CG1111: Engineering Principles and Practice I

Summary of Key Points for Week 2 to 5

NUS
National University of Singapore

Energy, Power & Efficiency

- Instantaneous power:

$$P = \frac{dW(t)}{dt}$$

- Efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{Total\ Loss}}$$

- C-rate of battery
  - A "1C" **rate** means that the discharge current will discharge the entire battery in 1 hour

Series vs Parallel Batteries

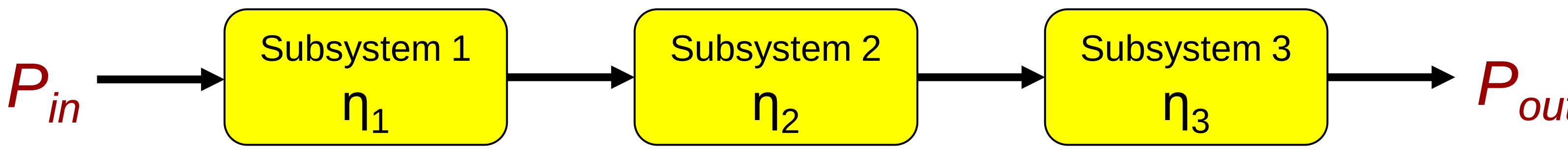
Two Batteries connected in Series:
Double the voltage but same capacity

Two Batteries connected in Parallel:
Double the capacity but same voltage

Power Requirements

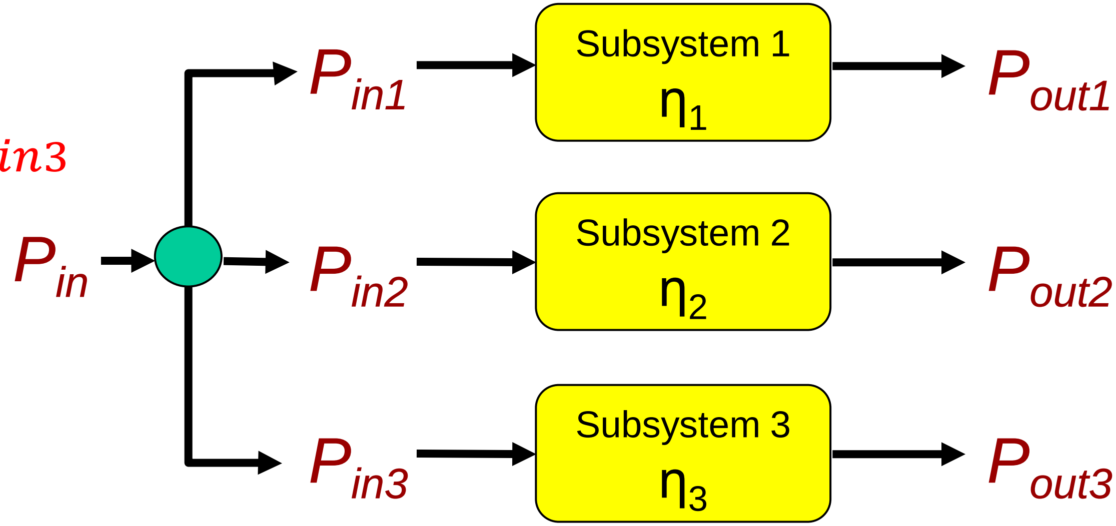
- Subsystems in Series

$$P_{in} = \frac{P_{out}}{\eta_1 . \eta_2 . \eta_3}$$

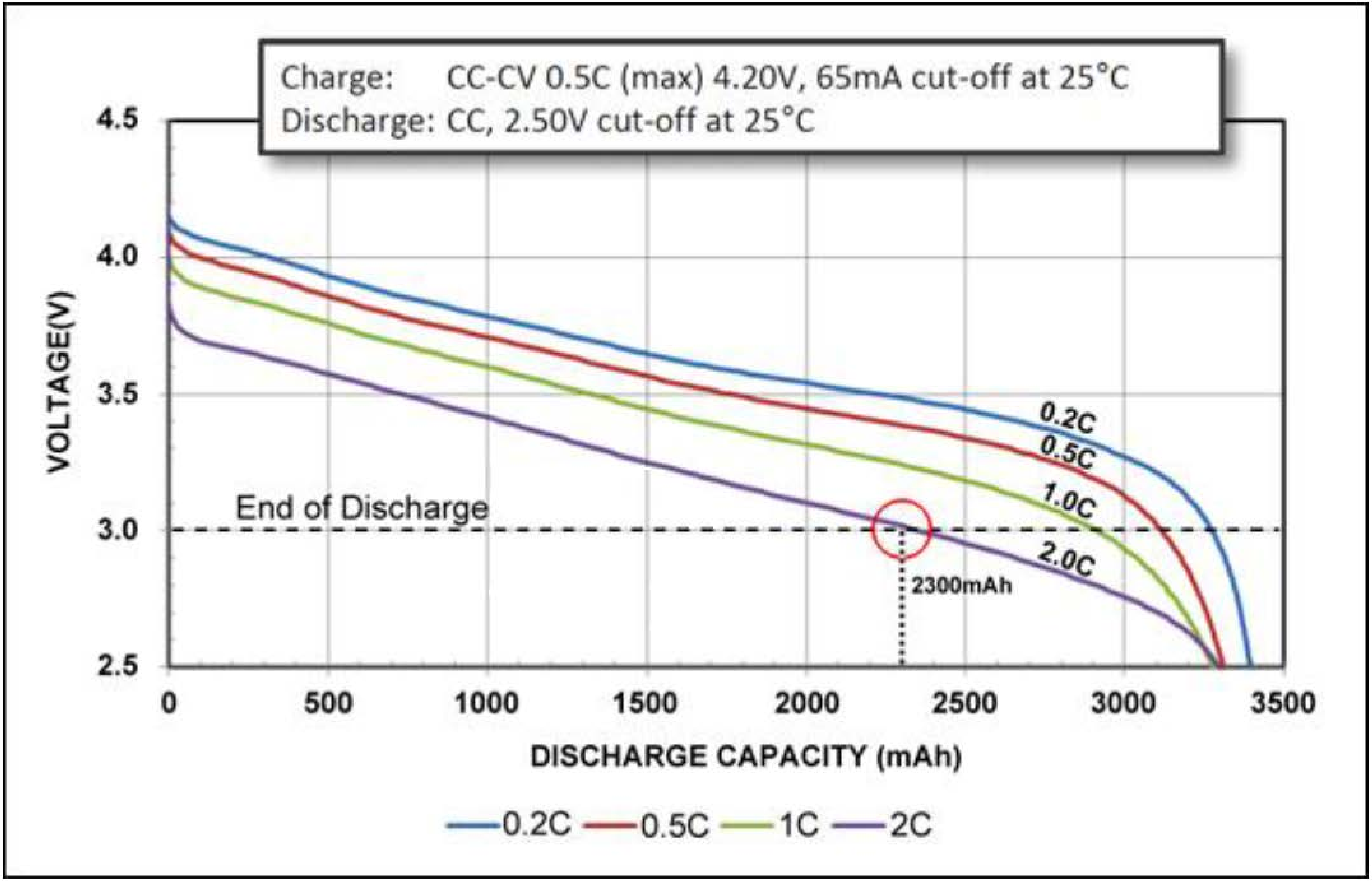


- Subsystems in Parallel

$$P_{in} = P_{in1} + P_{in2} + P_{in3}$$



Battery Discharging Characteristics



Battery Design Steps

1. Find mid-point voltage (V_{mp}) for given instantaneous C-rate
2. No. of batteries in series: $n_s = \lceil V_{op} / V_{mp} \rceil$, where
   V_{op} = desired operating voltage
3. Power requirements, $P_{in} = \Sigma\ (P_{out,i} / \eta_i)$ (parallel)
   or $P_{in} = P_{out} / (\Pi\ \eta_i)$ (series)
4. Load Capacity (Ah) = Load Energy / V_{BB} , where
   Load Energy (Wh) = $P_{in} \times$ (No. of operating hrs)
   Battery bank's voltage $V_{BB} = n_s \times V_{mp}$
5. Find single battery's capacity from given C-rate and allowed depth of discharge (usually corresponding to V_{eod} (knee))
6. No. of batts in parallel: $n_p = \lceil$Load Capacity / Battery Capacity$\rceil$
7. Total No. of batteries = $n_s * n_p$
8. Conclude design by calculating the actual Load Capacity of the battery pack.

Fundamentals of Electricity

- Ohm's Law
  - Empirical observation that $V \propto I$ for purely resistive element, and resistance is defined as V/I

- How is electrical power calculated?
  - $P = VI$ (always true regardless of type of element)
  - For resistors, we have $P = I^2R$ because $V = IR$
    - ✓ <u>Never, never</u> use $P = I^2R$ for non-resistive elements (e.g., LEDs)!!

KVL & KCL

- KCL (conservation of mass/charge)
  - The sum of all currents entering the node must be equal to the sum of all currents leaving the node
    - ✓ The “node” can be any enclosed surface drawn in any part of the circuit

- KVL (conservation of energy & power)
  - Around any closed loop, the sum of voltage drops must equal the sum of voltage rises

Series/Parallel Resistances

- Series: $R_{eq} = R_1 + R_2 + \ldots + R_N$

$$R_{eq} > R_i$$

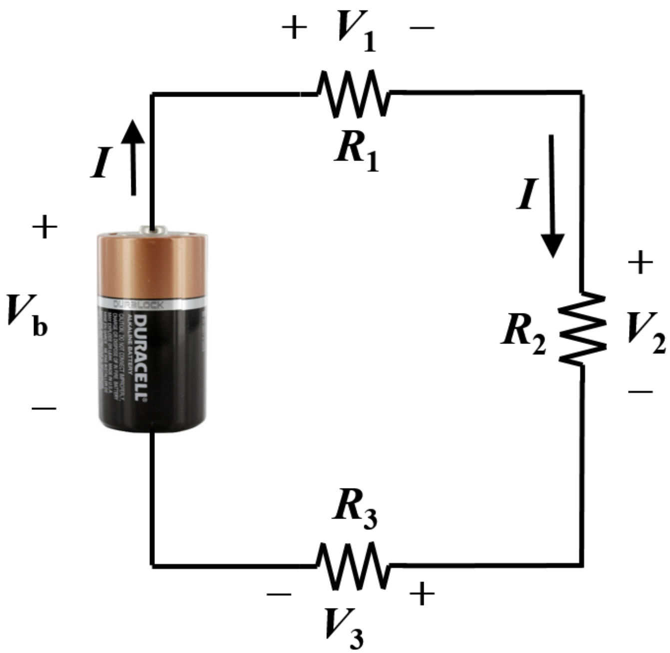
Resistances in series lead to increased resistance

- Parallel: $1/R_{eq} = 1/R_1 + 1/R_2 + \ldots + 1/R_N$

$$R_{eq} < R_i$$

Resistances in parallel lead to reduced resistance

Voltage Division Principle



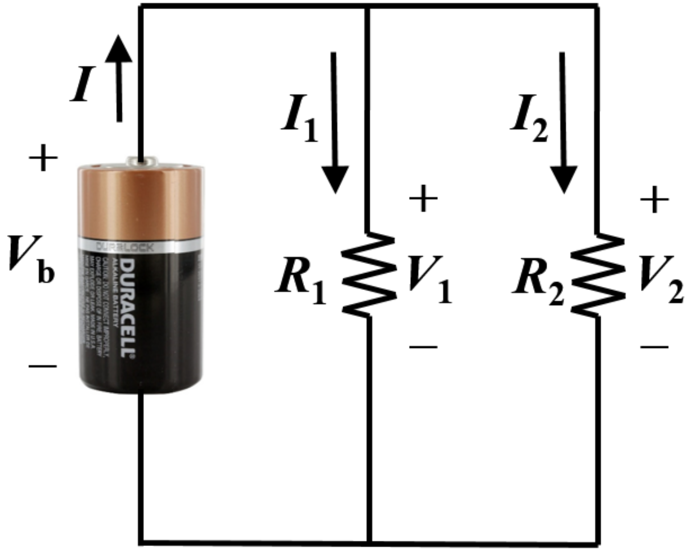
$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V_b$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_b$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} V_b$$

Works for any number of series resistors

Current Division Principle



$$I_1 = \frac{R_2}{R_1 + R_2} I$$

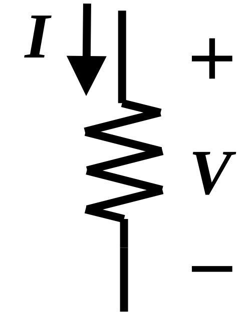
$$I_2 = \frac{R_1}{R_1 + R_2} I$$

Only works for **two** parallel branches

Passive vs Active Elements

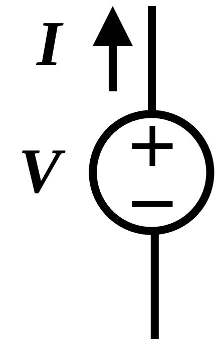
- Voltage has polarity, current has direction
  - –**Passive** elements (**consumes** power):
    voltage (potential) **drops** in the direction of the current

Note: Resistors are **always passive**



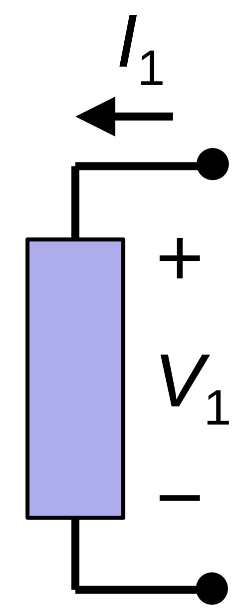
  - –**Active** elements (**delivers** power):
    voltage (potential) **rises** in the direction of the current

Current comes out from the '+' polarity
and the voltage is positive



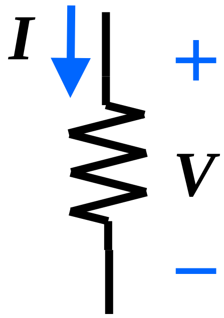
Passive Sign Convention

- Adopted when it is not clear whether an unknown element is <u>active</u> or <u>passive</u>
- <u>Assumes</u> all unknown elements are <u>passive</u>
  - Reference direction for <u>current</u> is assumed to <u>enter</u> the <u>positive voltage terminal</u> of the element
  - Significance:
    - ✓ If <u>power</u> is calculated to be <u>**positive**</u>, the element is <u>**passive**</u> (a load)
    - ✓ If <u>power</u> is calculated to be <u>**negative**</u>, the element is <u>**active**</u> (a source)



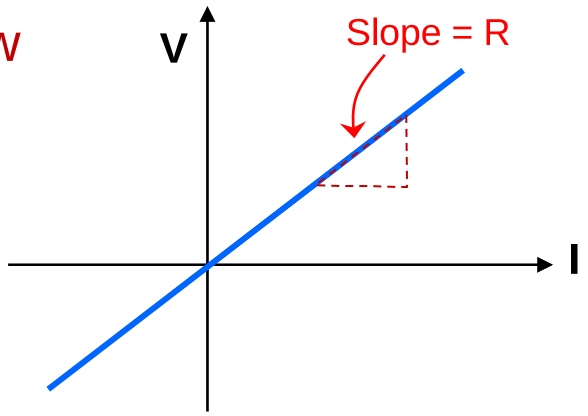
How to Label V & I for Resistors?

- A resistor consumes power:
  - Principle: In a resistor, the voltage (potential) must drop in the direction of the current



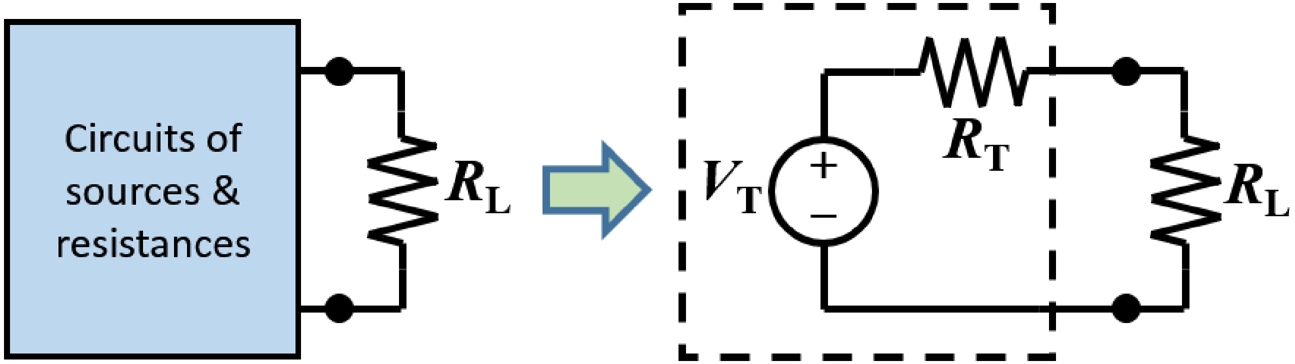
If the current's true direction or the voltage's true polarity is not obvious in the circuit, just pick & label either the current or the voltage, and then label the other one by following the above principle

You can only apply Ohm's Law if you label them this way!



Thevenin Equivalent Circuit

- Any circuit consisting of sources and resistances can be replaced by a Thevenin equivalent circuit

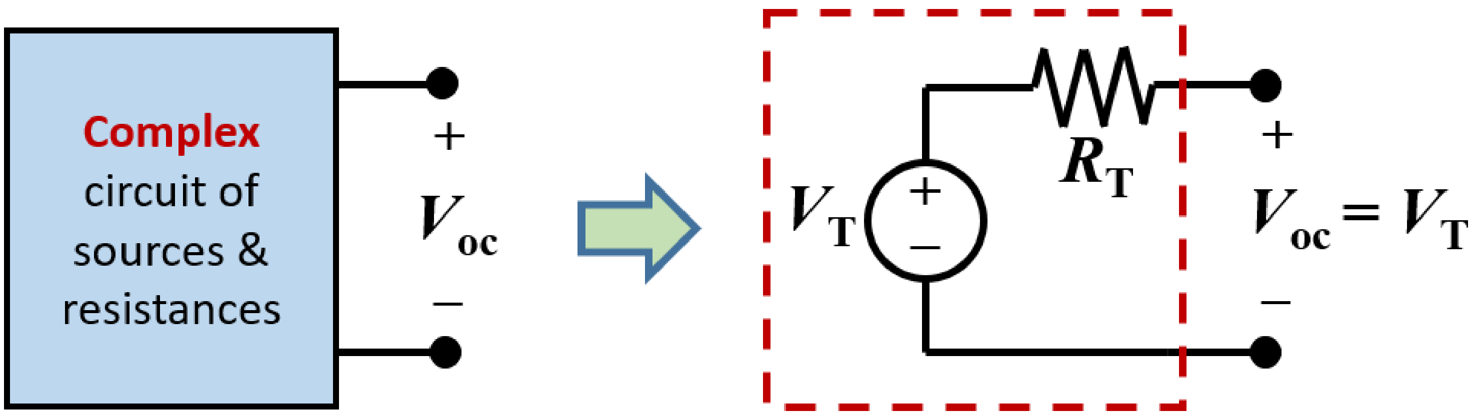


Thevenin equivalent circuit

The I-V behaviour of the Thevenin equivalent circuit is the same as that of the more complex circuit
$\rightarrow$ Very useful for circuit analysis!

Finding Thevenin Voltage

V_T can be obtained by finding V_{oc} across the 2 terminals of the complex circuit

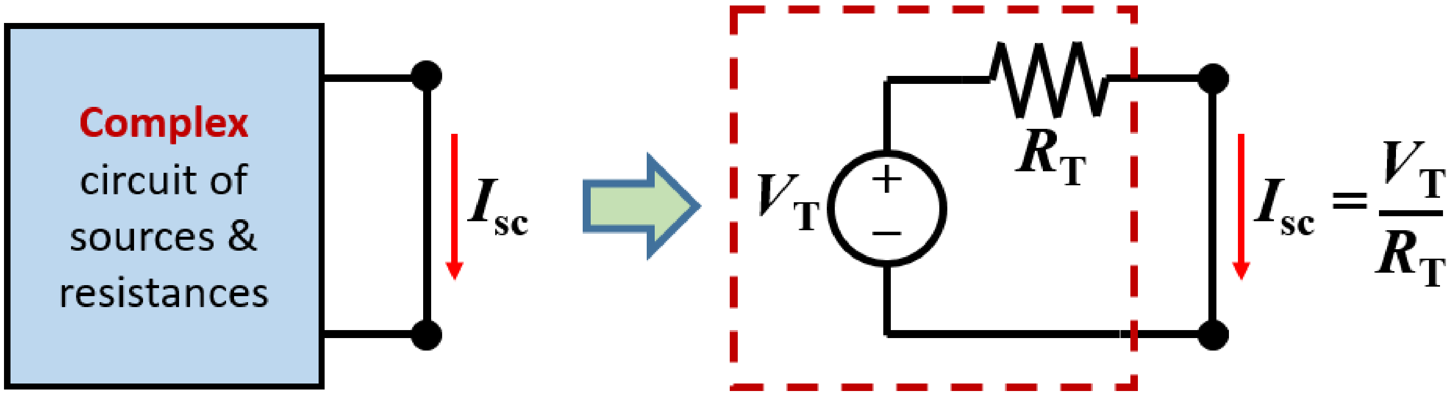


Thevenin equivalent circuit

Finding Thevenin Resistance

R_T can be obtained using Ohm's Law after finding I_{sc} across the 2 terminals of the complex circuit

$$R_T = \frac{V_T}{I_{sc}}$$

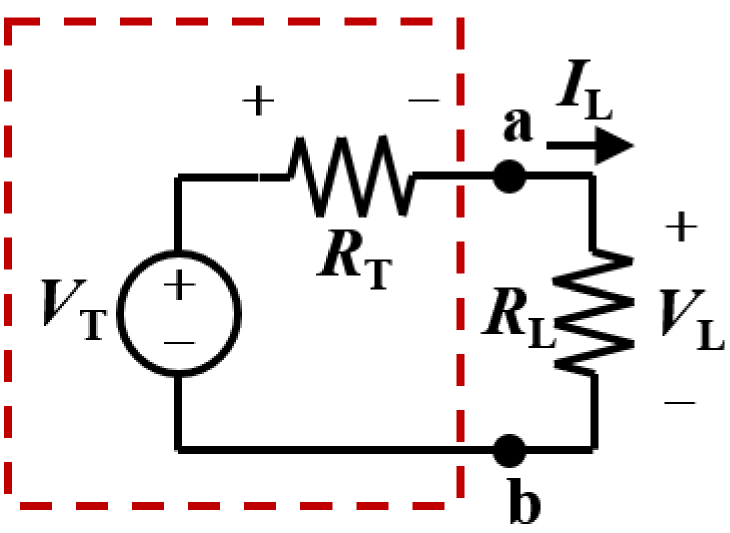


Thevenin equivalent circuit

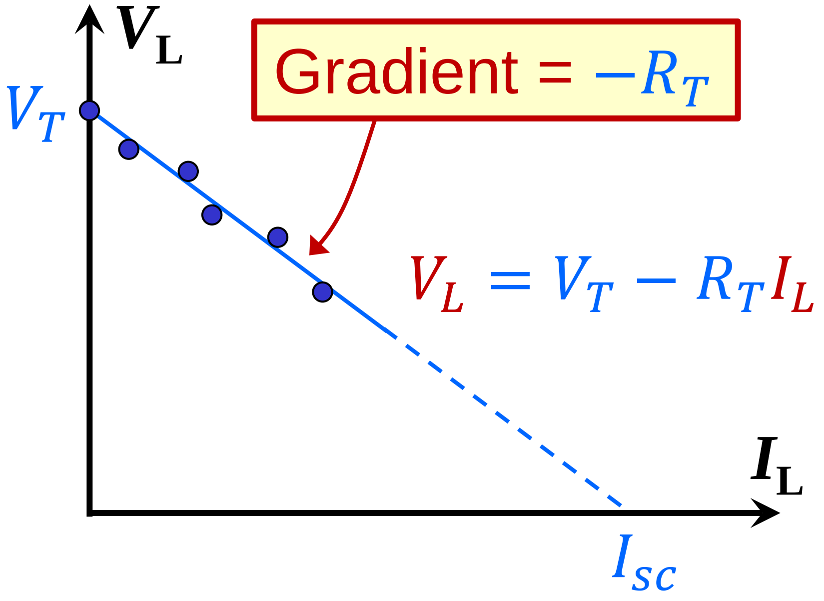
Experimental Method to Find R_T

- There is a risk that I_{sc} may be large

It is safer to connect the circuit to multiple resistive loads and then use graphical approach



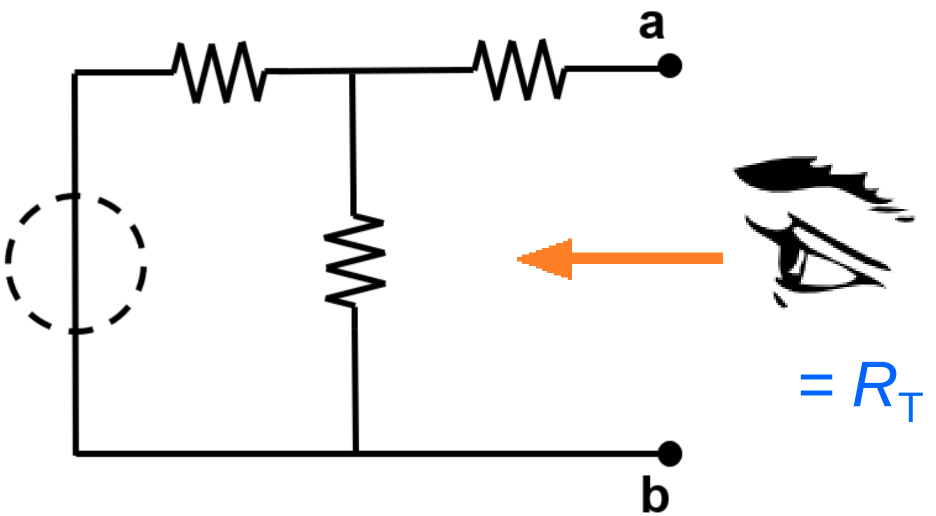
Thevenin equivalent circuit



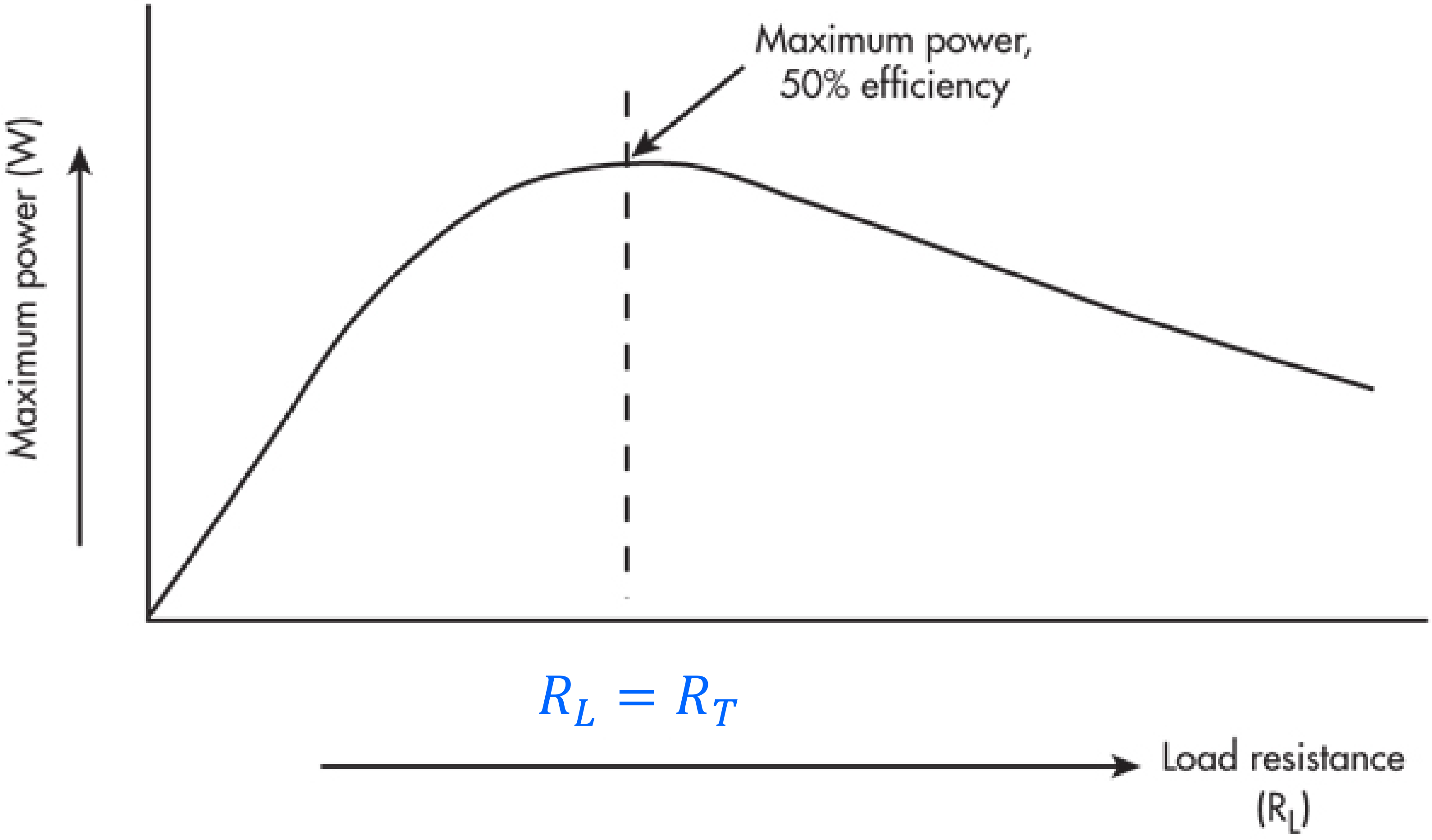
Another Analytical Method to Find R_T (Usually Easiest!)

- If the original electrical circuit consists of independent voltage sources only,
  - Replace all voltage sources with a short-circuit (0 V); this results in a purely resistive network

Then, find the equivalent resistance between the two terminals



Maximum Power Transfer from Thevenin Equivalent Circuit



$R_L = R_T$

Node Voltage Analysis Method

- Node voltage analysis
  - Most general method for analyzing circuits
- Basic idea:
  - Solve for all the unknown node voltages w.r.t. a reference node
  - Use them to calculate the voltage across any element, & the current passing through it

Node Voltage Analysis Method

- Identify the nodes with unknown voltages, & <u>minimize number of unknowns</u>

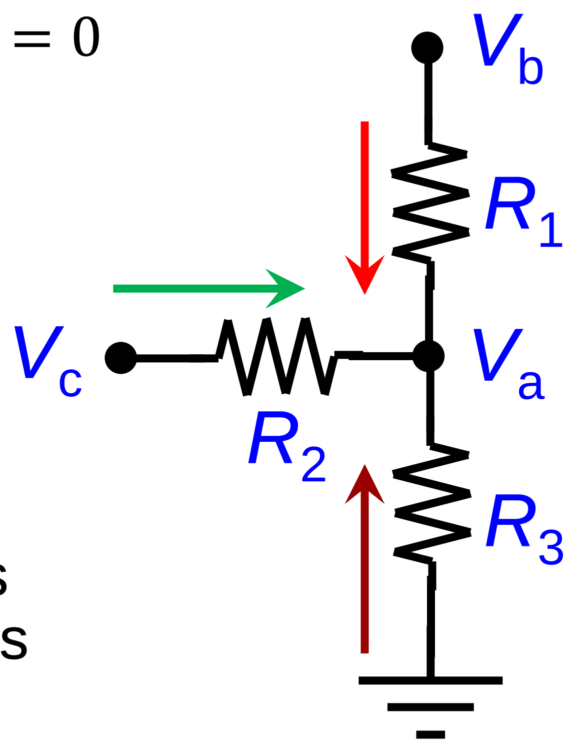
Note: The node must be adjacent to resistive branches only

- Apply KCL at the nodes, to obtain <u>as many equations</u> as the <u>number of unknown variables</u>

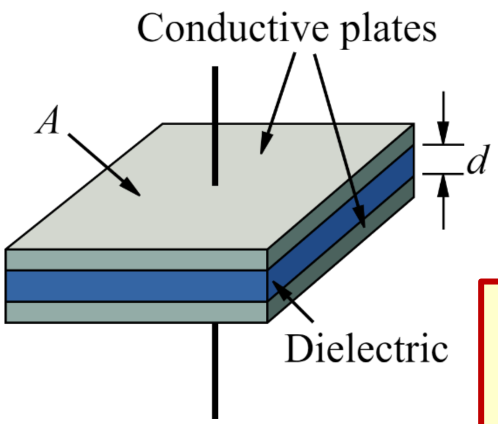
$$\frac{V_b - V_a}{R_1} + \frac{V_c - V_a}{R_2} + \frac{0 - V_a}{R_3} = 0$$

- <u>Rearrange</u> the equations to <u>consolidate the coefficients</u> <u>(via visual inspection)</u> for each unknown

- Solve the linear system of equations to obtain the unknown node voltages



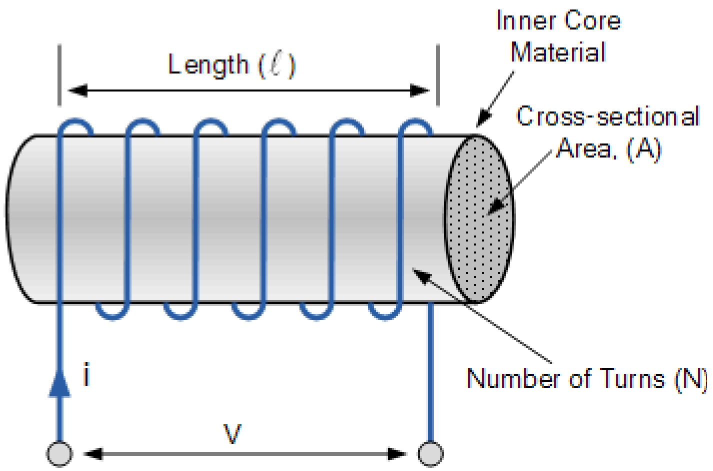
Duality of Capacitors & Inductors



ε: Permittivity of the dielectric
A: Cross-sectional area of the plate
d: Distance between the plates

$$\boxed{C = \frac{\varepsilon A}{d}} \quad \boxed{C = \frac{Q}{V}} \quad \boxed{i(t) = C\frac{dv(t)}{dt}} \quad \boxed{E = \frac{1}{2}CV^2}$$

$$\boxed{L = \frac{\mu_r \mu_0 N^2 A}{l}} \quad \boxed{L = \frac{\Phi}{I}} \quad \boxed{v(t) = L\frac{di(t)}{dt}} \quad \boxed{E = \frac{1}{2}LI^2}$$



μ_r: Relative permeability of core material
μ_0: Permeability of free space ($4\pi \times 10^{-7}$ Wb.A/m)
N: Number of turns in the coil
A: Cross-sectional area of core
l: Length of the core

Duality of Capacitors & Inductors

$$i(t) = C\frac{dv(t)}{dt}$$

Capacitor voltage cannot change instantaneously, and must be **continuous**

$$v_c(0^-) = v_c(0^+)$$

Capacitors in DC circuits behave as open−circuit at steady state

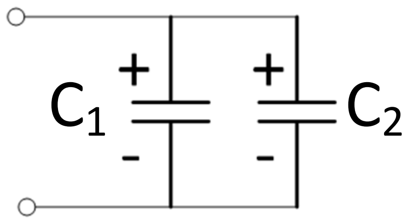
$$v(t) = L\frac{di(t)}{dt}$$

Inductor current cannot change instantaneously, and must be **continuous**

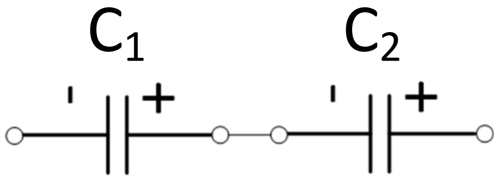
$$i_c(0^-) = i_c(0^+)$$

Inductors in DC circuits behave as short−circuit at steady state

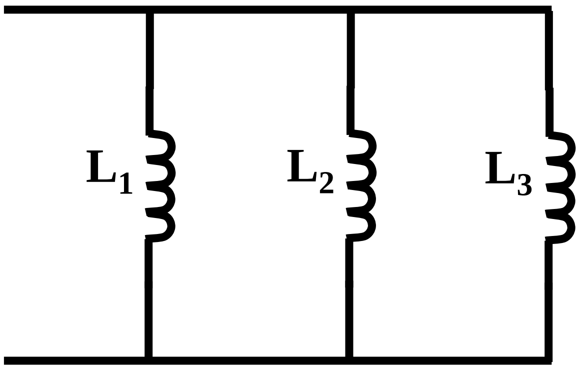
Duality of Capacitors & Inductors



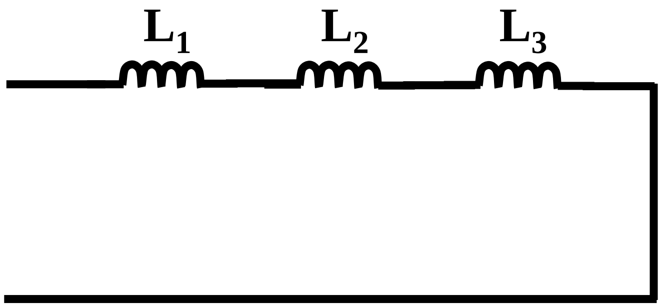
$$C_{eq} = C_1 + C_2$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$



$$L_{eq} = L_1 + L_2 + L_3$$

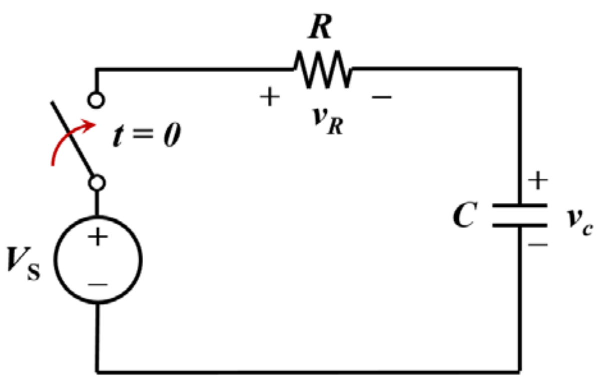
Duality of Capacitors & Inductors

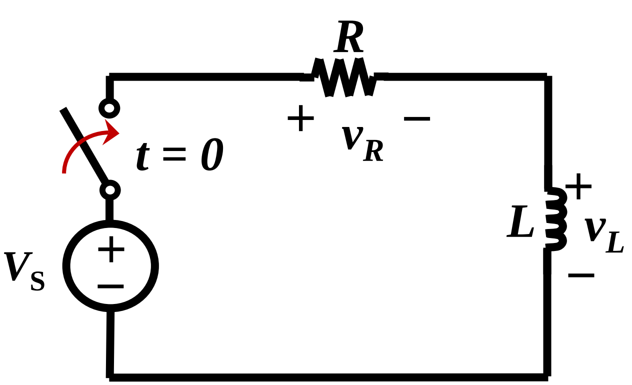
- A capacitor's **transient voltage** in a series RC circuit can be expressed as

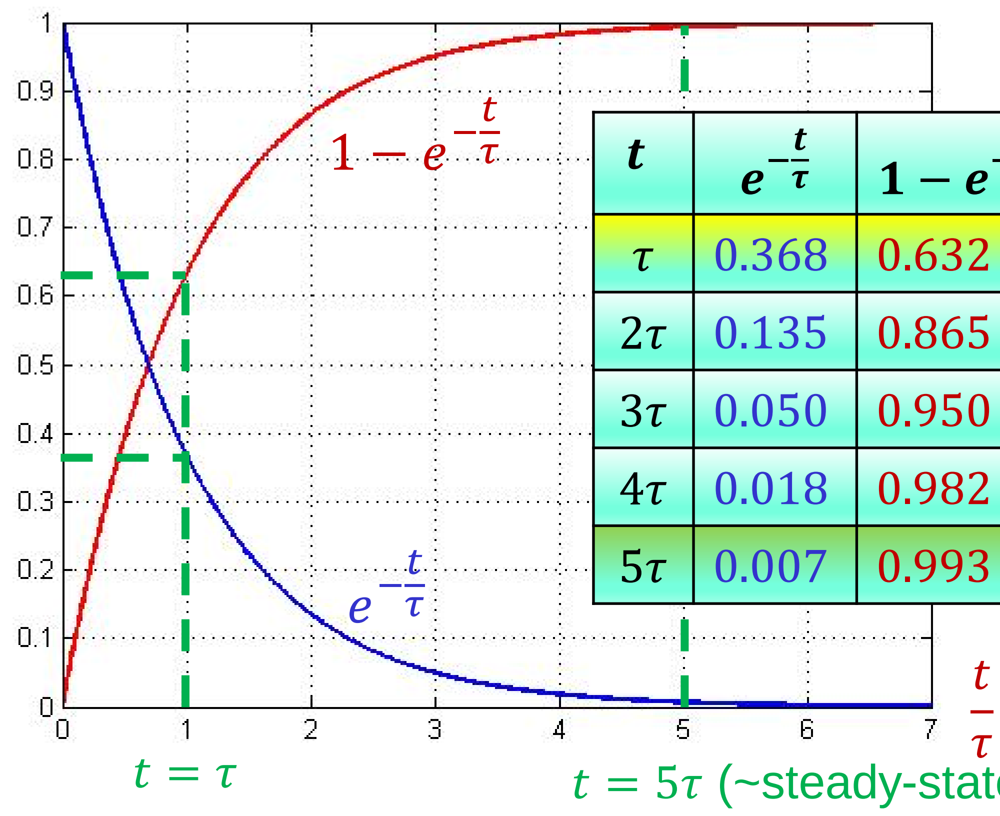
$$v_c(t) = v_c(0)e^{-\frac{t}{\tau}} + v_c(\infty)[1 - e^{-\frac{t}{\tau}}],\ \tau = RC$$

- An inductor's **transient current** in a series RL circuit can be expressed as

$$i_L(t) = i_L(0)e^{-\frac{t}{\tau}} + i_L(\infty)[1 - e^{-\frac{t}{\tau}}],\ \tau = L/R$$







t	$e^{-\frac{t}{\tau}}$	$1 - e^{-\frac{t}{\tau}}$
τ	0.368	0.632
2τ	0.135	0.865
3τ	0.050	0.950
4τ	0.018	0.982
5τ	0.007	0.993