Node Analysis

Branch voltage is the potential difference across the element in a branch.
Node voltage is the potential difference between the given node and another reference node / ground node

Since node voltages are defined with respect to a common reference, choice of reference ground is important

- ↳ maximum number of circuit elements connected to it
- ↳ connects to maximum number of voltage sources
- ↳ ground node that leads to more intuitive operations in circuit.

Node Method

① Select a reference / ground node. Define its potential to be 0V.

② Label potentials of remaining nodes wrt to ground node.
Nodes connected to ground through independent / dependent voltage sources should be labelled with the voltage of the source
Remaining unknowns labelled $e_1, e_2 \cdots e_n$

③ For each unknown node voltage, write KCL for that node
Use KVL and element laws to replace current with voltage & element parameters.

1 eqn for each unknown node voltage

④ Solve simultaneous equations

⑤ Back solve for branch voltage and currents.

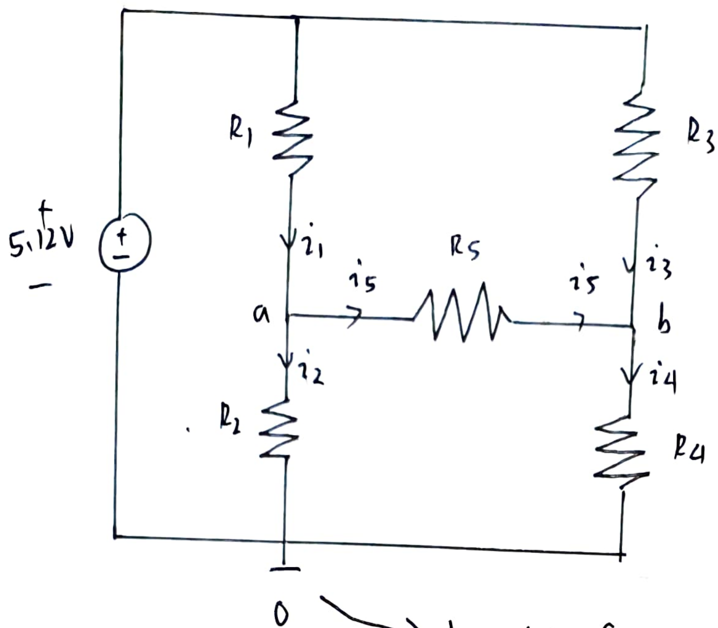
* general comments on equations produced by node method

If circuit is made of linear elements ⟹ source term enters equations as sums, not products

Gives rise to intuition for superposition for linear networks

Activity 1

5.12



best choice for ground node as it is directly connected to a voltage source and to many other elements too.

$\boxed{\text{2 unknown node voltages}}$

a, b

Measured voltage = $\boxed{5.12V}$

At node a: $i_1 = i_2 + i_5$

$$\frac{5.12 - a}{R_1} = \frac{a - 0}{R_2} + \frac{a - b}{R_5}$$

$$a\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right) - b\left(\frac{1}{R_5}\right) = \frac{5.12}{R_1} \qquad (1)$$

At node b: $i_5 + i_3 = i_4$

$$\frac{a-b}{R_5} + \frac{5.12 - b}{R_3} = \frac{b - 0}{R_4}$$

$$a\left(\frac{1}{R_5}\right) - b\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) = -\frac{5.12}{R_3} \qquad (2)$$

Matrix form

$$\underbrace{\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} & -\frac{1}{R_5} \\ \frac{1}{R_5} & -\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) \end{bmatrix}}_{\text{conductance matrix}} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{\text{col vector of unknown potential}} = \underbrace{\begin{bmatrix} \frac{5.12}{R_1} \\ -\frac{5.12}{R_3} \end{bmatrix}}_{\text{source matrix}}$$

$$\boxed{\bar{G}\bar{e} = \bar{S}\bar{s}}$$

Resistor	Measured Resistance (Ω)	calculated voltage (V)	calculated current (mA)	measured voltage (V)	measured current (mA
R_1	546	2.74	5.02	2.72	4.88
R_2	545	2.38	4.37	2.38	4.28
R_3	543	3.05	5.61	3.07	5.53
R_4	324	2.07	6.38	2.03	6.22
R_5	548	0.31	0.57	0.34	0.61

By solving simultaneous eqn
OR
multiplying both sides of matrix eqn by G^{-1}

$$\left.\right\} \quad a = 2.38V \qquad b = 2.07V$$

voltage across R_1 = $5.12V - a$
= $2.74V$

voltage across R_2 = a

voltage across R_3 = $5.12V - b$

voltage across R_4 = b

voltage across R_5 = $a - b$

current across $R_1 = \frac{5.12 - a}{R_1} = 5.02\ mA$

current across $R_2 = \frac{a}{R_2} = 4.37\ mA$

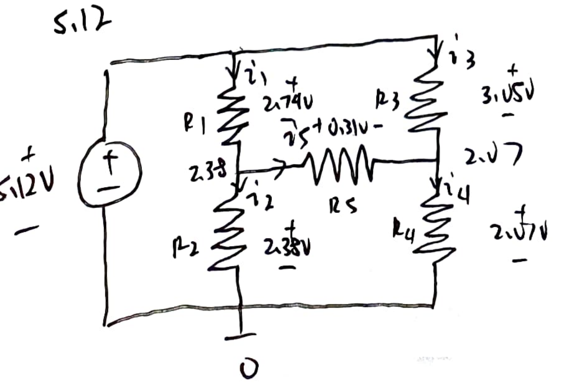
current across $R_3 = \frac{5.12 - b}{R_3} = 5.61\ mA$

current across $R_4 = \frac{b}{R_4} = 6.38\ mA$

current across $R_5 = \frac{a-b}{R_5} = 0.57\ mA$

All current and voltage have positive polarity, meaning original current flow & polarity were correct, or that I have super good intuition

5.12

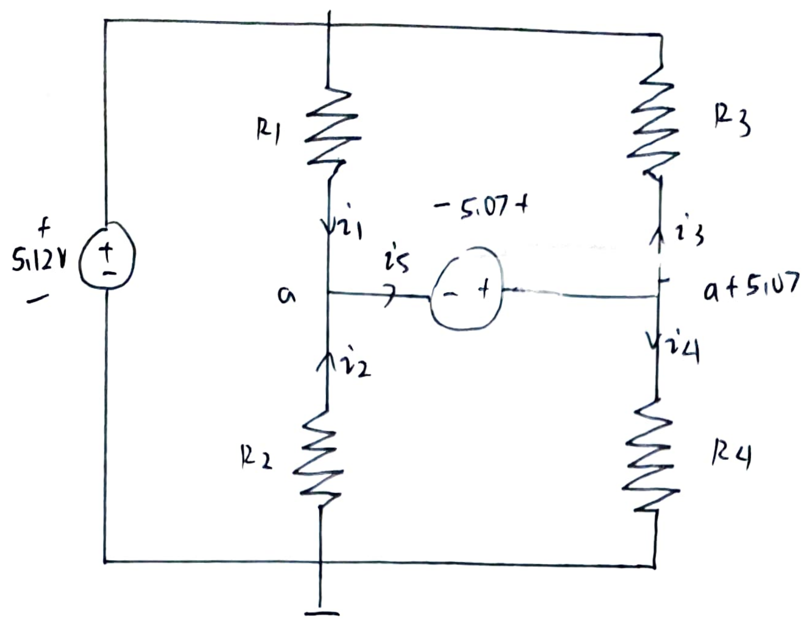


$i_1 = 5.02\ mA$
$i_2 = 4.37\ mA$
$i_3 = 5.61\ mA$
$i_4 = 6.38\ mA$
$i_5 = 0.57\ mA$

Measured readings follow closely with calculated values in table

Activity 2

5.12



only 1 unknown voltage a

$V_{laptop} = 5.12V$

$V_{arduino} = 5.07V$

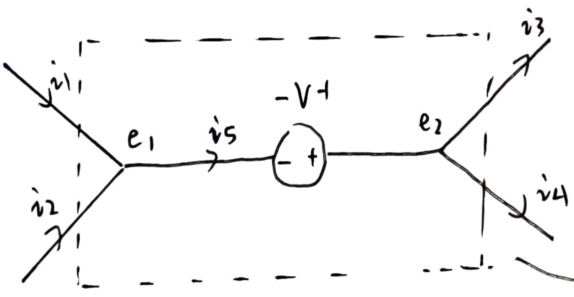
0 → still best choice for ground node

Floating independent voltage source

A floating independent voltage source is a voltage source that has neither terminal connected to ground, directly or indirectly through another independent voltage source

Node analysis fails as element laws for voltage sources does not relate branch current to branch voltage

⇒ unable to apply KCL to nodes of interest.



To derive KCL for nodes, draw a surface around floating source and nodes of interest → supernode

KCL for supernode : $\boxed{i_1 + i_2 = i_3 + i_4}$

$i_1 + i_2 = i_s$

$i_s = i_3 + i_4$

} eliminate i_s

current into = current coming out

node voltages are also labelled in terms of 1 of the nodes within super node

$$\boxed{e_2 = e_1 + V}$$

Evaluating supernode : $i_1' + i_2' = i_3' + i_4'$

$$\frac{5.12 - a}{R_1} + \frac{0 - a}{R_2} = \frac{a + 5.07 - 5.12}{R_3} + \frac{a + 5.07 - 0}{R_4}$$

$$a\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{5.12}{R_1} + \frac{0.05}{R_3} - \frac{5.07}{R_4}$$

By substituting in values of R , $\boxed{a = -0.71\,V}$

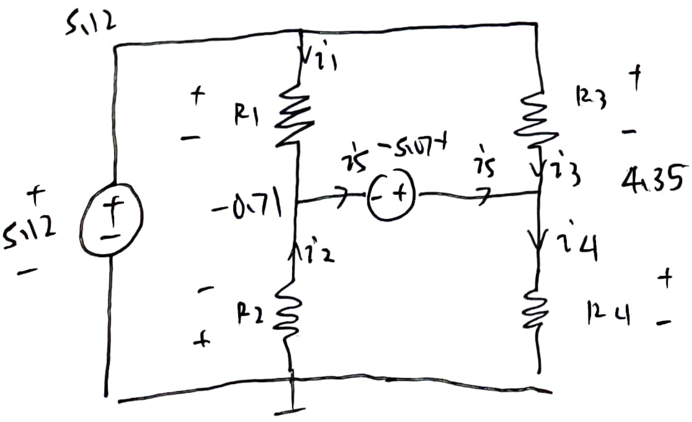
Element	Measured resistance(Ω)	calculated voltage (V)	calculated current (mA)	measured voltage (V)	measured current (mA
R_1	546	5.34	10.7	5.82	10.5
R_2	545	-0.71	1.32	-0.71	1.31
R_3	543	0.77	-1.42	0.75	-1.41
R_4	324	4.35	13.4	4.34	13.2
5.07V Arduino	–	–	12.0	–	11.8

~ voltage and current calculated in same way as Activity 1 ~

for Arduino source, current calculated using KCL

$$i_1 + i_2' = \boxed{i_5'} = i_3' + i_4' \quad (\text{supernode})$$

only i_3 current should be reversed



$i_1 = 10.7\,mA$

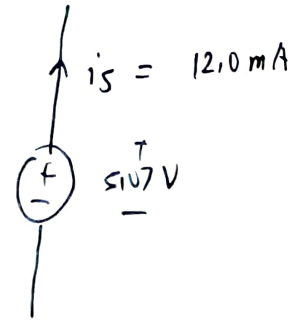
$i_2 = 1.32\,mA$

$i_3 = 1.42\,mA$ (direction corrected)

$i_4 = 13.4\,mA$

$i_5' = 12.0\,mA$

This time, measured readings follow $\boxed{\text{super}}$ closely to calculated values,



Arduino source

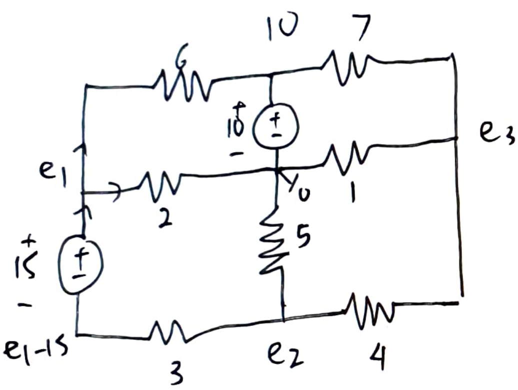
As voltage rises in the same direction of current

it supplies power that delivers current

(active element)

$P = VI$ is positive

Challenge



ground node in the centre

$$e_1: \quad \frac{e_2 - e_1 + 15}{3} = \frac{e_1}{2} + \frac{e_1 - 10}{6}$$

$$e_1 = \frac{e_2}{3} + \frac{20}{3} \qquad (1)$$

$$e_2: \quad \frac{e_1 - 15 - e_2}{3} = \frac{e_2}{5} + \frac{e_2 - e_3}{4}$$

$$\frac{47}{60} e_2 = \frac{e_1}{3} + \frac{e_3}{4} - 5 \qquad (2)$$

$$e_3: \quad \frac{10 - e_3}{7} = \frac{e_3}{1} + \frac{e_3 - e_2}{4}$$

$$\frac{39}{28} e_3 = \frac{e_2}{4} + \frac{10}{7} \qquad (3)$$

solving

$$e_1 = \frac{1955}{367}$$

$$e_2 = -\frac{1475}{367}$$

$$e_3 = \frac{335}{1101}$$