CG1111: Engineering Principles and Practice I

Principles of Capacitors

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Learning Outcomes

Capacitors:

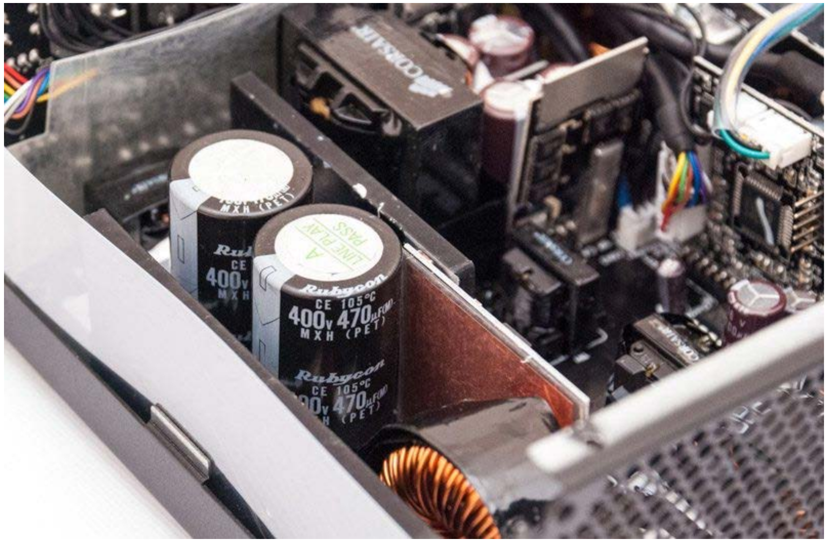
- Be aware of some of their applications
- Appreciate their basic principles
- Appreciate their energy storage capability

Application: Camera Flash

- Phone Camera Flash: Very high intensity for a short duration
- Using a special circuit, its internal capacitor is charged to 100s of volts, & discharge

As a Filter in DC Power Supply

- Removes ripples in DC voltage output



Capacitive Touch Screen

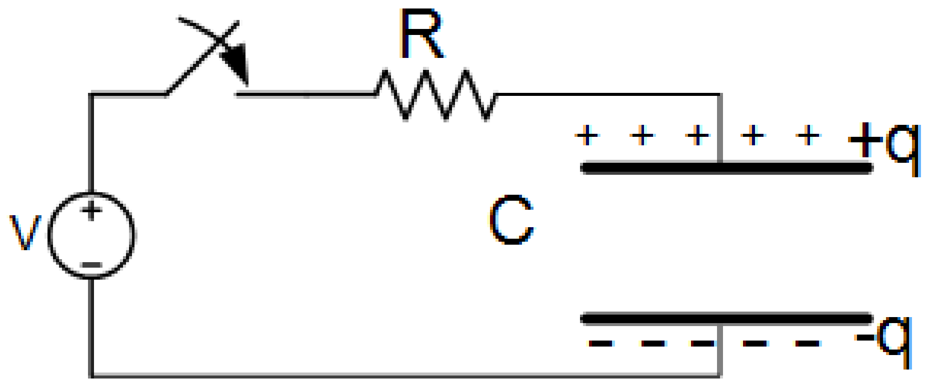
- Currently, many smartphones' touchscreens are built using principles of capacitance.

Examples of Capacitors

Credit: Picture from Wikipedia

Principle Construction of Capacitor

- Constructed by separating two conducting surfaces (electrodes) by a thin layer of insulating material (dielectric)



- When connected to a DC voltage source, opposite charges develop in the 2 surfaces

Capacitance

- The capacitance *C* describes a capacitor's capability to store electric charge when a voltage is applied

- Formally defined as amount of charge stored per volt:

$$\boxed{C = \frac{Q}{V}}$$

- Unit of capacitance is Farad (F)
  (1 F is equivalent to 1 C per volt – large!)

Practical Capacitors

- Most commonly found capacitors today are specified in pF, nF, and μF

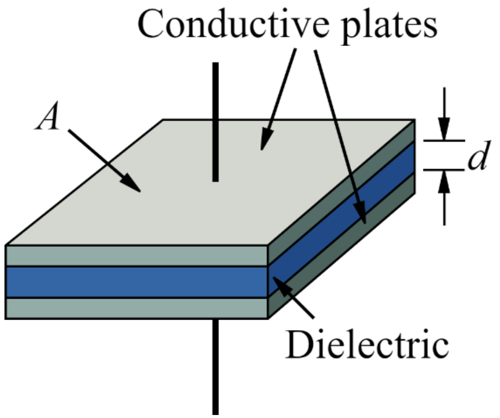
- However, there are super-capacitors today that can attain values greater than 1 F

- The trend is towards achieving increasingly large capacitance values $\rightarrow$ new applications!

Capacitance of Plate Capacitor

- In the case of the parallel-plate capacitor, the capacitance can be determined using

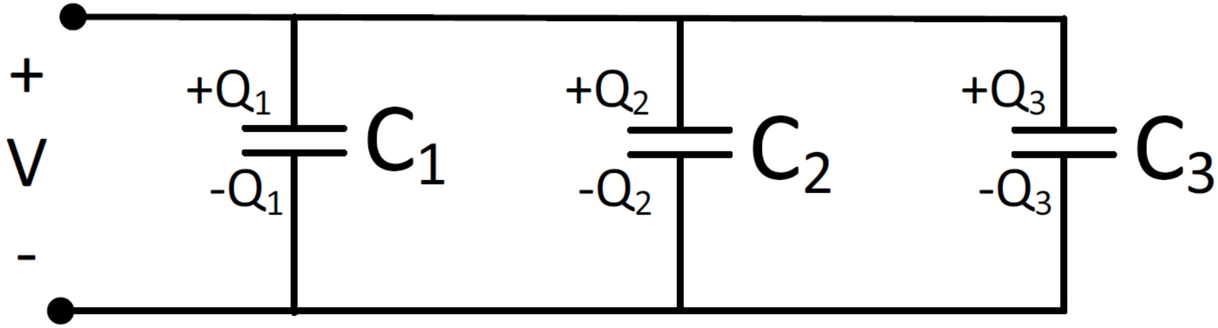
$\boxed{C = \frac{\varepsilon A}{d}}$, where $\varepsilon = \varepsilon_r \varepsilon_0$

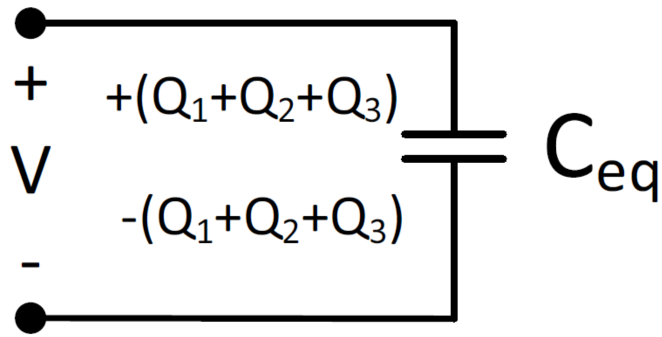


- ε : Permittivity of dielectric
- ε_r : Relative permittivity
- ε_0 : Permittivity of free space (8.85×10^{-12} F/m)
- A: Overlap area of conductor
- d: Distance between plates

Material	ε_r
Vacuum	1
Paper	3.85
Glass	4.7
Titanium dioxide	86

Capacitances in Parallel

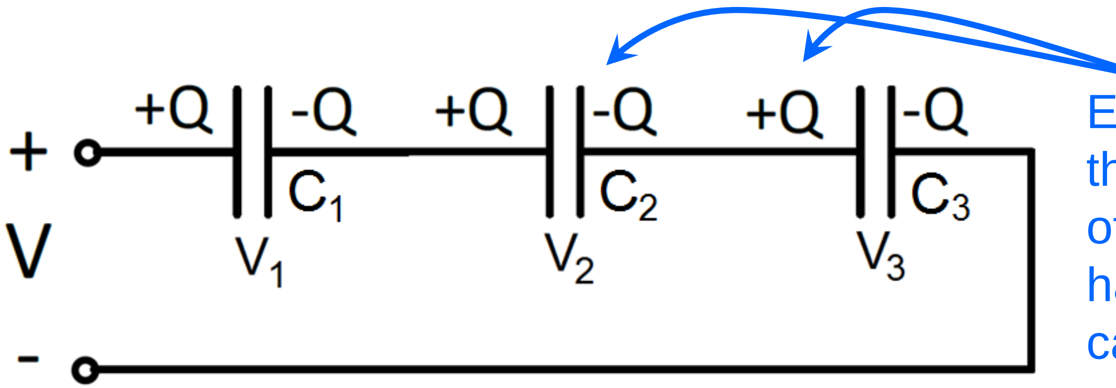




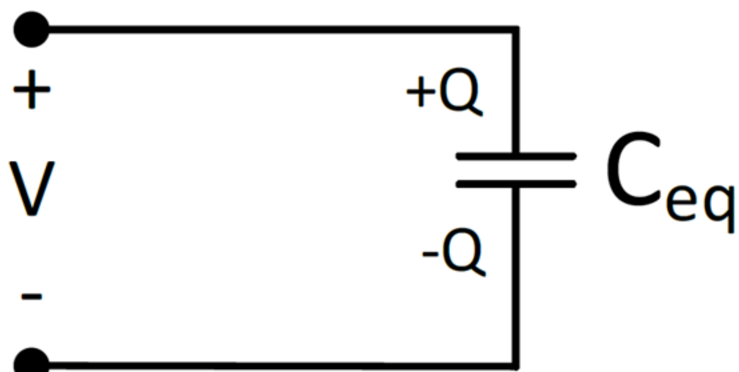
- Voltage is the same for all three capacitors, but charge different
- Total $Q_{total} = Q_1 + Q_2 + Q_3$
  $= C_1V + C_2V + C_3V$
  $= (C_1 + C_2 + C_3)V$
  $= C_{eq}V$

Hence, $\boxed{C_{eq} = C_1 + C_2 + C_3}$

Capacitances in Series



Equal in magnitude because the charge stored by a plate of any of the capacitors must have come from the adjacent capacitor's plate



- Charge is the same for all three capacitors, but voltage different
- From KVL:

$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Hence, $\boxed{\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}}$

Current-Voltage Relationship

- Capacitor has current flowing only when the stored electric charge is changing (voltage also changing)
- Since $i(t) = \frac{dq(t)}{dt}$, and $q(t) = Cv(t)$, we have

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

- At steady state, when the voltage is stable, its current will be 0

Hence, capacitors in DC circuits behave as open−circuit at steady state

Capacitor's Voltage Cannot Change Instantaneously!

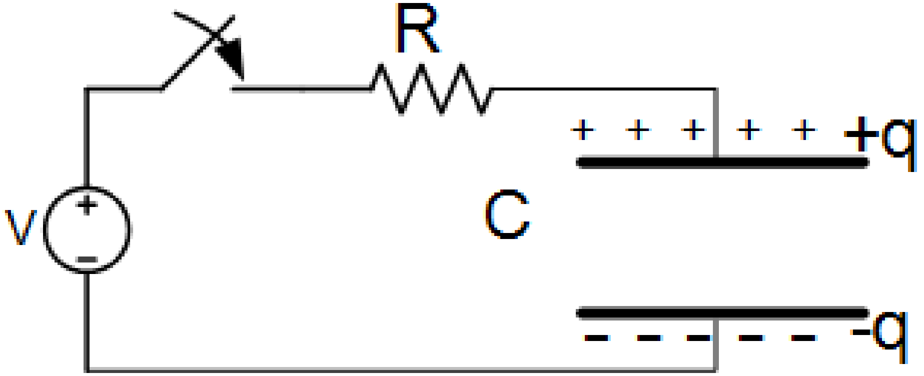
- From previous slide, $i(t) = C\frac{dv(t)}{dt}$
- If voltage changes instantaneously from v_1 to v_2 (i.e., in zero time duration), then $\frac{dv(t)}{dt} = \infty$

  $\rightarrow$ Impossible because it requires $i(t) = \infty$

> Hence, capacitor voltage cannot change instantaneously, and must be continuous

- Useful fact for our circuit analysis later...

Energy Storage



- When a capacitor gets charged, the energy is stored in its electric field
- The stored energy can be expressed in terms of the work done in moving the charges to set up the field

Capacitor's Energy Equation

- Recall: voltage is a measure of the energy transferred (work done) per unit charge
- Hence, work done in moving charge dq from one plate to another:

$$dw = v\,dq$$

- For a capacitor that already has voltage V, the stored charge is $\mathrm{Q} = CV$
- Hence, work done to attain Q is

$$W = \int_0^Q v\,dq = \int_0^Q \frac{q}{C}\,dq = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \boxed{\frac{1}{2}CV^2}$$

THANK YOU