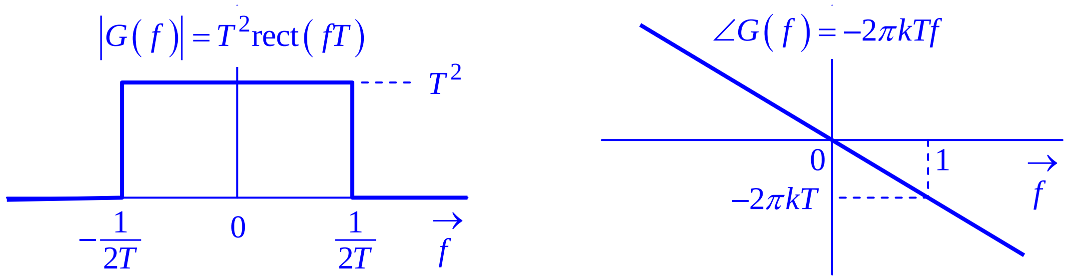
ANSWER KEY

Q.1 (a) $G(f) = T^2 \text{rect}(fT) e^{-j2\pi kTf}$

(b)



(c) $g(0) = 0$

Q.2 (a) DC gain $= K/a$ and Corner frequency $= a$

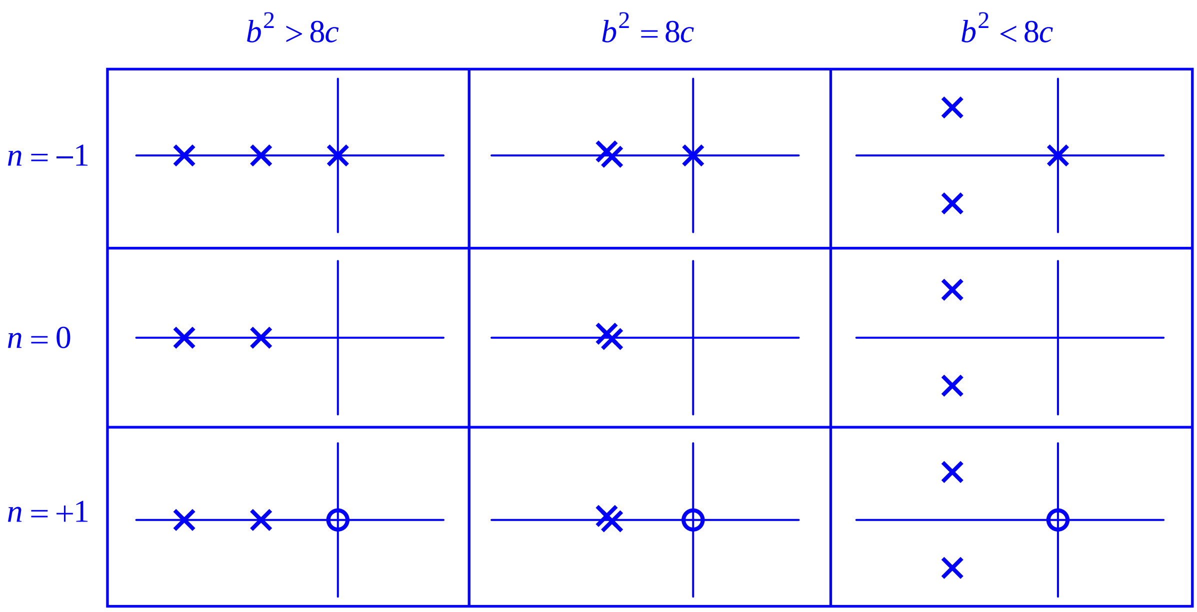
(b) $K = a = \omega_1/10$

(c) $y(t) = 1.791 \times 10^{-1} \cos(20t - 5.711°) + 1.342 \times 10^{-2} \cos(400t - 63.43°)$

Q.3 Given: $K,\ b,\ c > 0$ and $n \in \{-1, 0, 1\}$

(a)
$n = -1: \rightarrow$ 3 poles, 1 integrator
$n = 0\ : \rightarrow$ 2 poles
$n = +1: \rightarrow$ 2 poles, 1 zero, 1 differentiator

(b)



(c) $n = 0$ and $1 \rightarrow$ BIBO stable

$n = -1 \rightarrow$ Marginally stable

(d) $\zeta = \dfrac{b}{\sqrt{8c}}$ Assume $b,\ c \neq 0$ $\rightarrow \begin{cases} b > \sqrt{8c} & \rightarrow \text{overdamped} \\ b = \sqrt{8c} & \rightarrow \text{critically damped} \\ b < \sqrt{8c} & \rightarrow \text{underdamped} \end{cases}$

(e)

Low frequency asymptote:

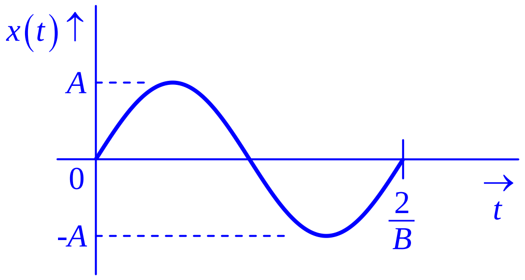
$$\underset{s\to 0}{\tilde{H}(s)} = \frac{Ks^n}{\cancel{2s^2} + \cancel{bs} + c} \simeq \frac{K}{c}s^n \rightarrow \begin{cases} n = -1: & \text{Integrator with a gain of } K/c \\ n = 0: & \text{DC gain of } K/c \\ n = +1: & \text{Differentiator with a gain of } K/c \end{cases}$$

High frequency asymptote:

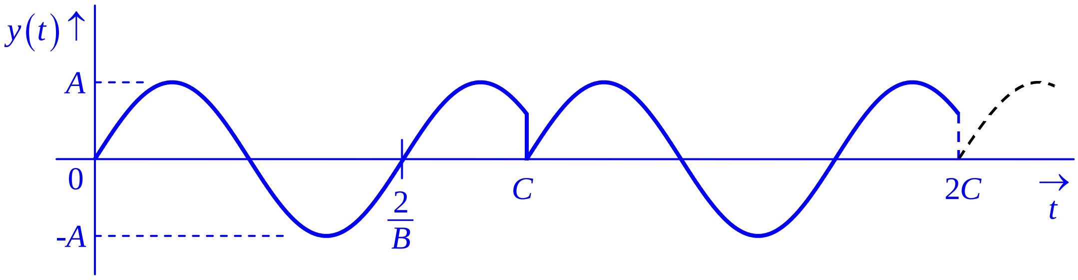
$$\underset{s\to\infty}{\tilde{H}(s)} = \frac{Ks^n}{2s^2 + \cancel{bs} + \cancel{c}} \simeq \frac{K}{2}\frac{s^n}{s^2} \rightarrow \begin{cases} n = -1: & \text{Cascade of 3 integrators with a combined gain of } K/2 \\ n = 0: & \text{Cascade of 2 integrators with a combined gain of } K/2 \\ n = +1: & \text{An Integrator with a gain of } K/2 \end{cases}$$

Q.4

(a)



(b) Period of $y(t) = C$.



(c) $c_k = \dfrac{1}{C}G\left(\dfrac{k}{C}\right) = \dfrac{A}{j2}\left[\operatorname{sinc}(k - 0.5BC)e^{-j\pi(k-0.5BC)} - \operatorname{sinc}(k + 0.5BC)e^{-j\pi(k+0.5BC)}\right]$

(d) DC value of $x(t) = 0$

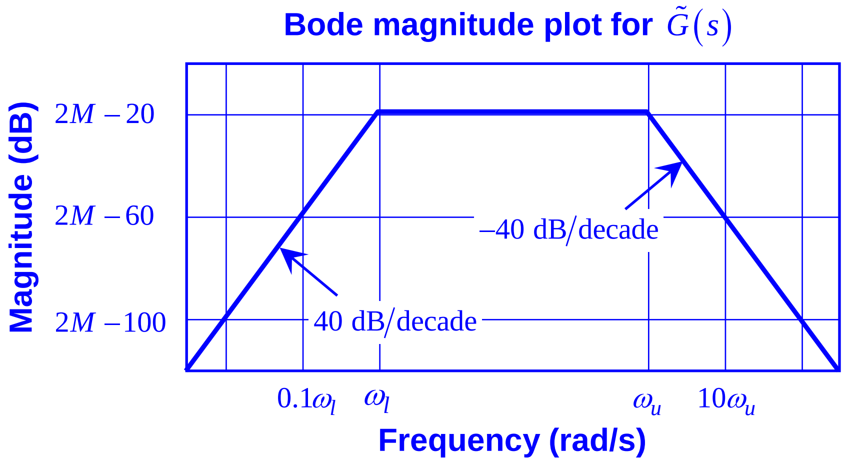
DC value of $y(t) = c_0 = A\operatorname{sinc}(0.5BC)\sin(0.5\pi BC)$

Q.5 (a) $L = 0,\quad a = \dfrac{1}{10^{M/20}\omega_u},\quad b = \dfrac{\omega_l + \omega_u}{10^{M/20}\omega_u},\quad c = \dfrac{\omega_l}{10^{M/20}}$

(b)(i) $A = \dfrac{5\times 10^{M/20}}{\omega_l + \omega_u}\sqrt{\dfrac{\omega_u}{\omega_l}}\qquad B^\circ = -30^\circ$

(b)(ii) $A = \dfrac{5\times 10^{M/20}}{\omega_l}\qquad B^\circ = 60^\circ$

(c)(i)



(c)(ii) Low-frequency asymptotic values of the phase response of $\tilde{G}(s) = +180^\circ$

High-frequency asymptotic values of the phase response of $\tilde{G}(s) = -180^\circ$

(c)(iii) Filter delay of $\tilde{G}(s)$ is twice that of $\tilde{H}(s)$

Q.6 (a) $X(f) = \frac{AT}{2}\left[\text{rect}\left(\frac{f - 0.5B}{1/T} \right) + \text{rect}\left(\frac{f + 0.5B}{1/T} \right) \right]$

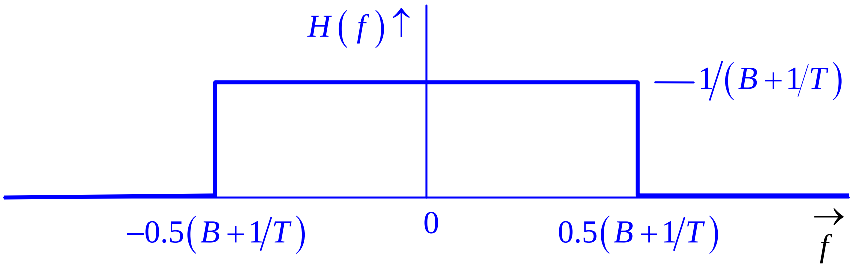
(b) If $B > \frac{1}{T}$: $x(t)$ is <u>bandpass</u> with bandwidth $1/T$ Hz

Nyquist sampling frequency $= B + 1/T$ Hz.

If $B \le \frac{1}{T}$: $x(t)$ is <u>lowpass</u> with bandwidth $0.5(B + 1/T)$ Hz

Nyquist sampling frequency $= B + 1/T$ Hz.

(c)



(d) If $B > \frac{1}{T}$: YES, because $x(t)$ is <u>bandpass</u>.

In this case $x(t)$ has a center frequency of $B/2$ Hz, and a bandwidth of $1/T$ Hz. Use the formulas given in the lecture notes to determine the lowest possible sampling frequency.

If $B \le \frac{1}{T}$: NO, because $x(t)$ is <u>lowpass</u>.