CG1111 - Solution for Additional Practice Questions on DC Motors

1) $K_t = 15 \text{ mNm/A}$

$R = 2.7\Omega$

$V_m = 12V$

(a) Maximum speed possible = no-load speed

Assuming zero friction, $I_{no\text{-}load} = 0 A$

Since $V_m = RI + E_b$,

$\therefore V_m = E_b$ at no-load

$\therefore V_m = E_b = K_b \omega = K_t \omega$

$$\Rightarrow \omega = \frac{V_m}{K_t} = \frac{12}{15 \times 10^{-3}} = \boxed{800 \text{ rad/s}}$$

(b) Maximum torque occurs at zero speed.

At $\omega = 0$, Back emf $E_b = 0$

$\therefore V_m = R I_{max}$

$$\therefore \tau_{max} = K_t I_{max} = K_t \left(\frac{V_m}{R}\right) = \frac{15 \times 10^{-3} \times 12}{2.7}$$

$$= \boxed{66.7 \text{ mNm}}$$

(c) $P_{mech} = \tau\omega$, $\tau = K_t I$, $I = \frac{V_m}{R} - \frac{K\omega}{R}$

$$\Rightarrow P_{mech} = \frac{K_t V_m \omega}{R} - \frac{K_t^2 \omega^2}{R}$$

$$\therefore \frac{dP_{mech}}{d\omega} = \frac{K_t V_m}{R} - \frac{2K_t^2 \omega}{R}$$

When $\frac{dP_{mech}}{d\omega} = 0$, $V_m = 2K_t\omega \Rightarrow \omega = \frac{V_m}{2K_t}$

$$\therefore \text{ Max } P_{mech} = \frac{1}{R}\left[K_t V_m\left(\frac{V_m}{2K_t}\right) - K_t^2\left(\frac{V_m}{2K_t}\right)^2\right]$$

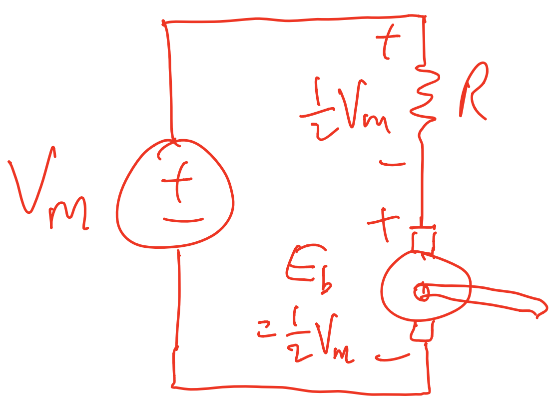
$$= \frac{1}{R}\left[\frac{V_m^2}{2} - \frac{V_m^2}{4}\right]$$

$$= \frac{V_m^2}{4R} = \frac{12^2}{4\times 2.7} = \boxed{13.3W}$$

Note: When max P_{mech} is produced,

$$\omega = \frac{V_m}{2K_t} \Rightarrow E_b = \frac{V_m}{2}$$

i.e., when Back emf $= \frac{1}{2}V_m$



This is similar to the maximum power transfer's condition for Thevenin equivalent circuit, where $\frac{1}{2}$ the supply voltage is across the Thevenin resistance, while the other half is across the load.

2) $V_m = 15\ V$

$N_{no-load} = 3750\ RPM \Rightarrow \omega_{no-load} = 392.7\ rad/s$

$I_{stall} = 0.75\ A$

(a) At no load, $V_m = E_b = K_t\, \omega_{no-load}$

$$\therefore K_t = \frac{V_m}{\omega_{no-load}} = \frac{15}{392.7} = \boxed{38.2\ mNm/A}$$

$$V_m = R\, I_{stall}$$

$$\Rightarrow R = \frac{V_m}{I_{stall}} = \frac{15}{0.75} = \boxed{20\ \Omega}$$

(b) With unknown load, $N_{load} = 2500\ RPM$

$$\Rightarrow \omega_{load} = 261.8\ rad/s$$

$$E_b = K_b\, \omega_{load} = K_t\, \omega_{load} = 10\ V$$

$$I = \frac{V_m - E_b}{R} = \frac{15-10}{20} = 0.25\ A$$

$$\tau = K_t I = \boxed{9.55\ mNm}$$

$$P_{load} = \tau\, \omega_{load} = \boxed{2.5\ W}$$

3) $V_m = 12\,V$, $\tau_{2500} = 10\ mNm$

$\tau_{1200} = 20\ mNm$

$2500\,RPM \Rightarrow 261.8\ rad/s$

$1200\,RPM \Rightarrow 125.7\ rad/s$

$\tau = KI$, $E_b = K\omega = V_m - IR$

$$\Rightarrow \omega = \frac{V_m}{K} - \frac{IR}{K}$$

$$= \frac{V_m}{K} - \frac{R\tau}{K^2}$$

(a)

$$261.8 = \frac{12}{K} - \frac{R(10\times10^{-3})}{K^2} \quad —(1)$$

$$125.7 = \frac{12}{K} - \frac{R(20\times10^{-3})}{K^2} \quad —(2)$$

(1) — (2);

$$\frac{R}{K^2}(10\times10^{-3}) = 136.07$$

$$\Rightarrow \frac{R}{K^2} = 13607 \quad —(3)$$

Sub (3) into (1), $K = \boxed{30.2\ mNm/A}$

(b) From ③, we get $R = \boxed{12.4\,\Omega}$

(c) When stalled, $V_m = IR$

$$\therefore I_{stall} = \frac{V_m}{R} = \boxed{0.968A}$$

$$\therefore \tau_{stall} = K I_{stall}$$

$$= \boxed{29.2 \text{ mNm/A}}$$

(d) At no load, $V_m = E_b = K\omega$

$$\therefore \omega_{no-load} = \frac{V_m}{K} = \boxed{397.4 \text{ rad/s}}$$

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4) $V_m = 24V$, $K = 60 \text{ mNm/A}$, $R = 50\,\Omega$

$N = 2500 \text{ RPM} \Rightarrow \omega = 261.8 \text{ rad/s}$

(a) $$I = \frac{V - K\omega}{R} = 0.166 \text{ A}$$

$$\therefore \tau = KI = \boxed{9.95 \text{ mNm}}$$

(b) At 60% duty cycle,

$$V_{60\%} = 0.6 \times 24 = 14.4\,V$$

Same load $\Rightarrow$ same torque $\Rightarrow$ same current

$$\omega = \frac{V - IR}{K}$$

$$= \boxed{101.8 \text{ rad/s}}$$