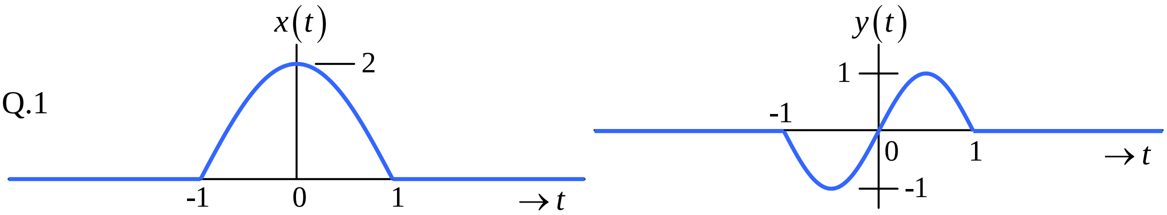
CG2023 TUTORIAL 3 (PROBLEMS)

Q.1 A half-cosine pulse $x(t)$ and a sine pulse $y(t)$ are shown in Fig.Q.1.

Fig.Q.1

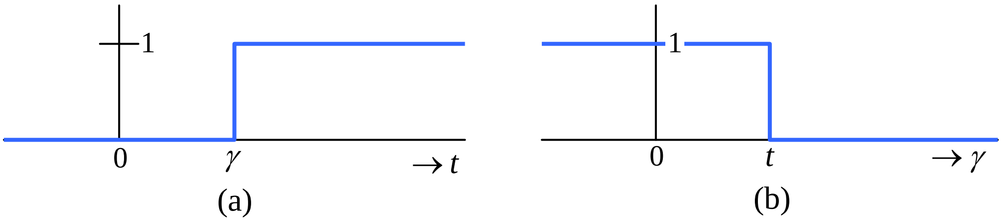


(a) Derive the spectrum of $x(t)$ using the forward Fourier transform equation and show how the derivation can be simplified by applying relevant Fourier transform properties.

(b) Using the results of Part-(a), determine the spectrum of $y(t)$.

Q.2 (a) Show that Fig.Q.2(a) and Fig.Q.2(b) are plots of the same function $u(t-\gamma)$, where $u(\cdot)$ denotes the unit step function.

Fig.Q.2



(b) Evaluate $\left[\cos(t)u(t)\right] \ast u(t)$ where $\ast$ denotes convolution.

Q.3 Fig.Q.3 shows the plot of a triangular pulse $x(t)$.

Determine the magnitude and phase spectra of $x(t)$. Hence, or otherwise, find the energy spectral density and total energy of $\dfrac{dx(t)}{dt}$.

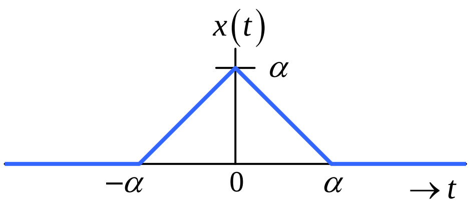


Fig.Q.3

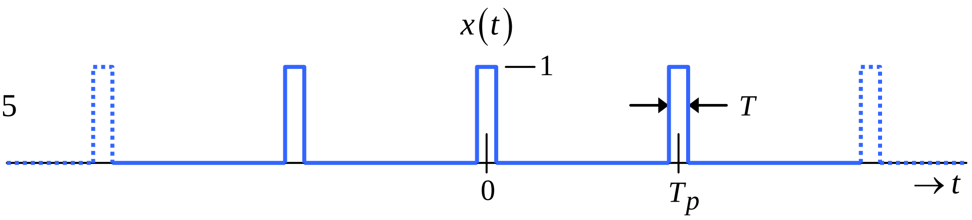
Q.4 The spectrum of a low-pass energy signal $x(t)$ is given by $X(f) = \exp(-\alpha|f|)$ where α is a positive constant.

(a) The 99% energy containment bandwidth of a signal is defined as the smallest bandwidth that contains at least 99% of the total signal energy. Find the 99% energy containment bandwidth of $x(t)$?

(b) Find the 3dB bandwidth of $x(t)$. How many percent of the total energy of $x(t)$ does its 3dB bandwidth contain?

Q.5 A military lookout tower uses a laser pointer as a make-shift signaling device to communicate with a base camp. The laser pointer's built-in ON-OFF pushbutton switch is replaced by an electronic switch which is activated by a signal $x(t)$. The output of the laser pointer has the form $y(t) = x(t) \cdot \mu \cos(2\pi f_c t)$ where μ and f_c are the amplitude and frequency of the laser beam when $x(t)$ has a value of 1. Unless there is an incident, the laser pointer continuously sends short pulses of light, spaced at regular interval, back to the base camp to indicate a 'No Incident' situation. The $x(t)$ used for signaling 'No Incident' is shown in Fig.Q.5.

Fig.Q.5



(a) Derive the power spectral density, $P_x(f)$, of $x(t)$.

(b) What is the average power of $x(t)$?

(c) Suppose the parameters of the system is so chosen such that $y(t)$ is periodic with period T_p and that each of the pulses of $x(t)$ spans an integer number of cycles of $\cos(2\pi f_c t)$, i.e., $f_c T = \text{integer}$. What is the average power of the laser output $y(t)$.

Supplementary Problems

These problems are for self practice.

S.1 Find the Fourier transform of each of the following signals:

(a) $x(t) = \cos(2\pi f_c t)u(t)$ (b) $x(t) = \sin(2\pi f_c t)u(t)$

(c) $x(t) = \exp(-\alpha t)\cos(2\pi f_c t)u(t);\ \ \alpha > 0$ (d) $x(t) = \exp(-\alpha t)\sin(2\pi f_c t)u(t);\ \ \alpha > 0$

Given: $\Im\{\exp(-\alpha t)u(t)\} = \dfrac{1}{\alpha + j2\pi f}$

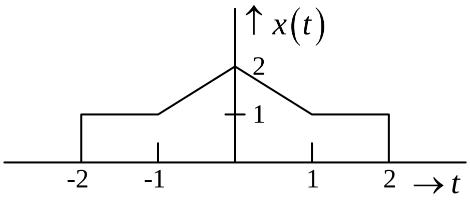
Answer: (a) $X(f) = \frac{1}{4}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \dfrac{jf}{2\pi\left(f_c^2 - f^2\right)}$

(b) $X(f) = \frac{j}{4}\left[\delta(f + f_c) - \delta(f - f_c)\right] + \dfrac{f_c}{2\pi\left(f_c^2 - f^2\right)}$

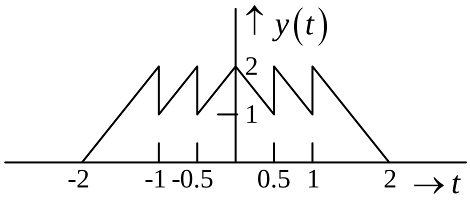
(c) $X(f) = \dfrac{\alpha + j2\pi f}{\left[\alpha^2 - 4\pi^2\left(f^2 - f_c^2\right)\right] + j4\alpha\pi f}$

(d) $X(f) = \dfrac{2\pi f_c}{\left[\alpha^2 - 4\pi^2\left(f^2 - f_c^2\right)\right] + j4\alpha\pi f}$

S.2 Find the Fourier transform of each of the following signals:



(a)



(b)

Answer: (a) $X(f) = 4\text{sinc}(4f) + \text{sinc}^2(f)$

(b) $Y(f) = 8\text{sinc}^2(2f) - 2\text{sinc}(2f) - \text{sinc}(f)$

S.3 Given: $\Im\{x(t)\} = \text{rect}(\pi f)$. Find the value of $\int_{-\infty}^{\infty} |y(t)|^2\, dt$ if $y(t) = \dfrac{dx(t)}{dt}$.

Answer: $1/(3\pi)$

S.4 Given: $\Im\left\{\dfrac{\pi}{\alpha}\exp(-2\pi\alpha|t|)\right\} = \dfrac{1}{\alpha^2 + f^2}$. Determine the 99% energy containment bandwidth for the signal $x(t) = \dfrac{1}{\alpha^2 + t^2}$.

Answer: $0.366/\alpha$

S.5 With $\omega = 2\pi f$, show that $\delta(f) = 2\pi\delta(\omega)$ or $\delta(\omega) = \dfrac{1}{2\pi}\delta(f)$.

S.6 The Fourier Transform pair in **cyclic** frequency is given by

$$\underbrace{X(f)=\int_{-\infty}^{\infty}x(t)\exp(-j2\pi ft)\,dt}_{\text{Forward FT}} \quad \text{and} \quad \underbrace{x(t)=\int_{-\infty}^{\infty}X(f)\exp(j2\pi ft)\,df}_{\text{Inverse FT}} .$$

Show that the Fourier transform pair in **angular** frequency is given by

$$\underbrace{\tilde{X}(\omega)=\int_{-\infty}^{\infty}x(t)\exp(-j\omega t)\,dt}_{\text{Forward FT}} \quad \text{and} \quad \underbrace{x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{X}(\omega)\exp(j\omega t)\,d\omega}_{\text{Inverse FT}}$$

where $\tilde{X}(\omega)=X(f)\Big|_{f=\frac{\omega}{2\pi}}$.

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition****.*

Selected solved-problems: 5.19-to-5.27, 5.32, 5.34, 5.40, 5.42, 5.42, 5.57

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.
