ANSWER KEY

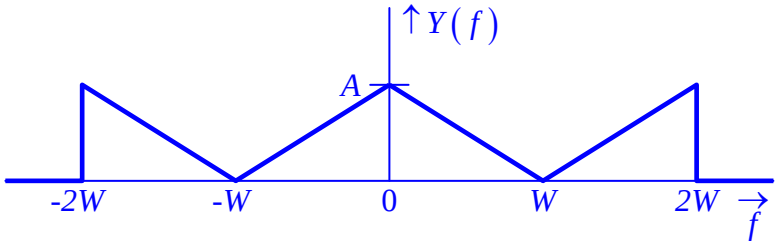
Q.1 (a) $x(t) = A\,\mathrm{rect}\left(\dfrac{t - W/2}{W}\right) - A\,\mathrm{rect}\left(\dfrac{t - 3W/2}{W}\right)$

(b) $y(t) = x(t) \ast \sum_{n=-\infty}^{\infty} \delta(t - 2nW)$

(c) $X(f) = j2AW\,\mathrm{sinc}(Wf)\sin(W\pi f)e^{-j2W\pi f}$

(d) Average power of $y(t) = A^2 = W^2$

Q.2 (a)



(b) Nyquist sampling frequency $= 2 \times 2W = 4W$

(c)(i) $f_1 = 2W$

(c)(ii) $f_s = 6W$

Q.3 (a) Low-frequency asymptotic slope of $\left|\tilde{H}(s)\right| = 20$ dB/decade

Low-frequency asymptotic value of $\angle\tilde{H}(s) = 90°$

High-frequency asymptotic slope of $\left|\tilde{H}(s)\right| = 0$ dB/decade

High-frequency asymptotic slope of $\angle\tilde{H}(s) = 0°$

(b) Yes. EXPLAIN WHY?.

(c) $y(t) = 50\left|\tilde{H}(j5)\right| \cdot \cos\left(5t + 90° + \angle\tilde{H}(j5)\right)$

where $\left|\tilde{H}(j5)\right| = \dfrac{5C\sqrt{25 + D^2}}{\sqrt{\left(E^2 - 25\right)^2 + 210^2}}$ and $\angle\tilde{H}(j5) = \tan^{-1}\left(\dfrac{D}{-5}\right) - \tan^{-1}\left(\dfrac{210}{E^2 - 25}\right)$

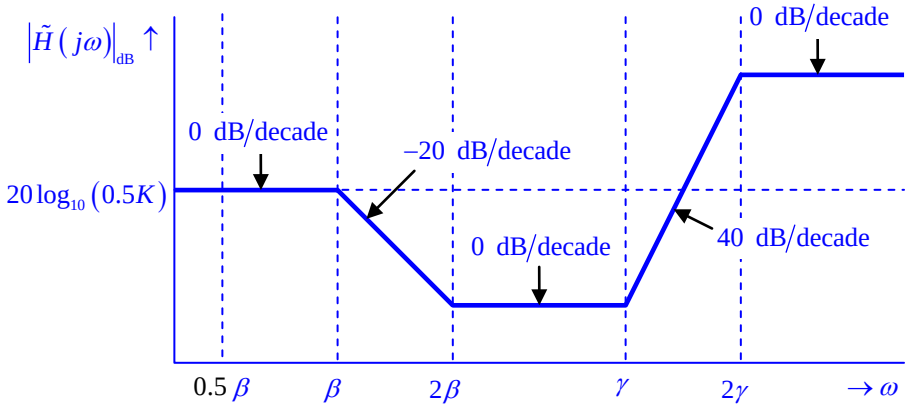
Q.4 (a) 4 poles and 2 zeros (b) $K_{dc} = 10^{H_{\mathrm{dB}}/20}$

(c) $\tilde{H}(s) = \dfrac{50000K_{dc}(s+6)^2}{3(s+0.3)(s+50)(s+200)^2}$

Q.5 (a) DC gain $= 0.5K$

(b) $\zeta = \dfrac{39600}{2\gamma} \rightarrow$ 2^{nd}-order factor is $\begin{cases} \text{critically damped if } \zeta = 1 \\ \text{underdamped if } 0 < \zeta < 1 \end{cases}$

(c) $\left.\begin{array}{l} \text{The largest value of } \beta = 20 \times (90 + 9 + 99) = 3960 \\ \text{The smallest value of } \gamma = 100 \times 99 = 9900 \end{array}\right\} \rightarrow \gamma > 2\beta \quad (\textit{for all student numbers})$



(d) $y(t) \simeq \dfrac{0.5K\beta}{\omega_o}\sin(\omega_o t)$.

Q.6 (a) $S(f) = \sum_k A\dfrac{T_1}{T_2}\text{sinc}\left(k\dfrac{T_1}{T_2}\right)\delta\left(f - \dfrac{k}{T_2}\right)$

$X(f) = B\ \text{tri}\left(\dfrac{f}{10^3}\right)e^{-j\pi f \times 10^{-3}}$

(b) $Y(f) = \sum_k AB \times \dfrac{T_1}{T_2}\text{sinc}\left(k\dfrac{T_1}{T_2}\right)\text{tri}\left(\dfrac{f - k/T_2}{10^3}\right)e^{-j\pi\left(f - \frac{k}{T_2}\right)\times 10^{-3}}$

(c) Maximal value of $T_2 = 0.5 \times 10^{-3}$ second.

(d) Bandwidth of lowpass filter $= 10^3$ Hz.
Passband gain of lowpass filter $= \dfrac{50}{A}$.
