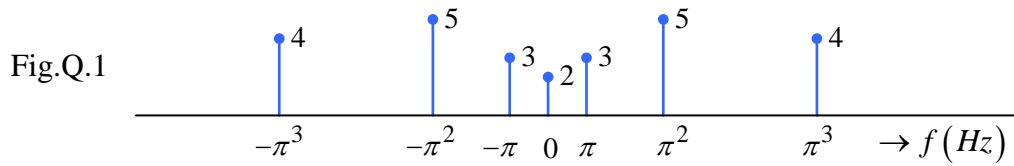


CG2023 TUTORIAL 2 (PROBLEMS)

Q.1 The discrete-frequency spectrum of a signal $x(t)$ is shown in Fig.Q.1.



- (a) What is the dc value of $x(t)$?
- (b) Is $x(t)$ a power or energy signal?
- (c) What is the Fourier series expansion of $x(t)$?

Q.2 Determine the Fourier series coefficients of each of the following periodic signals.

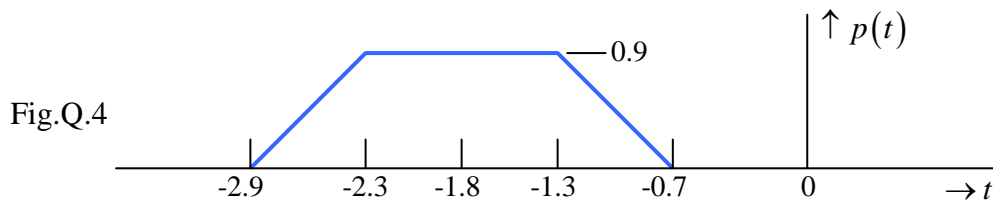
- (a) $x(t) = 6\sin(12\pi t) + 4\exp(j(8\pi t + \pi/4)) + 2$
- (b) $x(t) = 0.5(|\sin(\pi t)| + \sin(\pi t))$

Q.3 Let $x(t) = x_1(t) + x_2(t)$ where $x_1(t)$ and $x_2(t)$ are arbitrary periodic signals of finite periods T_1 and T_2 , respectively. State the condition(s) for $x(t)$ to be periodic. If T is the period of $x(t)$, express T_1 and T_2 in terms of T .

Q.4 Determine the Fourier series coefficients of

$$x(t) = \sum_{n=-\infty}^{\infty} 2p(t - 1.6n)$$

where $p(t)$ is given in Fig.Q.4.



Q.5 Consider the signal $x(t) = \cos(3\pi t)$ and define

$$y(t) = \sum_{k=-\infty}^{\infty} c_k \exp(j2\pi kt)$$

where $c_k = \int_{-0.5}^{0.5} x(t) \exp(-j2\pi kt) dt$. Sketch $x(t)$ and $y(t)$. Show all the important dimensions in your sketches.

Supplementary Problems

These problems are for self practice.

S.1 Consider a rectified sine wave signal $x(t)$ defined by

$$x(t) = |\sin(\pi t)|.$$

- (a) Sketch $x(t)$ and find its fundamental period.
- (b) Find the complex exponential Fourier series of $x(t)$.
- (c) Find the trigonometric Fourier series of $x(t)$.

Answer: (a) *period = 1*

$$(b) \quad x(t) = -\frac{2}{\pi} \sum_{k=-\infty}^{\infty} \frac{1}{4k^2 - 1} \exp(j2\pi kt) \quad (c) \quad x(t) = \frac{2}{\pi} - \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} \cos(2\pi kt)$$

S.2 Find the complex exponential Fourier series of a periodic signal $x(t)$ defined by

$$x(t) = t^2; \quad -\pi < t < \pi \quad \text{and} \quad x(t + 2\pi) = x(t).$$

Answer: $c_k = \begin{cases} 2(-1)^k / k^2; & k \neq 0 \\ \pi^2 / 3; & k = 0 \end{cases}$

S.3 Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental frequency, ω_p , and period, T_p .

$$(a) \quad x(t) = \cos(3.2t) + \sin(1.6t) + \exp(j2.8t) \quad (b) \quad x(t) = \cos(4t) + \sin(\pi t)$$

Answer: (a) *periodic, $\omega_p = 0.4$ rad/s, $T_p = 5\pi$ s*

(b) *non-periodic*

S.4 Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period, T_p and average power, P .

$$(a) \quad x(t) = \cos(2t + 0.25\pi) \quad (b) \quad x(t) = \cos^2(t) \\ (c) \quad x(t) = \cos(2\pi t)u(t) \quad (d) \quad x(t) = \exp(j\pi t)$$

Answer: (a) *periodic, $T_p = \pi$, $P = 0.5$*

(b) *periodic, $T_p = \pi$, $P = 0.375$*

(c) *non-periodic*

(d) *periodic, $T_p = 2$, $P = 1$*

Below is a list of solved problems selected from Chapter 5 of Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition.

Selected solved-problems: 5.4-to-5.13

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems.