CG1111: Engineering Principles and Practice I

Principles & DC Transient of Inductors

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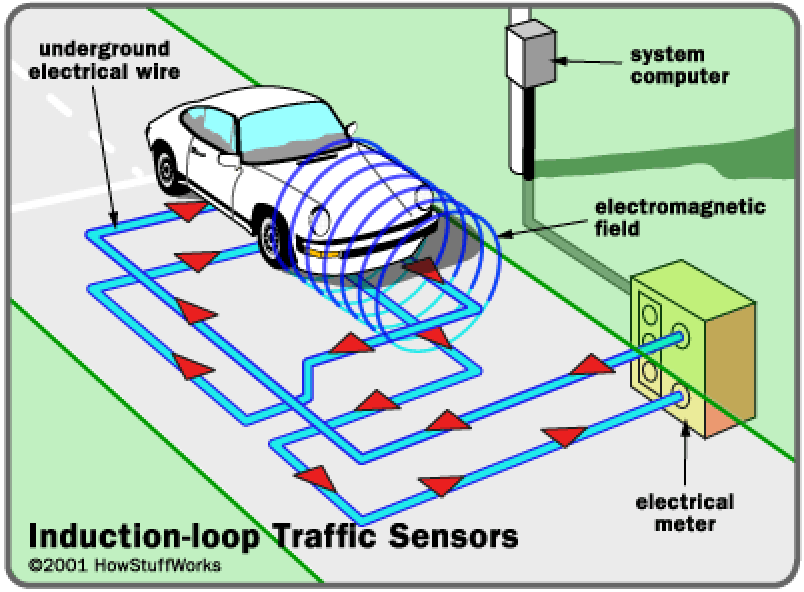
Learning Outcomes

Inductors:

- Be aware of some of their applications
- Appreciate their basic principles
- Appreciate their energy storage capability

Application: Traffic Sensor

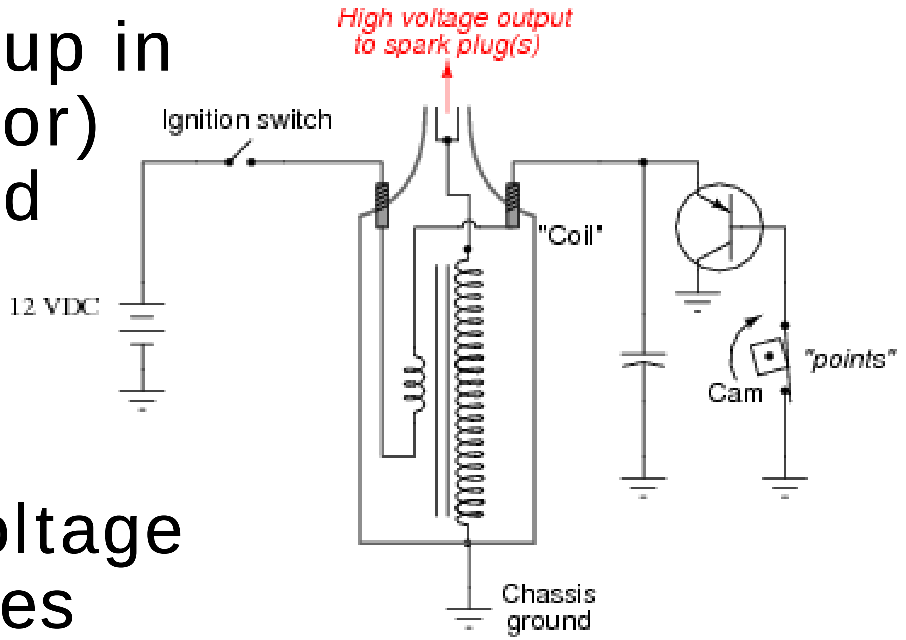


Credit: Picture from HowStuffWorks.com

- Many traffic junctions have wire coil sensors embedded underneath the roads
- When a vehicle is above the coil, its inductance increases due to the car's steel
- The change in inductance is detected by a circuit

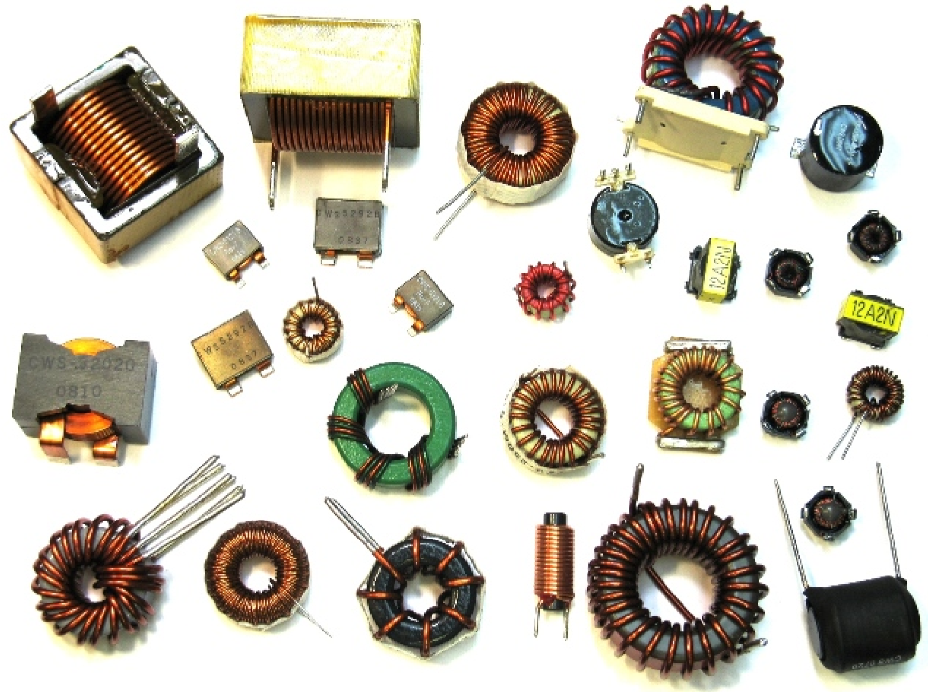
Application: Ignition System for Spark Plugs

- Allows current is built up in ignition coil (an inductor) $\rightarrow$ stores magnetic field energy
- When current is interrupted, a large voltage occurs across electrodes
- Fuel-air mixture becomes ionized & an electrically conductive channel develops $\rightarrow$ spark!



Generates a spark to ignite fuel-air mixture in petrol car's engine

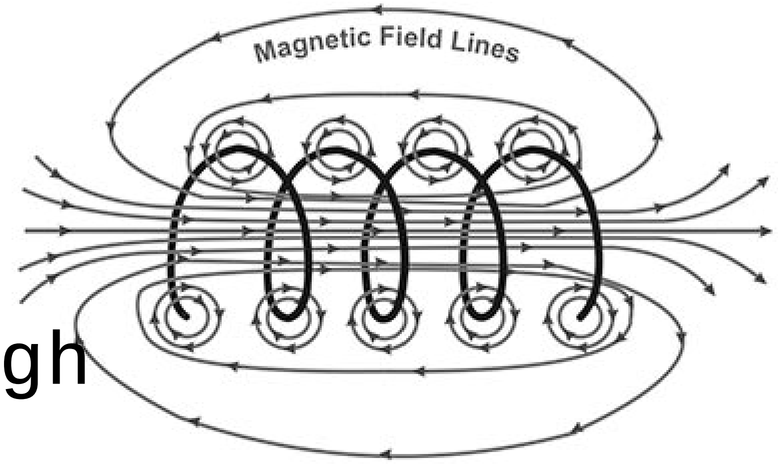
Examples of Inductors



Credit: Picture from Autodesk.com

Principle Construction of Inductor

- Constructed by coiling a wire in some form



Credit: Picture from Autodesk.com

- A current flowing through the coil produces a magnetic flux Φ that is proportional to this current

- The proportionality constant is called the "Inductance": $\Phi = Li$

Faraday's Law

- A voltage is induced in a coil when the magnetic flux varies with time

$$v = \frac{d\Phi}{dt} = \frac{d(Li)}{dt}$$

- Since *L* is assumed to be a constant:

$$\boxed{v = L\frac{di}{dt}}$$

Steady-state Voltage

- From $v = L\frac{di}{dt}$ we see that inductor has voltage only when its current is changing

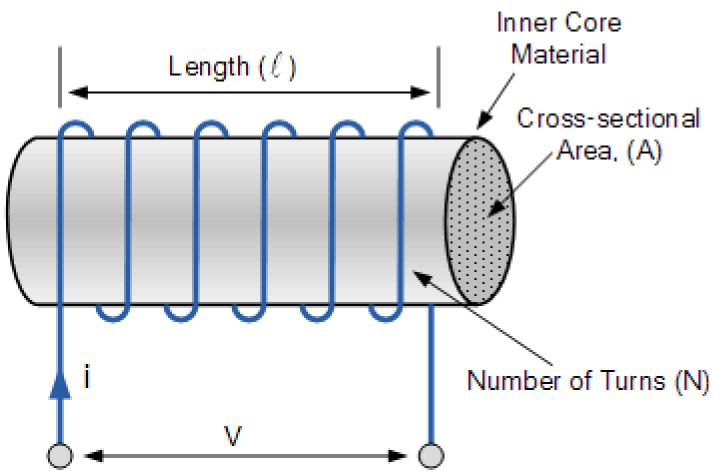
- At steady state, when the current is stable, its voltage will be 0

Hence, inductors in DC circuits behave as short−circuit in steady state

Formula for Inductance

- A coil's inductance can be determined using

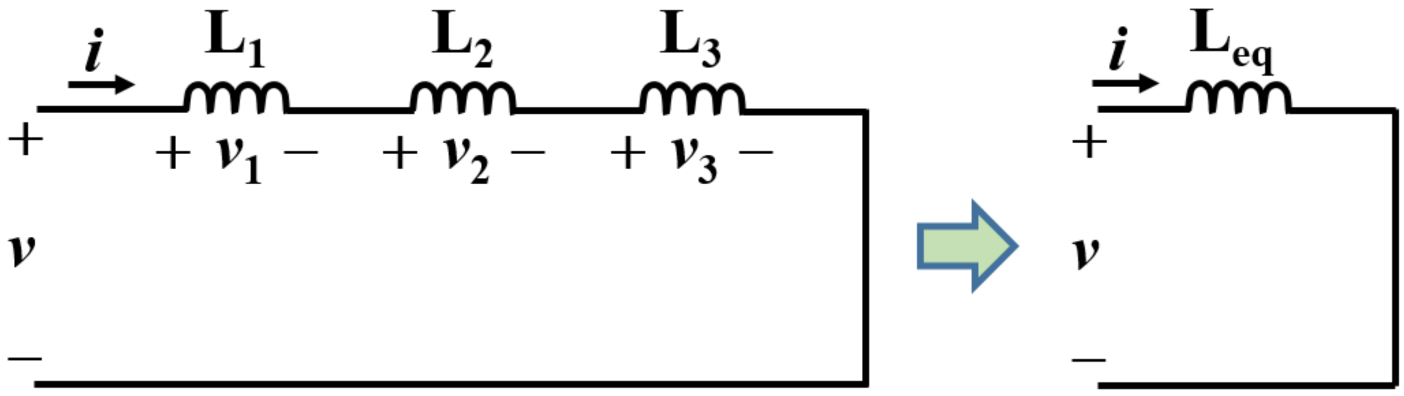
$$L = \frac{\mu_r \mu_0 N^2 A}{l}$$, where



- μ_r : Relative permeability of core material
- μ_0 : Permeability of free space (4π x 10^{-7} Wb.A/m)
- N: Number of turns in the coil
- A: Cross-sectional area of core
- l: Length of the core

Material	μ_r
Vacuum	1
Carbon steel	100
Iron	5000

Inductances in Series

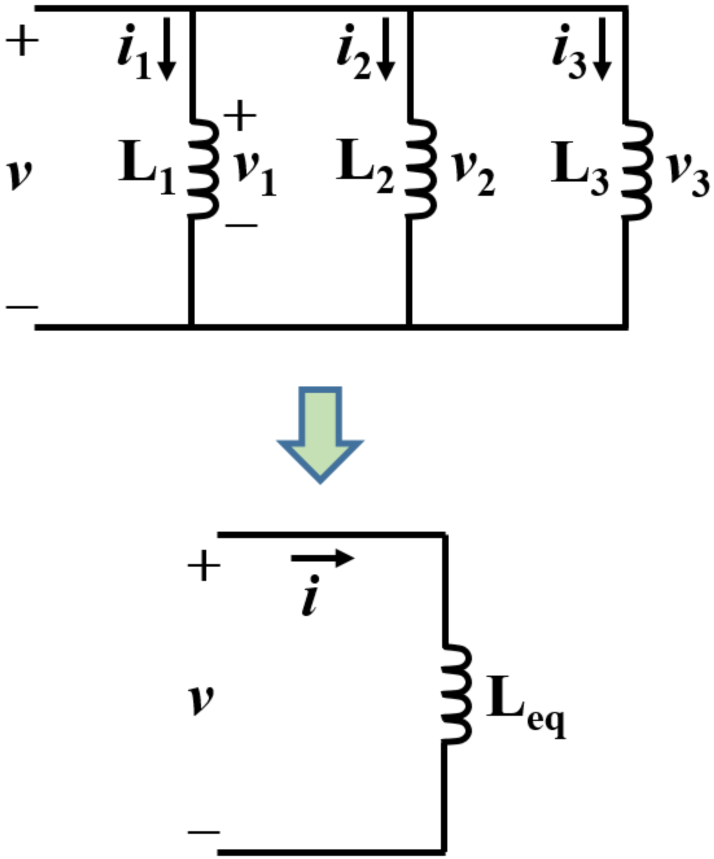


- Current is the same for all three inductors, but voltage different
- From KVL: $v = v_1 + v_2 + v_3$

$$\Rightarrow \qquad L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$$

Hence, $\boxed{L_{eq} = L_1 + L_2 + L_3}$

Inductances in Parallel



- Voltage is the same for all three inductors, but current different
- Since $v_n = L_n \frac{di_n}{dt}$
$$\Rightarrow \frac{di_n}{dt} = \frac{v_n}{L_n} = \frac{v}{L_n}$$
- From KCL: $i = i_1 + i_2 + i_3$
Hence, $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt}$
- $\therefore \frac{v}{L_{eq}} = \frac{v}{L_1} + \frac{v}{L_2} + \frac{v}{L_3}$

Hence, $\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$

Energy Storage

- When current passes through an inductor, energy is stored in its magnetic field
- The stored energy can be expressed in terms of the work done in establishing the magnetic field
- Instantaneous power $P = iv = iL\frac{di}{dt}$
- Hence, work done is

$$W = \int_0^T P\,dt = \int_0^T iL\frac{di}{dt}dt = L\int_0^I i\,di = \boxed{\frac{1}{2}LI^2}$$

Inductor's Current Cannot Change Instantaneously!

- From earlier slide, $v(t) = L\frac{di(t)}{dt}$
- If current changes instantaneously from i_1 to i_2 (i.e., in zero time duration), then $\frac{di(t)}{dt} = \infty$

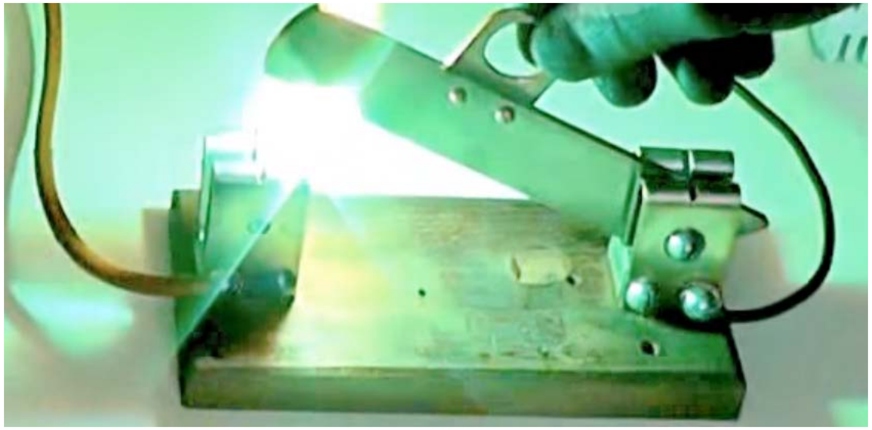
  $\rightarrow$ Impossible because it requires $v(t) = \infty$

Hence, inductor current cannot change instantaneously, and must be continuous

- This reluctance to change is because of the energy stored in the inductor's magnetic field

What Happens if We Try to Break Inductor's Current?

- Not possible to open a switch in 0 time in real life
- The inductor's voltage becomes extremely large, but not ∞ (e.g., how spark plug works)
- The air around can breakdown causing a flash arc (see https://www.youtube.com/watch?v=Zez2r1RPpWY)



Inductor's Transient Current in a Series RL Circuit

- Following the same procedure as transient voltage analysis for capacitors, we can also derive an inductor's current when it is in a series RL circuit:

$$\boxed{i_L(t) = i_L(0)e^{-\frac{t}{\tau}} + i_L(\infty)[1 - e^{-\frac{t}{\tau}}],\ \tau = \frac{L}{R}}$$

THANK YOU