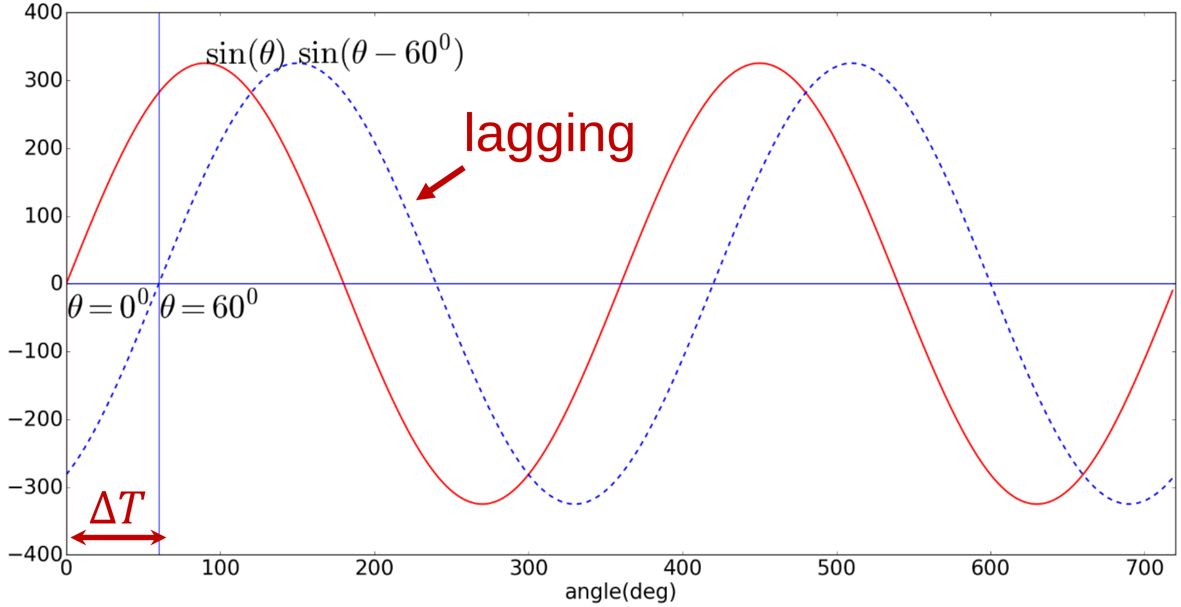
CG1111: Engineering Principles and Practice I

Debrief and Tutorial for Week 7

NUS
National University of Singapore

Principles of AC Circuits

Sinusoidal waveform



$$v(t) = V_m \cos(\omega t \pm \emptyset)$$

$$\emptyset = \frac{\Delta T}{T} \times 360°$$

'+' if leading
'–' if lagging

V_m: Amplitude (or peak)

ω: Angular frequency in rad/s

$\emptyset$: Phase angle

$$\omega = 2\pi f$$

$$T = \frac{1}{f}$$

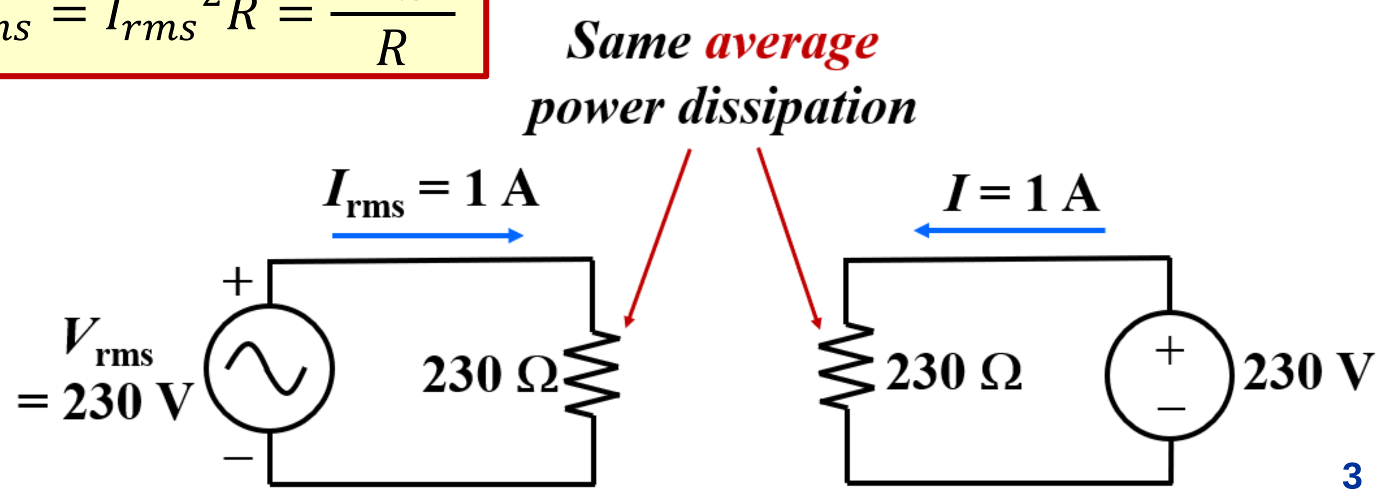
Root-Mean-Square (RMS)

- Significance of rms value:
  - They are the equivalent values of the DC voltage & current that would have the **same average power dissipation** in a **resistive load**
  - So that you can apply the same formula as DC!
  - Average power dissipation of resistive load in AC:

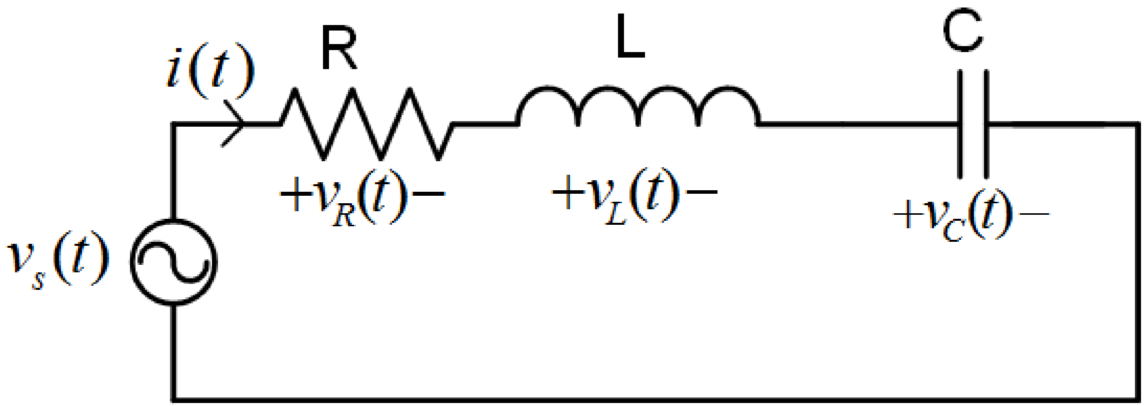
$$\boxed{P = V_{rms} \times I_{rms} = I_{rms}{}^2 R = \frac{V_{rms}{}^2}{R}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$



AC Circuit Analysis in Time Domain?

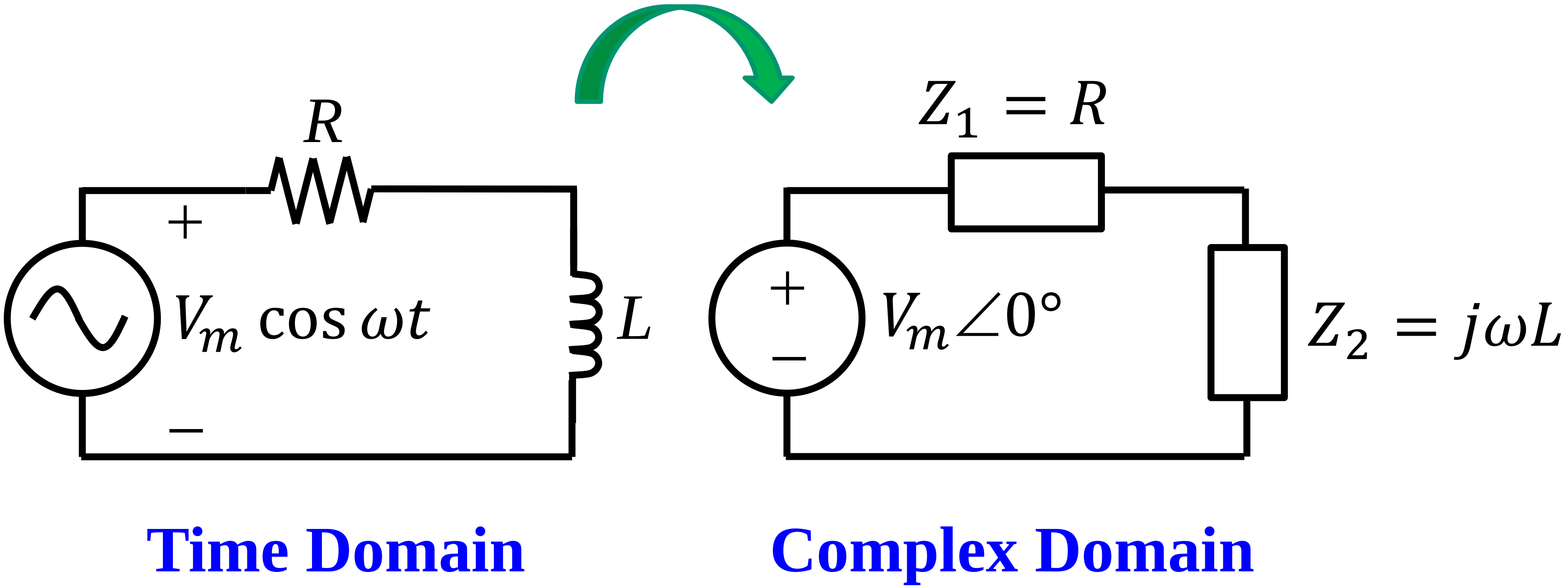


- In AC circuits, inductors/capacitors result in differential equations:

$$V_m \cos(\omega t) = v_R(t) + v_L(t) + v_C(t) = iR + L\frac{di}{dt} + \frac{\int i\, dt}{C}$$

$\rightarrow$ difficult to solve in time-domain!

AC Circuit Analysis



Can then solve using DC circuit analysis techniques:

- KVL, KCL, Ohm's Law, Potential divider principle, current division principle, Thevenin equivalent, NVA, series equivalent impedance, parallel equivalent impedance, etc.

AC Circuit Analysis with Phasors & Impedances

We must work with KVL & KCL in Phasor form

Steps:

1. Replace voltage sources with their phasors (all must have same frequency)
2. Replace R, L, C elements with their impedances
3. Analyse circuit using DC circuit analysis techniques (work within complex domain)
4. Convert final results back to time-domain

Phasors

- Sinusoidal voltage:

$$V_m \cos(\omega t + \theta)$$

- Phasor:

$$V_m \angle \theta$$

Note:
Another common practice is to represent phasors using the RMS value instead of the magnitude. In that case, the phasor will be written as $\frac{V_m}{\sqrt{2}} \angle \theta$.

Note:

- Phasor is just a definition. It leads to mathematical convenience, but has no physical significance

Impedances

- For resistance: $\boxed{R}$

- For inductor: $\boxed{j\omega L = \omega L \angle 90°}$

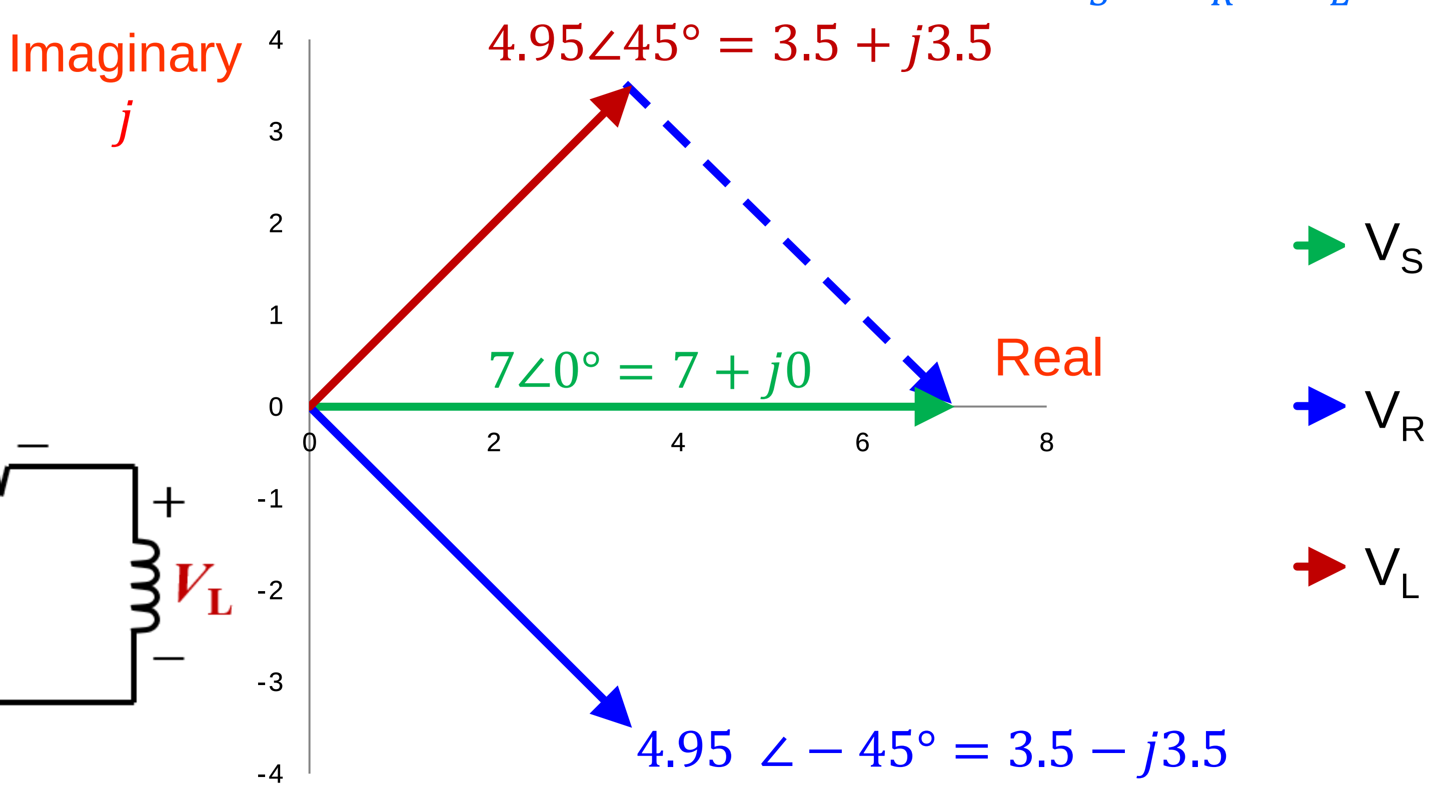
- For capacitor: $\boxed{\dfrac{1}{j\omega C} = \dfrac{-j}{\omega C} = \dfrac{1}{\omega C} \angle -90°}$

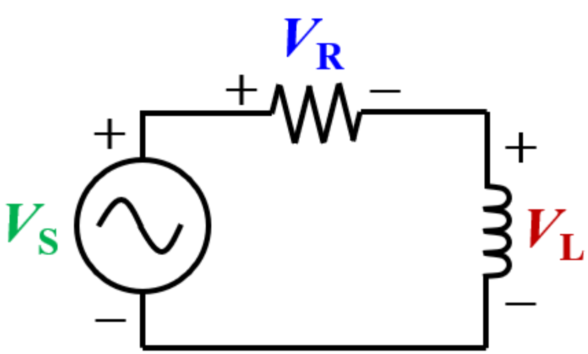
Illustration: KVL Applies for Phasors

Voltage	Magnitude	Phase Angle	Real	Imaginary
V_S	7	0°	7	0
V_R	4.95	−45°	3.5	−3.5
V_L	4.95	45°	3.5	3.5

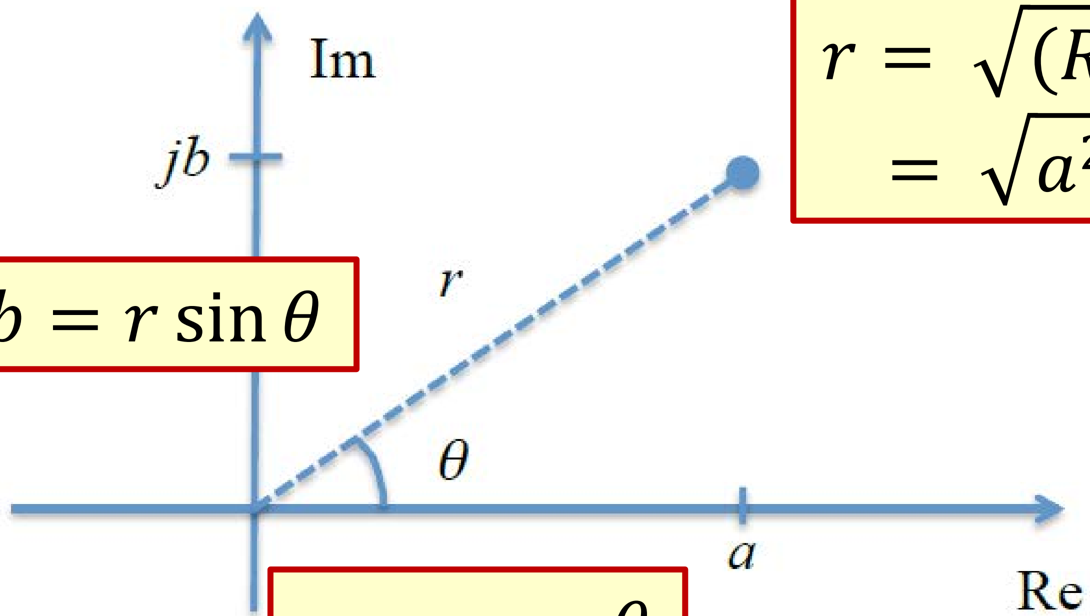
In AC circuits with R, L, C elements, because of the phase differences, we can only apply KVL in complex domain:

$$V_S = V_R + V_L$$





Relationship Among Magnitude, Phase, Real & Imaginary Parts



$$r = \sqrt{(Re)^2 + (Im)^2} = \sqrt{a^2 + b^2}$$

$$b = r \sin\theta$$

$$\theta = \tan^{-1}\left(\frac{Im}{Re}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

$$a = r \cos\theta$$

Phasor Division & Multiplication

- Division: $$\boxed{\frac{A\angle\theta_1}{B\angle\theta_2} = \frac{A}{B}\angle(\theta_1 - \theta_2)}$$

- Multiplication:

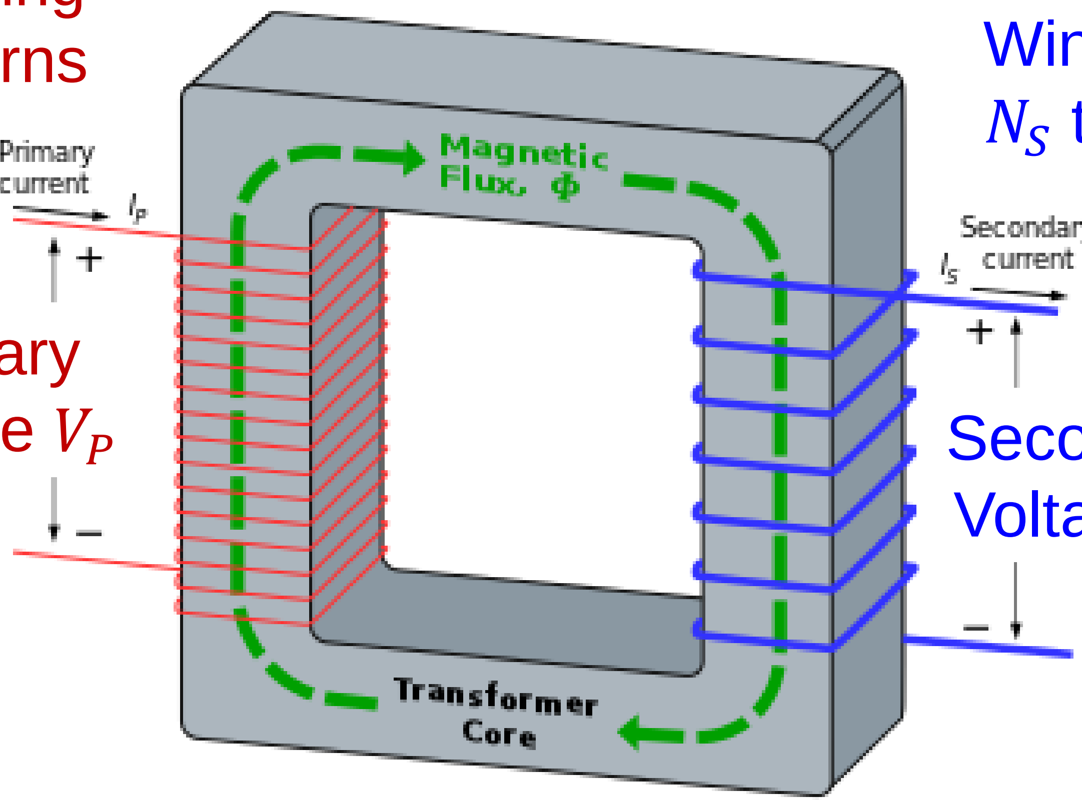
$$\boxed{A\angle\theta_1 \times B\angle\theta_2 = AB\angle(\theta_1 + \theta_2)}$$

Step-Up/Down Transformer

Primary Winding N_P turns

Primary Current I_P

Primary Voltage V_P



Secondary Winding N_S turns

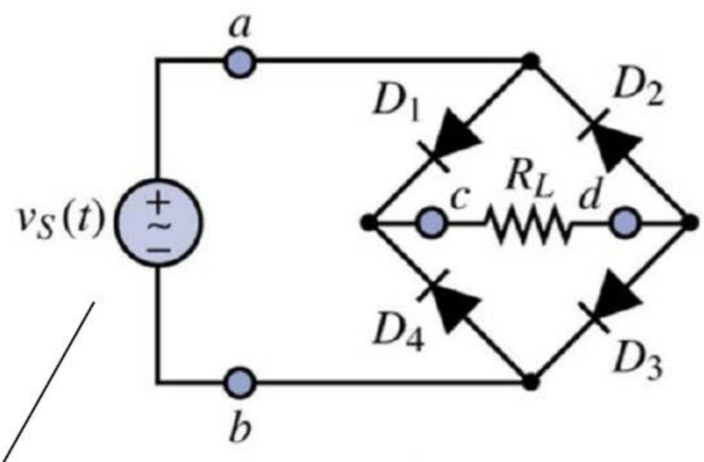
Secondary Current I_S

Secondary Voltage V_S

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

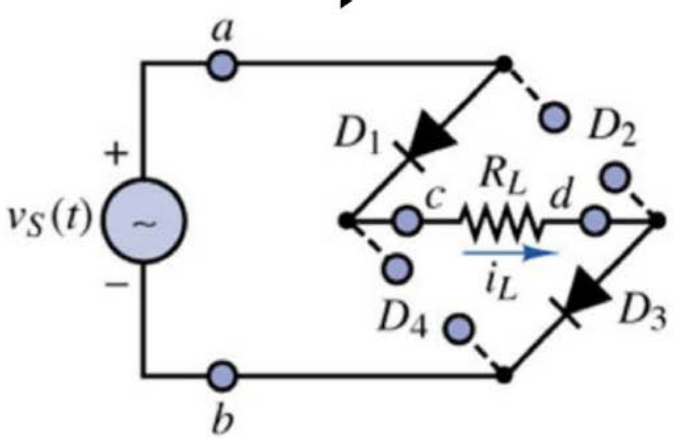
$$\frac{I_S}{I_P} = \frac{N_P}{N_S}$$

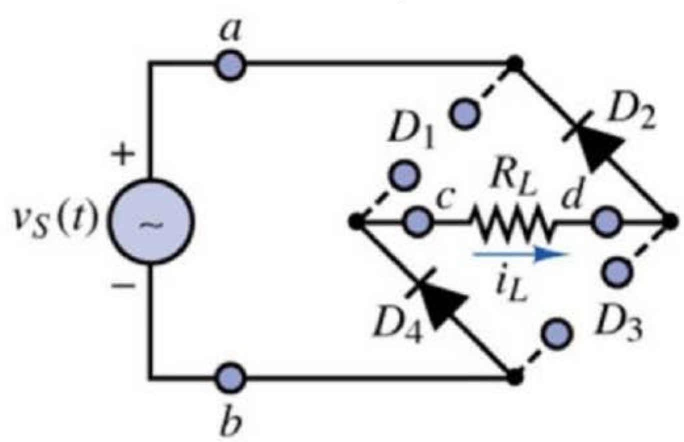
Why Rectifier?

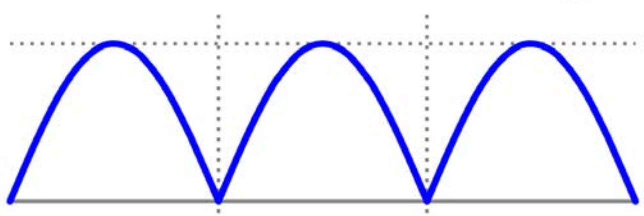


Positive
half-cycle

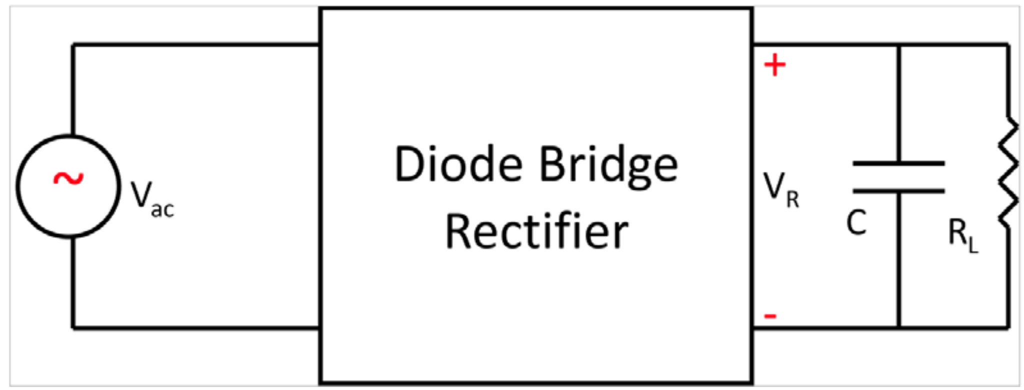
Negative
half-cycle

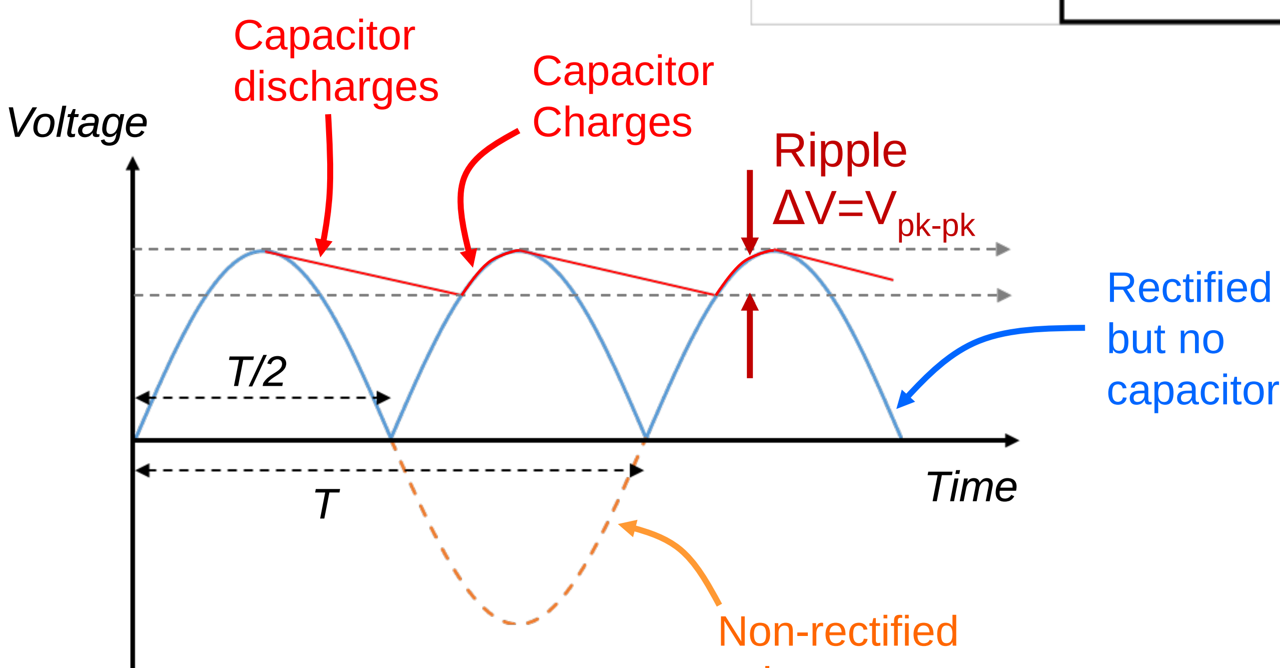






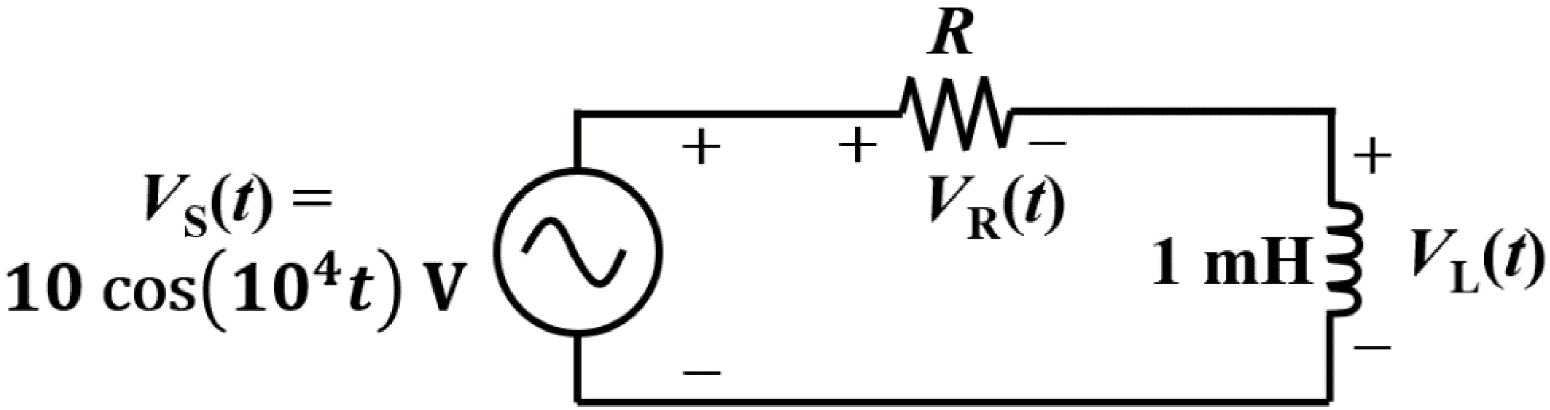
Why Filter Capacitor?





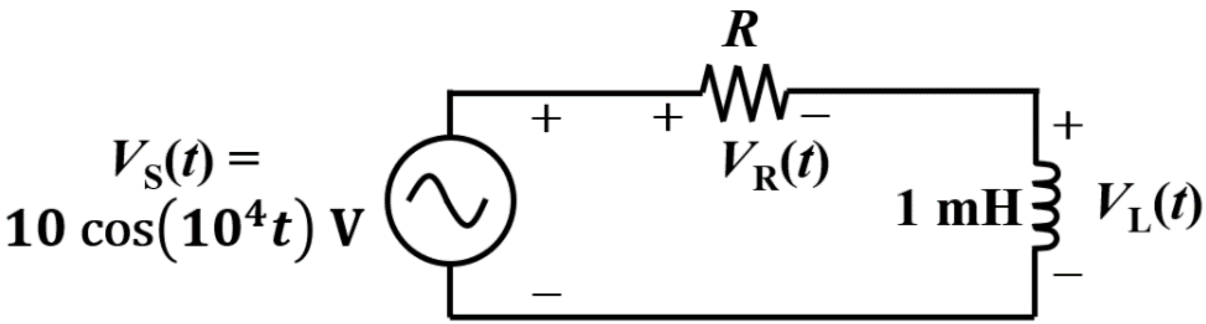
Question 1a

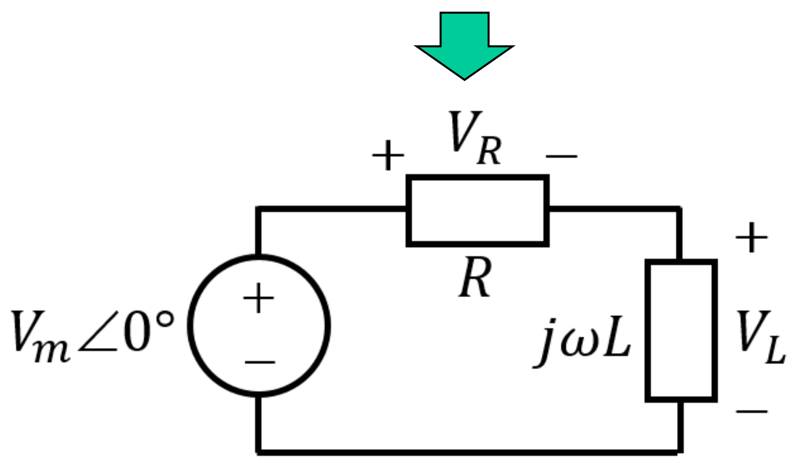
- Find the expression for $V_R(t)$



Solution to Q1a

1) Replace voltage source by phasor
2) Replace circuit components by impedances
3) Use DC circuit laws (KVL here) to solve

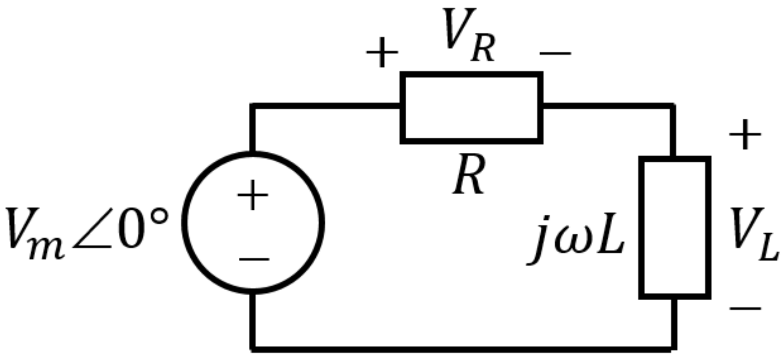




$$V_m = 10$$

$$\omega = 10^4$$

Solution to Q1a



Easier to work in phasors for multiplication and division

Applying voltage divider rule:

$$V_R = \frac{R}{R + j\omega L} V_m \angle 0° = \frac{RV_m \angle 0°}{(\sqrt{R^2 + \omega^2 L^2}) \angle \theta} = \frac{RV_m \angle -\theta}{(\sqrt{R^2 + \omega^2 L^2})}$$

where

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

Therefore,

$$V_R(t) = \frac{RV_m}{(\sqrt{R^2 + \omega^2 L^2})} \cos(\omega t - \theta)$$

Question/Solution Q1b

- What is the *R* that would cause the phase difference between $V_S(t)$ & $V_R(t)$ to be 45°?

Phasors of $V_S(t)$ & $V_R(t)$:

$$V_S = V_m \angle 0°, \qquad V_R = \frac{R V_m \angle -\theta}{\left(\sqrt{R^2 + \omega^2 L^2}\right)}$$

where

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

From $\theta = 45°$,

$$\frac{\omega L}{R} = \tan 45° = 1 \Rightarrow R = \omega L = 10^4 \times 0.001 = 10\ \Omega$$

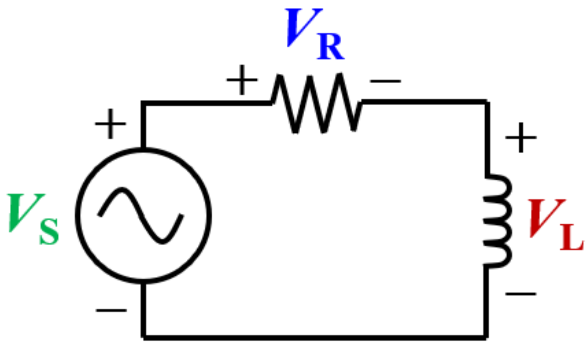
Question/Solution Q1c

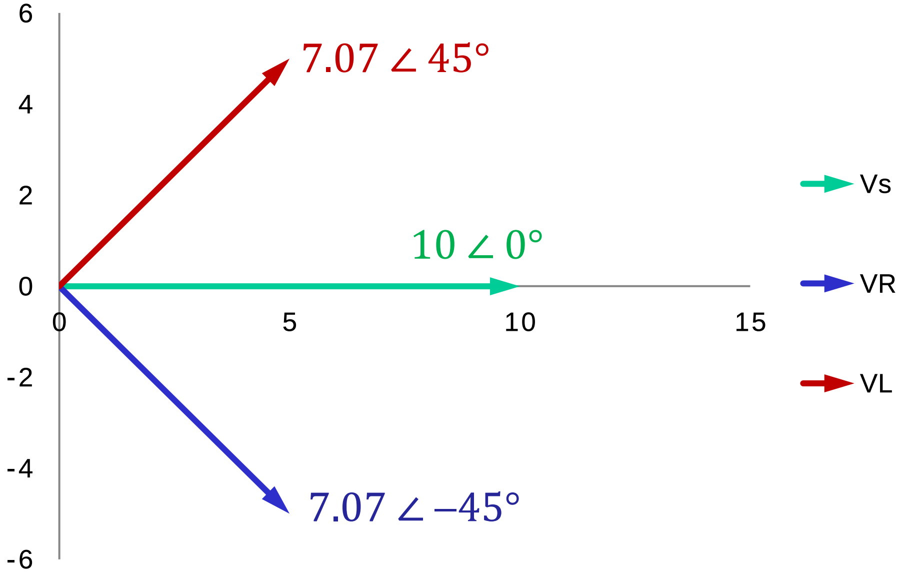
- Draw the phasor diagram for V_S, V_R & V_L

$$V_R = \frac{R \times V_m \angle 0°}{(\sqrt{R^2 + \omega^2 L^2}) \angle 45°} = \frac{10 \times 10 \angle (0° - 45°)}{(\sqrt{10^2 + 10^8 10^{-6}})} = 7.07 \angle -45°$$

$$V_L = \frac{\omega L \angle 90° \times V_m \angle 0°}{(\sqrt{R^2 + \omega^2 L^2}) \angle 45°} = \frac{10^4 \times 10^{-3} \times 10 \angle (90° - 45°)}{(\sqrt{10^2 + 10^8 10^{-6}})} = 7.07 \angle 45°$$

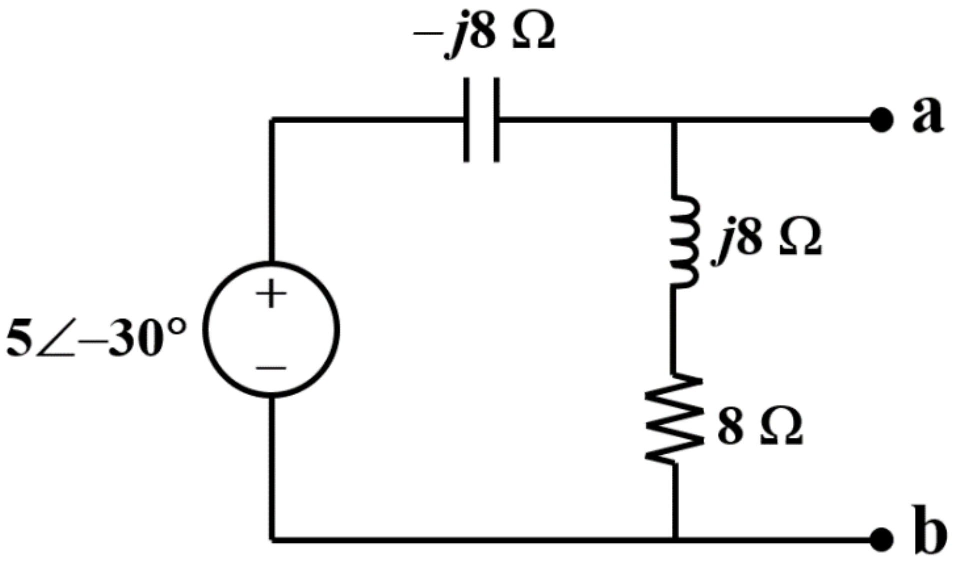
$$V_S = V_m \angle 0° = 10 \angle 0°$$



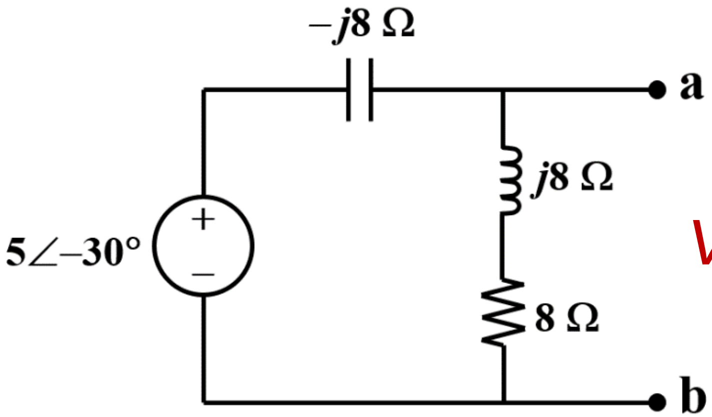


Question 2

- Find the Thevenin equivalent seen across terminals **a** and **b**:



Solution to Q2



V_{Th} = Open-circuit voltage across a & b

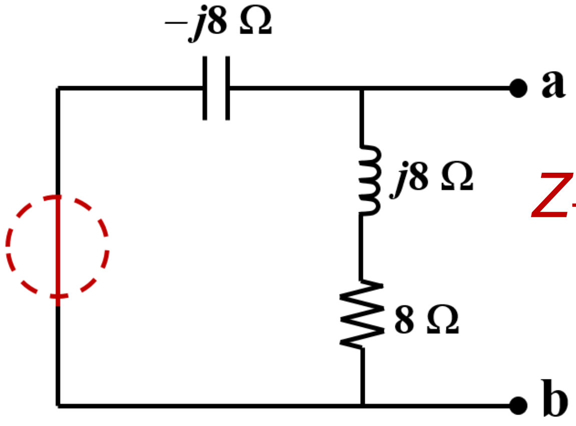
Applying voltage divider rule:

$$V_{Th} = \frac{8 + j8}{8 + j8 - j8} \times 5\angle{-30°} = (1 + j) \times 5\angle{-30°}$$

$$= \left[\sqrt{1^2 + 1^2}\angle \tan^{-1}\left(\frac{1}{1}\right)\right] \times 5\angle{-30°}$$

$$= 5\sqrt{2}\angle(45° - 30°) = 5\sqrt{2}\angle 15°$$

Solution to Q2



Z_{Th} = Impedance seen across a & b after putting voltage source to 0

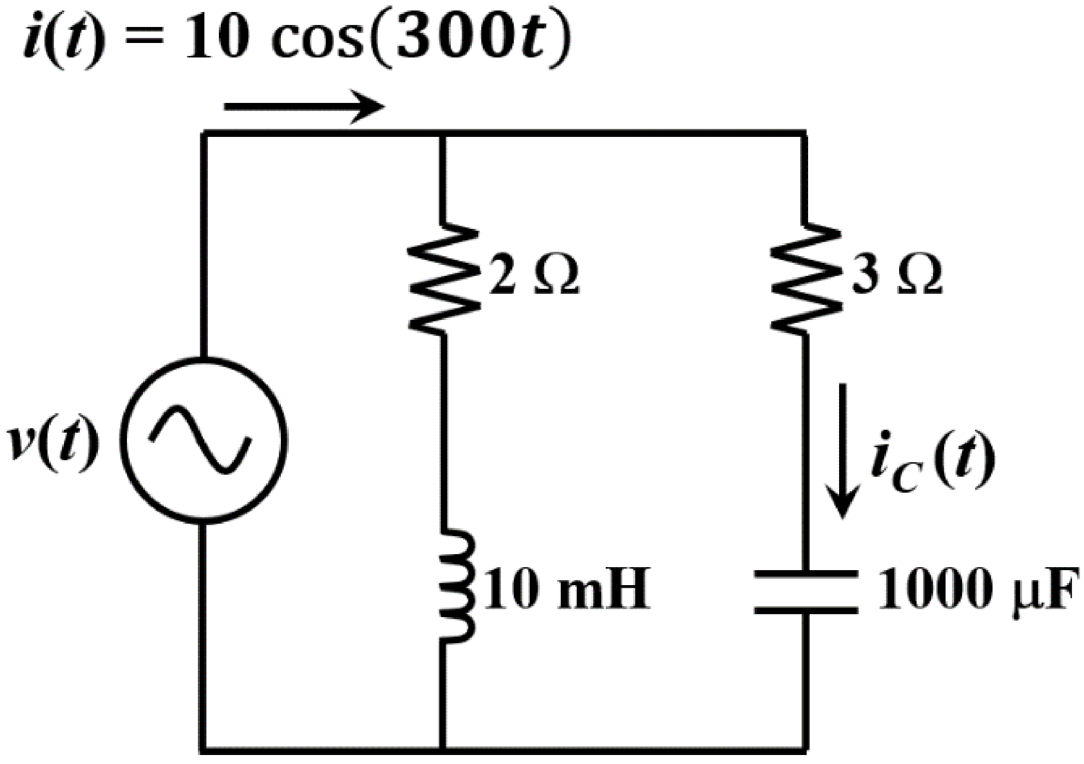
$$Z_{Th} = \frac{(-j8) \times (8 + j8)}{(-j8) + (8 + j8)} = \frac{(-j8) \times (8 + j8)}{8}$$

$$= (-j) \times (8 + j8)$$

$$= -j8 - j^2 8 = -j8 + 8\ \Omega$$

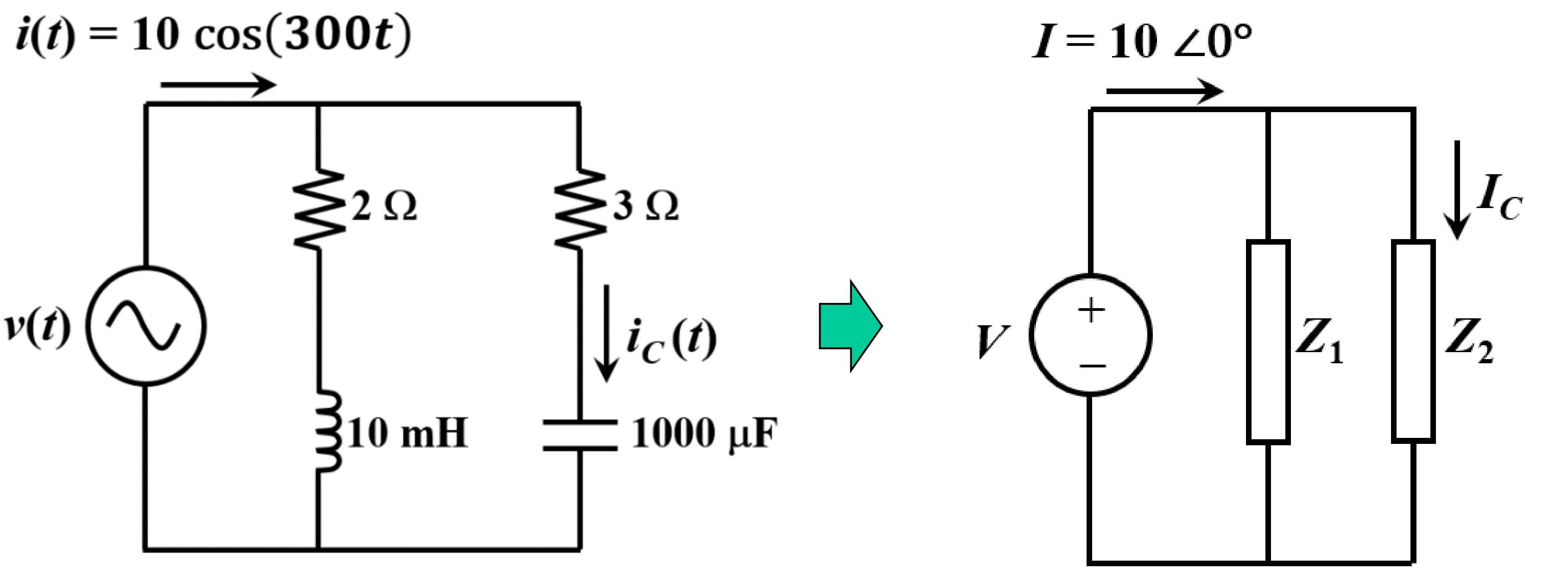
Question 3a

- Find the voltage $v(t)$

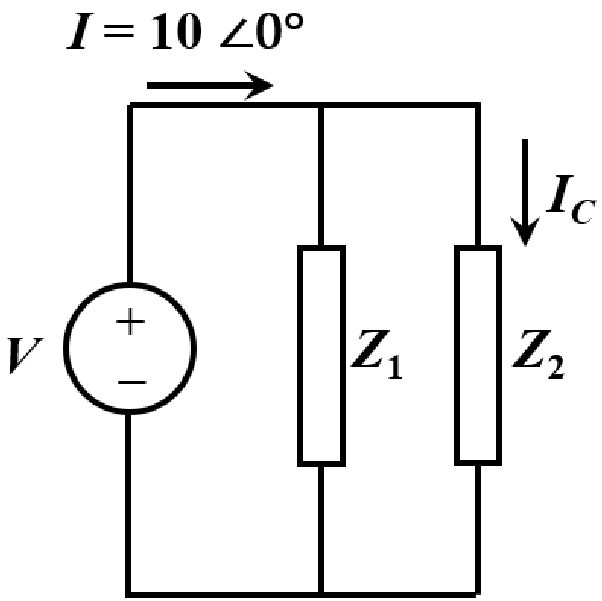


Solution to Q3a

1) Replace current by phasor
2) Replace circuit components by impedances
3) Use DC circuit laws (Ohms Law here) to solve



Solution to Q3a



$\omega = 300\ \text{rad/s}$

$Z_1 = 2 + j\omega L = 2 + j(300)(0.01)$
$= 2 + j3$

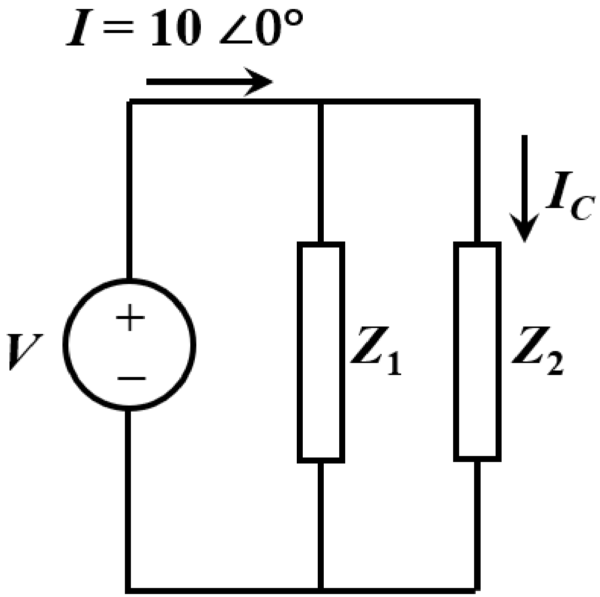
$Z_2 = 3 - \frac{j}{\omega C} = 3 - \frac{j}{300(1000 \times 10^{-6})}$
$= 3 - j\frac{10}{3}$

$Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = 3.23\ \angle 12.1°$ (parallel impedances)

$V = 3.23\ \angle 12.1° \times 10\ \angle 0° = 32.3\ \angle 12.1°$ (Ohm's Law)

$v(t) = 32.3 \cos(300t + 12.1°)\ \text{V}$

Question/Solution to Q3b



- Find the current $i_C(t)$
- Use current division principle!

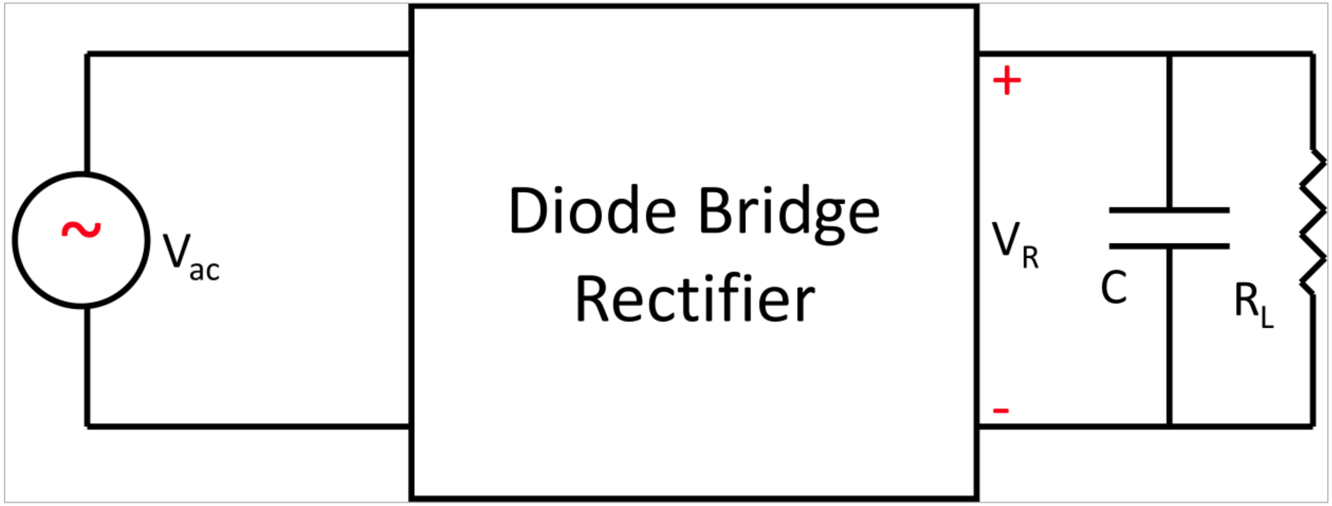
$$I_C = \frac{Z_1}{Z_1 + Z_2} \times I = \frac{(2 + j3) \times 10}{2 + j3 + 3 - j\dfrac{10}{3}} = 7.2\,\angle 60.1^\circ$$

$$i_C(t) = 7.2\cos(300t + 60.1^\circ)\ A$$

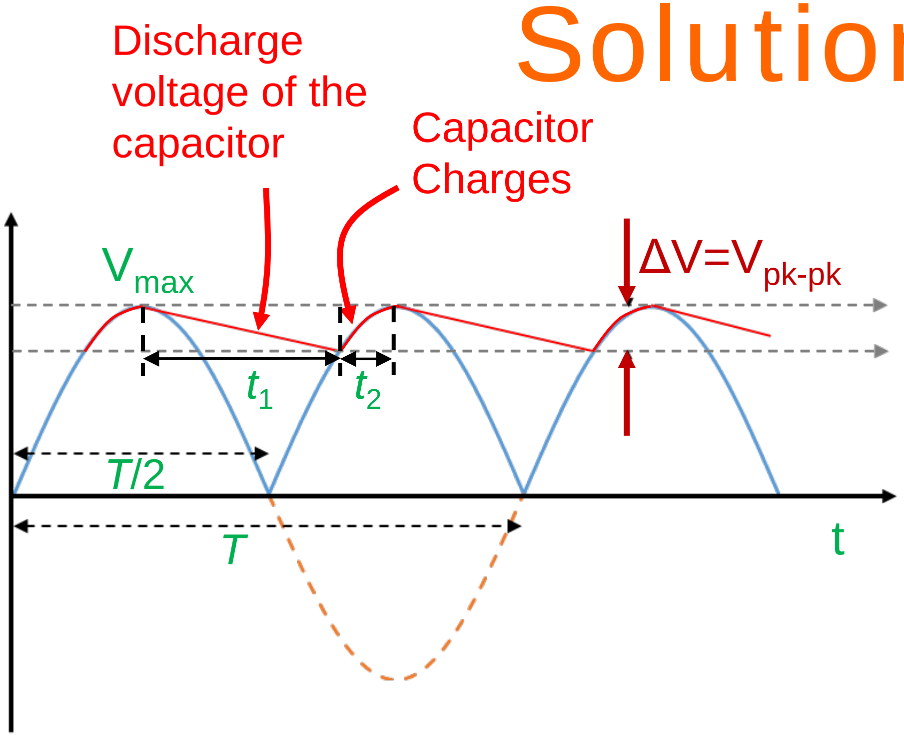
Question 4

Derive an expression for the voltage ripple at load R_L, given that:

- Average load voltage is V_{Load}
- AC power supply's frequency is f_s.



Solution to Q4



- Let V_{Load}: average load voltage
- Average load current is given by

$$I_L = \frac{V_{Load}}{R_L}$$

- Since it is a full-wave diode bridge rectifier, the pattern repeats every *T*/2

- Let ΔV be the peak-to-peak ripple voltage
- $\Delta V = \frac{\Delta Q}{C}$ (from capacitance's definition)
- For small ΔV, $t_1 \approx T/2$. Since $i(t) = \frac{dQ}{dt}$, average current $I_L \approx \frac{\Delta Q}{T/2}$.
- Hence $\Delta V \approx \frac{I_L * T/2}{C} = \frac{V_{Load}}{R_L} * \frac{1}{2f_s} * \frac{1}{C}$

Question 5

- Average current: 0.2 A
- Average voltage: 15 V
- AC source's frequency: 50 Hz
- Required peak-to-peak ripple $\Delta V \leq 0.5$ V
- Assume: Ideal diodes with no voltage drop
- Find the minimum value of the filter capacitor needed

Solution to Q5

- $\Delta V = \frac{V_{Load}}{2f_S R_L C} = \frac{I_L}{2f_S C}$

- $I_L = 0.2\ \text{A},\ f_s = 50\ \text{Hz},$ and we need $\Delta V \leq 0.5\ V$

Therefore,

$$0.5 \geq \frac{I_L}{2f_s C}$$

$$C \geq \frac{I_L}{2f_s(0.5)} = \frac{0.2}{2 * 50 * 0.5} = 4\ \text{mF}$$