CG1111 Engineering Principles and Practice I

Principles of Capacitors and Inductors

(Week 5, Studio 2) – Activity #3 for Homework

Activity #3: Understanding the transient behaviour of an RL circuit (60 mins)

Note: This activity is not graded. Note that you need your 555 timer circuit from Activity #2.

Introduction:

- As you have learnt in the e-lecture, an inductor's current cannot change instantaneously. Hence, when there is a disturbance to the circuit (e.g., the closing or opening of a switch) at time $t = 0$, the inductor current $i_L(t = 0^-)$ must be equal to $i_L(t = 0^+)$.

- If the circuit has been in steady-state long before time $t = 0$, we can use steady-state circuit analysis to find $i_L(t = 0^-)$ by assuming that an inductor is a short-circuit. Likewise, as time $t \to \infty$ after the disturbance to the circuit, we can also use steady-state circuit analysis to find $i_L(t = \infty)$ by assuming that an inductor is a short-circuit.

- For the case of a **series RL circuit**, the transient current $i_L(t)$ for time $t \geq 0$ is given by:
$$i_L(t) = i_L(0)e^{-\frac{t}{\tau}} + i_L(\infty)\left(1 - e^{-\frac{t}{\tau}}\right), \tau = \frac{L}{R}$$

- Similar to the case of RC circuits as you have seen in Activity #1, you need to translate an RL circuit into a series RL circuit before you can apply the above transient current equation. This can be achieved by finding the Thevenin equivalent circuit connected to the inductor.

- It is important to realize that practical inductors have resistances, and we can usually model a practical inductor as an ideal inductor (with inductance L) in series with a resistor (with resistance R_L as a result of the **coil's resistance**).

- Figure 1 shows a circuit consisting of a practical inductor in series with a resistor.

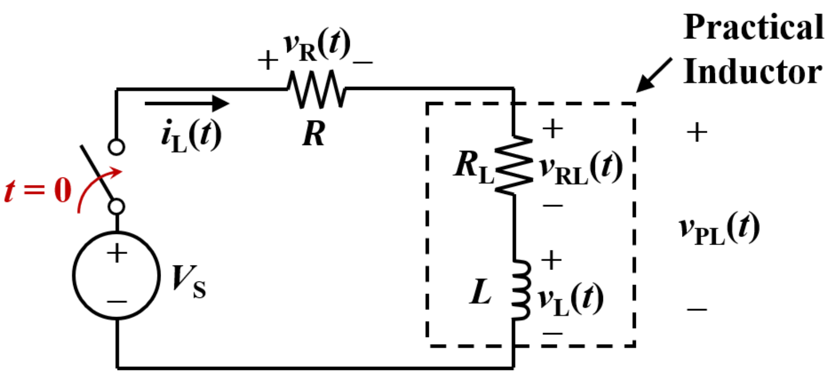


Figure 1: A circuit consisting of a practical inductor in series with a resistor.

- In this activity, you will be exploring the transient behaviour of a series RL circuit. Through the measurements taken from your experiment, you will try to estimate the inductance of the given inductor, and verify it against the inductor's nominal value.

Procedure:

1. For the circuit shown in Figure 1, what are the values of $i_L(t = 0^-)$ and $i_L(t = 0^+)$? What is the value of $i_L(t = \infty)$? Write down the expression for $i_L(t)$ for time $t \geq 0$. Pay close attention to what is the correct expression to use as the time-constant τ. (Hint: It is no longer $\frac{L}{R}$ for this circuit.)

2. When we measure the voltage across a practical inductor using an oscilloscope, we are actually measuring $v_{PL}(t)$, which includes not just the voltage across the ideal inductor $v_L(t)$, but also the voltage across the coil's resistance $v_{RL}(t)$. Keeping that in mind, write down the expression for the practical inductor's voltage $v_{PL}(t)$ for time $t \geq 0$ in terms of V_S, $i_L(t)$, and R. (Hint: Use Ohm's Law and KVL.)

3. Figure 2 below shows how the current $i_L(t)$ and the practical inductor's voltage $v_{PL}(t)$ change w.r.t. time for $t \geq 0$. Show that $v_{PL}(\tau) = V_S - R\frac{0.632\, V_S}{R+R_L}$.

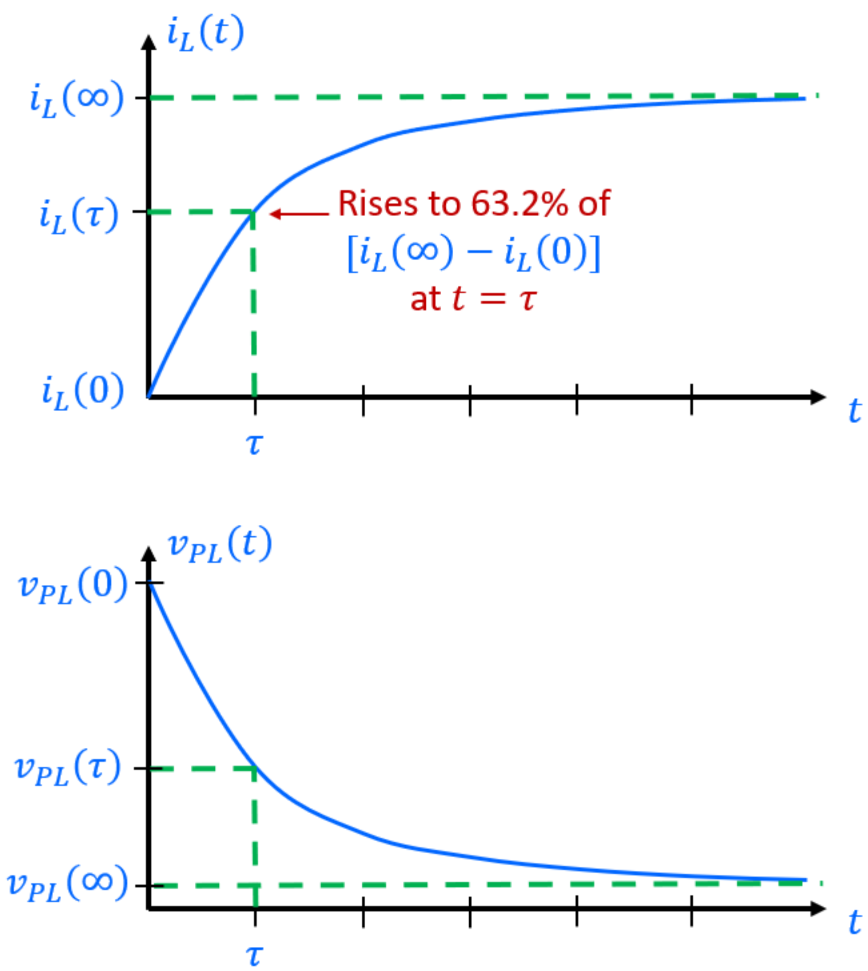


Figure 2: current $i_L(t)$ and practical inductor's voltage $v_{PL}(t)$ w.r.t. time t.

4. You will now perform experiments to verify the above behaviour. Construct the circuit shown in Figure 3 below. You will be using a 470 Ω resistor in series with a 0.82 mH practical inductor. Take some time now to measure the actual resistance of your 470 Ω resistor, as well as the resistance R_L of your inductor using the multimeter. Write down the values in your learning journal.

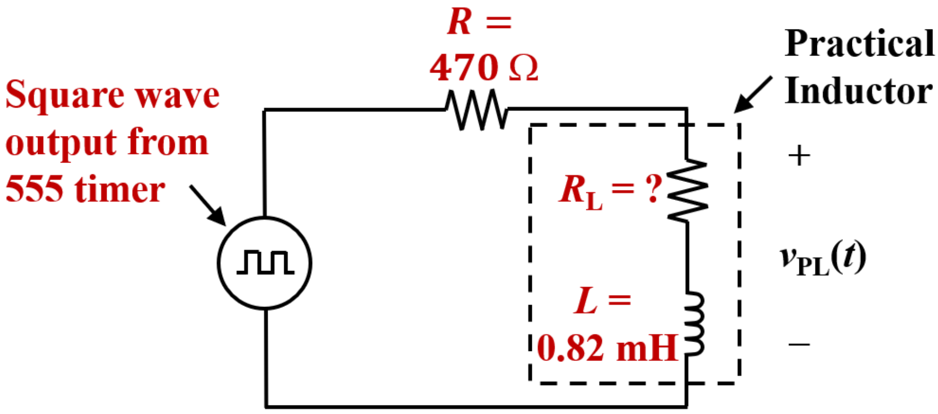


Figure 3: Experimental circuit used for verifying the transient behaviour of series RL circuit.

5. Note that in this experiment, we are using the square wave output from the 555 timer circuit in Activity #2 to emulate the connection and disconnection of a voltage source. The half-wave period of the square waveform (~7 ms) is much higher than the time constant of your RL circuit (~microseconds), so it resembles steady-state every time before the voltage level switches. The reason we are not using a physical switch for this experiment is because you will not be able to properly close a switch manually in microseconds!

6. Connect the mini-grabber of your BitScope's CHA (Channel A) to the output from the 555 timer, and the mini-grabber of CHB (Channel B) to the "+" polarity of $v_{PL}(t)$. Remember that all the grounds in the circuit must be common (the '–' terminal of your USB breakout cable, the GNDs of your BitScope probes, the '– ' polarity of your inductor's $v_{PL}(t)$, and also the grounds of your 555 timer circuit). Turn on the power to your 555 timer circuit, connect the BitScope to your computer, and run BitScope DSO. Turn on the power to BitScope.

7. Turn on Channel B in the "Channel Controls (7)" panel (see Appendix for the positions of the control panels). Change the voltage offset type of both CHA and CHB to "REF" (refer to Activity #1 Step 9 if you have forgotten how to do this). Lower the 0 V line of both channels by 2 V so that the top part of the waveforms do not get clipped off later. You can leave the **voltage per division** unchanged as **1 V/Div** for both channels.

8. In the "Trigger Controls (4)" panel, left-click on the trigger voltage level, and drag your mouse slightly to change its value to around **100 mV**. This sets the voltage level at which your BitScope will be triggered to capture the waveform.

9. In the "Timebase Control (6)" panel, set the timescale to **1 us/Div**. Turn off "REPEAT" as we will perform a single waveform capture. Next, right-click on "POST" and select 25%. This causes BitScope to buffer 25% more data in the moments before the trigger activates.

10. You are now ready to capture the rising edge of the square waveform (CHA), and the corresponding inductor's voltage $v_{PL}(t)$ (CHB). Left-click once on "TRACE" button inside the "Timebase Control (6)" panel. More than frequent, the rising edge of the square waveform may have a small spike before it stabilizes at the steady voltage level. When this happens, it does not closely match the turning on of a DC voltage source. Instead, we would like to try repeating the waveform capture until we get a good sample that does not have an obvious spike. This can be accomplished by clicking the "TRACE" button repeatedly until you are satisfied with the captured waveform (each click captures one sample). Figure 4 below shows an acceptable waveform.

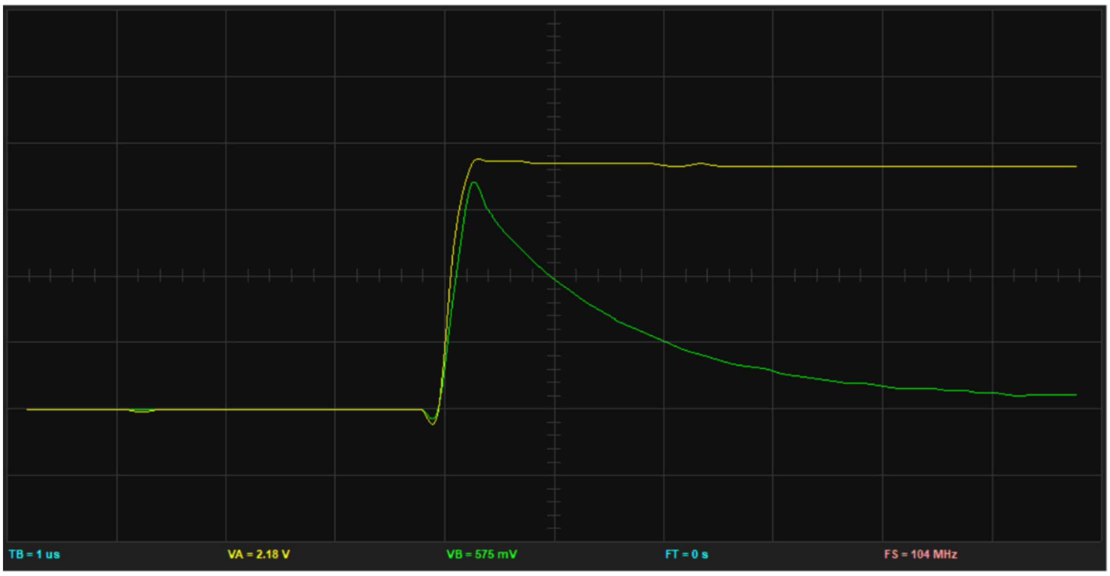


Figure 4: A good sample of the rising edge of the square wave and inductor's voltage $v_{PL}(t)$.

11. When you are satisfied with the captured waveform, turn on the cursors in the "Scope Selectors (2)" panel. Use the horizontal orange cursor to measure the amplitude of the square waveform. This is taken to be the DC voltage supply's V_S (see Figure 1). Using the expression given in Step 3, calculate $v_{PL}(\tau)$.

12. Next, drag the horizontal orange cursor to the level of the calculated $v_{PL}(\tau)$. Use the vertical cursors to measure the time constant τ (refer to the $v_{PL}(\tau)$ graph in Figure 2). You can assume that time $t = 0$ starts from the moment the yellow line shoots above 0 V. Time $t = \tau$ is the point where the green line falls back to $v_{PL}(\tau)$. Figure 5 below shows an example of how τ is measured.

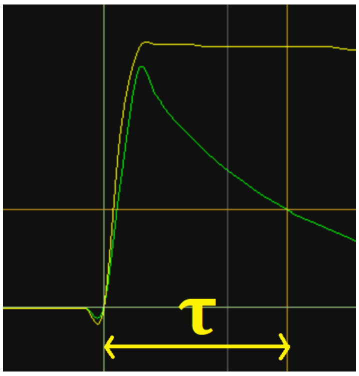
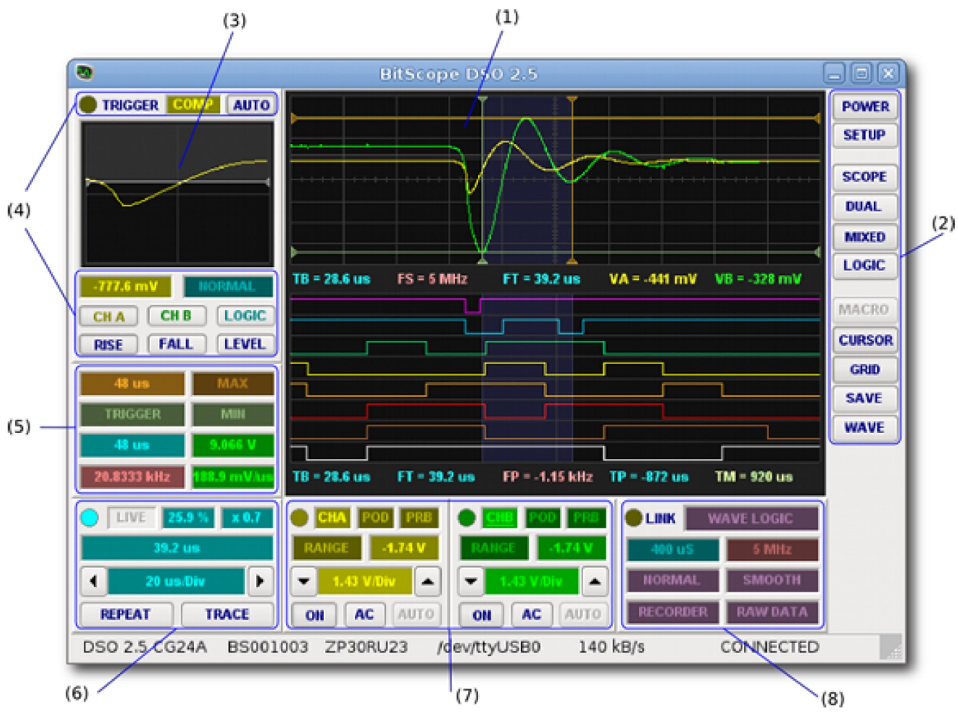


Figure 5: Example of how to measure τ.

13. Now that you have measured the value of the time constant τ, you can use the expression for τ that you have written down in Step 1 to calculate the value of inductance L. Verify this against the nominal value of 0.82 mH for the inductor you have used in this experiment. <u>Note:</u> The tolerance colour band of this inductor is silver, meaning that its tolerance is 10%.

14. Do a screen capture of your BitScope DSO showing the waveforms and the cursor measurements, and paste it into your learning journal.

APPENDIX: BITSCOPE DSO'S INTERFACE



ID	FEATURE	DESCRIPTION
(1)	Main Display	Waveform, logic and spectrum displays, measurements and cursors.
(2)	Scope Selectors	Virtual instruments, scope tools, presets, cursors, graticule etc.
(3)	Trigger Windows	Shows trigger levels, analog and logic waveforms at the trigger.
(4)	Trigger Controls	Controls trigger setup and displays trigger waveform and data.
(5)	Cursor Measurements	X and Y cursor values, voltage, time and rate measurements.
(6)	Timebase Control	Timebase, Zoom and Time Focus control parameters.
(7)	Channel Controls	Controls input source, range, vertical position and scaling.
(8)	Capture Control	Capture sample rate, duration, frame rate and display modes.