CG2023 Signals & Systems

AY2018/19-2

Midterm Quiz (Close Book)

Date: 7 March 2019 Time Allowed: 1.5 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **FOUR (4)** questions and comprises **TWELVE (12)** printed pages.
2. Answer all 4 questions. Each question carries 10 marks.
3. This is a closed book quiz. You are allowed to bring **ONE (1)** crib sheet of A4 size.
4. Programmable and/or graphic calculators are not allowed.
5. Tables of formulas are given on Pages 11 & 12 which you may detach for easy reference.
6. Write your **answers** in the spaces indicated in this question paper. Attachment is not allowed.
7. Write your **name, matric number** and **seat number** in the spaces indicated below.

Name :______________________________

Matric # :______________________________

Seat # :______________________________

Q.1 A signal is modeled by $x(t) = 4\text{rect}(4t - 8)$ and its Fourier transform is denoted by $X(f)$.

(a) Find the expression of $X(f)$. (2 marks)

(b) Let $y(t) = 2x(t)\cos(800\pi t)$.

i Find the Fourier transform, $Y(f)$, of $y(t)$. (3 marks)

ii. What are the energy and 1$^{\text{st}}$-null bandwidth of $y(t)$? (5 marks)

Q.1 ANSWER

Q.1 ANSWER ~ continued

Q.2 The spectrum of a signal $x(t)$ is given by $X(f) = 2e^{-j\pi f}\left[\mathrm{sinc}(f) - 2\mathrm{sinc}(2f)e^{-j3\pi f}\right]$.

(a) Find $x(t)$ in terms of $'rect'$ functions. (2 marks)

(b) Find the energy of $x(t)$. (2 marks)

(c) Let $y(t) = x(t) \ast \sum_{n=-\infty}^{\infty} \delta(t-6n)$ where '$\ast$' denotes convolution.

i Sketch $y(t)$ with proper labelling. (2 marks)

ii. Find the Fourier series coefficients and the average power of $y(t)$. (4 marks)

Q.2 ANSWER

Q.2 ANSWER ~ continued

Q.3 The discrete-frequency spectrum of a signal $x(t)$ is shown in Figure Q3.

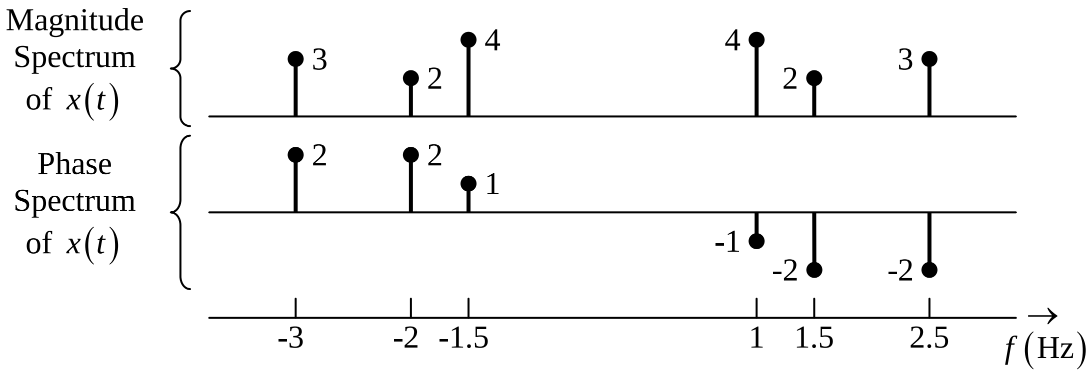

Figure Q3

(a) Is $x(t)$ is a (real or complex), (energy or power) and (periodic or aperiodic) signal? Provide a reason for each of your answers. (3 marks)

(b) What is the average power of $x(\alpha t)$ where α is an arbitrary non-zero real constant? Justify your answer. (4 marks)

(c) Following the format used in Figure Q3, sketch the spectrum of $x(-t)$. (3 marks)

Q.3 ANSWER

Q.3 ANSWER ~ continued

Q.4 Consider the signal $q(t)$ shown in Figure Q4(a).

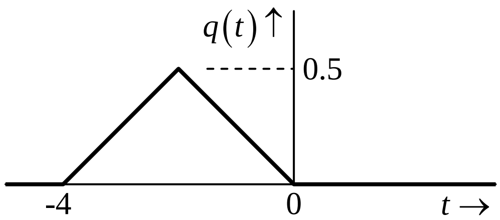

Figure Q4(a)

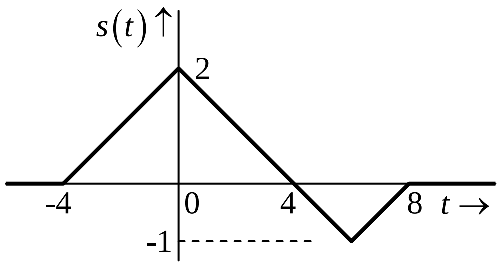

Figure Q4(b)

(a) Draw the spectrum of $q(t)$ with proper labelling. (3 marks)

(b) Find the energy spectral density and the energy of $q(t)$. (3 marks)

(c) Let $s(t)$ be as defined in Figure Q4(b). Find the spectrum of $s(t)$ using the results from Part (a). (4 marks)

Q.4 ANSWER

Q.4 ANSWER ~ continued

This page is reserved for marks entry. Anything written on this page will not be graded.

Question #	Marks	Remarks
1		
2		
3		
4		
Total Marks		

Fourier Series: $$\begin{cases} c_k = \frac{1}{T}\int_{\tilde{t}}^{\tilde{t}+T} x(t)\exp(-j2\pi k\, t/T)\,dt \\ x(t) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi k\, t/T) \end{cases}$$

Fourier Transform: $$\begin{cases} X(f) = \int_{-\infty}^{\infty} x(t)\exp(-j2\pi ft)\,dt \\ x(t) = \int_{-\infty}^{\infty} X(f)\exp(j2\pi ft)\,df \end{cases}$$

FOURIER TRANSFORMS OF BASIC FUNCTIONS

	$x(t)$	$X(f)$
Constant	K	$K\delta(f)$
Unit Impulse	$\delta(t)$	**1**
Unit Step	$u(t)$	$\frac{1}{2}\left[\delta(f)+\frac{1}{j\pi f}\right]$
Sign (or Signum)	$\operatorname{sgn}(t)$	$\frac{1}{j\pi f}$
Rectangle	$\operatorname{rect}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}(fT)$
Triangle	$\operatorname{tri}\left(\frac{t}{T}\right)$	$T\operatorname{sinc}^2(fT)$
Sine Cardinal	$\operatorname{sinc}\left(\frac{t}{T}\right)$	$T\operatorname{rect}(fT)$
Complex Exponential	$\exp(j2\pi f_o t)$	$\delta(f-f_o)$
Cosine	$\cos(2\pi f_o t)$	$\frac{1}{2}[\delta(f-f_o)+\delta(f+f_o)]$
Sine	$\sin(2\pi f_o t)$	$-\frac{j}{2}[\delta(f-f_o)-\delta(f+f_o)]$
Gaussian	$\exp\left(-\frac{t^2}{\alpha^2}\right)$	$\alpha\pi^{0.5}\exp(-\alpha^2\pi^2 f^2)$
Comb	$\sum_{m=-\infty}^{\infty}\delta(t-mT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta\left(f-\frac{k}{T}\right)$

FOURIER TRANSFORM PROPERTIES

	Time-domain	**Frequency-domain**
Linearity	$\alpha x_1(t)+\beta x_2(t)$	$\alpha X_1(f)+\beta X_2(f)$
Time scaling	$x(\beta t)$	$\frac{1}{\lvert\beta\rvert}X\left(\frac{f}{\beta}\right)$
Duality	$X(t)$	$x(-f)$
Time shifting	$x(t-t_o)$	$X(f)\exp(-j2\pi ft_o)$
Frequency shifting (Modulation)	$x(t)\exp(j2\pi f_o t)$	$X(f-f_o)$
Differentiation in the time-domain	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Multiplication in the time-domain	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty}X_1(\zeta)X_2(f-\zeta)\,d\zeta$ or $X_1(f)*X_2(f)$
Convolution in the time-domain	$\int_{-\infty}^{\infty}x_1(\zeta)x_2(t-\zeta)\,d\zeta$ or $x_1(t)*x_2(t)$	$X_1(f)X_2(f)$
Integration in the time-domain	$\int_{-\infty}^{t}x(\tau)\,d\tau$	$\frac{1}{j2\pi f}X(f)+\frac{1}{2}X(0)\delta(f)$
		$\frac{1}{j2\pi f}X(f)$ **if** $X(0)=0$

Trigonometric Identities

$\exp(\pm j\theta) = \cos(\theta) \pm j\sin(\theta)$	$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
$\cos(\theta) = 0.5\left[\exp(j\theta) + \exp(-j\theta)\right]$	$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
$\sin(\theta) = -0.5j\left[\exp(j\theta) - \exp(-j\theta)\right]$	$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$
$\sin^2(\theta) + \cos^2(\theta) = 1$	
$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$	$\sin(\alpha)\sin(\beta) = 0.5\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$
$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$	$\cos(\alpha)\cos(\beta) = 0.5\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$
$\sin^2(\theta) = 0.5\left[1 - \cos(2\theta)\right]$	$\sin(\alpha)\cos(\beta) = 0.5\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$
$\cos^2(\theta) = 0.5\left[1 + \cos(2\theta)\right]$	$C\cos(\theta) - S\sin(\theta) = \sqrt{C^2 + S^2}\cos\left[\theta + \tan^{-1}(S/C)\right]$

Complex Unit (j) ➔ $\left(j = \sqrt{-1} = e^{j\pi/2} = e^{j90^\circ}\right)$ $\left(-j = \dfrac{1}{j} = e^{-j\pi/2} = e^{-j90^\circ}\right)$ $\left(j^2 = -1\right)$

Definitions of Basic Functions

Rectangle:

$$\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1; & -T/2 \le t < T/2 \\ 0; & \text{elsewhere} \end{cases}$$

Triangle:

$$\operatorname{tri}\left(\frac{t}{T}\right) = \begin{cases} 1 - |t|/T; & |t| \le T \\ 0; & |t| > T \end{cases}$$

Sine Cardinal:

$$\operatorname{sinc}\left(\frac{t}{T}\right) = \begin{cases} \dfrac{\sin(\pi t/T)}{\pi t/T}; & t \ne 0 \\ 1; & t = 0 \end{cases}$$

Signum:

$$\operatorname{sgn}(t) = \begin{cases} 1; & t \ge 0 \\ -1; & t < 0 \end{cases}$$

Unit Impulse:

$$\delta(t) = \begin{cases} \infty; & t = 0 \\ 0; & t \ne 0 \end{cases} \qquad \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

Unit Step:

$$u(t) = \begin{cases} 1; & t \ge 0 \\ 0; & t < 0 \end{cases}$$