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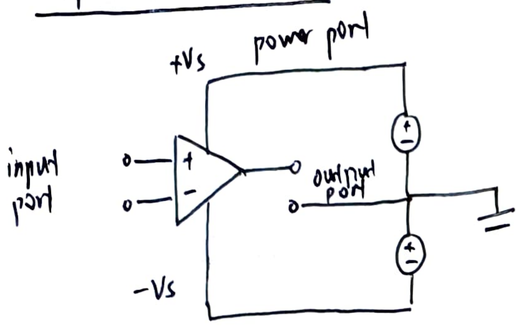
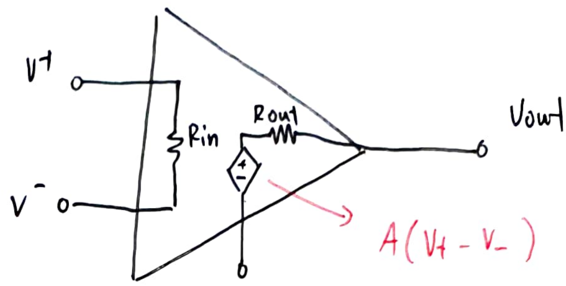
Group 4B

Week 9 Studio 1

13/10/2020

Operational Amplifier



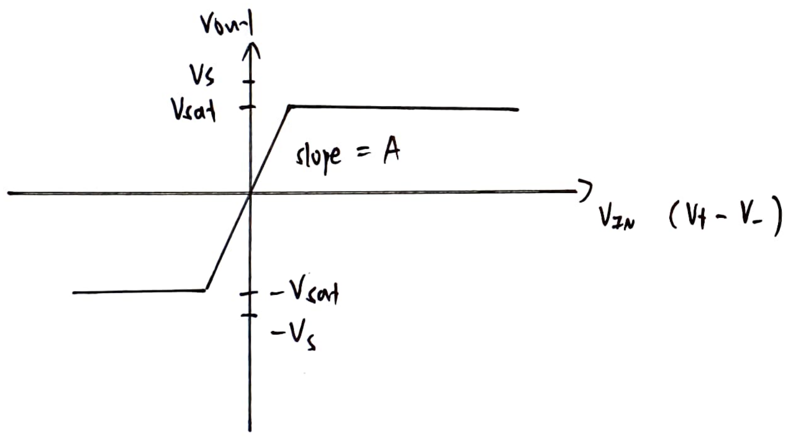


Ideally, input resistance $R_{in} \rightarrow \infty$

output resistance $R_{out} \rightarrow 0$

A (open loop voltage gain) $\rightarrow \infty$ (10^5)

No saturation



$V_{out} = A(V_+ - V_-)$

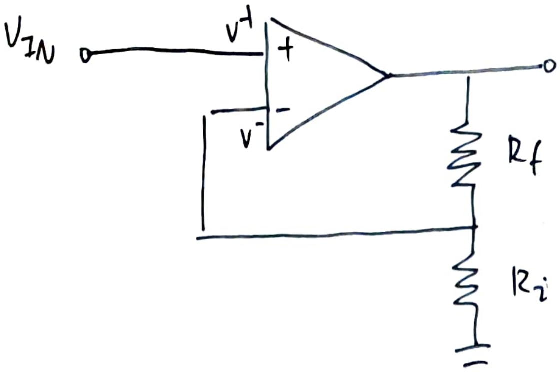
V_{sat} is determined by the power supply voltage and is usually ~1V lower than V_s

Op Amp Golden Rules

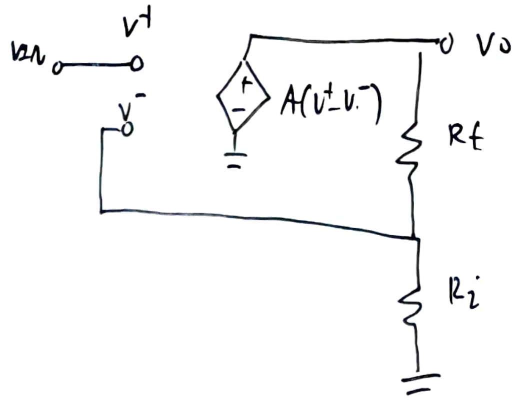
① output always tries to make voltage difference $(V_+ - V_-)$ between the inputs 0 (negative feedback!)

② inputs draw no current as ideal op-amp has input resistance $\rightarrow \infty$

Non-Inverting Amplifier



$\equiv$



$$V_o = A(v^+ - v^-)$$

$$= A\left(V_{IN} - \frac{R_i}{R_i + R_f} V_o\right)$$

$$V_o\left(1 + \frac{AR_i}{R_i + R_f}\right) = AV_{IN}$$

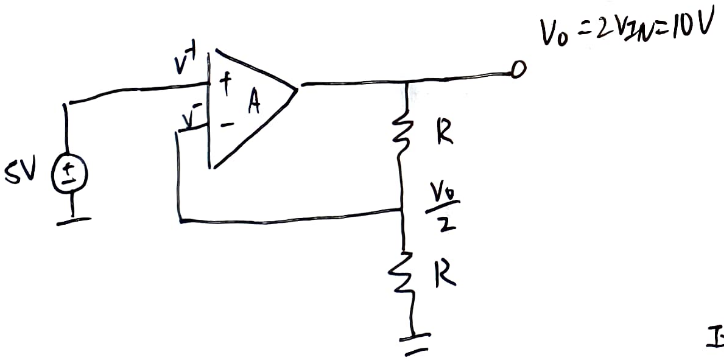
$$V_o = \frac{AV_{IN}}{\cancel{1+}\frac{AR_i}{R_i+R_f}} \approx \frac{\cancel{A}V_{IN}}{\frac{\cancel{A}R_i}{R_i+R_f}} \approx \frac{R_i+R_f}{R_i} V_{IN}$$

$1 << \frac{AR_i}{R_i + R_f}$ for large A

$$\approx \boxed{1 + \frac{R_f}{R_i}} V_{IN}$$

gain

Negative feedback



let $R_f = R_i = R$

and $V_{IN} = 5V$

$$V_o = \left(1 + \frac{1}{1}\right) V_{IN} = 10V$$

If we perturb the circuit, $\uparrow V_o$ to $12V$

then $v^- = \frac{V_o}{2} = 6V$

$V_o = A(5-6) \Rightarrow \downarrow V_o$

tries to oppose change and self correct

This negative feedback is due to a portion of output being fed back into v^- input

Under negative feedback

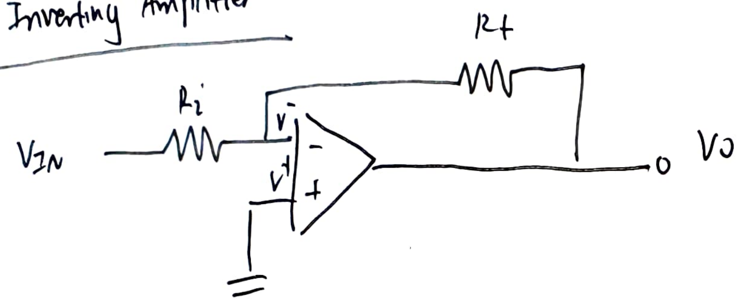
$$v^+ - v^- = \frac{V_o}{A} = \frac{\frac{R_i + R_f}{R_i} V_{IN}}{A} \to 0 \quad \text{when } A \text{ is large}$$

$v^+ \approx v^-$ (golden rule 1)

$\left.\begin{array}{l} i^+ \approx 0 \\ i^- \approx 0 \end{array}\right\}$ (golden rule 2)

yields easier analysis method under negative feedback

Inverting Amplifier



easier analysis

$v^+ \approx v^- = 0$, $i^+ \approx 0$, $i^- \approx 0$

$$\frac{V_{IN} - 0}{R_i} = \frac{0 - V_o}{R_f} \Rightarrow \frac{V_o}{V_{IN}} = -\frac{R_f}{R_i}$$

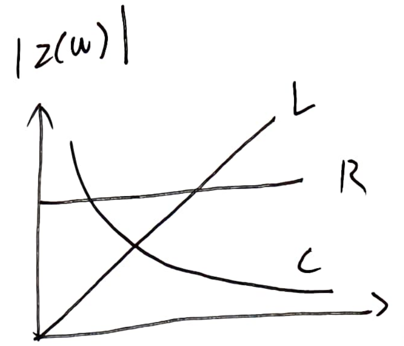
Impedence Model

At sinusoidal steady state

$Z_R = R$ (constant)

$Z_C = \frac{1}{j\omega C}$ (inverse)

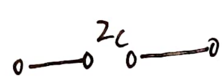
$Z_L = j\omega L$ (linear)



when $\omega = 0 \Rightarrow$ DC state

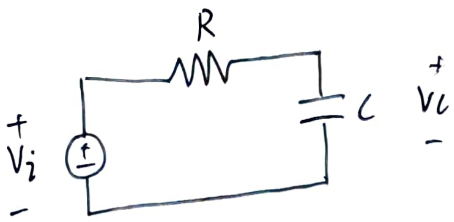
$Z_L = 0$ (short)

$Z_C = \infty$ (open)



Transfer function $H(\omega)$

take an RC circuit



By the impedence model:

$$V_C = \frac{Z_C}{Z_C + Z_R} V_i$$

$$= \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_i$$

multiply both sides by $j\omega C$

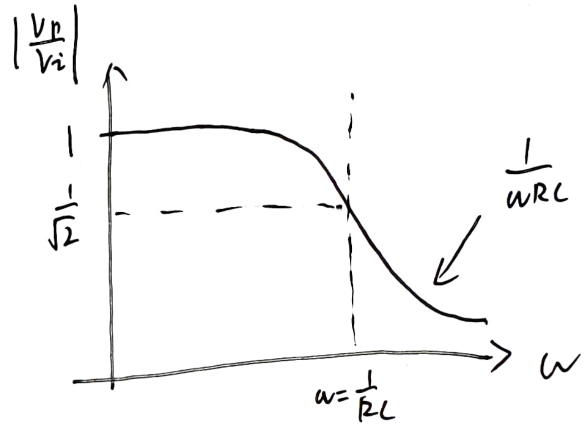
$$= \frac{1}{1 + j\omega CR} V_i$$

$$H(\omega) = \frac{V_C}{V_i} = \frac{1}{1+j\omega CR}$$

$$\left|\frac{V_C}{V_i}\right| = \left|\frac{1-j\omega CR}{1+\omega^2R^2C^2}\right|$$

$$= \sqrt{\frac{1^2+\omega^2R^2C^2}{(1+\omega^2R^2C^2)^2}}$$

$$= \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$



$\boxed{\text{LOW PASS FILTER}}$

low ω: $\approx \frac{1}{\sqrt{1}} \approx 1$

high ω: $\approx \frac{1}{\omega RC}$ (inversely proportional)

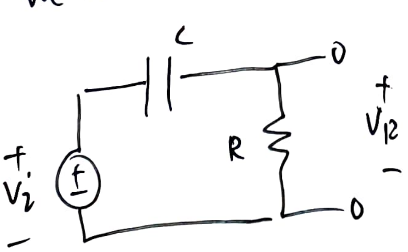
$\omega = \frac{1}{RC}$: $\frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$ (break frequency)

$$\omega = 2\pi f = \frac{1}{RC}$$

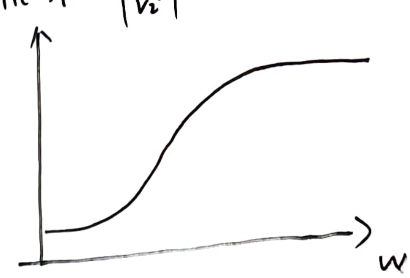
$$f = \frac{1}{2\pi RC}$$

Filters

we can build other filters by combining impedences.



$|H(\omega)| = \left|\frac{V_R}{V_i}\right|$



$\boxed{\text{high pass filter}}$

Intuitively (without equations)

low ω: capacitor acts as open

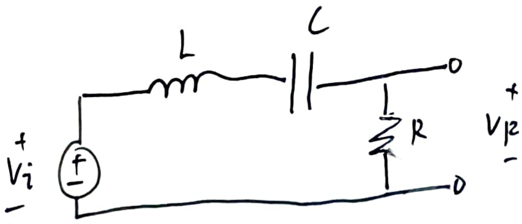
$\left|\frac{V_R}{V_i}\right| \approx 0$

high ω: capacitor acts as short

$\left|\frac{V_R}{V_i}\right| \approx 1$

In first order circuits, time constant is of interest $\omega = \frac{1}{RC}$

RLC circuit (2nd order)



$$\frac{V_R}{V_i} = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}$$

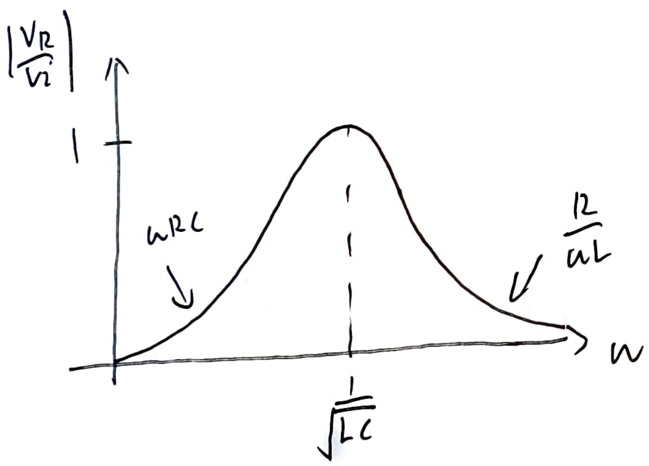
$$= \frac{j\omega C R}{1 + j\omega C R - \omega^2 L C}$$

$$\left|\frac{V_R}{V_i}\right| = \frac{\omega R C}{\sqrt{(1-\omega^2 LC)^2 + (\omega R C)^2}}$$

low ω : $\approx \dfrac{\omega R C}{\sqrt{(1-0)^2 + 0^2}} \approx \omega R C$ (linear)

high ω : $\approx \dfrac{\omega R C}{\omega^2 L C} \approx \dfrac{R}{\omega L}$ (inversely proportional)

$\omega = \dfrac{1}{\sqrt{LC}}$: $\dfrac{\omega R C}{\sqrt{(\omega R C)^2}} \approx 1$ (resonance)



Intuitively

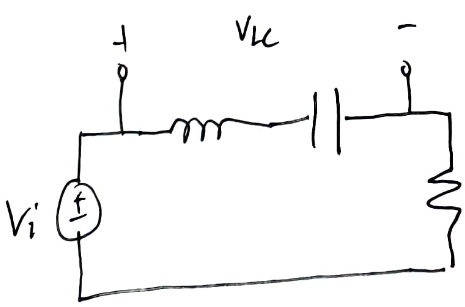
low ω $\Rightarrow$ capacitor acts as open (blocks low freq)

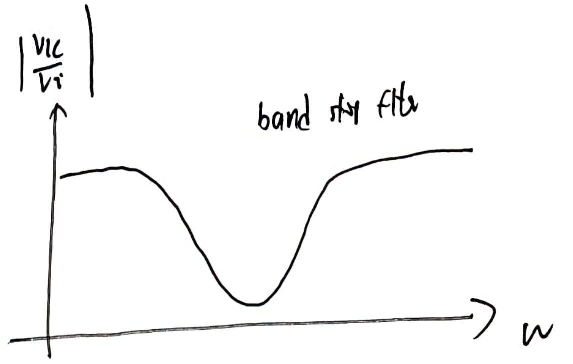
high ω $\Rightarrow$ inductor acts as open (blocks high freq)

At resonance $\Rightarrow$ $Z_L + Z_C \approx 0$

V_i only sees R !

$\boxed{\text{Band Pass Filter}}$





just swap terminal across L & C

Activity 1

1.

V_{in} (V^+)	V_{ref} (V^-)	V_{out}
+2.5V	0V	5V
+3.5V	+3.75V	0V
+1.25V	−0.5V	5V
−3V	−1V	0V
−2.5V	+2.5V	0V

For ideal comparator with no feedback resistor

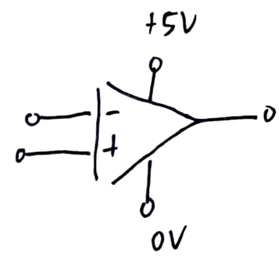
$V^+ > V^- \Rightarrow V_{CC}$ (ideal op-amp)

$V^+ < V^- \Rightarrow V_{EE}$ (ideal op-amp)

since there is only a single 5V source

$V_{CC} = 5V$

$V_{EE} = 0V$



2. If a dual power supply of +5V and −5V is used the output voltage values would be $\begin{cases} +5V & \text{if } V^+ > V^- \\ -5V & \text{if } V^+ < V^- \end{cases}$

6. $V_{ref} = \frac{R_2}{R_1+R_2} 5V$

setting R_1 to be $1k\Omega$, $1 + \frac{R_1}{R_2} = \frac{5}{1.6}$

$R_2 \simeq 470\,\Omega$ (use a variable resistor)

9. A square waveform is observed as the output.
when V_{in} generated from the bitscope exceeds $V_{ref} \approx 1.6V$, output $= +V_{sat} \approx 3.9V$
else, $V_{out} = -V_{sat}$ which is consistent with the behaviour of a comparator
where the output is either HIGH or LOW depending on the difference between the input

10. still using R_1 as a $1k\Omega$ resistor

when $V_{ref} = 0.7V \Rightarrow R_2 \approx 160\,\Omega$

when $V_{ref} = 2.5V \Rightarrow R_2 \approx 1000\,\Omega$

11. When Vref is small, the time when Vin > Vref is much longer when Vref is closer to the 3.3V peak to peak voltage of Vin, the time when Vin > Vref is much shorter

The output is similar to pulse width modulation

Activity 2

1. derive the voltage gain $\left(\frac{V_{out}}{V_+}\right)$ equation

By applying the op-amp golden rules or analysis method under negative feedback $\Rightarrow$ $V^+ \approx V^-$ and input draws no current

$$\frac{0 - V^+}{1000} = 0 + \frac{V^+ - V_{out}}{1000}$$

$$\frac{V_{out}}{1000} = \frac{2V^+}{1000} \Rightarrow \frac{V_{out}}{V^+} = \boxed{2} \rightarrow \text{gain with current resistance values}$$

9.

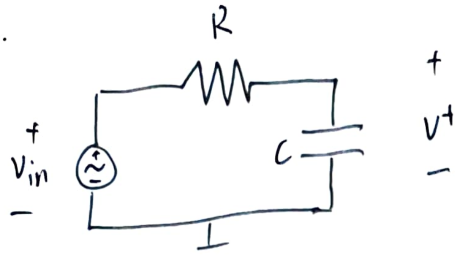
Frequency (Hz)	Vin (V)	Vout (V)	Gain $\left(\frac{V_{out}}{V_{in}}\right)$	Gain in dB
200	1.60	3.23	2.02	6.11
500	1.60	3.21	2.01	6.06
1000	1.60	3.09	1.93	5.71
1500	1.60	2.98	1.86	5.39
2000	1.60	2.82	1.76	4.91
3000	1.60	2.49	1.56	3.86
5000	1.60	1.88	1.18	1.44
10000	1.60	1.09	0.68	-3.35
20000	1.60	0.58	0.36	-8.87
50000	1.60	0.25	0.16	-15.92

10. Plot gain in dB vs frequency

11. cut-off frequency when $\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}}$ maximum gain = $-3dB$ from initial gain is somewhere between 3000 to 5000 Hz

From the excel plot, the cut off frequency is around 4000Hz.

12.



from 1.) $V_{out} = 2V^+$

By impedence model:

$$v^+ = \frac{Z_C}{Z_C + Z_R} V_{in}$$

$$= \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} V_{in}$$

$$\frac{v^+}{V_{in}} = \frac{1}{1 + Rj\omega C}$$

$$\left|\frac{v^+}{V_{in}}\right| = \left|\frac{1(1 - j\omega RC)}{(1 + j\omega RC)(1 - j\omega RC)}\right|$$

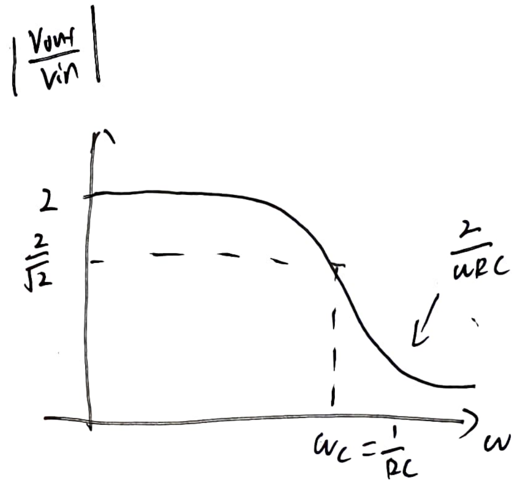
$a + bi \Rightarrow r = \sqrt{a^2 + b^2}$

$$= \left|\frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2}\right|$$

$$= \sqrt{\left(\frac{1}{1 + \omega^2 R^2 C^2}\right)^2 + \left(\frac{-\omega RC}{1 + \omega^2 R^2 C^2}\right)^2}$$

$$= \sqrt{\frac{1 + \omega^2 R^2 C^2}{(1 + \omega^2 R^2 C^2)^2}} = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = 2\left|\frac{v^+}{V_{in}}\right| = \frac{2}{\sqrt{1 + \omega^2 R^2 C^2}}$$



low ω: $\approx \frac{2}{\sqrt{1 + 0}} \approx 2$

high ω: $\approx \frac{2}{\sqrt{\omega^2 R^2 C^2}} \approx \frac{2}{\omega RC}$

when $\omega = \frac{1}{RC}$: $\frac{2}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} \cdot 2$ (cut off point)

$\omega_c = 2\pi f_c \Rightarrow f_c = \frac{1}{2\pi RC}$

comparing with the theoretical value obtained using $f_c = \frac{1}{2\pi RC} = 4080\text{Hz}$, the cut off frequency from the excel plot of around 4000 Hz, the 2 values agree strongly.

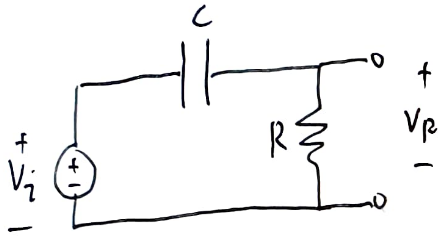
13. $2000 = \dfrac{1}{2\pi R C}$

$$C = \frac{1}{2\pi(2000)(390000)}$$

$$= 204\,pF$$

<u>challenge</u>

2. Design a high pass filter with an RC circuit



$$\frac{V_R}{V_i} = \frac{R}{R + \frac{1}{j\omega C}}$$

$$= \frac{j\omega RC}{j\omega RC + 1}$$

$$\left|\frac{V_R}{V_i}\right| = \left|\frac{(j\omega RC)(1 - j\omega RC)}{1 + \omega^2R^2C^2}\right|$$

$$= \left|\frac{\omega^2R^2C^2 + j\omega RC}{1+\omega^2R^2C^2}\right|$$

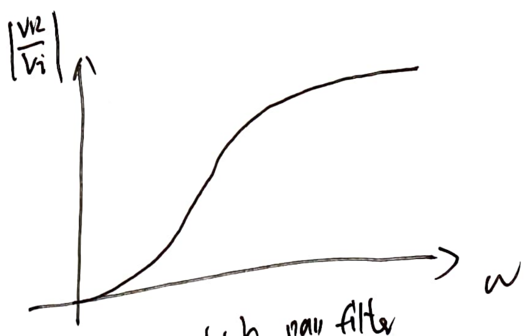
$$= \sqrt{\frac{(\omega^2R^2C^2)^2 + \omega^2R^2C^2}{(1+\omega^2R^2C^2)^2}}$$

$$= \sqrt{\frac{\omega^2R^2C^2}{1+\omega^2R^2C^2}}$$

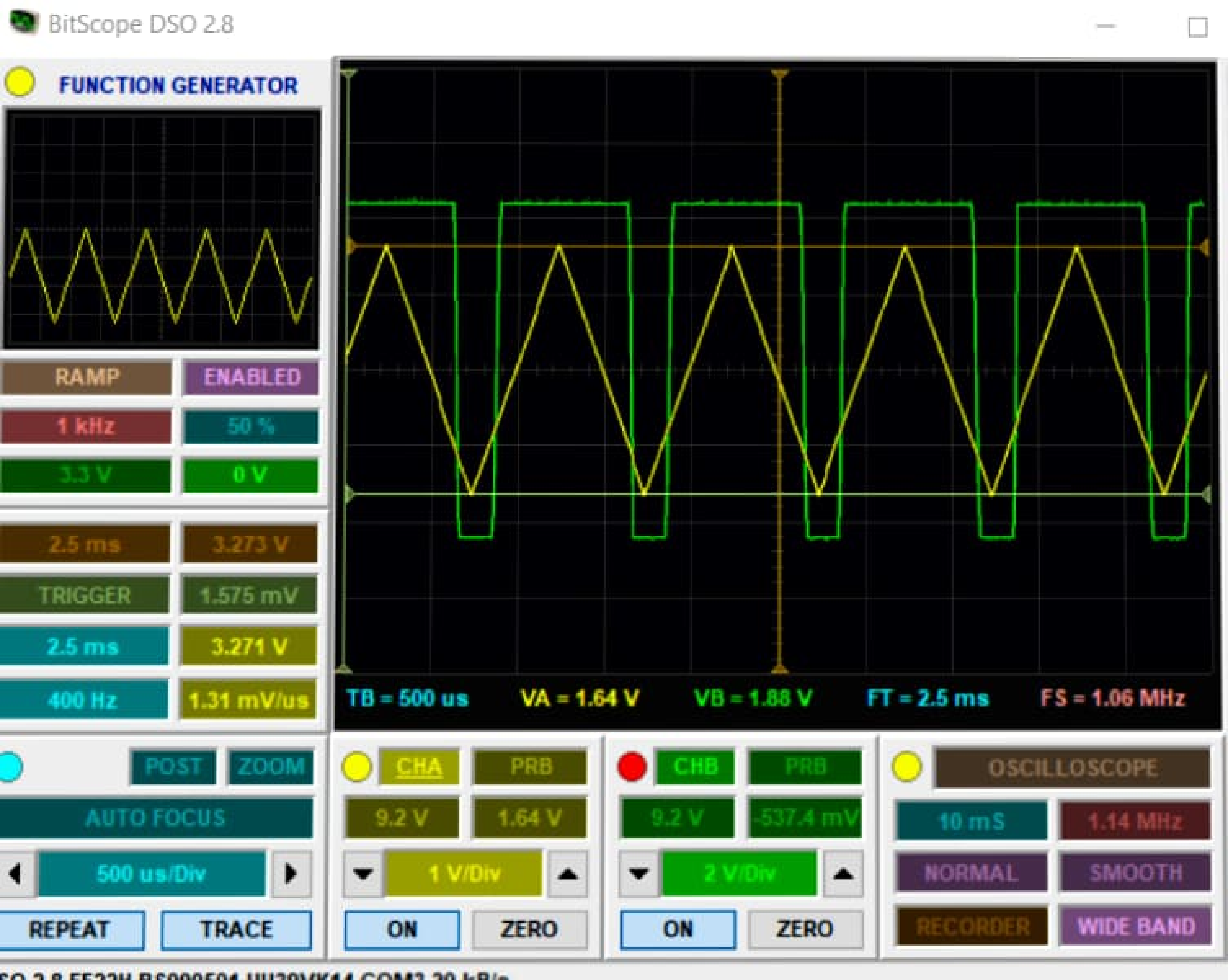
low ω: $\approx \sqrt{\frac{0}{1+0}} \approx 0$

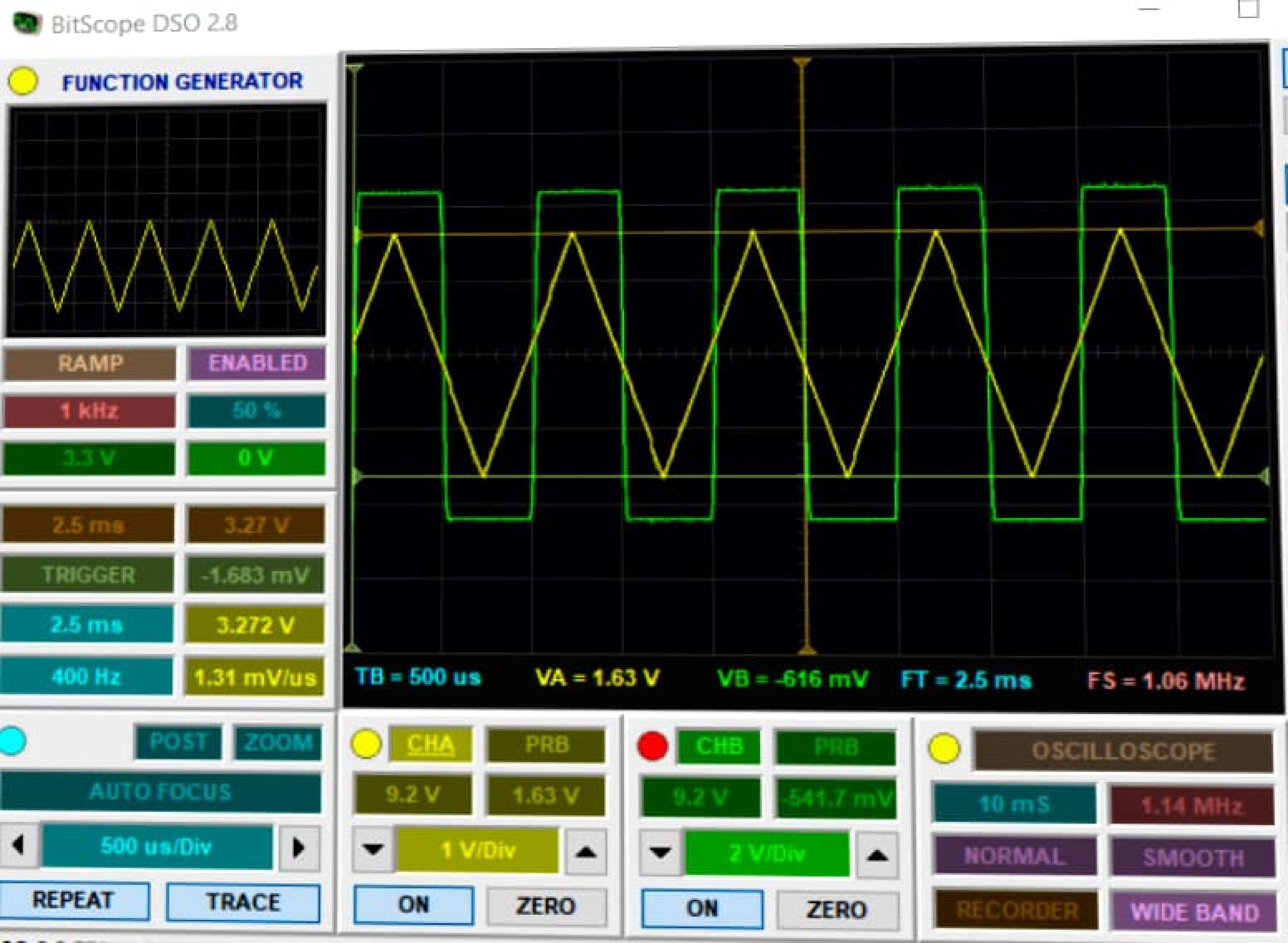
high ω: $\approx \sqrt{\frac{\omega^2R^2C^2}{\omega^2R^2C^2}} \approx 1$

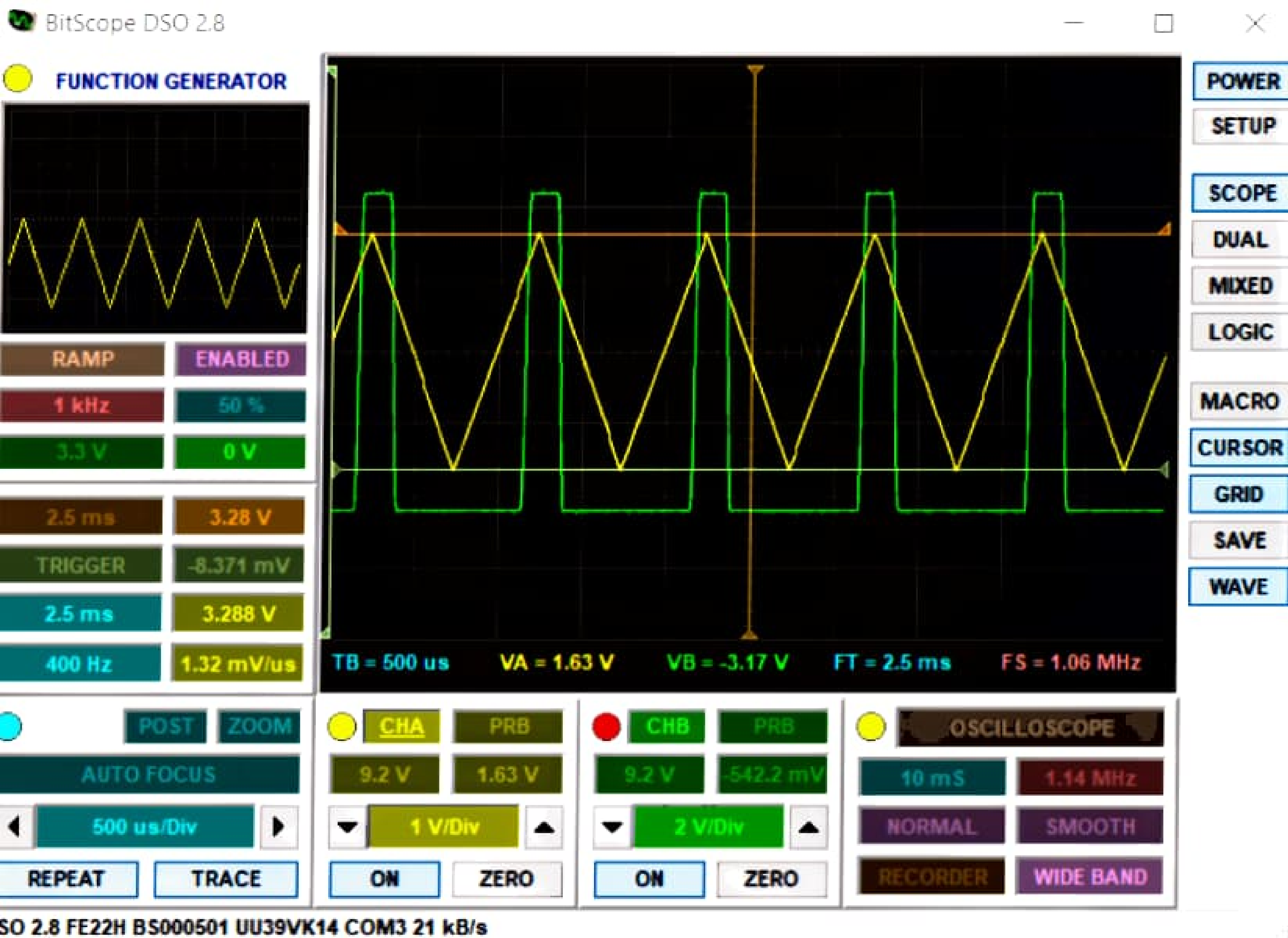
$\omega = \frac{1}{RC}$: $\approx \sqrt{\frac{1}{1+1}} = \frac{1}{\sqrt{2}}$ (cut-off)



high pass filter


BitScope DSO 2.8
FUNCTION GENERATOR
RAMP
ENABLED
1 kHz
50 %
3.3 V
0 V
2.5 ms
3.273 V
TRIGGER
1.575 mV
2.5 ms
3.271 V
400 Hz
1.31 mV/us
TB = 500 us
VA = 1.64 V
VB = 1.88 V
FT = 2.5 ms
FS = 1.06 MHz
POST
ZOOM
AUTO FOCUS
500 us/Div
REPEAT
TRACE
CHA
PRB
9.2 V
1.64 V
1 V/Div
ON
ZERO
CHB
PRB
9.2 V
-537.4 mV
2 V/Div
ON
ZERO
OSCILLOSCOPE
10 mS
1.14 MHz
NORMAL
SMOOTH
RECORDER
WIDE BAND


BitScope DSO 2.8
FUNCTION GENERATOR
RAMP
ENABLED
1 kHz
50 %
3.3 V
0 V
2.5 ms
3.27 V
TRIGGER
-1.683 mV
2.5 ms
3.272 V
400 Hz
1.31 mV/us
TB = 500 us
VA = 1.63 V
VB = -616 mV
FT = 2.5 ms
FS = 1.06 MHz
POST
ZOOM
AUTO FOCUS
500 us/Div
REPEAT
TRACE
CHA
PRB
9.2 V
1.63 V
1 V/Div
ON
ZERO
CHB
PRB
9.2 V
-541.7 mV
2 V/Div
ON
ZERO
OSCILLOSCOPE
10 mS
1.14 MHz
NORMAL
SMOOTH
RECORDER
WIDE BAND


BitScope DSO 2.8
FUNCTION GENERATOR
RAMP
ENABLED
1 kHz
50 %
3.3 V
0 V
2.5 ms
3.28 V
TRIGGER
-8.371 mV
2.5 ms
3.288 V
400 Hz
1.32 mV/us
TB = 500 us
VA = 1.63 V
VB = -3.17 V
FT = 2.5 ms
FS = 1.06 MHz
POWER
SETUP
SCOPE
DUAL
MIXED
LOGIC
MACRO
CURSOR
GRID
SAVE
WAVE
POST
ZOOM
AUTO FOCUS
500 us/Div
REPEAT
TRACE
CHA
PRB
9.2 V
1.63 V
1 V/Div
ON
ZERO
CHB
PRB
9.2 V
-542.2 mV
2 V/Div
ON
ZERO
OSCILLOSCOPE
10 mS
1.14 MHz
NORMAL
SMOOTH
RECORDER
WIDE BAND
SO 2.8 FE22H BS000501 UU39VK14 COM3 21 kB/s

Gain in Db
6.11
6.06
5.71
5.39
4.91
3.86
1.44
-3.35
-8.87
-15.92