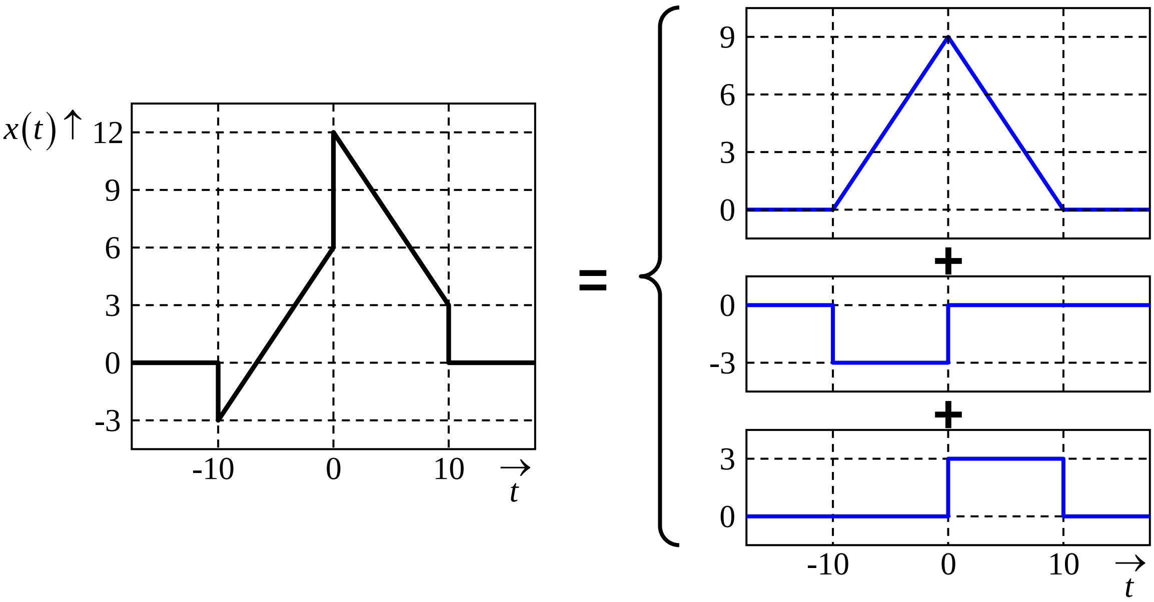
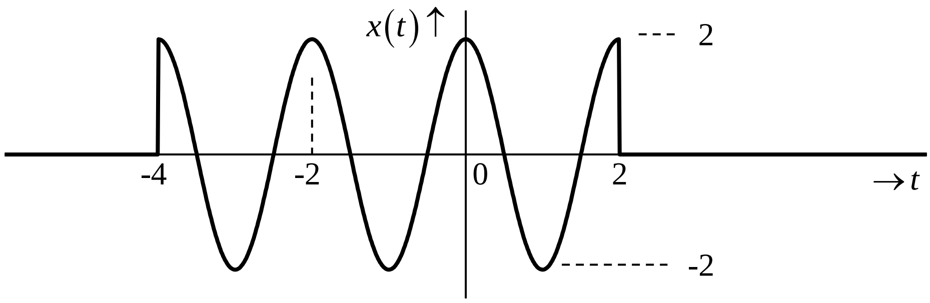
CG2023 ASSIGNMENT 1 (Temporal Operations on Signals)
<u>SOLUTIONS</u>

1. $x(t)=A\,\mathrm{tri}\left(\frac{t}{\alpha}\right)+B\,\mathrm{rect}\left(\frac{t-b}{\beta}\right)+C\,\mathrm{rect}\left(\frac{t-c}{\chi}\right)=9\,\mathrm{tri}\left(\frac{t}{10}\right)-3\,\mathrm{rect}\left(\frac{t+5}{10}\right)+3\,\mathrm{rect}\left(\frac{t-5}{10}\right)$

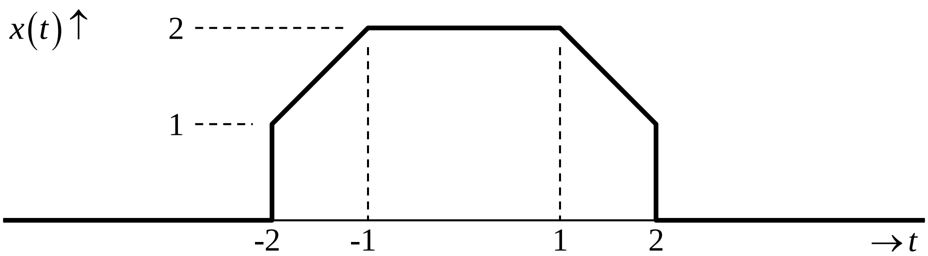
   Therefore, $A=9$ $\;-B=C=3$ $\;b=-c=5$ and $\alpha=\beta=\chi=10$



2. (a) $x(t)=2\cos(\pi t)u(t+4)u(2-t)$.



   (b) $x(t)=\mathrm{rect}(0.25t)+2\mathrm{tri}(0.5t)-\mathrm{tri}(t)$



3. Given: $\begin{cases} x(t) = -3 + j4 = 5e^{j\tan^{-1}(4/(-3))} = 5e^{j2.2143} \\ y(t) = \sqrt{2}e^{j0.25\pi} = 1.4142e^{j0.7854} = 1 + j \end{cases}$

(a) $x(t) - y(t) = -3 + j4 - (1 + j) = -4 + j3$

$$\left|x(t) - y(t)\right| = \sqrt{4^2 + 3^2} = 5$$

$$\angle\left[x(t) - y(t)\right] = \tan^{-1}\left(\frac{3}{-4}\right) \cong 2.5 \text{ rad}$$

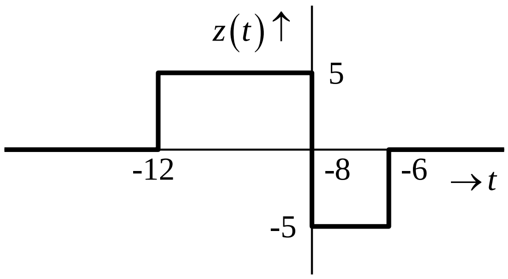
(b)

$$x(t)y(t) = 5e^{j2.2143} \times 1.4142e^{j0.7854} = 7.071e^{j2.9997}$$
$$= 7.071\left[\cos(2.9997) + j\sin(2.9997)\right] \cong -7 + j1$$

$$\frac{x(t)}{y(t)} = \frac{5e^{j2.2143}}{1.4142e^{j0.7854}} = 3.5356e^{j1.4289}$$
$$= 3.5356\left[\cos(1.4289) + j\sin(1.4289)\right] \cong 0.5 + j3.5$$

4. $z(t) = 5w\left(-\frac{(t+8)}{2}\right)$.

$z(t)$ is $w(t)$ time-expanded by a factor of 2, time-reversed, time-advanced by 8 units, and amplified by a factor of 5.



5. $y(t)$ is $x(t)$ time-contracted by a factor of $\frac{3}{2}$, time-reversed, time-delayed by 6 units, and amplified by a factor of $\frac{3}{2}$, i.e. $y(t) = \frac{3}{2}x\left(-\frac{3}{2}(t-6)\right)$

6. $$y(t) = 3\text{rect}\left(\frac{t}{8}\right) \ast \left[2\,\text{tri}\left(\frac{t}{12}\right) \times \sum_n \delta(t - 6n)\right] = 3\text{rect}\left(\frac{t}{8}\right) \ast \left[\delta(t+6) + 2\delta(t) + \delta(t-6)\right]$$
$$= 3\text{rect}\left(\frac{t+6}{8}\right) + 6\text{rect}\left(\frac{t}{8}\right) + 3\text{rect}\left(\frac{t-6}{8}\right)$$

$$\int_{-\infty}^{\infty} y(t)dt = \int_{-\infty}^{\infty}\left[3\text{rect}\left(\frac{t+6}{8}\right) + 6\text{rect}\left(\frac{t}{8}\right) + 3\text{rect}\left(\frac{t-6}{8}\right)\right]dt = (3\times 8) + (6\times 8) + (3\times 8) = 96$$