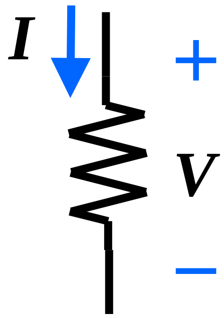
CG1111:
Engineering Principles and Practice I

Debrief and Tutorial for Week 4

NUS
National University of Singapore

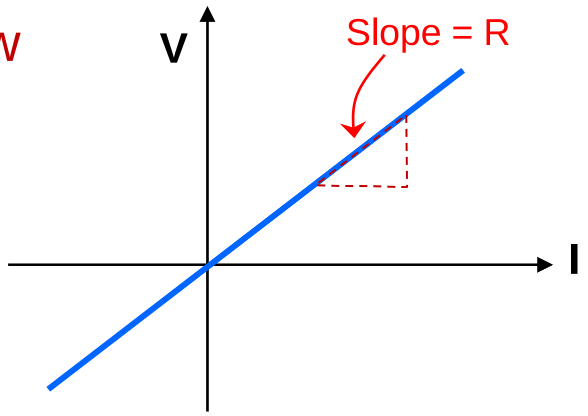
How to Label V & I for Resistors?

- A resistor consumes power:
  - Principle: In a resistor, the voltage (potential) must drop in the direction of the current



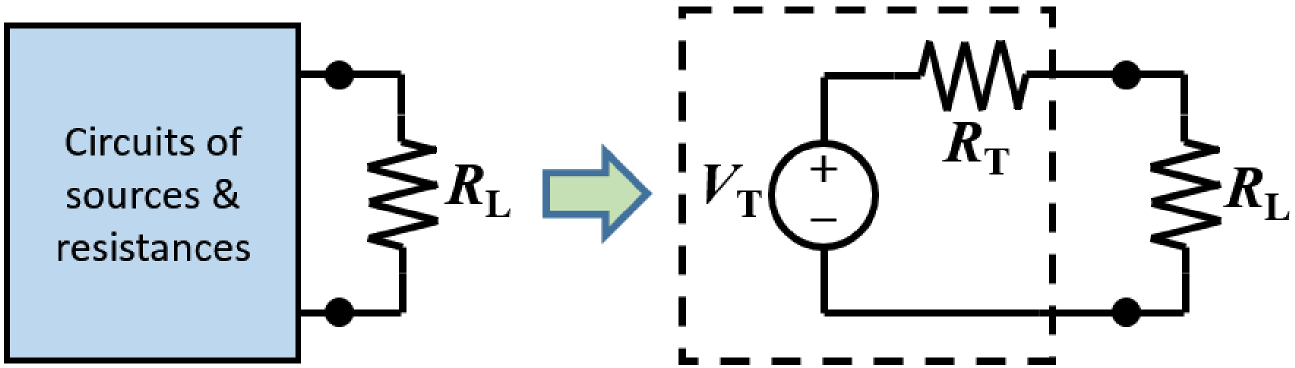
If the current's true direction or the voltage's true polarity is not obvious in the circuit, just pick & label either the current or the voltage, and then label the other one by following the above principle

You can only apply Ohm's Law if you label them this way!



Thevenin Equivalent Circuit

- Any circuit consisting of sources and resistances can be replaced by a Thevenin equivalent circuit

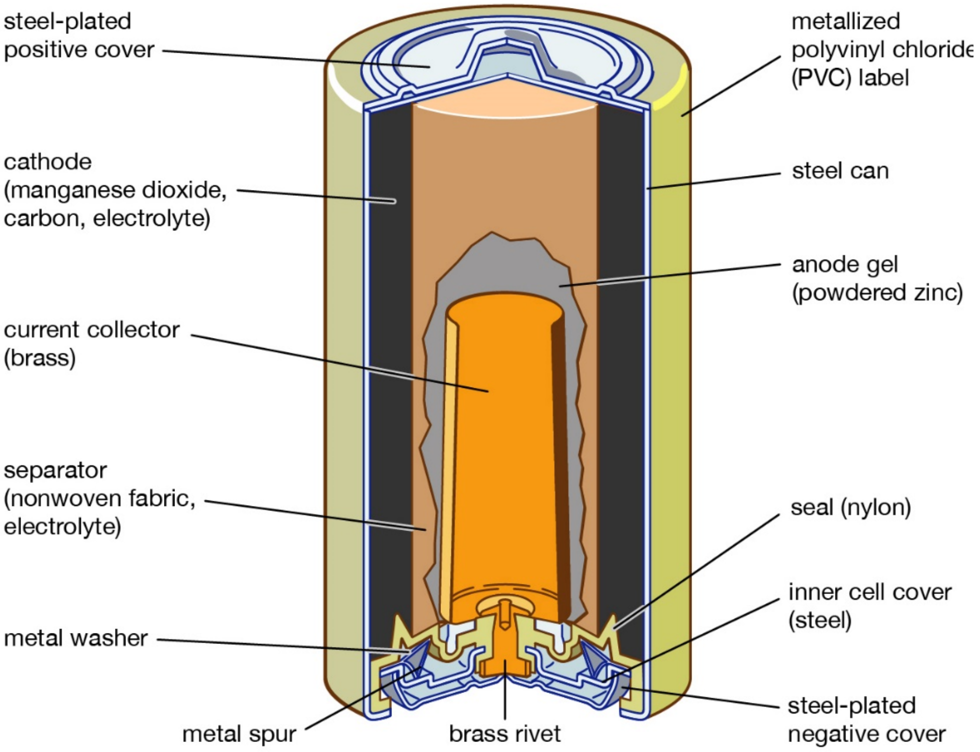


Thevenin equivalent circuit

The I-V behaviour of the Thevenin equivalent circuit is the same as that of the complex circuit
→ Very useful for circuit analysis!

Battery's Equivalent Circuit

It is just a model!!



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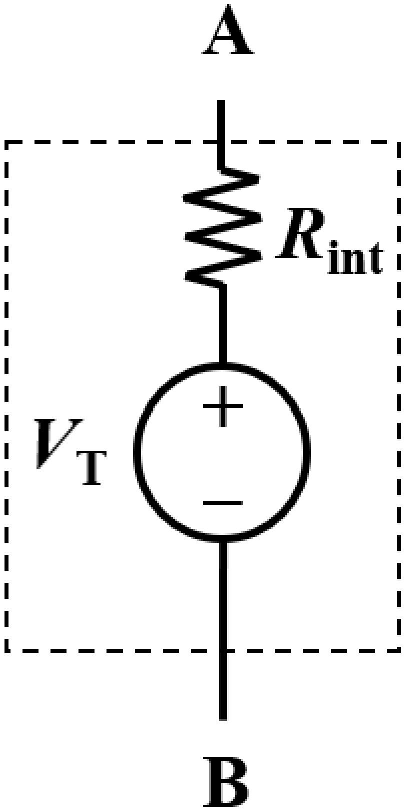
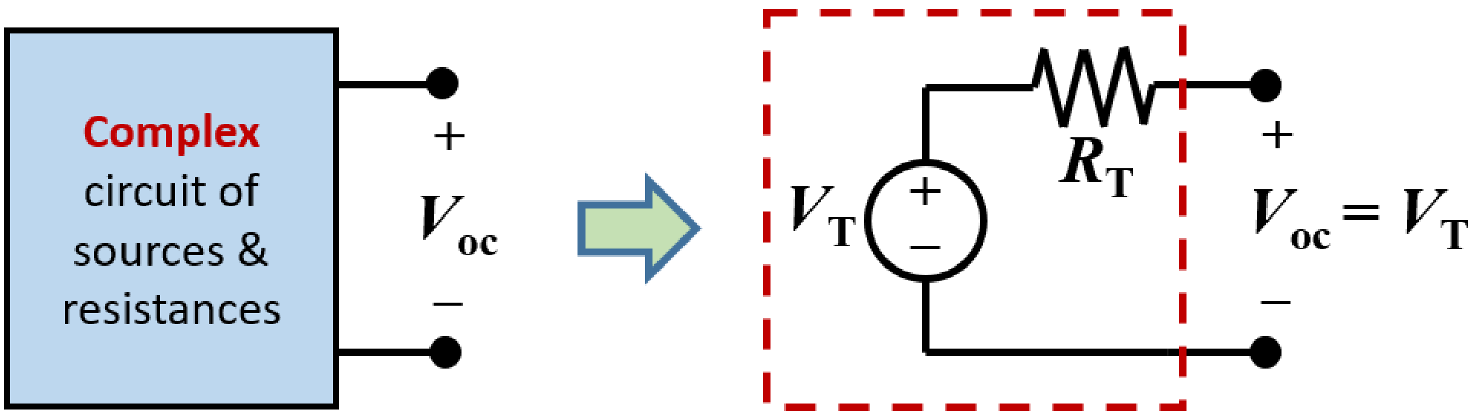


- Battery's equivalent circuit is actually a Thevenin equivalent circuit!

Finding Thevenin Voltage

V_T can be obtained by finding V_{oc} across the 2 terminals of the complex circuit

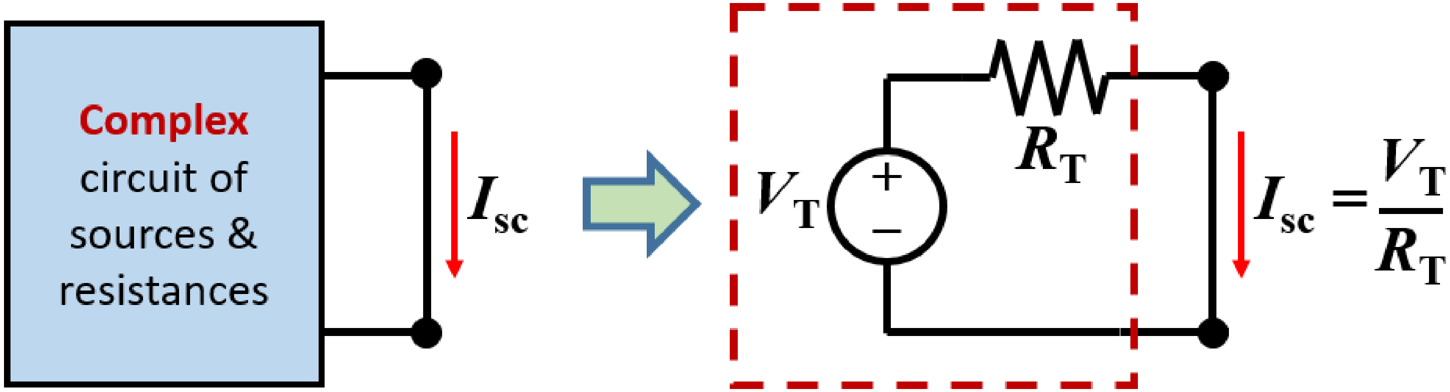


Thevenin equivalent circuit

Finding Thevenin Resistance

R_T can be obtained using Ohm's Law after finding I_{sc} across the 2 terminals of the complex circuit

$$R_T = \frac{V_T}{I_{sc}}$$

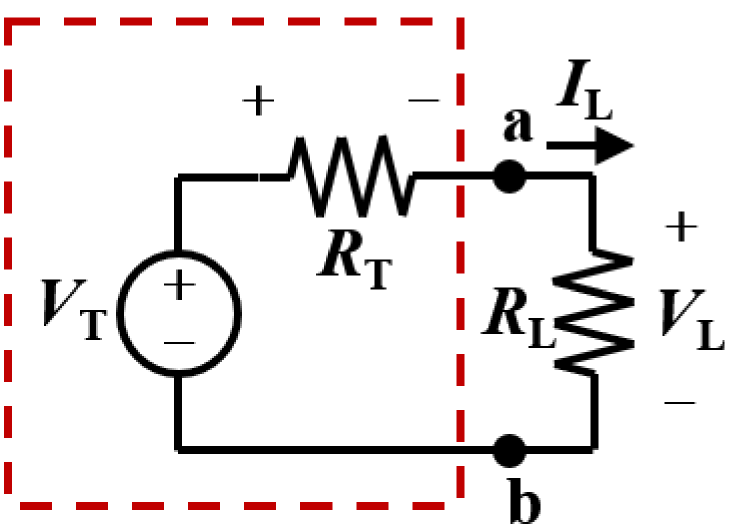


Thevenin equivalent circuit

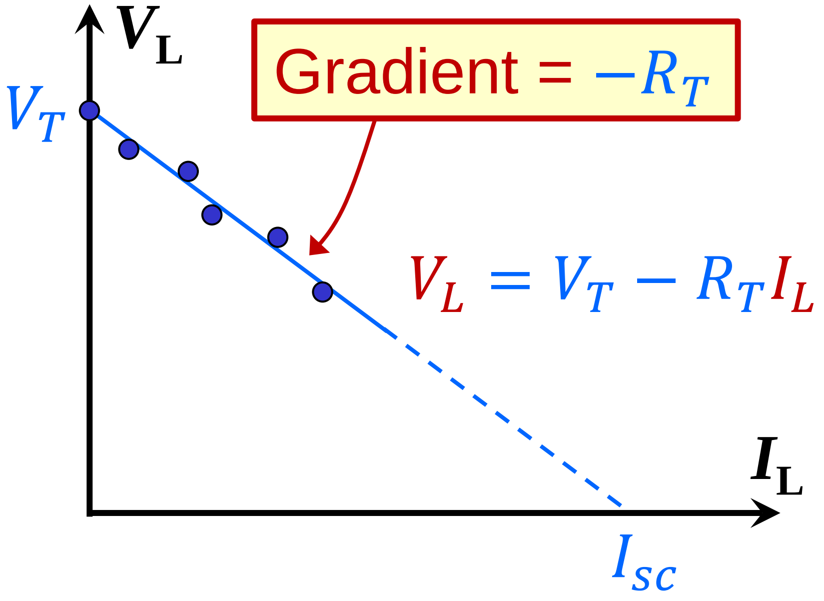
Experimental Method to Find R_T

- There is a risk that I_{sc} may be large

It is safer to connect the circuit to multiple resistive loads and then use graphical approach



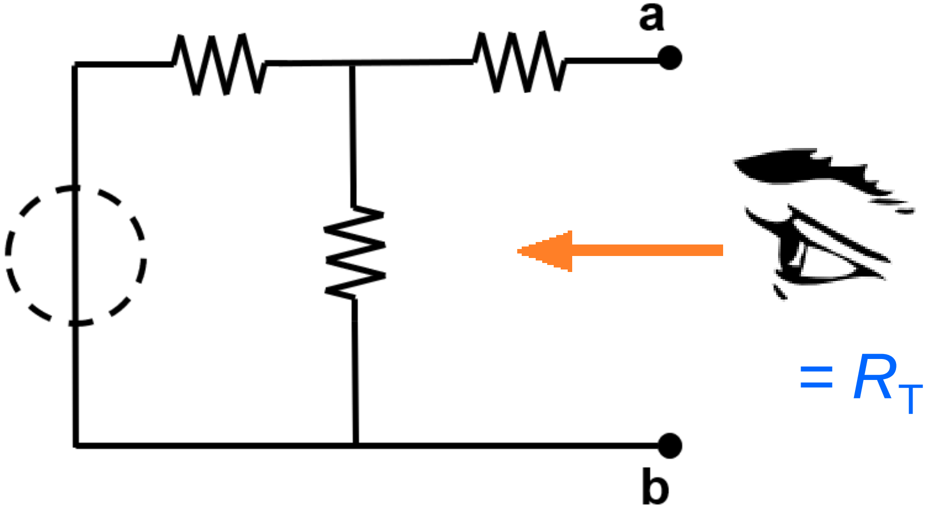
Thevenin equivalent circuit



Another Analytical Method to Find R_T (Usually Easiest!)

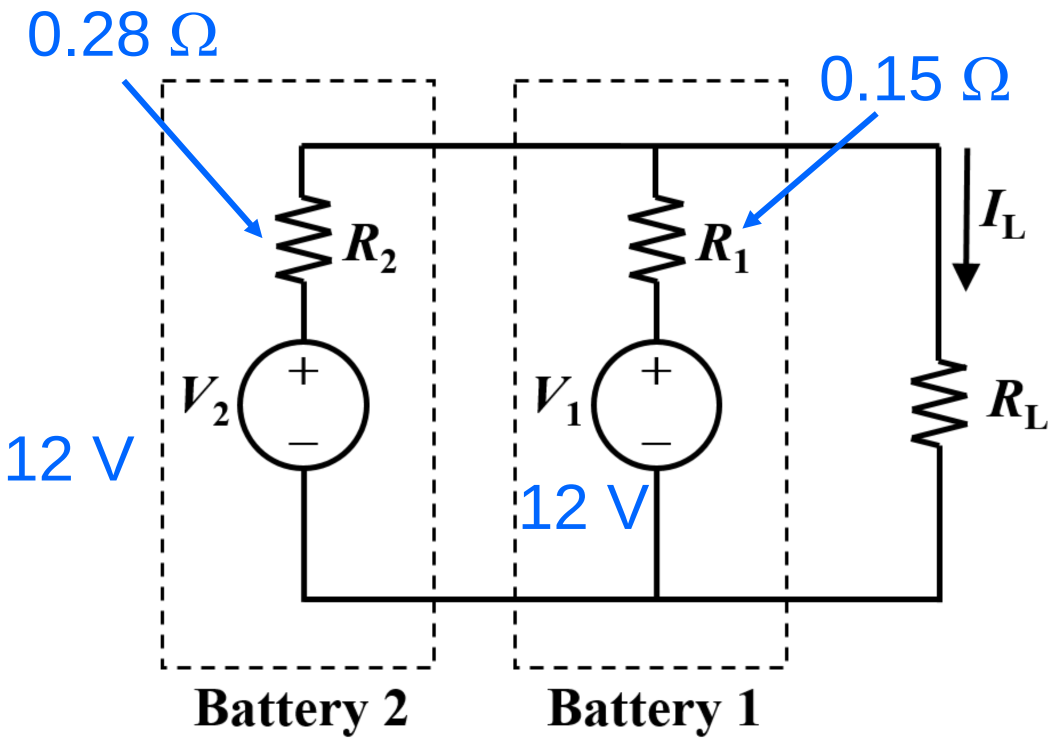
- If the original electrical circuit consists of independent voltage sources only,
  - Replace all voltage sources with a short-circuit (0 V); this results in a purely resistive network

Then, find the equivalent resistance between the two terminals

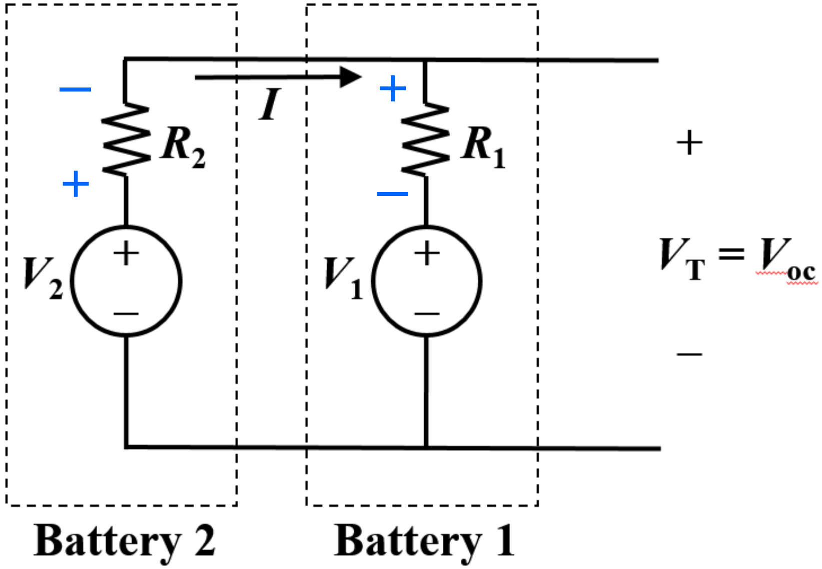


Question 1

(i) Find Thevenin equivalent circuit seen by R_L



Solution to Q1(i)



- Using KVL in the loop without load:

$$V_2 = R_2\, I + R_1\, I + V_1$$

$$12 = (0.28 + 0.15)I + 12$$

$$I = 0$$

- Therefore, $V_T = 12$ V

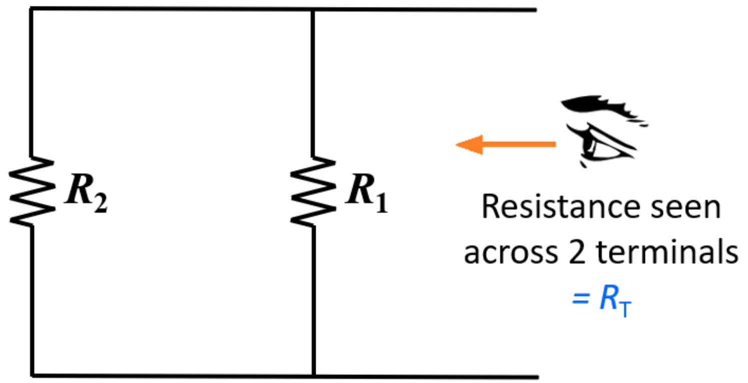
Solution to Q1(i)

To find R_T, put V_1 and V_2 to 0, and calculate resistance as seen across the two terminals:

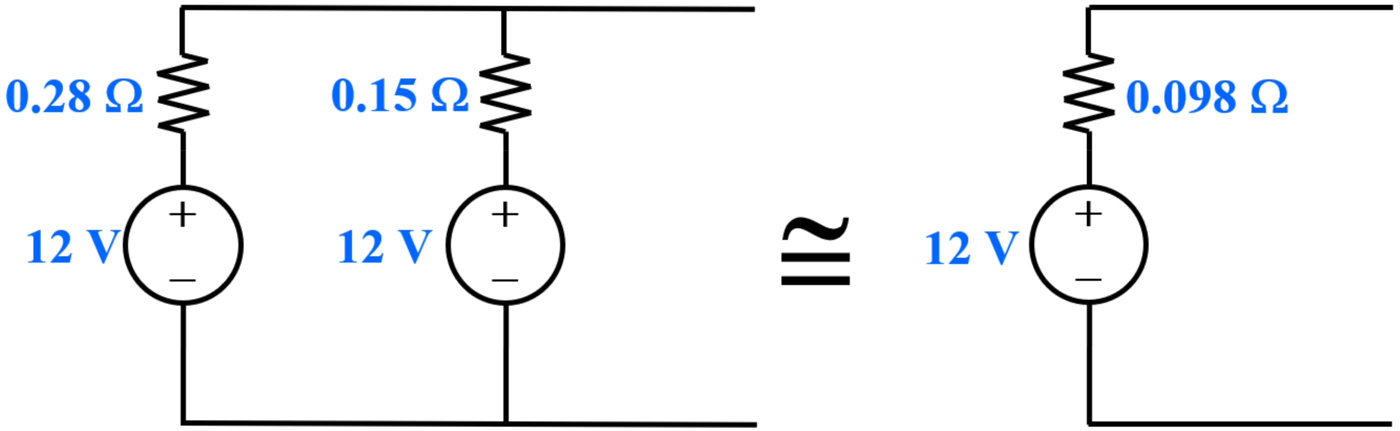
R_T $= R_1 \,//\, R_2$

$= 0.15 \,//\, 0.28$

$= 0.098\ \Omega$



Hence,

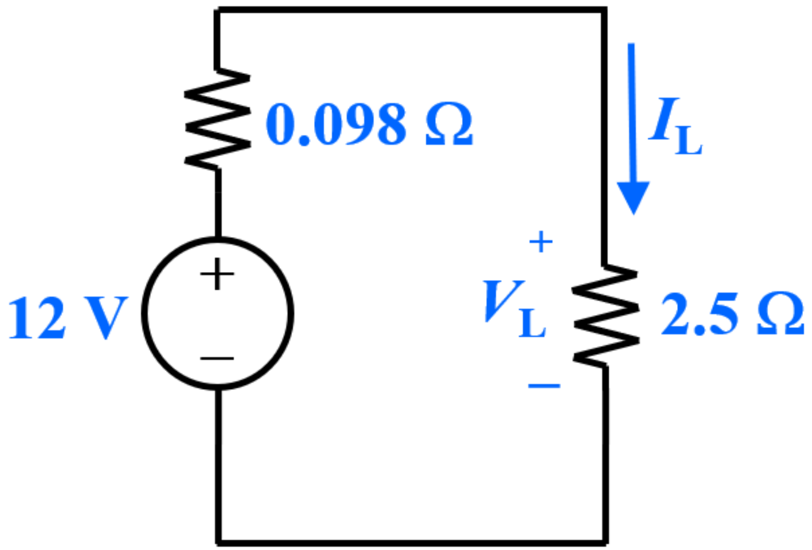


Solution to Q1(ii)

If load R_L = 2.5 Ω, find I_L and V_L

$$I_L = \frac{12}{0.098 + 2.5} = 4.62\ \text{A}$$

$$V_L = \frac{2.5}{0.098 + 2.5} \times 12 = 11.5\ \text{V}$$



Solution to Q1(iii)

- Is load voltage V_L higher or lower compared to using just one of the two batteries alone?

- Using battery 1 alone:

$$V_L = \frac{2.5}{0.15 + 2.5} \times 12 = 11.3\ \mathrm{V}$$

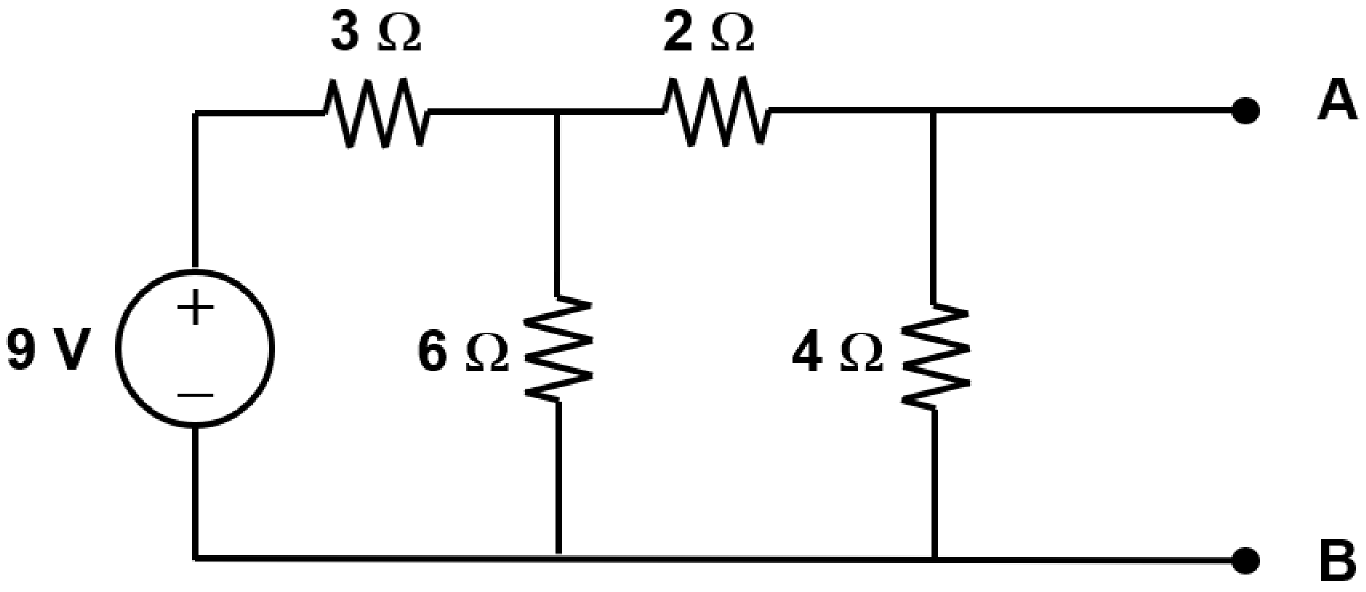
- Using battery 2 alone:

$$V_L = \frac{2.5}{0.28 + 2.5} \times 12 = 10.8\ \mathrm{V}$$

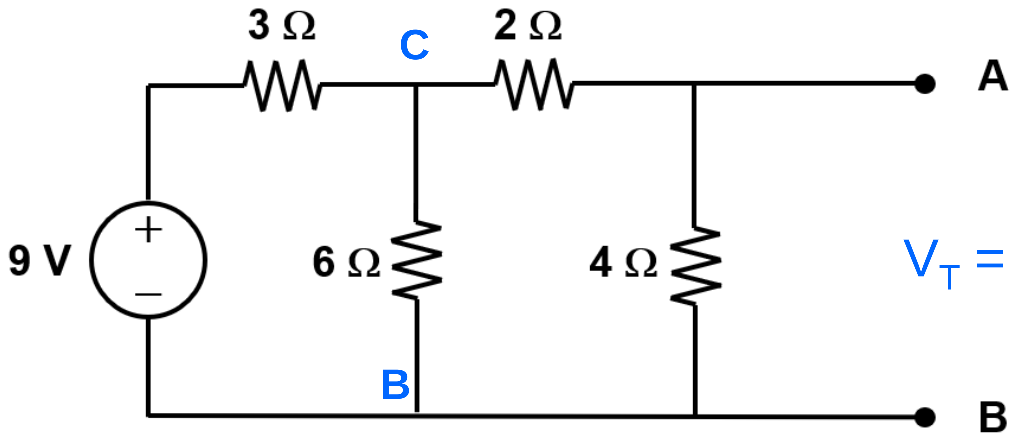
- Load voltage V_L is higher compared to using any one of the two batteries alone

Question 2

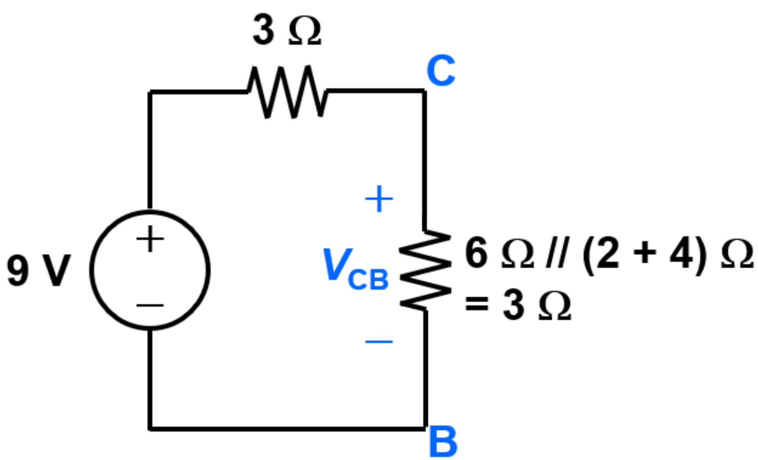
Find Thevenin equivalent circuit seen across A & B:



Solution to Q2



$V_T = V_{AB}$

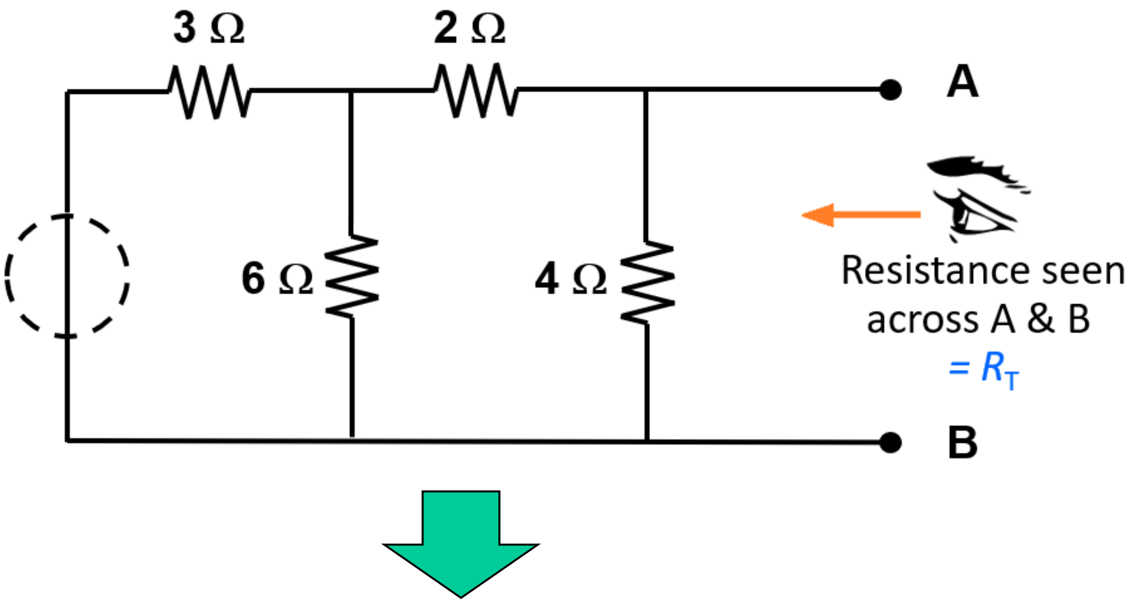


- $V_{CB} = \frac{3}{3+3} \times 9 = 4.5\ V$

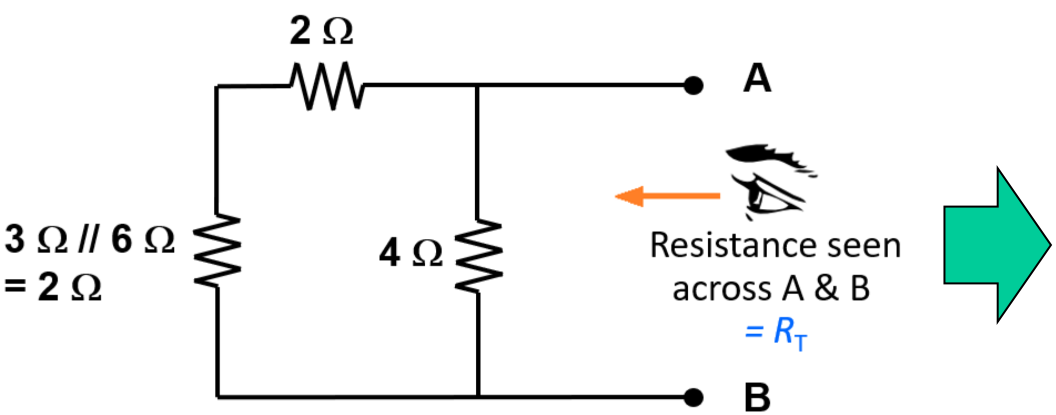
- Hence,

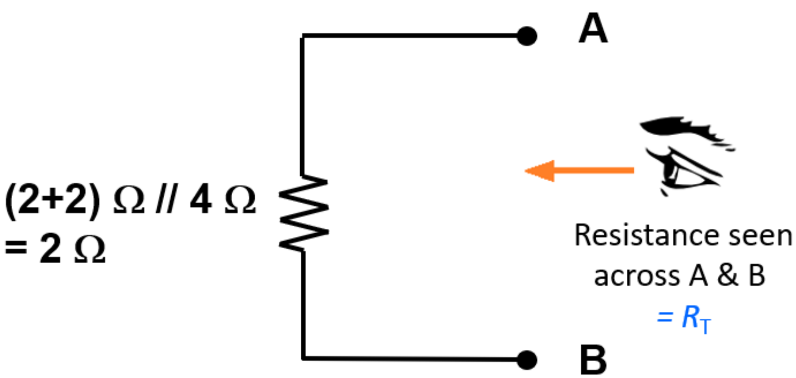
$$V_T = V_{AB}$$
$$= \frac{4}{4+2} \times 4.5 = 3\ V$$

Solution to Q2



Put the voltage source to 0 V (becomes essentially just a wire)

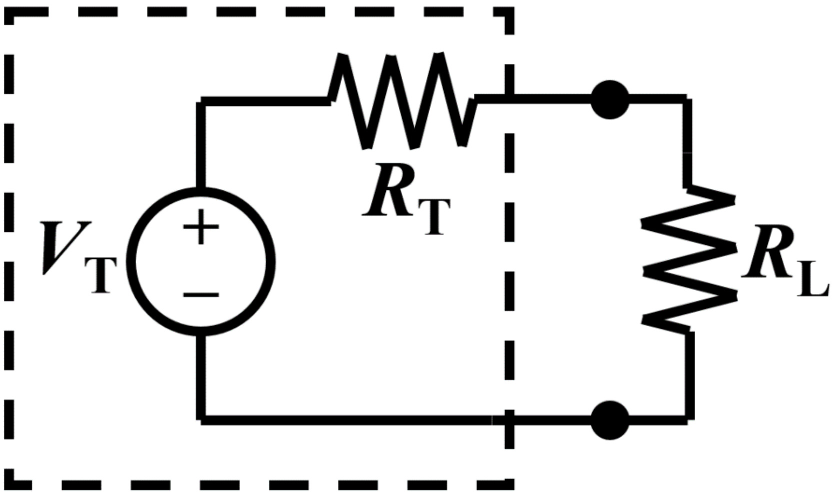




Hence, $R_T = 2\ \Omega$

Question 3

- For the Thevenin equivalent circuit shown below, derive the value of R_L that causes maximum power transfer from the source to the load R_L.

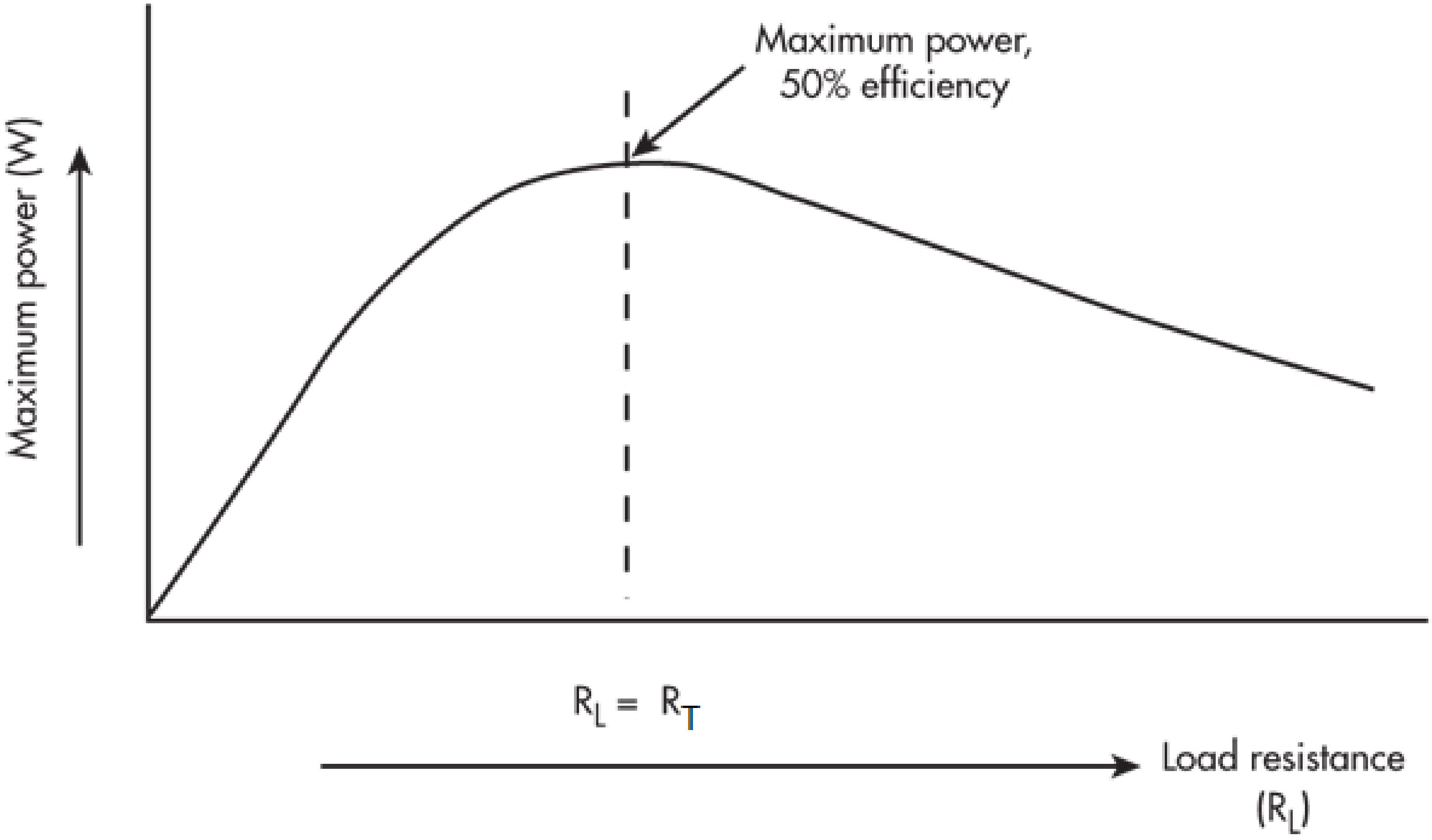


Thevenin equivalent circuit

Solution to Q3

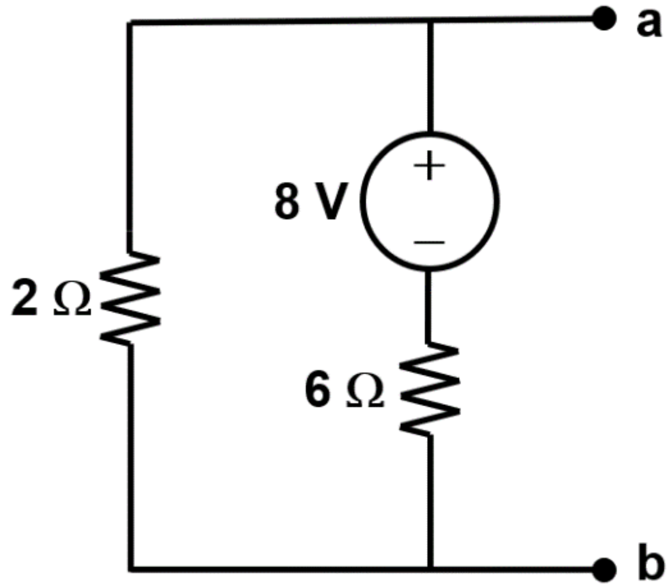
- $i = \frac{V_T}{R_T+R_L}$
- $P_L = i^2 R_L = \left(\frac{V_T}{R_T+R_L}\right)^2 R_L$
- The value of R_L which gives maximum P_L is the value for which $\frac{dP_L}{dR_L} = 0$
- $\frac{dP_L}{dR_L} = \frac{V_T^2(R_T+R_L)^2 - 2V_T^2 R_L(R_T+R_L)}{(R_T+R_L)^4} = 0$
- $V_T^2(R_T + R_L)^2 - 2V_T^2 R_L(R_T + R_L) = 0$
- The above equation is true when $R_L = R_T$

Solution to Q3



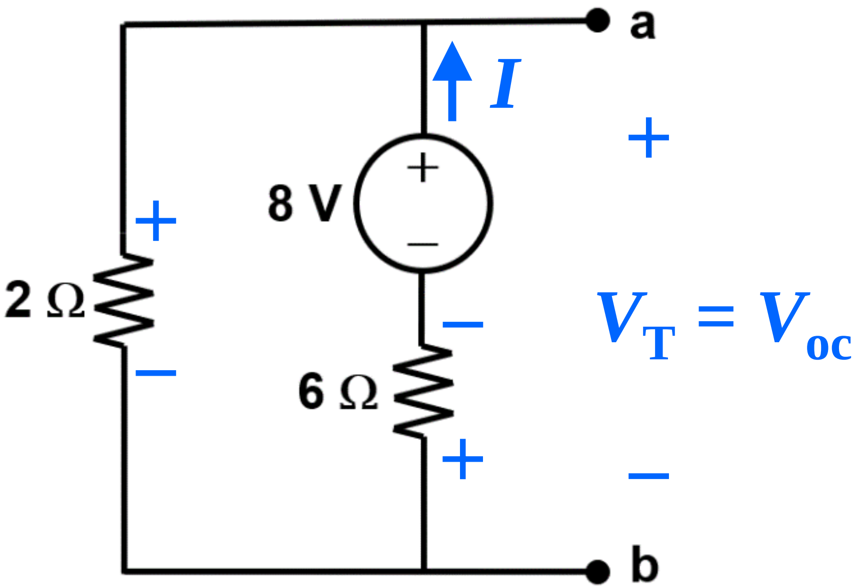
Question 4

Determine the value of R_L to be placed across nodes a and b, in order for it to draw maximum power. Calculate this power.



Solution to Q4

- Find Thevenin equivalent circuit!



$$I = \frac{8}{2+6} = 1\ \mathrm{A}$$

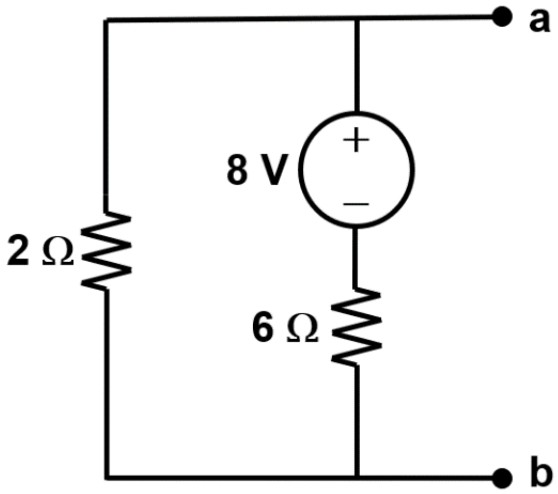
$$\begin{aligned} V_T &= V_{oc} \\ &= 8 - (6 \times 1) \\ &= 2\ \mathrm{V} \end{aligned}$$

$$\begin{aligned} R_T &= 6 \,//\, 2 \\ &= 1.5\ \Omega \end{aligned}$$

How do we label the voltage polarity across a resistor (a passive element)?

Label it in such a way that the potential decreases in the labeled direction of current.

Solution to Q4



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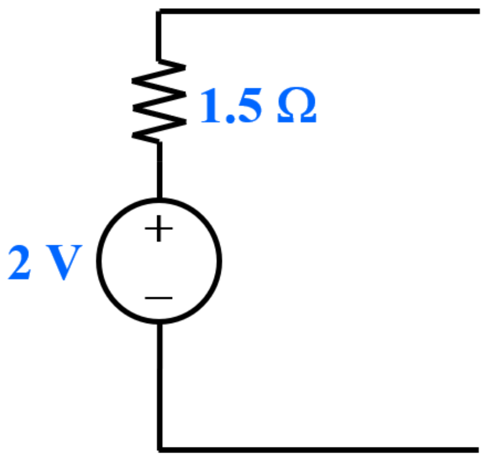

Load R_L that draws maximum power:

$$R_L = R_T = 1.5\ \Omega$$

Maximum power:

$$P = (I_L)^2 R_L = \left(\frac{2}{1.5 + 1.5}\right)^2 \times 1.5 = 0.67\ \mathrm{W}$$