CG1111: Engineering Principles and Practice I

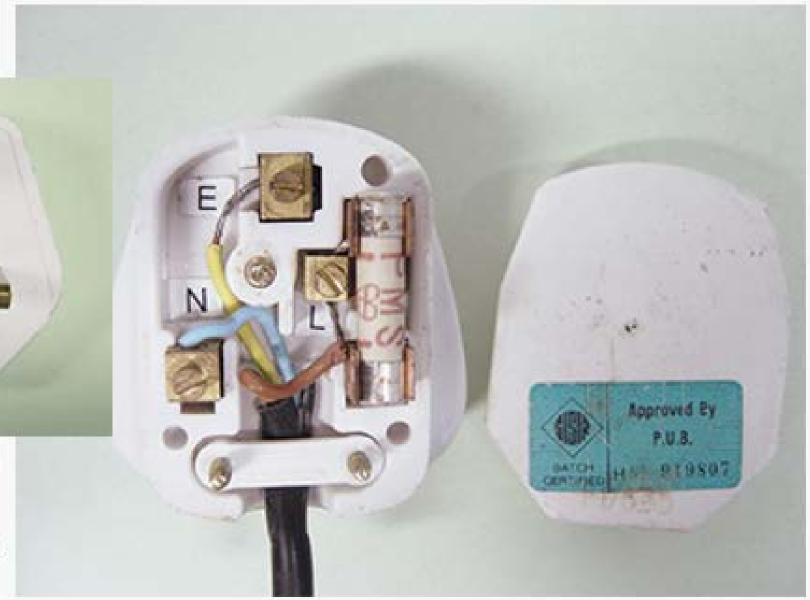
Principles of AC Circuits

National University of Singapore

Learning Outcomes

- Familiarize with the common power supply distribution at home/office
- Understand the properties of sinusoidal signals:
  - Amplitude, RMS, frequency, angular frequency, and phase
- Understand AC circuit analysis technique using phasor and impedance
  - Transformation to complex domain

3-Pin Power Plug

Parts of Plug	Image
Terminal hole marking • L ('Live', also known as 'Phase') • N ('Neutral') • E ('Earth', also known as 'Protective') Core colour of flexible cable • L: Brown • N: Blue • E: Green and yellow Fuses • A 13 Ampere plug should be fitted with a 13 Ampere fuse	 

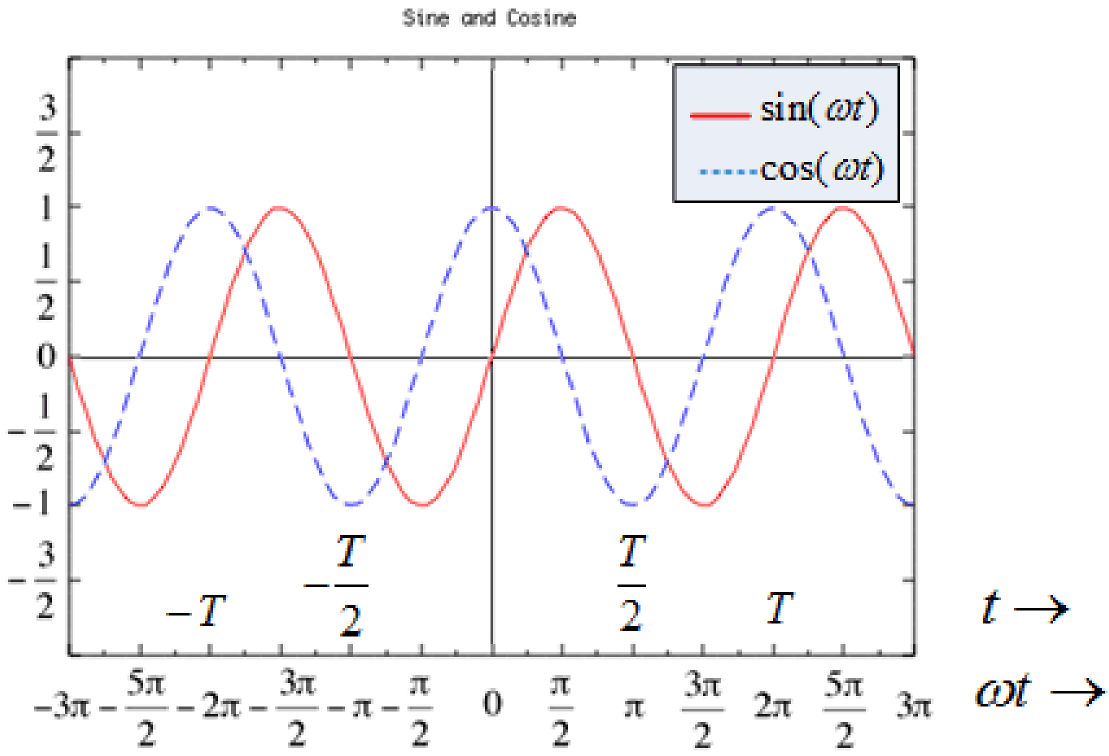
- Between the Live (L) and Neutral (N), there is a voltage source whose value is a sine/cosine function of time
- Frequency: 50 Hz
- Some appliances may transform it internally to DC

Sinusoidal Waveform

General expression of sinusoidal voltage:

$$v(t) = V_m \cos(\omega t + \emptyset)$$

V_m: Peak voltage
ω: Angular frequency in rad/s
$\emptyset$: Phase angle



Note:

- $\omega = 2\pi f$, where f is the frequency in Hz, i.e., number of cycles per sec
- $T = \frac{1}{f}$ is the time period which the waveform repeats itself

Root Mean Square (RMS)

- The RMS values of AC voltage and current, are the equivalent values of the DC voltage & current that would have the same <u>average</u> power dissipation in a resistive load

- Average power dissipation of resistive load in AC:

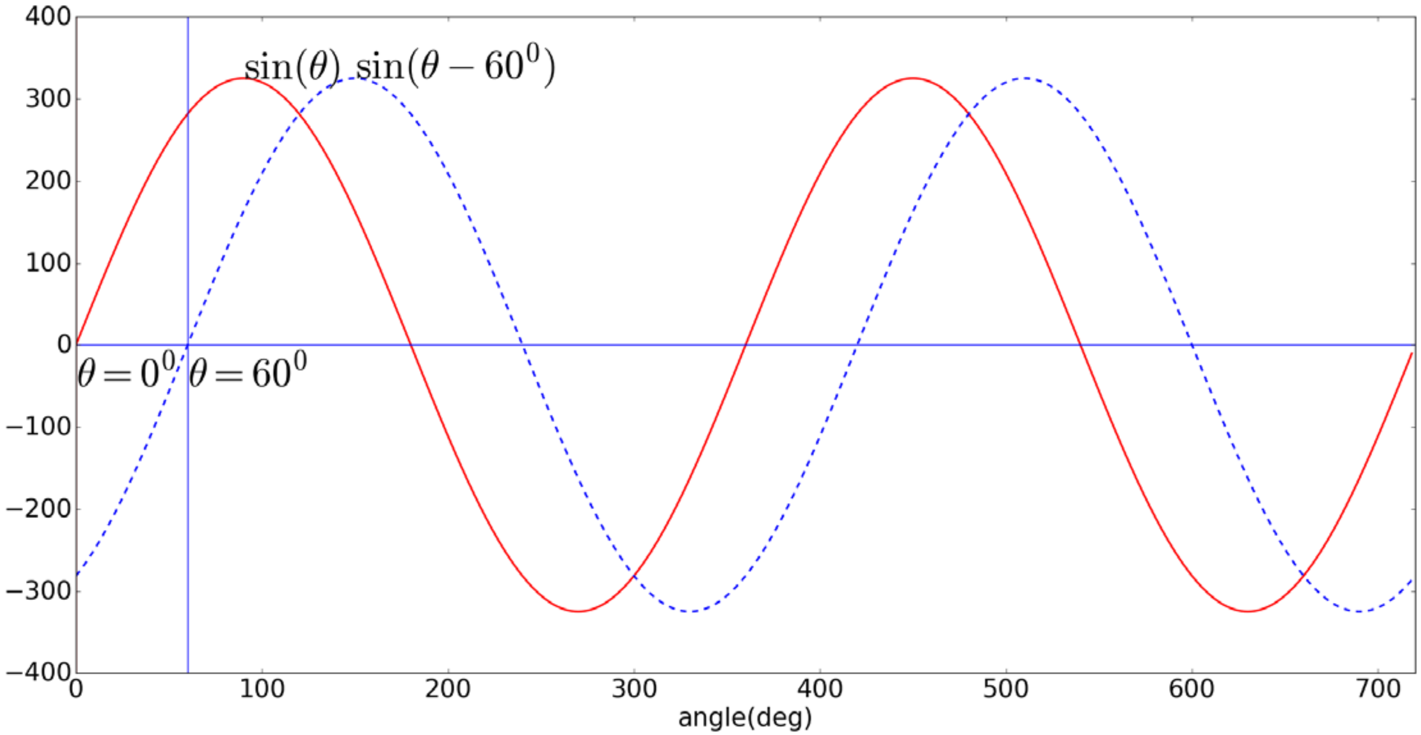
$$P = V_{rms} \times I_{rms} = {I_{rms}}^2 R = \frac{{V_{rms}}^2}{R}$$

- Singapore's mains electricity: 230 V rms

- Definition of RMS: $x_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$

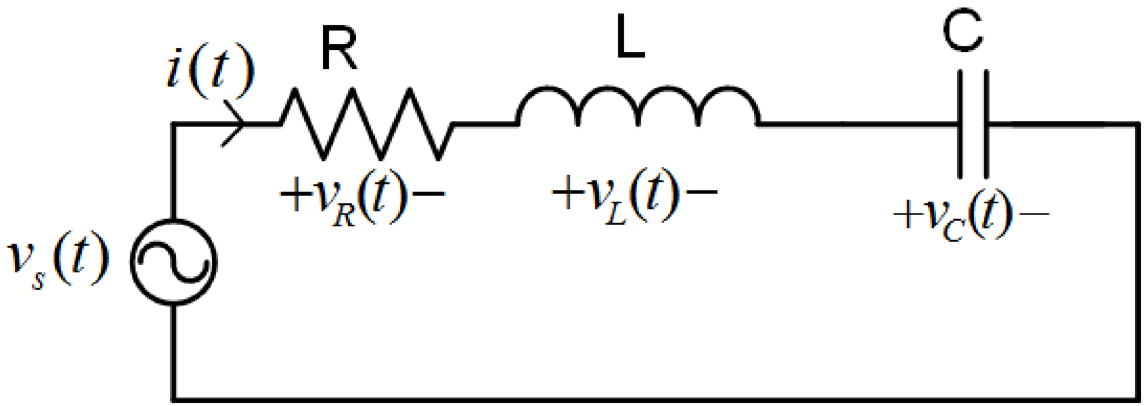
- For sinusoidal waveform, $v(t) = V_m \cos(\omega t + \emptyset)$, the relationship between V_{rms} and V_m is $\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$

Phase Difference Between Signals of the Same Frequency



- For signals with the same frequency, the relative position in their cycle is a measure of their phase difference
- In the above example, the blue (dotted) signal is lagging (behind) the red (solid) signal by 60°

AC Circuit Analysis in Time Domain?



- In DC circuits, inductors/capacitors are treated as short/open at steady state $\rightarrow$ result in resistive circuits

- In AC circuits, however, they will result in differential equations:

$$V_m \cos(\omega t) = v_R(t) + v_L(t) + v_C(t) = iR + L\frac{di}{dt} + \frac{\int i\, dt}{C}$$

$\rightarrow$ difficult to solve in time-domain!

AC Circuit Analysis in Complex Domain

- Transform differential equations in time-domain into algebraic equations in complex-domain!
- Two mathematical terms:
  - Phasor: Voltage & Current sinusoids are represented by complex numbers called “Phasors”
  - Impedance: R-L-C are represented by their complex resistances called “Impedances”
- This converts the AC circuit to an equivalent resistive circuit with DC sources
  - Can then use DC circuit analysis techniques!

Phasors

- Sinusoidal voltage:

$$V_m \cos(\omega t + \theta)$$

- Phasor:

$$V_m \angle\theta$$

Note:
Another common practice is to represent phasors using the RMS value instead of the magnitude. In that case, the phasor will be written as $\frac{V_m}{\sqrt{2}} \angle\theta$.

Note:

- Phasor is just a definition. It leads to mathematical convenience, but has no physical significance

Phasor Examples

- $325\cos(300t + 45°) \rightarrow 325\angle 45°$

- $10\cos(300t - 60°) \rightarrow 10\angle{-60°}$

- $20\sin(300t + 75°) \rightarrow 20\cos(300t + 75° - 90°)$
  $\rightarrow 20\angle{-15°}$

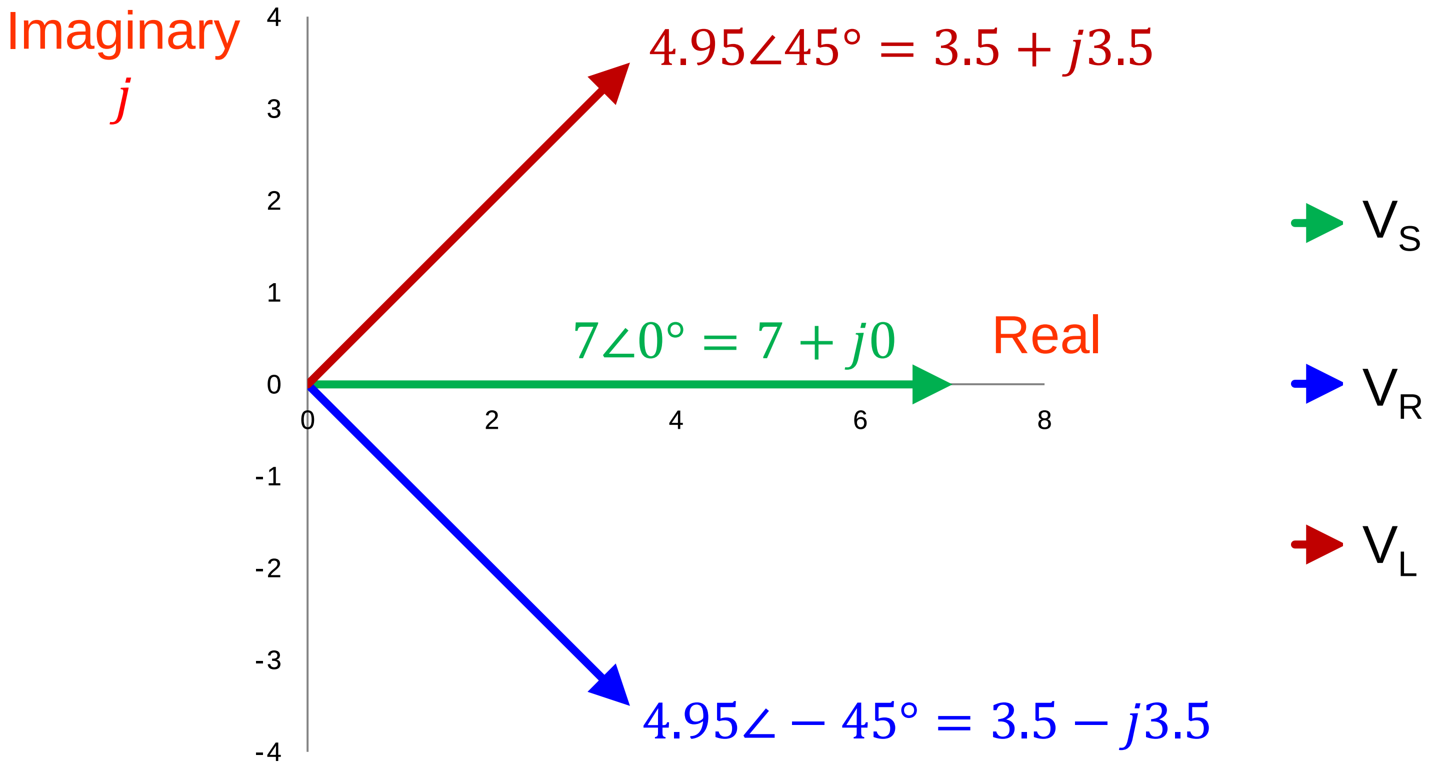
- All signals must have the same frequency
- All must be converted to sine or cosine (consistent) before taking phase angle

Phasor (Polar ↔ Rectangular Form)

$$r\angle\theta = x + jy$$

where $\quad x = r\cos\theta$

$\quad\quad\quad\; y = r\sin\theta$



Impedance of Inductor

- Suppose inductor current is

$$i(t) = I_m \cos(\omega t)$$
$$\to I_m \angle 0^{\circ} \text{ (Phasor)}$$

- Then its voltage is

$$v(t) = L\frac{di}{dt} = -\omega L I_m \sin(\omega t) = \omega L I_m \cos(\omega t + 90^{\circ})$$
$$\to \omega L I_m \angle 90^{\circ} \text{(Phasor)}$$

- Impedance of inductor is

$$Z_L = \frac{\text{voltage phasor}}{\text{current phasor}} = \frac{\omega L I_m \angle 90^{\circ}}{I_m \angle 0^{\circ}} = \omega L \angle 90^{\circ} = \boxed{j\omega L}$$

Impedance of Capacitor

- Suppose capacitor voltage applied is

$$v(t) = V_m \cos(\omega t)$$
$$\rightarrow V_m \angle 0^\circ \text{ (Phasor)}$$

- Then its current is

$$i(t) = C\frac{dv}{dt} = -\omega C V_m \sin(\omega t) = \omega C V_m \cos(\omega t + 90^\circ)$$
$$\rightarrow \omega C V_m \angle 90^\circ \text{(Phasor)}$$

- Impedance of capacitor is

Also equal to $\frac{1}{\omega C}\angle -90^\circ$

$$Z_C = \frac{\text{voltage phasor}}{\text{current phasor}} = \frac{V_m \angle 0^\circ}{\omega C V_m \angle 90^\circ} = \frac{1}{\omega C \angle 90^\circ} = \boxed{\frac{1}{j\omega C} = \frac{-j}{\omega C}}$$

Impedance of Resistor

- Suppose resistor current is

$$i(t) = I_m \cos(\omega t)$$
$$\rightarrow I_m \angle 0^\circ \text{ (Phasor)}$$

- Then its voltage is

$$v(t) = Ri = RI_m \cos(\omega t)$$
$$\rightarrow RI_m \angle 0^\circ \text{(Phasor)}$$

- Impedance of resistor is

$$Z_R = \frac{\text{voltage phasor}}{\text{current phasor}} = \frac{RI_m \angle 0^\circ}{I_m \angle 0^\circ} = R \angle 0^\circ = \boxed{R}$$

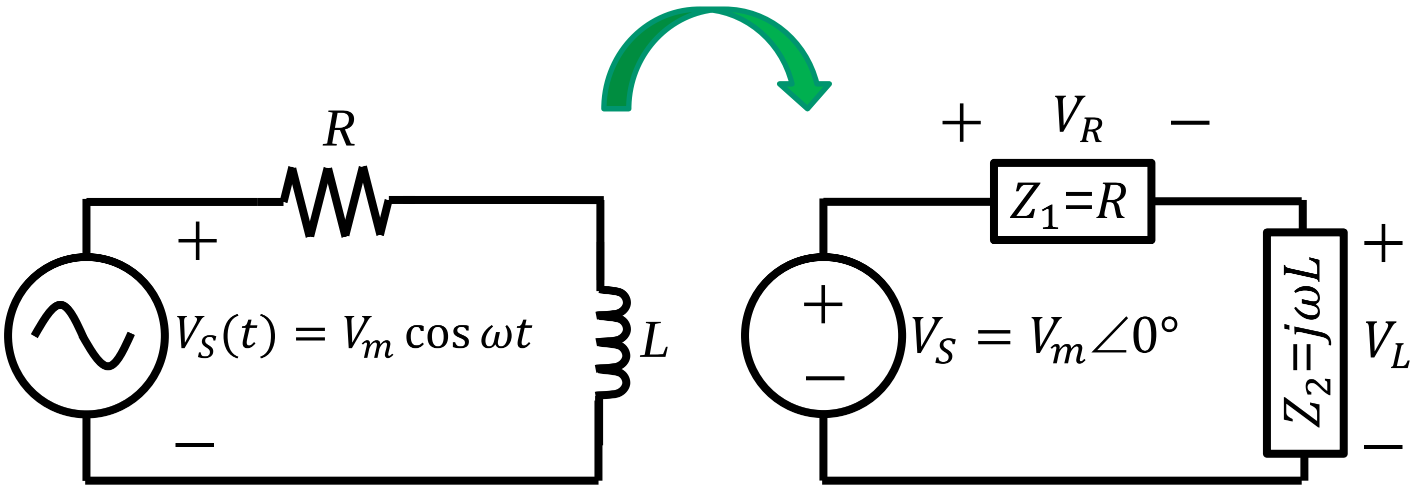
AC Circuit Analysis with Phasors & Impedances

Must work with KVL & KCL in Phasor form

Steps:

1. Replace voltage sources with their phasors (all must have same frequency)
2. Replace R, L, C elements with their impedances
3. Analyse circuit using DC circuit analysis techniques (work within complex domain)
4. Convert final results back to time-domain

AC Circuit Analysis Example



Time Domain **Complex Domain**

$$V_L = \frac{Z_2}{Z_1 + Z_2} V_m\angle 0° = \frac{j\omega L}{R + j\omega L} V_m\angle 0° = \frac{\omega L \angle 90° \times V_m \angle 0°}{\sqrt{R^2 + \omega^2 L^2} \angle \tan^{-1}(\frac{\omega L}{R})}$$

$$= \frac{\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle [90° - \tan^{-1}(\frac{\omega L}{R})]$$

Therefore, the inductor's voltage in time domain is:

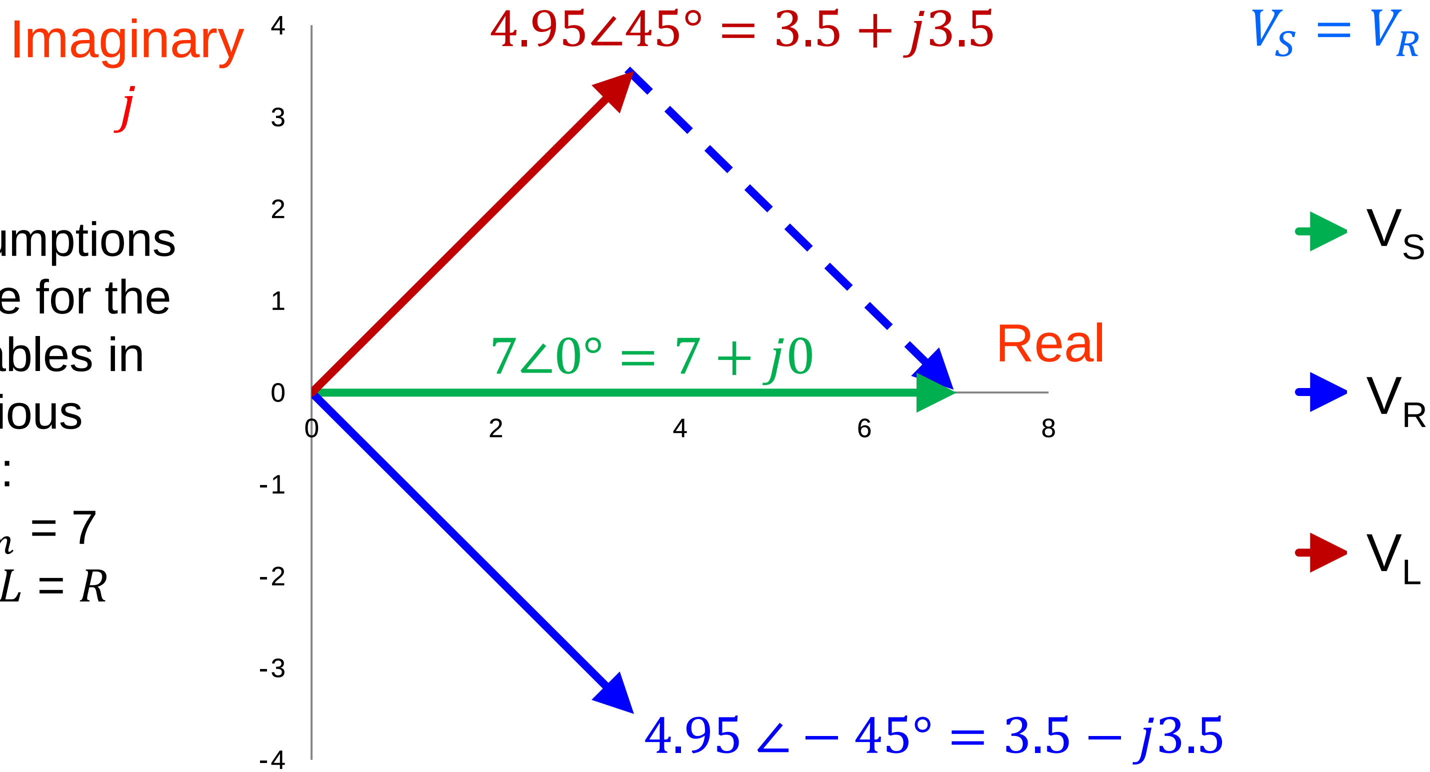
$$V_L(t) = \frac{\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos[\omega t + 90° - \tan^{-1}(\frac{\omega L}{R})]$$

Illustration: KVL Applies for Phasors

Voltage	Magnitude	Phase Angle	Real	Imaginary
V_S	7	0°	7	0
V_R	4.95	−45°	3.5	−3.5
V_L	4.95	45°	3.5	3.5

In AC circuits with R, L, C elements, because of the phase differences, we can only apply KVL in complex domain:

$$V_S = V_R + V_L$$



Assumptions made for the variables in previous slide:

- V_m = 7
- ωL = R

THANK YOU