CG1111: Engineering Principles and Practice I

Debrief and Tutorial for Week 11

(Filters and Signal Processing Basics)

National University of Singapore

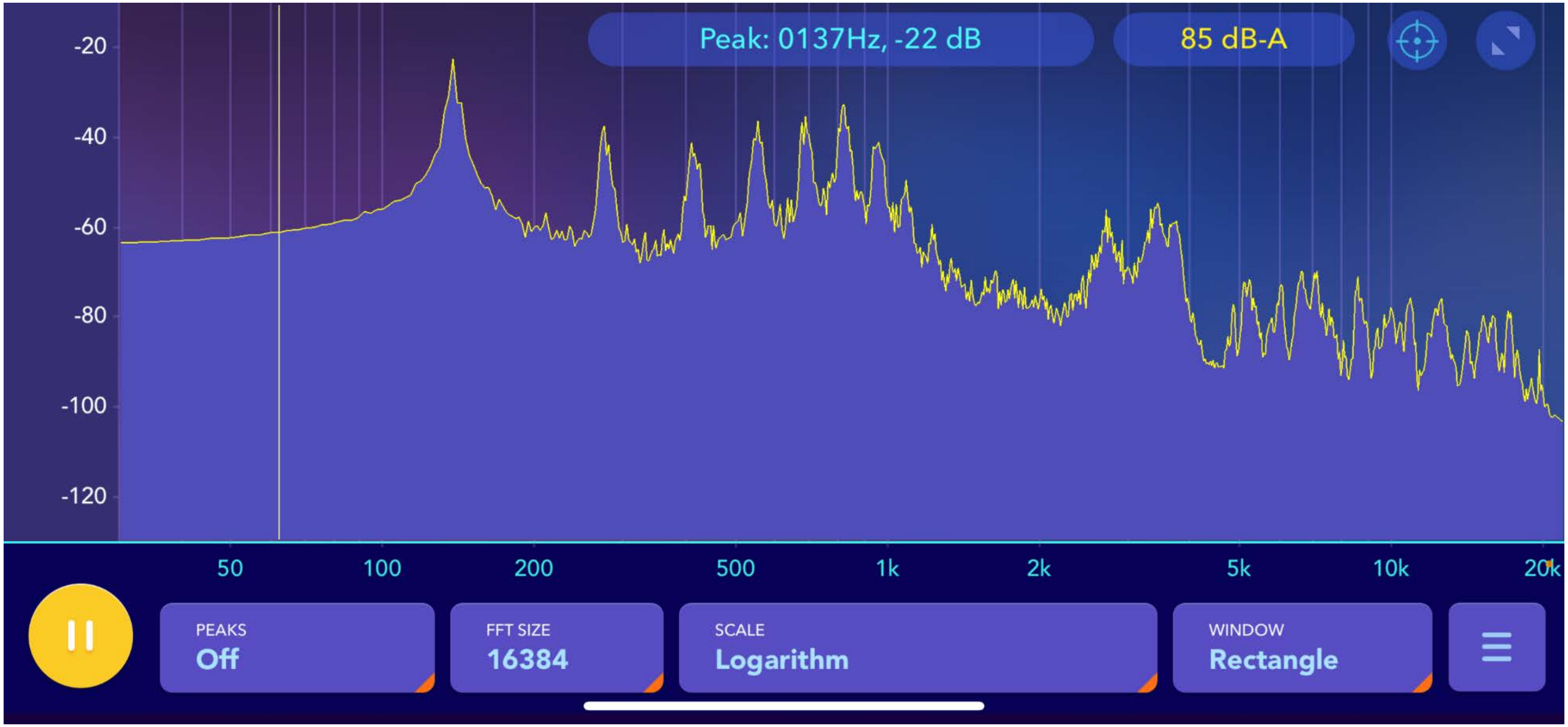
Spectral Analysis

- Any function of time can be described as a sum of sine waves, each with different amplitudes and frequencies

- **Spectral analysis** investigates the distribution of a signal's frequency components

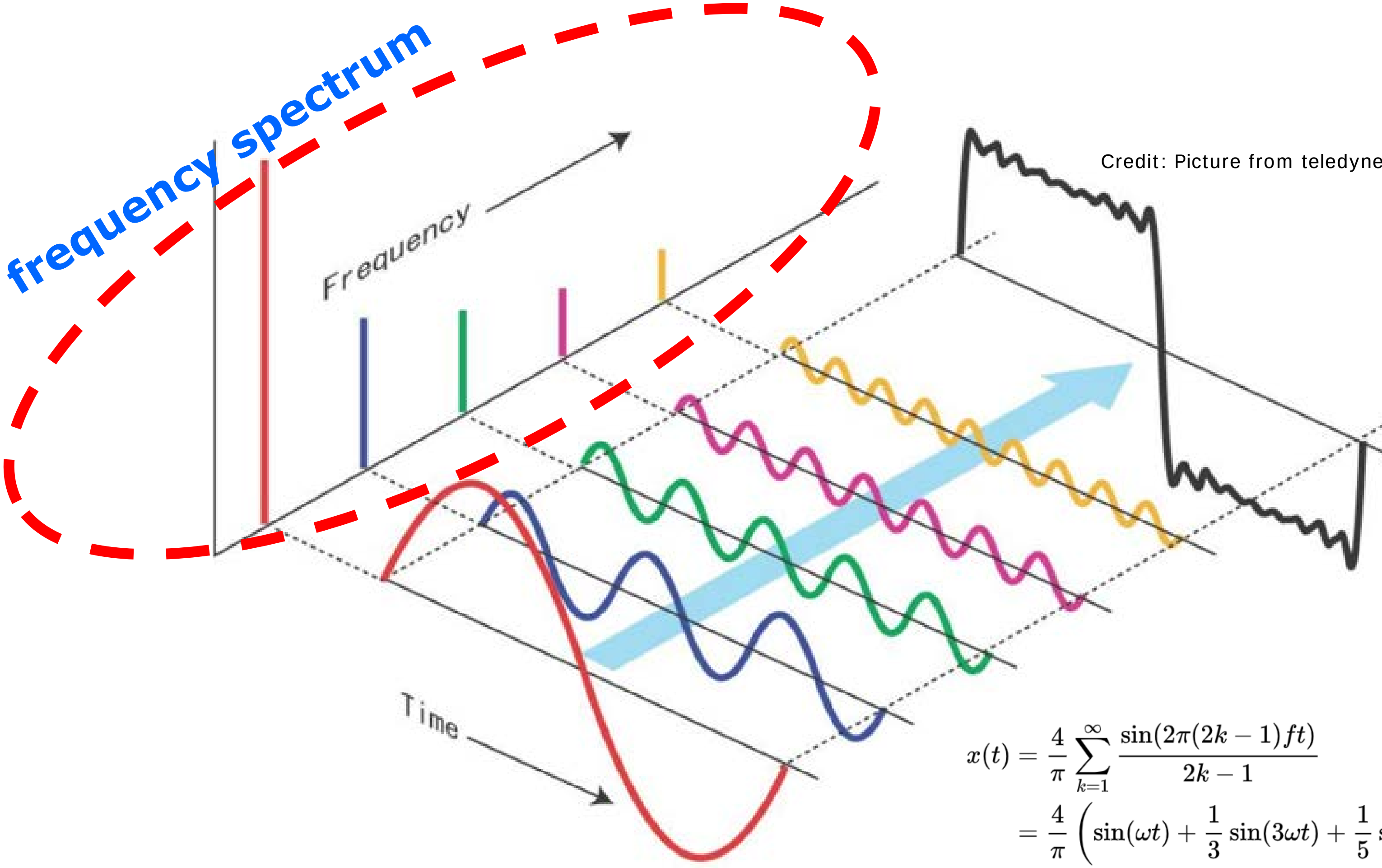
- The plot of a signal's frequency components and their corresponding magnitudes is called "**frequency spectrum**"

Example of Frequency Spectrum



The frequency spectrum of a particular audio tone

Oscilloscope Can Help Us Perform Spectral Analysis

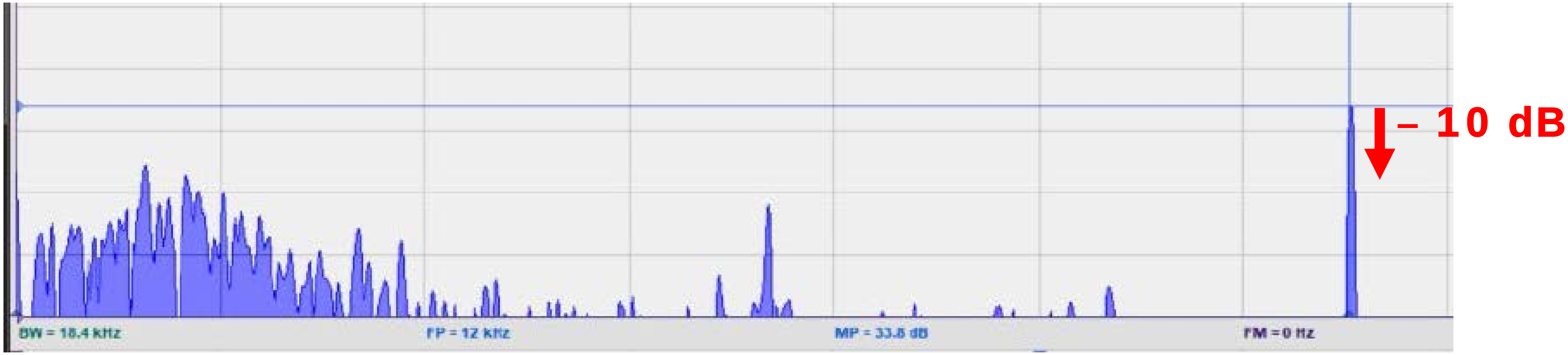


Credit: Picture from teledynelecroy.com

$$x(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2\pi(2k-1)ft)}{2k-1}$$
$$= \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \dots\right)$$

- A square wave (triangular waveform too) can be decomposed into an infinite sum of sinusoidal waves

Filters Can Help Suppress (Attenuate) Undesirable Frequencies



Question 1

- An audio song from a CD is passed through a low-pass filter with cut-off frequency of 4000 Hz to obtain the signal $Y(t)$.

- What is the minimum sampling rate that we can use to sample $Y(t)$ without degrading its quality significantly?

Solution to Q1

- The audio spectrum ranges from 20 Hz to 20 kHz

- After applying low-pass filter with 4 kHz cut-off frequency, the frequencies beyond 4 kHz are attenuated considerably (they are now less than 50% of their original power)

- The minimum sampling rate is 8 kHz (although it is still better to sample at higher frequencies, e.g., 16 kHz, to minimize aliasing effects)

Aliasing can still occur from the attenuated frequencies, creating random noise which might fall within the desired 20 Hz - 4 kHz range

Question 2

A microphone is recording an audio song with a frequency range of the 5th octave.

The song is corrupted by a 12 kHz noise.

This recording needs to be sampled by an Arduino Uno for controlling the loudness of the speaker connected to the audio system.

Solution to Q2a

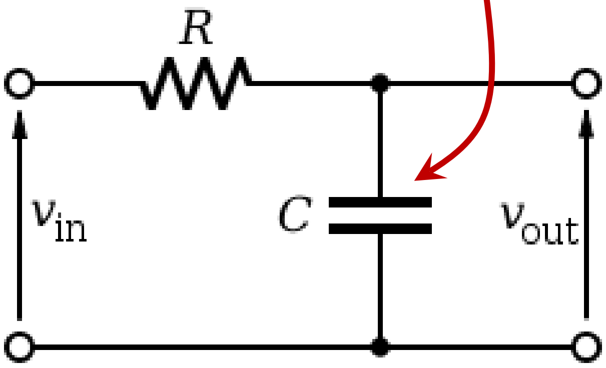
a) Design an analog low-pass filter to suppress the 12 kHz noise by at least 15 dB

Since it is not mentioned if the filter has to be active or passive, we can design either one. The design for the RC circuit will be the same.

The active filter (e.g., using an additional non-inverting amplifier) will have an additional gain factor associated with it.

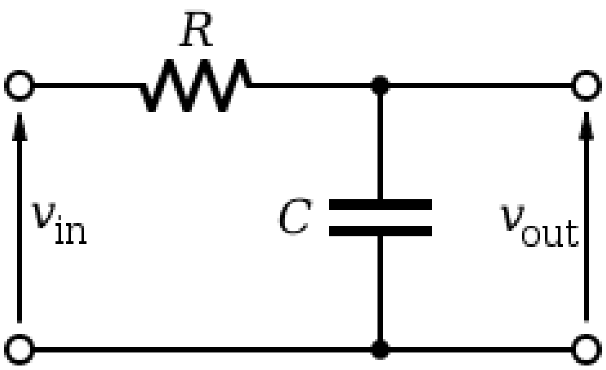
Let us design a passive filter

The capacitor's impedance reduces as frequency increases



Solution to Q2a

- Here V_{out} = Voltage across the capacitor
- We can use the voltage divider rule to find the voltage gain



$$\Rightarrow V_{\text{out}} = \left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R}\right] V_{\text{in}}$$

$$\Rightarrow \frac{V_{\text{out}}}{V_{\text{in}}} = \left[\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R}\right] = \left[\frac{1}{1+jR\omega C}\right] = \frac{1\angle 0^\circ}{\sqrt{1^2+(R\omega C)^2}\angle\theta^\circ}$$

$$\Rightarrow \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \left[\frac{1}{\sqrt{1+(R\omega C)^2}}\right] = \underbrace{20\log_{10}1}_{\substack{\text{Passband gain} \\ \text{(= 0 dB)}}} - \underbrace{20\log_{10}\sqrt{1+(R\omega C)^2}}_{\substack{\text{Change in gain with } \omega \\ \text{(we want –15 dB at 12 kHz)}}}\,\text{dB}$$

Magnitude

Solution to Q2a

A gain reduction of 15 dB at f=12 kHz means:

$$-20\log_{10}\sqrt{1+(\omega CR)^2}\Big|_{f\,=\,12\text{ kHz}} = -15\text{ dB}$$

Hence, $\sqrt{1+(\omega CR)^2} = 10^{15/20}$ when f = 12 kHz

Our low-pass filter needs to have:

$$RC = \frac{\sqrt{10^{(15/20)2}-1}}{2\pi \times 12000} = 73.4\ \mu\text{s}$$

If we choose *C* = 10 nF, we get *R* = 7.34 kΩ

Solution to Q2b

b) What is the minimum sampling rate required for processing the audio song?

- 5^{th} Octave: 523 Hz to 987 Hz (C5 to B5) (not required to remember)

- The maximum frequency in this octave is 987 Hz

➔ The minimum sampling frequency will be 2×987 Hz = 1974 Hz

Solution to Q2c

c) If we need to design a digital low-pass filter to suppress the 12 kHz noise (instead of an analog low-pass filter), what is the minimum sampling rate required?

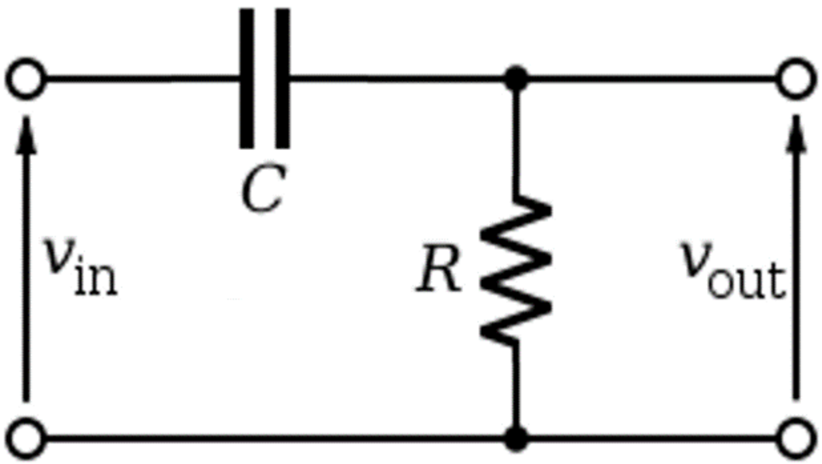
If the low-pass filter is digital, we need to consider that the maximum frequency in the signal is 12 kHz

➔ The minimum sampling frequency will be 2×12 kHz = 24 kHz

(Otherwise the noise will have aliasing after sampling, and end up as some other frequencies which may be difficult to suppress)

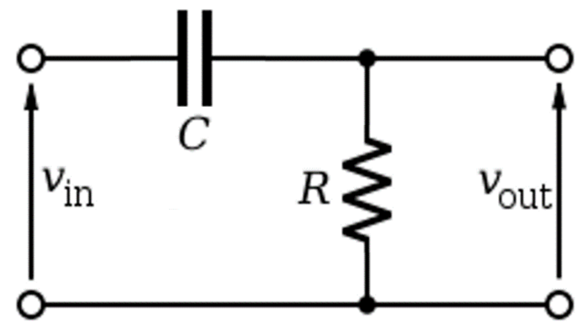
Question 3

For the following passive first-order high-pass filter, show that its cutoff frequency is given by $f_H = \frac{1}{2\pi CR}$



Solution to Q3

- Using voltage divider rule to find the voltage gain:



$$\Rightarrow V_{\text{out}} = \left[\frac{R}{R + \frac{1}{j\omega C}}\right] V_{\text{in}}$$

$$\Rightarrow \frac{V_{\text{out}}}{V_{\text{in}}} = \left[\frac{R}{R + \frac{1}{j\omega C}}\right] = \left[\frac{1}{1 - \frac{j}{\omega CR}}\right] = \frac{1\angle 0^\circ}{\sqrt{1^2 + \left(\frac{1}{\omega CR}\right)^2}\angle\theta^\circ}$$

$$\Rightarrow \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \left[\frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}}\right] = \underbrace{20\log_{10} 1}_{\substack{\text{Passband gain} \\ (=\ 0\ \text{dB})}} \underbrace{- 20\log_{10}\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}\ \text{dB}}_{\substack{\text{Change in gain with } \omega \\ (-3\ \text{dB occurs at } f_H)}}$$

Magnitude

Solution to Q3

A gain reduction of 3 dB at $f = f_H$ means:

$$-20\log_{10}\sqrt{1+\left(\frac{1}{\omega CR}\right)^2}\,\Bigg|_{f=f_H} = -3\text{ dB}$$

Hence, $\sqrt{1+\left(\frac{1}{\omega CR}\right)^2} = 10^{3/20}$ when $f = f_H$,

which gives $\frac{1}{\omega CR} = \frac{1}{2\pi f_H CR} = 1 \;\rightarrow\; f_H = \frac{1}{2\pi CR}$