} = 0$

$$V_L = V_{TH} - R_{TH} I_L$$

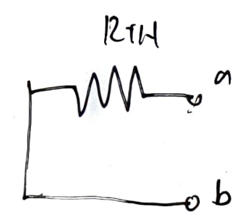



$\boxed{\text{gradient} = -R_{TH}}$

OR

## Analytical method

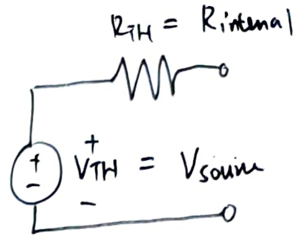
find equivalence resistance after setting all internal sources to 0



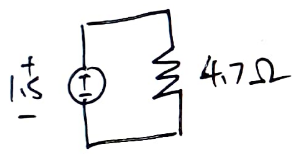

resistance seen across a & b = $R_{TH}$

# Activity 1

A battery loses energy in the form of heat when used, hence cannot be modelled as a pure voltage source. We typically model a battery as a voltage source with a resistance in series, called the internal resistance. This corresponds to the simplest case of a Thevenin equivalent circuit.




Power resistors used have a 0.5W rating.
The resistor power rating is the amount of heat a resistive element can dissipate for an indefinite period of time w/o degrading performance.
It is also a safety measure to prevent a short due to heat melting components in the resistor.




If we connect one of the power resistors provided / smallest resistance eg 4.7 Ω and assume negligible internal resistance from battery, power dissipated by resistor $= \frac{1.5V^2}{4.7\Omega} = 0.479$ W

which is just barely below 0.5W rating and way above 0.25W

$V_{open, Duracell} = 1.57$ V

$V_{open, Energizer} = 1.58$ V

By Ohm's law, $\boxed{I_R = \frac{V_R}{R}}$

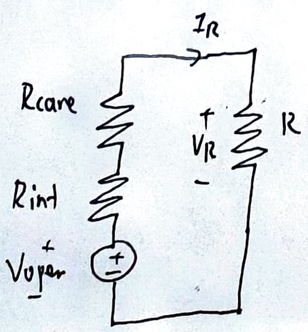
for calculated current $I_R$

~ insert table and plot from excel ~

* should not use multimeter to measure current as current may be high due to low resistance, fuse might blow

when resistance is infinite, no current will pass through → open circuit.

4.3) we should not take voltage directly across the battery inside the battery case as the case might have non-negligible resistance

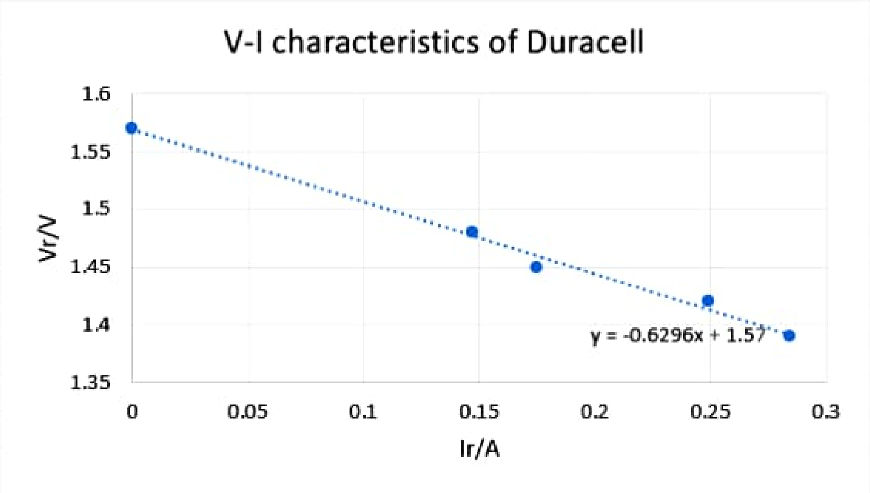


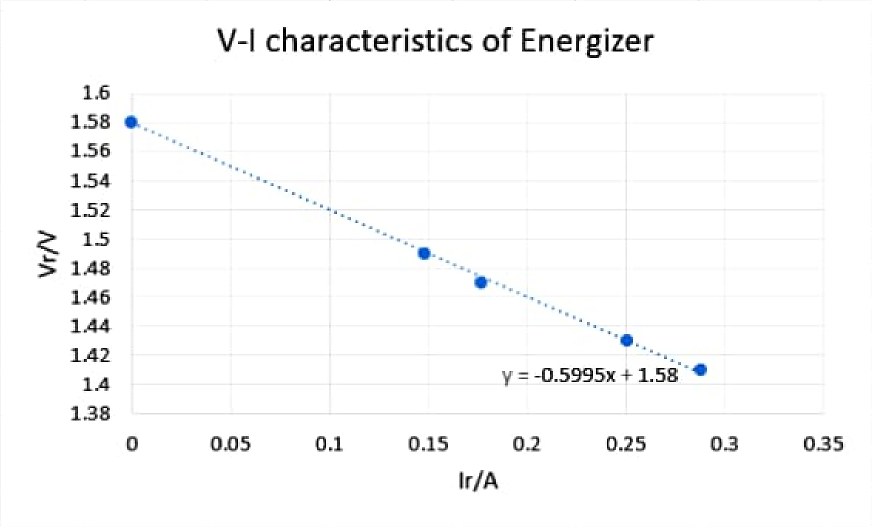

By KVL: $V_{open} - V_R - I_R R_{case} - I_R R_{int} = 0$

$V_R = V_{open} - \boxed{I_R R_{case}} - I_R R_{int}$

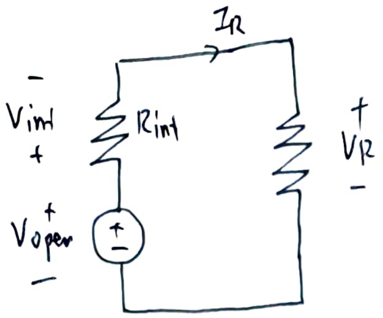
extra term which causes $V_R$ to be lower when connected outside of case compared to inside

| | | | Duracell | | Energizer | |
|---|---|---|---|---|---|---|
| nominal res /Ω | | measured res /Ω | Vr / V | Ir / A | Vr / V | Ir /A |
| inf (open circuit) | | inf | 1.57 | 0 | 1.58 | 0 |
| 10 | | 10.1 | 1.48 | 0.147 | 1.49 | 0.148 |
| 8.2 | | 8.3 | 1.45 | 0.175 | 1.47 | 0.177 |
| 5.6 | | 5.7 | 1.42 | 0.249 | 1.43 | 0.251 |
| 4.7 | | 4.9 | 1.39 | 0.284 | 1.41 | 0.288 |



A practical voltage source can be modelled as a series connection of an ideal voltage source $V_{open}$ and internal resistance $R_{int}$.




label voltages based on passive sign convention where current $I_R$ enters positive voltage terminal.

except $V_{open}$ which is obviously the source / active element

By KVL : $V_{open} - V_R - V_{int} = 0$

$$\boxed{V_R = V_{open} - I_R R_{int}}$$

The line through the data points should be linear as $V_{open}$ and $R_{int}$ are fixed, hence the derived equation shows that $V_R$ and $I_R$ are linearly related

when the line cuts the y-axis, $I_R = 0$ $\Rightarrow$ $V_R = V_{open}$ ( y-intercept should be set to $V_{open}$)

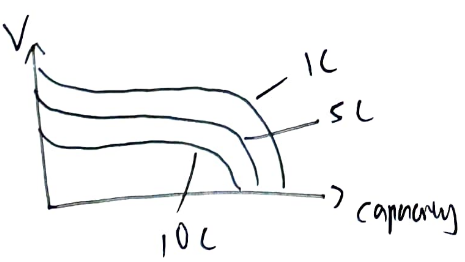
comparing both equations, $-R_{int}$ should be the gradient of the line

Duracell : $V_R = 1.57 - 0.6296\, I_R$ $\boxed{R_{int} \approx 0.630\,\Omega}$ Duracell

Energizer : $V_R = 1.58 - 0.5995\, I_R$ $\boxed{R_{int} \approx 0.600\,\Omega}$ Energizer

Analysis

when current drawn from the battery increases, the battery's terminal voltage $V_R$ drops. This behaviour can be related to battery discharge graphs where batteries with higher C-rate have lower mid point voltage

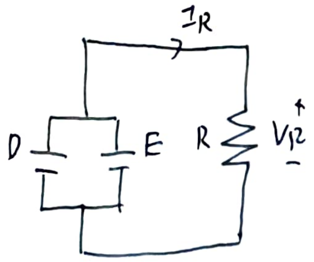



higher C-rate, higher current drawn

A good battery should have low internal resistance to reduce power dissipated within battery and to be able to deliver higher current on demand ( more in Activity 2 analysis )

shorted terminal $\Rightarrow$ $V_R = 0$ $\quad I_R = \dfrac{1.57\,V}{0.630\,\Omega} = \boxed{2.49\,A}$ $\quad$ Instantaneous $P = V_{open} \cdot I_R = \boxed{3.91\,W}$

## Activity 2




$V_{open}$ of combined power source $= 1.57V$

By Ohm's law, $I_R = \frac{V_R}{R}$

for each calculated current $I_R$

~ insert table and plot from excel ~

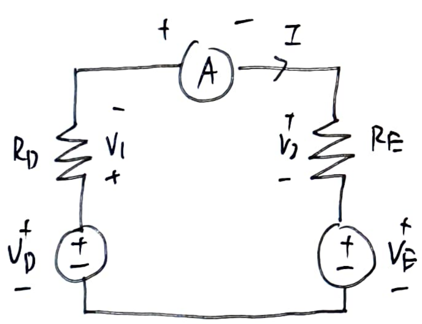
similar to activity 1, if we take the parallel batteries as a thevenin equivalent with $V_{open}$ connected in series with $R_{int}$

$$V_R = V_{open} - I_R R_{int}$$

From the graph: $V_R = 1.57 - 0.3962\, I_R$

estimated $R_{int}$ for combined power source $= 0.396\,\Omega$

which seems reasonable since it is lower than both individual $R_{int}$ for Duracell & Energizer batteries alone




By KVL : $V_D - V_E - R_E I - R_D I = 0$

$$I = \frac{V_D - V_E}{R_D + R_E}$$

If the 2 batteries have different open circuit voltage, current will flow from battery with higher voltage to battery with lower voltage.

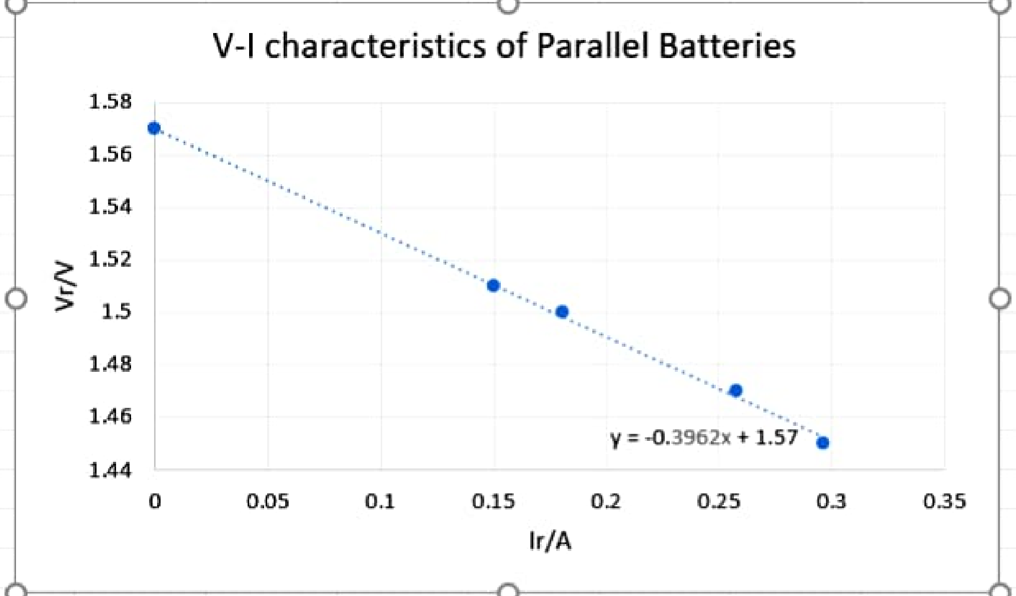
measured current $I = -1.7\,mA$

calculated $I = \frac{1.57V - 1.58V}{0.630\,\Omega + 0.600\,\Omega} = -8.13\,mA$

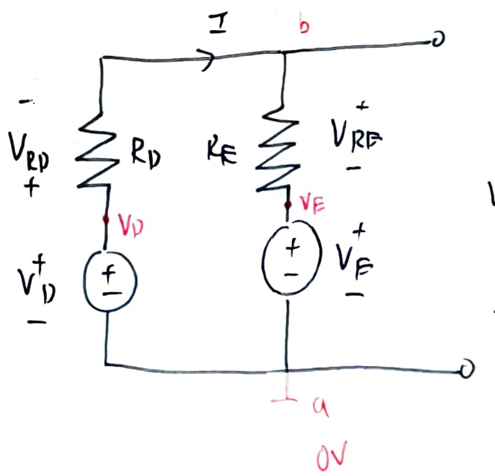
} difference could be high due to small difference in open voltage compared to internal resistance

* At least current in same direction ✓

| | | Parallel Batteries | |
|---|---|---|---|
| nominal res /Ω | measured res /Ω | Vr / V | Ir / mA |
| inf (open circuit) | inf | 1.57 | 0 |
| 10 | 10.1 | 1.51 | 0.15 |
| 8.2 | 8.3 | 1.5 | 0.181 |
| 5.6 | 5.7 | 1.47 | 0.258 |
| 4.7 | 4.9 | 1.45 | 0.296 |



Deriving Thevenin Equivalent of parallel power source




By KVL : $V_D - V_E - V_{RE} - V_{RD} = 0$

$$V_D - V_E = IR_E + IR_D$$

$$I = \frac{V_D - V_E}{R_D + R_E}$$

Taking node a to be ground, $V_{TH}$ = voltage across nodes a & b

$$V_{TH} = V_E + V_{RE}$$

$$= V_E + IR_E$$

$$= \boxed{V_E + \frac{V_D - V_E}{R_D + R_E} R_E}$$

also equivalent to $V_D - V_{RD}$

$$= V_D - \frac{V_D - V_E}{R_D + R_E} R_D$$

$R_{TH}$ can be found by setting all internal voltage sources to shorts $V = 0$

$$R_{TH} = R_D \parallel R_E$$

$$= \boxed{\frac{R_D R_E}{R_D + R_E}}$$

Using $R_D$ and $R_E$ obtained from Activity 1,

$$V_{TH} = 1.58V + \frac{1.57V - 1.58V}{0.630\Omega + 0.600\Omega} \times 0.600\Omega$$

$$= 1.575V$$

comparing with $V_{open}$ = 1.57V obtained from step 1, the 2 values agree with each othe

$$R_{TH} = \frac{0.630\Omega \times 0.600\Omega}{0.630\Omega + 0.600\Omega} = 0.307\Omega$$

comparing with $R_{int}$ from plot, $R_{int}$ = 0.396 Ω, which also agree... to a certain extent

Both $R_{TH}$ and $R_{int}$ should be lower than the individual $R_D$ & $R_E$ ✓✓

Comparing $R_{TH}$ to original $R_D$ and $R_E$

$R_{TH} = 0.307\,\Omega$ , $R_D = 0.630\,\Omega$

$R_E = 0.600\,\Omega$

$R_{TH}$ is lower than each individual internal resistance $R_D$ and $R_E$

$P_{LOSS} = I^2R \propto R$

2 Advantages of having lower internal resistance

1. Less power is dissipated by the battery's internal resistance when current flows
voltage drop due to internal resistance from $V_{open}$ is also lower } more efficient

2. low internal resistance delivers higher current on demand when a load
is connected, hence higher power is delivered to component $P = I^2R$

Precautions when choosing cells

1. open circuit voltage between the cells must be similar or a high current
might flow from the cell with higher voltage to cell with lower voltage
especially when internal resistance of batteries are low

$$I = \frac{V_D - V_E}{R_D + R_E}$$

This would cause energy to be wasted