NATIONAL UNIVERSITY OF SINGAPORE

CG2023 – SIGNALS AND SYSTEMS

(Semester II: AY2019/2020)

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This paper contains **SIX (6)** questions and comprises **FIVE (5)** printed pages.

2. Answer **ALL** questions in both Sections A and B.

3. The maximum mark for this paper is 80.

4. This is an **OPEN BOOK** e-Examination. You may not consult, discuss and/or collaborate with any other person regarding the exam.

5. Write your answer on A4 size paper with black or dark blue color ink. Write the page number and the question number on the top-left corner of each page. Start your answer to a question on a new page. Write the values of the parameters that you have generated at the beginning of each question..

6. Your written answers should be clear and readable. Severe readability issues may be penalized by mark.

7. At the end of the examination, scan your answer pages (in page number sequence) and combine them with the filled and signed ECE-declaration-form.pdf into **ONE (1)** PDF file using [your matric number]-CG2023.pdf as the filename. Upload it within 20 minutes from the time the examination ended. Suggestions on how you may e-submit your answer have already been communicated to you in advance.

   IMPORTANT: **A mark of zero will be assigned to submissions that are late or do not conform to the stipulated submission format. If you submit multiple PDF files, only the first submitted file will be graded.**

8. You are expected to be familiar with the adhere to the NUS code of student conduct (http://nus.edu.sg/osa/resources/code-of-student-conduct), and comply with all other rules for the assessments, and be aware that failure to comply may result in disciplinary action against you.

SECTION A

Answer ALL questions in this section (Each question carries 10 marks)

Q.1 Let signal $g(t) = \text{sinc}\left(\frac{t}{T}\right) \ast \text{sinc}\left(\frac{t-kT}{T}\right)$ where "$\ast$" denotes convolution.

Generate the values for T and k using the parameter generator for Question_1. Use these values to answer the following questions in Q.1.

(a) Find the Fourier transform, $G(f)$, of $g(t)$. (4 marks)

(b) Find $\left|G(f)\right|$, $\angle G(f)$ and sketch them with proper labelling. (3 marks)

(c) What is the value of $g(0)$? (3 marks)

Q.2 Signal $x(t) = s(t) + n(t)$ where $s(t) = A\cos(\omega_0 t)$ and $n(t) = \cos(\omega_1 t)$. A student wants to suppress the amplitude of $n(t)$ using a lowpass filter which is a LTI system with transfer function $\tilde{H}(s) = \frac{K}{s+a}$. Here K and a are positive real parameters to be designed.

Generate the values for A, ω_0 and ω_1 using the parameter generator for Question_2. Use these values to answer the following questions in Q.2.

(a) Find the DC gain (in dB) and corner frequency (in rad/s) of the lowpass filter in terms of K and a. (3 marks)

(b) Propose a suitable value for a and indicate the corresponding value of K based on the straight-line Bode plot such that the amplitude of $n(t)$ is suppressed by at least 20 dB and the amplitude of $s(t)$ is not affected.

(4 marks)

(c) If $a = \frac{\omega_1}{2}$ and $K = A$, find the output $y(t)$ of the lowpass filter when $x(t)$ is the input. (3 marks)

(Round your answers to 4 significant figures.)

Q.3 The transfer function of a LTI system is given by

$$\tilde{H}(s) = \frac{Ks^n}{2s^2 + bs + c}.$$

Generate the values for K, n, b and c using the parameter generator for Question_3. Use these values to answer the following questions in Q.3.

(a) How many poles, zeros, integrators and differentiators does the system have? (2 marks)

(b) Draw an adequately labeled pole-zero map for the system. (2 marks)

(c) Is the system BIBO stable, marginally stable or unstable? (1 marks)

(d) Find the damping ratio of the system and state whether the system is overdamped, critically damped, underdamped or undamped. (2 marks)

(e) State concisely the high-frequency and low-frequency asymptotic behavior of the LTI system. (3 marks)

Q.4 Consider the signal $x(t) = A\sin(B\pi t)$ and define $y(t) = \sum_k c_k e^{j2\pi kt/C}$ where c_k are the Fourier series coefficients of $y(t)$ defined as $c_k = \frac{1}{C}\int_0^C x(t)e^{-j2\pi kt/C}dt$ with k being integers.

Generate the values for A, B and C using the parameter generator for Question_4. Use these values to answer the following questions in Q.4.

(a) Sketch $x(t)$ for one period. (2 marks)

(b) Find the fundamental period of $y(t)$ and sketch $y(t)$ for two periods. (2 marks)

(c) Find the Fourier Series coefficients c_k of $y(t)$. (4 marks)

(d) What are the DC values of $x(t)$ and $y(t)$? (2 marks)

SECTION B

Answer ALL questions in this section (Each question carries 20 marks)

Q.5 The filter used by a radio amateur to limit the bandwidth of his transmission has a transfer function given by

$$\tilde{H}(s) = \frac{s+L}{as^2+bs+c}$$

where L, a, b and c are constants. The Bode magnitude plot for $\tilde{H}(s)$ is shown in Figure Q5.

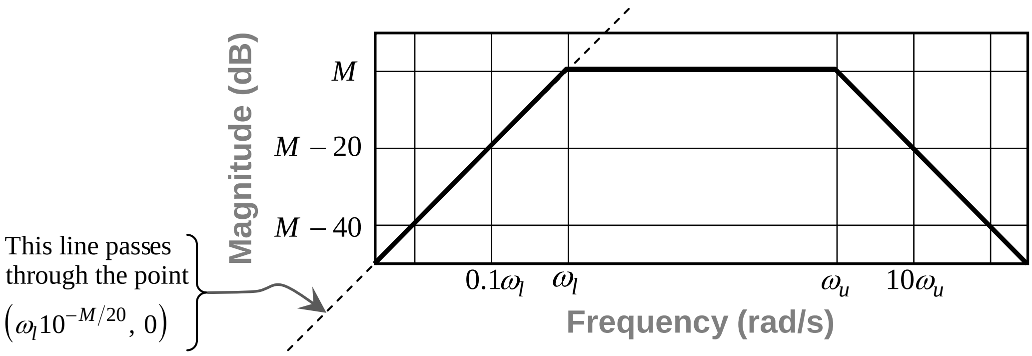

Figure Q5

Generate the values for M, ω_l and ω_u using the parameter generator for Question_5. Use these values to answer the following questions in Q.5.

(a) Find the values of L, a, b and c. (5 marks)
(Round your answers to 4 significant figures.)

(b) At steady-state, the filter $\tilde{H}(s)$ produces an output $y(t) = A\omega_o \cos(\omega_o t + B^\circ)$ when the input is $x(t) = 5\cos(\omega_o t - 30^\circ)$.

i. Find the values of A and B° if $\omega_o^2 = \omega_l \omega_u$. (3 marks)

ii. Find the approximate values of A and B° if $\omega_o << 0.1\omega_l$. (3 marks)

(Round your answers to 4 significant figures.)

(c) Due to some performance issues, the original filter $\tilde{H}(s)$ is replaced by a new filter $\tilde{G}(s) = 0.1\tilde{H}^2(s)$.

i. Sketch the Bode magnitude plot for $\tilde{G}(s)$. Label your sketch adequately. (3 marks)

ii. Find the low-frequency and high frequency asymptotic values of the phase response of $\tilde{G}(s)$. (3 marks)

iii. In switching from $\tilde{H}(s)$ to $\tilde{G}(s)$, by how much is the filter delay increased or decreased? (3 marks)

Q.6 The variation in the temperature of an industrial equipment is given by

$$x(t) = A\,\mathrm{sinc}\left(\frac{t}{T}\right)\cos(B\pi t).$$

Generate the values for A, T and B using the parameter generator for Question_6. Use these values to answer the following questions in Q.6.

(a) Find the spectrum of this temperature variation $x(t)$. (4 marks)

(b) Find the bandwidth and Nyquist sampling frequency of $x(t)$. (4 marks)

(c) A signal $x_s(t)$ is formed by sampling the signal $x(t)$ at the Nyquist sampling frequency. Sketch the frequency response $H(f)$ of an ideal low pass filter needed for perfect reconstruction of $x(t)$ from $x_s(t)$. (6 marks)

(d) Can the signal $x(t)$ be sampled at a frequency lower than the Nyquist sampling frequency if perfect reconstruction is required? If "Yes", find the lowest sampling frequency and sketch the spectrum of the original signal and the sampled signal. If "No", explain why.

(6 marks)

END OF PAPER