CG1111: Engineering Principles and Practice I

Debrief and Tutorial for Week 8

NUS
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Characteristics of DC Motors

Torque of shaft: $$\boxed{T_{\text{shaft}} = K_t\, I_m \ \ [\mathrm{N.m}]}$$

Back emf: $$\boxed{E_b = K_e\, \omega \ \ [\mathrm{V}]}$$

For PMDC motor: $$\boxed{K_t = K_e}$$

Note: $$\boxed{\omega = 2\pi \times \frac{RPM}{60} \ \ [\mathrm{rad/s}]}$$

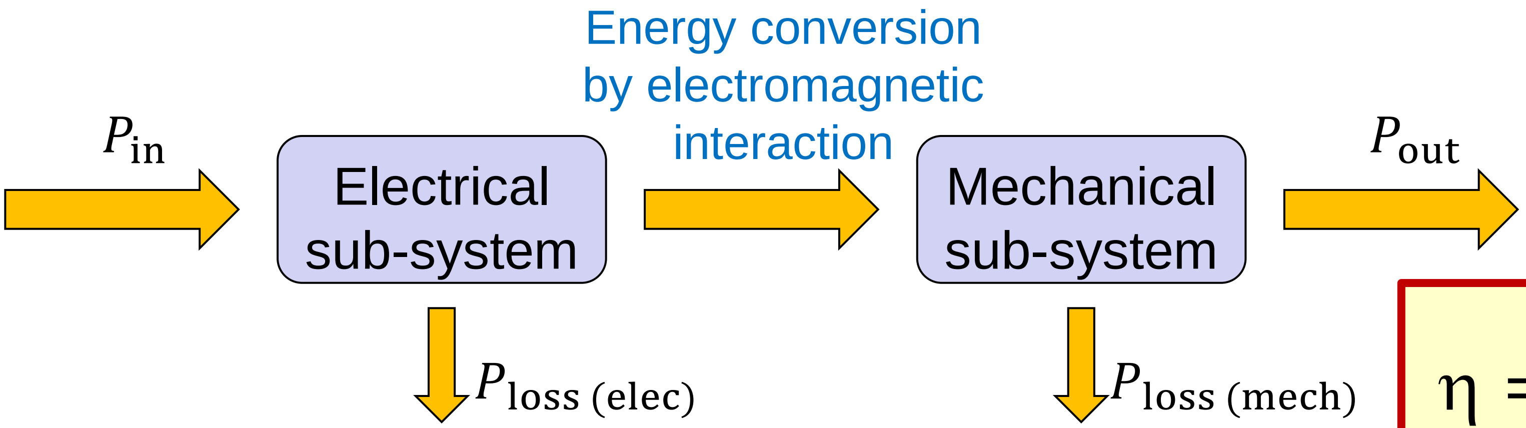
Power

- Mechanical power at motor shaft:

$$\boxed{P_{\text{out}} = T_{\text{shaft}}\,\omega \quad [\text{W}]}$$

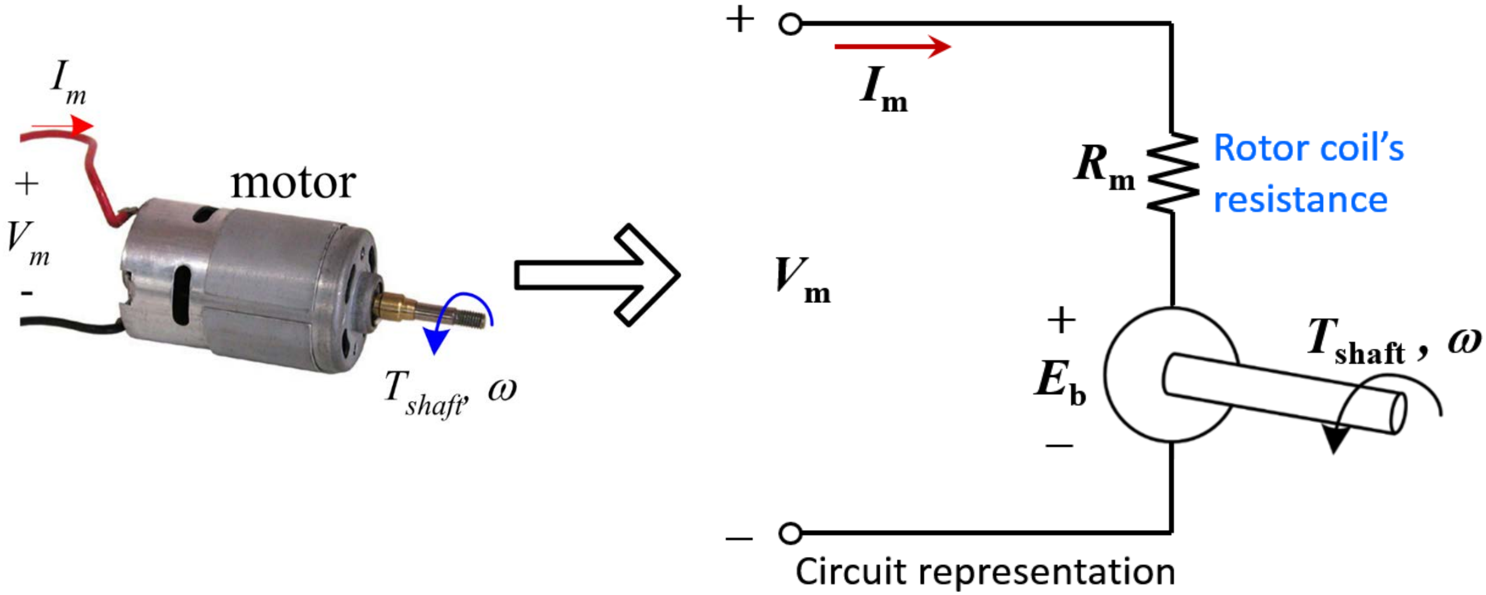
- Electrical power supplied to motor:

$$\boxed{P_{\text{in}} = V_m\, I_m \quad [\text{W}]}$$



$$\boxed{\eta = \frac{P_{\text{out}}}{P_{\text{in}}}}$$

Circuit Representation: PMDC Motor



- From the circuit: $$\boxed{I_m = \frac{V_m - E_b}{R_m}}$$

- Since $E_b = K_e\,\omega$, we have: $$\boxed{I_m = \frac{V_m}{R_m} - \frac{K_e\,\omega}{R_m}}$$

Basic Properties of PMDC Motor

Rearranging: $$\boxed{\omega = \frac{V_m}{K_e} - \frac{R_m\, I_m}{K_e}}$$

- For a fixed load (i.e., fixed T_{shaft}, which implies fixed I_m since $T_{\text{shaft}} = K_t\, I_m$):

Shaft speed ω can be increased by increasing motor voltage V_m

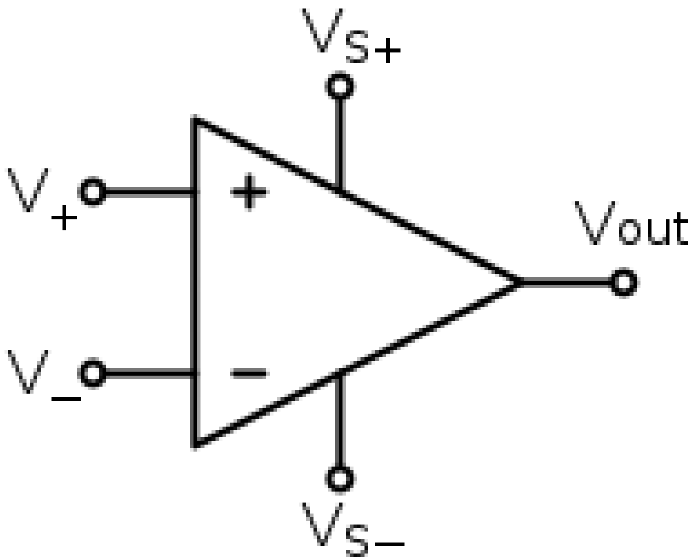
- For a fixed voltage, if T_{shaft} increases, I_m increases, and hence ω decreases:

Shaft speed ω decreases with increasing load T_{shaft}

Op-Amp Terminals

Positive Power Supply (Positive Rail)

Non-Inverting Input Terminal

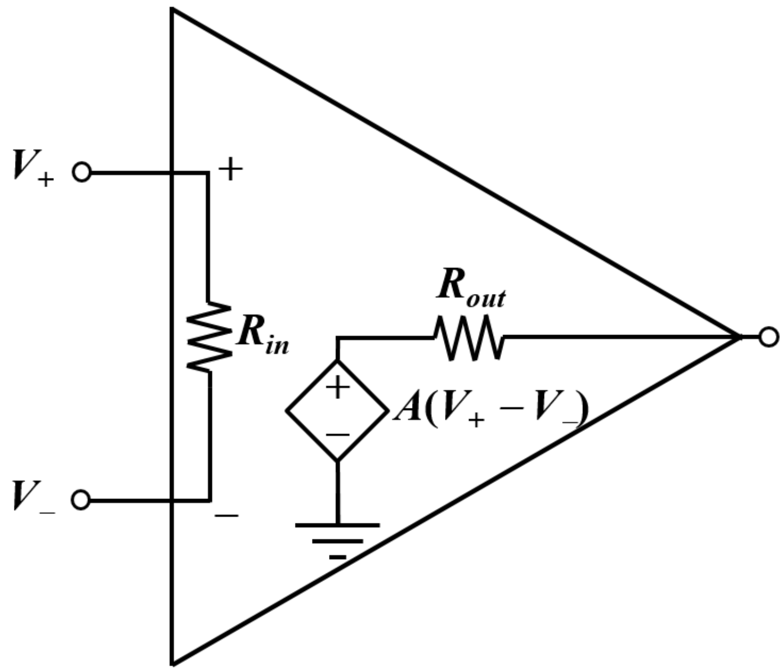


Output Terminal

Inverting Input Terminal

Negative Power Supply (Negative Rail)

Op-Amp Equivalent Circuit



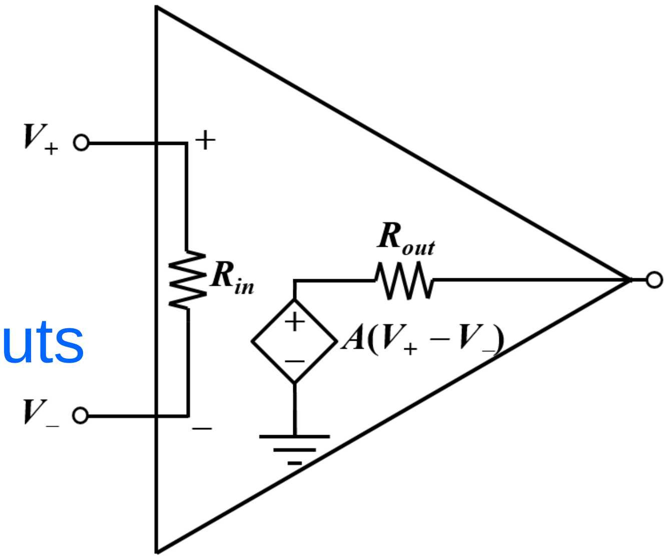
- *A* is the open-loop voltage gain
  - –It is very large, approaching infinity
- R_{in} is the input impedance & R_{out} is the output impedance

Typical Op Amp Parameters

Parameter	Variable	Typical Ranges	Ideal Values
Voltage Gain	A	10^5 to 10^8	∞
Input Impedance	R_{in}	10^5 to 10^8 Ω	∞ Ω
Output Impedance	R_{out}	10 to 100 Ω	0 Ω
Supply Voltage	V_{S+} or V_{cc} V_{S-} or V_{EE} or $-V_{cc}$	5 to 30 V –30 to 0 V	N/A N/A

Op-amp Golden Rules

- Rule 1: In a closed loop, the output attempts to do whatever is necessary to make the voltage difference between the inputs zero
  - The voltage gain of a real op-amp is so high that a fraction of a mV difference between the V_+ & V_- inputs will swing the output to saturation

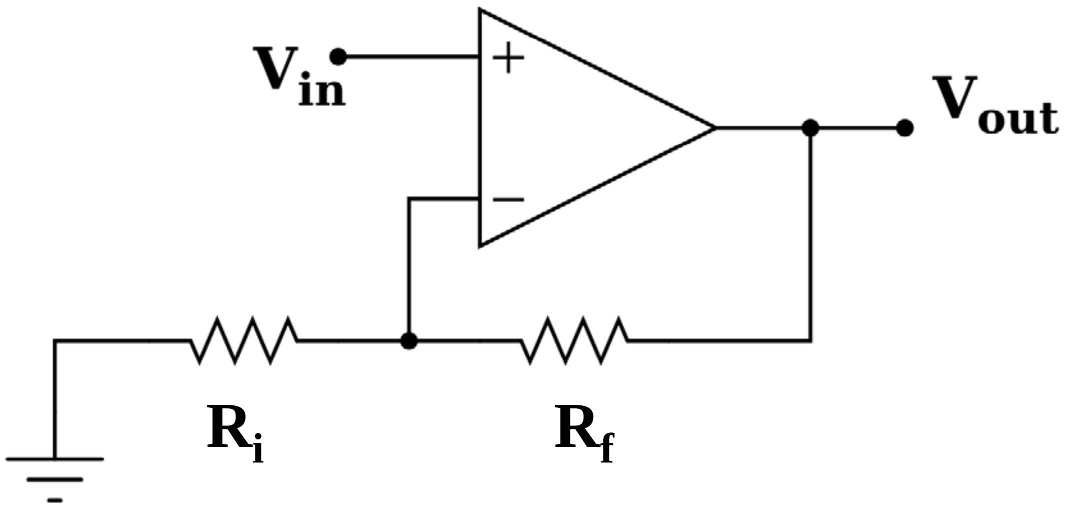


- Rule 2: The inputs draw no current
  - The ideal op-amp has infinite input impedance (R_{in}). Thus, the current drawn at the two input terminals ~0.

Non-Inverting Amplifier

- For an ideal op-amp, the non-inverting amplifier gain is given simply by

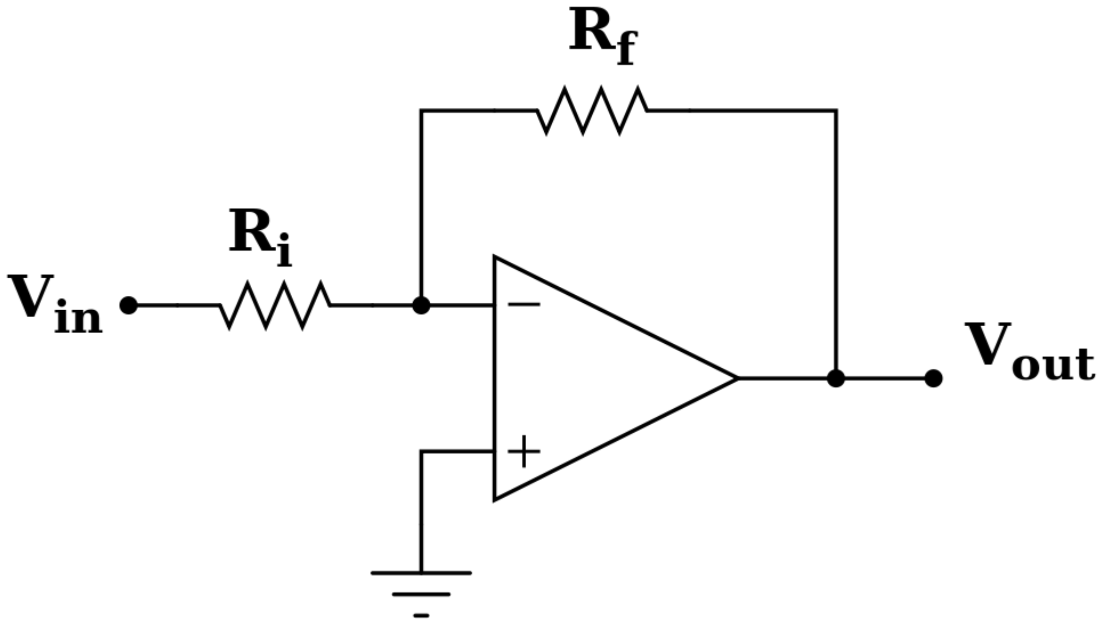
$$\boxed{\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_{\text{f}}}{R_{\text{i}}}}$$



Inverting Amplifier

- For an ideal op-amp, the inverting amplifier gain is given simply by

$$\boxed{\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_{\text{f}}}{R_{\text{i}}}}$$



Power Gain in decibels (dB)

- The Voltage Amplification (A_v) or Gain of a voltage amplifier/filter is given by:

$$\boxed{A_v = \frac{V_{\text{out}}}{V_{\text{in}}}}$$

- The voltage gain is commonly expressed in terms of the resulting power gain in dB:

$$\begin{aligned}\text{Power Gain (dB)} &= 10\log_{10}\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)^2 \text{ dB} \\ &= 20\log_{10}\left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| \text{ dB}\end{aligned}$$

Question 1

- PMDC motor, 12 V source
- No-load speed = 3800 RPM
- Stall torque = 30 mNm

- If now running at 2500 RPM, find
  a) I_m
  b) T_{shaft}
  c) E_b
  d) Electrical power consumed
  e) Shaft power
  f) Power loss in rotor coil

Solution to Q1

- First, find all unknown motor parameters

- 3800 RPM $\rightarrow \omega_{\text{no-load}} = 398\ \text{rad/s}$
- Recall that: $\boxed{I_m = \dfrac{V_m}{R_m} - \dfrac{K_e\,\omega}{R_m}}$

- At no-load, $I_m \approx 0$ since $T_{\text{shaft}} \approx 0$
  - Hence, $V_m \approx K_e\,\omega_{\text{no-load}} \rightarrow K_e \approx \dfrac{12}{398} = 30\ \dfrac{\text{mV}}{\text{rad/s}}$
  - For PMDC motor, $K_t = K_e = 30\ \dfrac{\text{mNm}}{\text{A}}$

- At stall condition, $T_{\text{stall}} = 30\ \text{mNm}$
  - $I_{\text{stall}} = \dfrac{T_{\text{stall}}}{K_t} = 1\ \text{A}$
  - Since $\omega_{\text{stall}} = 0,\ I_{\text{stall}} = \dfrac{V_m}{R_m} \rightarrow R_m = \dfrac{12\ \text{V}}{1\ \text{A}} = 12\ \Omega$

Solution to Q1

At 2500 RPM:

a) Find $I_{2500\ \mathrm{RPM}}$ and b) $T_{2500\ \mathrm{RPM}}$

$$\omega_{2500\ \mathrm{RPM}} = 261.8\ \mathrm{rad/s}$$

$$I_{2500\ \mathrm{RPM}} = \frac{V_m}{R_m} - \frac{K_e \omega_{2500\ \mathrm{RPM}}}{R_m}$$

$$= \frac{12}{12} - \frac{0.03 \times 261.8}{12} = 0.346\ \mathrm{A}$$

Therefore, $T_{2500\ \mathrm{RPM}} = K_t I_{2500\ \mathrm{RPM}}$

$$= 10.4\ \mathrm{mNm}$$

Solution to Q1

c) Find back emf $E_{2500\ \mathrm{RPM}}$

$$E_{2500\ \mathrm{RPM}} = K_e \omega_{2500\ \mathrm{RPM}} = 7.85\ \mathrm{V}$$

d) Find total electrical power consumed P_e

$$P_\mathrm{e} = V_\mathrm{m} \times I_{2500\ \mathrm{RPM}} = 4.15\ \mathrm{W}$$

Solution to Q1

e) Find shaft power P_{shaft}

$$P_{\mathrm{shaft}} = T_{2500\ \mathrm{RPM}} \times \omega_{2500\ \mathrm{RPM}} = 2.72\ \mathrm{W}$$

f) Find power loss in rotor coil

$$P_{\mathrm{loss}} = R_{\mathrm{m}} \times (I_{2500\ \mathrm{RPM}})^2 = 1.44\ \mathrm{W}$$

Question 2

- Same motor as Q1
- Load condition unchanged
  - (i.e., same torque as $T_{2500\ \mathrm{RPM}}$ previously)
- New desired speed = 1500 RPM

- Find:
  a) Rotor current I_{m} and the required V_{m}
  b) PWM duty cycle if DC source still 12 V
  c) T_{on} and T_{off} if PWM frequency = 5 kHz
  d) Electrical power consumed
  e) Power loss in rotor coil

Solution to Q2

a) Find current I_{m} and the required V_{m}

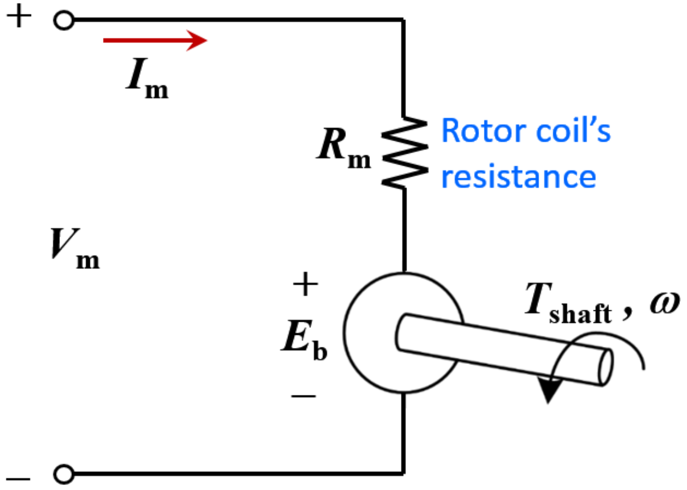
Same load means same torque as $T_{2500\ \mathrm{RPM}}$ previously, hence current remains unchanged at

$$I_{\mathrm{m}} = 0.346\ \mathrm{A}$$

$$\omega_{1500\ \mathrm{RPM}} = 157.1\ \mathrm{rad/s}$$

$$E_b = K_e\omega = 4.71\ \mathrm{V}$$

$$V_{\mathrm{m}} = R_{\mathrm{m}}I_{\mathrm{m}} + E_b = 8.86\ \mathrm{V}$$



Circuit representation

Solution to Q2

b) Find PWM duty cycle if DC source is 12 V

$$\text{Duty Cycle} = \frac{8.86}{12} = 73.8\%$$

c) T_{on} and T_{off} if PWM frequency = 5 kHz

$$T_{\mathrm{p}} = \frac{1}{5000} = 200\ \mu\text{s}$$

$$T_{\mathrm{on}} = 0.738 \times 200\ \mu\text{s} = 147.6\ \mu\text{s}$$

$$T_{\mathrm{off}} = T_{\mathrm{p}} - T_{\mathrm{on}} = 52.4\ \mu\text{s}$$

Solution to Q2

d) Find total electrical power consumed

$$P_{\mathrm{e}} = V_{\mathrm{m}} \times I_{\mathrm{m}} = 8.86 \times 0.346 = 3.07\ \mathrm{W}$$

e) Find power loss in rotor coil

$$P_{\mathrm{loss}} = R_{\mathrm{m}} \times (I_{\mathrm{m}})^2 = 1.44\ \mathrm{W}$$

(same as Q1!)

Question 3

- PMDC motor with fan
- Fan's load torque $T_L = 0.05\omega + 0.001\omega^2$
- $K_\mathrm{t} = 2.42\ \mathrm{Nm/A}$
- $R_\mathrm{m} = 0.2\ \Omega$
- 50 V DC power supply

- Find:

  Speed of fan

Solution to Q3

Recall that:

$$\boxed{T_{\text{shaft}} = K_t I_m}$$

$$\boxed{I_m = \frac{V_m}{R_m} - \frac{K_e \omega}{R_m}}$$

For PMDC motor: $\boxed{K_t = K_e}$

Therefore:

$$T_{\text{shaft}} = 2.42 \times \left(\frac{50}{0.2} - \frac{2.42}{0.2}\omega\right) = 605 - 29.3\omega$$

Solution to Q3

Since $T_{\text{shaft}} = T_L = 0.05\omega + 0.001\omega^2$

$$605 - 29.3\omega = 0.05\omega + 0.001\omega^2$$
$$\omega^2 + 29350\omega - 605000 = 0$$

Solution of quadratic equation:

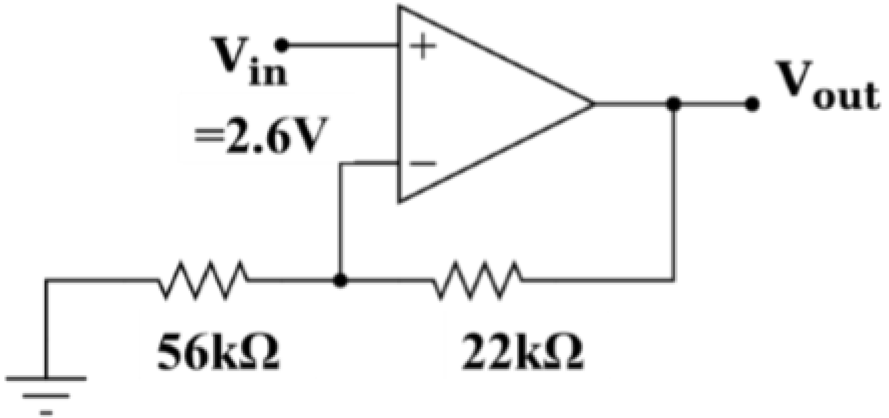
$$\omega = +20.6 \text{ or } -29371$$

Keeping the positive value, we have

$$\omega = 20.6 \text{ rad/s} \rightarrow 197 \text{ RPM}$$

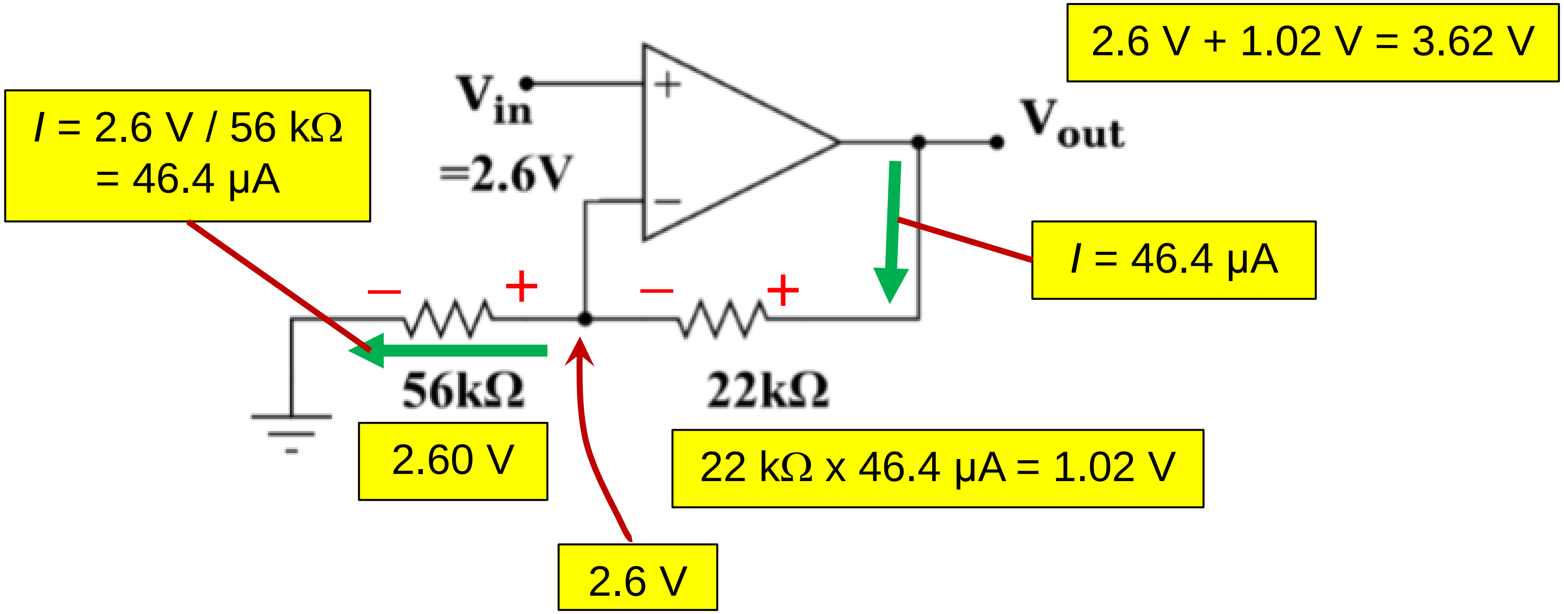
Question 4

- Calculate all voltage drops and currents in this circuit, and label the currents' directions & voltage polarities
- Calculate the overall voltage gain of this amplifier circuit (A_V), both as a ratio and as a figure in units of decibels (dB)



Solution to Q4

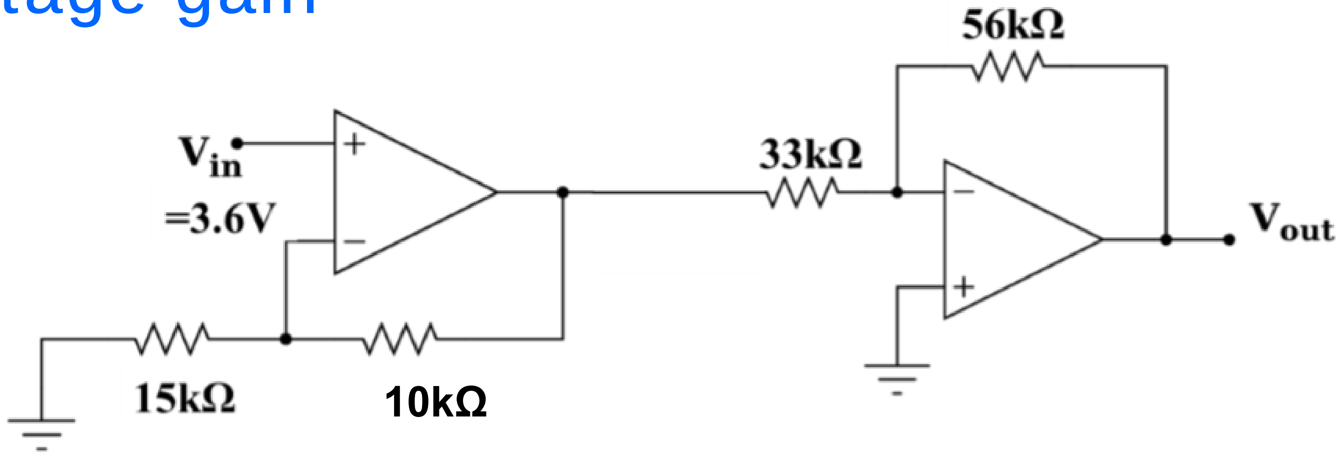
- Opamp's golden rule: V+ ≈ V–
- No current entering '+' & '–' inputs



- Gain $= \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{3.62}{2.6} = 1.39\,\mathrm{V/V}$ (Volt per Volt)
- Gain in dB = 20 $\log_{10}$ (1.39) = 2.86 dB

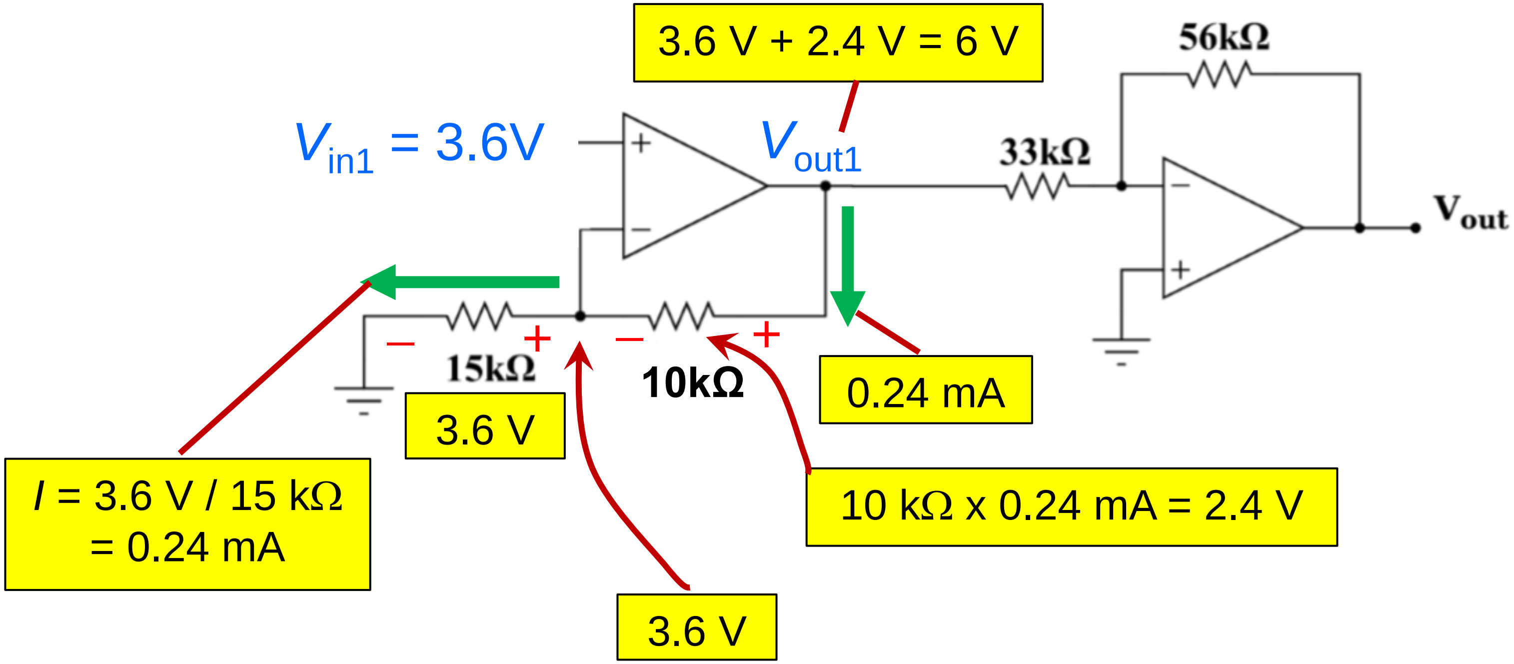
Question 5

- Calculate all voltage drops and currents in this circuit, and label the currents' directions & voltage polarities
- Calculate the voltage gain for each stage of this amplifier circuit (both as a ratio and in units of decibels), then calculate the overall voltage gain

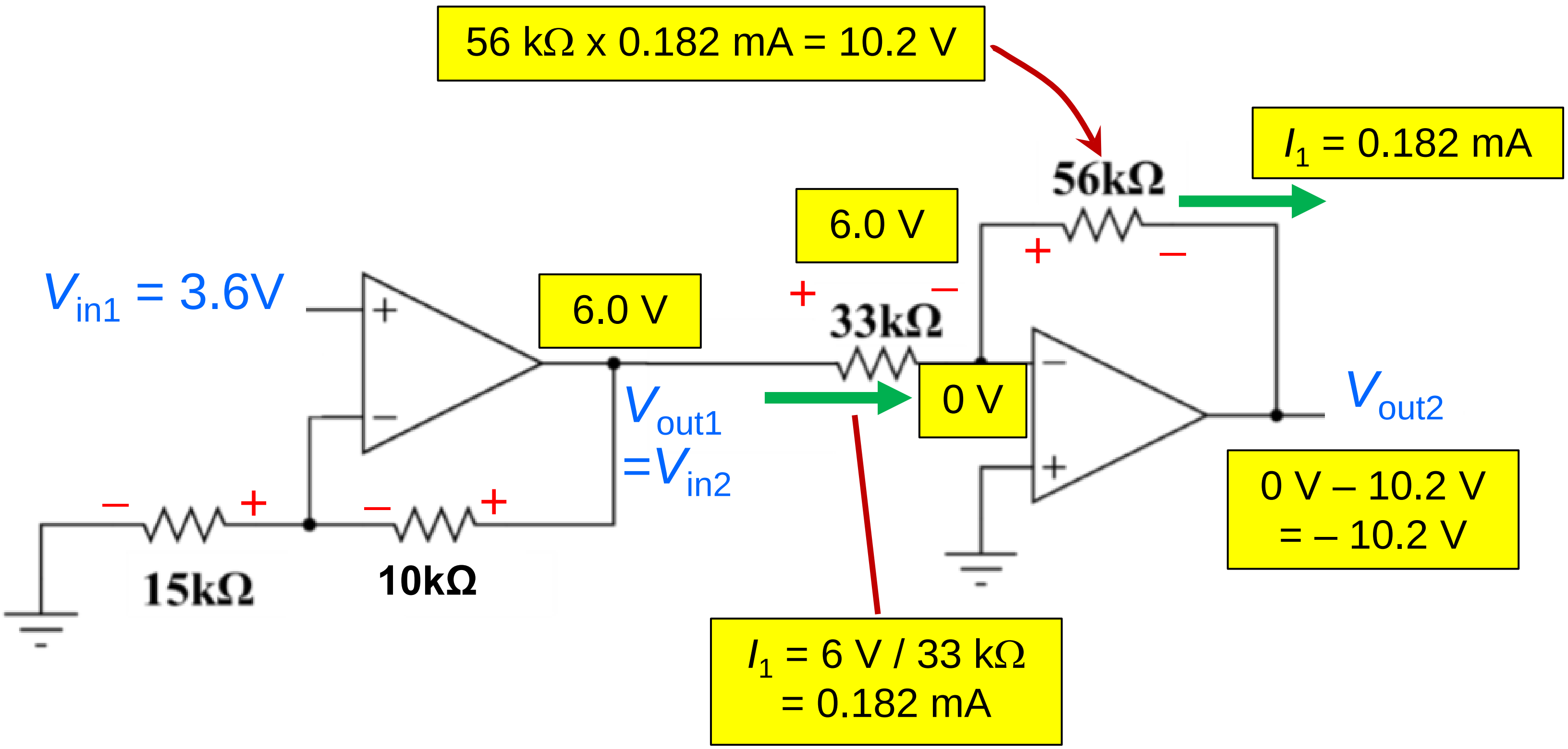


Solution to Q5

- Gain of 1st stage $= \dfrac{V_{\text{out1}}}{V_{\text{in1}}} = \dfrac{6}{3.6} = 1.67\ \text{V/V}$
- Gain in dB = 20 $\log_{10}$ (1.67) = 4.45 dB



Solution to Q5



- Gain of 2nd stage = $\frac{V_{\text{out2}}}{V_{\text{in2}}} = -\frac{10.2}{6}$ = –1.7 V/V
- Gain in dB = 20 $\log_{10}$ (|–1.7|) = 4.61 dB
- Overall voltage gain = 1.67 × (–1.7) = –2.84 V/V = 9.07 dB