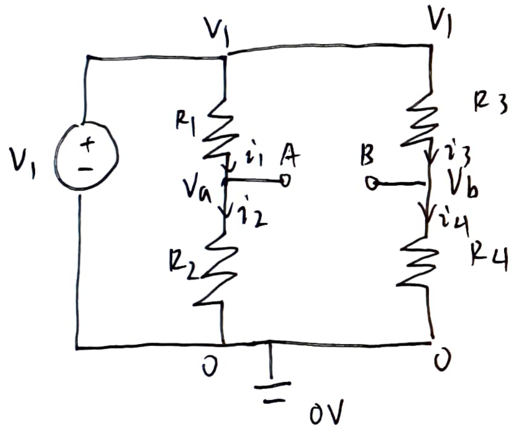
Week 3 Studio 2

Node Method

1. select a reference node, ground, defined to be 0V potential
2. label potential of remaining nodes wrt ground node. Nodes connected to ground via an independent source is labelled with the voltage of the source (with appropriate polarity)
3. Remaining unknowns labelled
4. write KCL for nodes with unknown voltage applying element laws / KVL.
5. solve for unknown potentials and back solve / substitute for branch voltage / current

Activity 1



Applying KCL at node V_a : $i_1 = i_2$

$$\frac{V_1 - V_a}{R_1} = \frac{V_a - 0}{R_2}$$

$$V_a\left(\frac{1}{R_2} + \frac{1}{R_1}\right) = \frac{V_1}{R_1}$$

$$V_a = \frac{R_2}{R_1 + R_2} V_1$$

OR by quoting voltage divider principle :

$$V_a = \frac{R_2}{R_1 + R_2} V_1$$

$$V_b = \frac{R_4}{R_3 + R_4} V_1$$

Applying KCL at node V_b : $i_3 = i_4$

$$\frac{V_1 - V_b}{R_3} = \frac{V_b - 0}{R_4}$$

$$V_b = \frac{R_4}{R_3 + R_4} V_1$$

$$V_{AB} = V_a - V_b = V_1\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right)$$

not equivalent
reverse polarities.

$$V_{BA} = V_b - V_a = V_1\left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right)$$

Suppose $R_1, R_2, R_3 = R$

$R_4 = R + \Delta R$

$$V_{AB} = V_1 \left(\frac{R}{R+R} - \frac{R+\Delta R}{R+R+\Delta R} \right)$$

$$= V_1 \left(\frac{1}{2} - \frac{1+\frac{\Delta R}{R}}{1+1+\frac{\Delta R}{R}} \right)$$

divide both numerator and denominator by R

$$= V_1 \left(\frac{1}{2} - \frac{1+x}{2+x} \right)$$

let $\frac{\Delta R}{R}$ be x

$$= \left(\frac{-x}{4+2x} \right) V_1$$

$$4V_{AB} + 2xV_{AB} = -xV_1$$

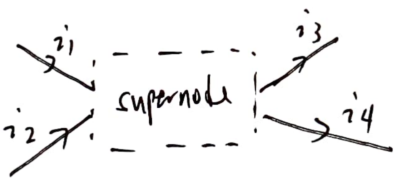
$$x(2V_{AB} + V_1) = -4V_{AB}$$

$$\boxed{x = \frac{\Delta R}{R} = \frac{-4V_{AB}}{2V_{AB} + V_1}}$$

Supernodes

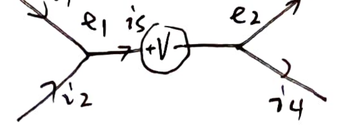
treat the group of elements as a black box / super node

current going into the supernode = current coming out of super node (KCL)



$$i_1 + i_2 = i_3 + i_4$$

super nodes are usually used when node analysis fails due to a floating independent voltage source



$$e_2 = e_1 - V$$

Activity 2

$V_1 = 5.12\ V$

from activity 1 : $\frac{\Delta R}{R} = \frac{-4V_{AB}}{2V_{AB}+V_1}$

At reference value $R_1, R_2, R_3, R_4 = 500$

$V_{AB} = 1mV$

$\Delta R = \frac{-4V_{AB}}{2V_{AB}+V_1} \cdot R$

* minimise as best as you can

$\Delta R \approx KV_{AB}$ $K = -390.625$ (see derivation)

Resistance R_4 / Ω	Voltage V_{AB} / mV	ΔR using precise equation / Ω	ΔR using linear approximation / Ω
480 Ω	53	-20.28	-20.70
490 Ω	26	-10.05	-10.16
510 Ω	-25	9.86	9.77
520 Ω	-52	20.73	20.31

~ see sources of error in discussion behind ~

Linear approximation

when $\Delta R << R$, $\frac{\Delta R}{R} \approx 0$

from activity 1 : $V_{AB} = \left(\frac{-x}{4+2x}\right) V_1$ where $x = \frac{\Delta R}{R}$

If we allow the x in the denominator to → 0, we can obtain a linear approximation of ΔR

* we cannot let the numerator in the equation to → 0 as the equation $V_{AB} = 0 \cdot V_1$ would have no useful meaning

$$V_{AB} \approx \frac{-\frac{\;}{R}}{4} V_1$$

$$\Delta R \approx \frac{-4V_{AB} \cdot R}{V_1}$$

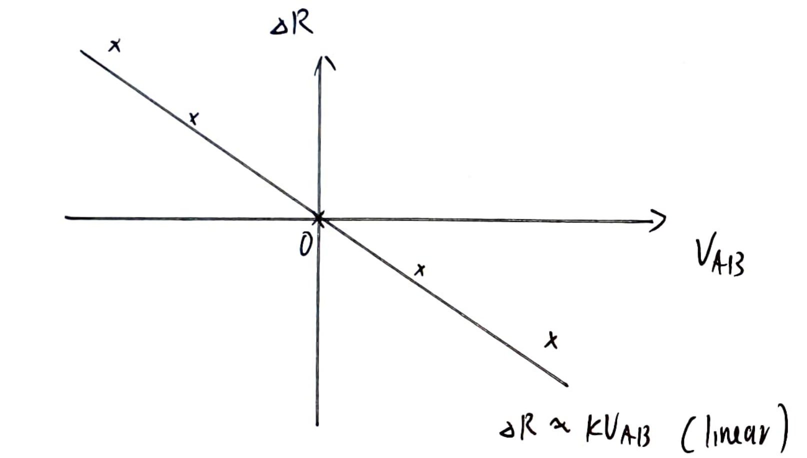
500Ω

$$\Delta R \approx kV_{AB} \qquad \text{where } k = -\frac{4R}{V_1} = -390.625\ \Omega/V$$

5.12V

~ fill in the table ~

Analysis and Observations



The equation from activity 1 is non linear, with values of ΔR curving away from the linear approximation as V_{AB} gets larger

comparing ΔR values obtained using the linear approx and precise equation, they follow closely for smaller values for $|V_{AB}|$ but diverge more when $|V_{AB}|$ is larger.

However, assuming we are operating in the range where ΔR is relatively small compared to R, using a linear approximation is very much appropriate

Sources of error

① getting each variable resistor to exactly 500 Ω / same value was challenging and there might be uncertainty in the readings too

② even if the readings on the ohmeter read 500 Ω, there might be uncertainty in the equipment measurement,

③ Initial V_{AB} for reference was not exactly 0 ≈ 1mV which would affect later measurements of V_{AB} when |ΔR| increases