National University of Singapore
Department of Electrical & Computer Engineering
CG2023 Signals & Systems

Experiment 1 : Spectrums and Sounds of Periodic Signals

Please read this manual carefully and complete the Prior Investigations before attending the laboratory session.

1. Introduction

All sounds are caused by vibrating objects that create pressure disturbances in the surrounding air. When the sound wave reaches the eardrum, it vibrates at the same frequency as the wave. The inner ear and the brain convert this vibration into a perceived sound. Since sound is a periodic pressure wave, a pure tone may be expressed mathematically as a sinusoidal waveform, i.e. $A\sin(2\pi f t + \phi)$ or $A\cos(2\pi f t + \phi)$. This means that, regardless of other characteristics such as amplitude or phase, a pure tone consists of only a single frequency. The pure tone is actually an artificial sound. Almost all sounds heard in the natural world are more complex waveforms i.e. it is a combination of multiple sinusoidal waves having different frequencies.

As stated by the Fourier Series Theorem, any periodic waveform can be produced by summing a series of sinusoidal waves with frequencies in a harmonic series and at specific phase relationships to each other. The lowest of these frequencies (the fundamental frequency) determines the pitch of the tone, which is perceived by the human hearing. Two periodic signals with identical periods but different shapes will, therefore, sound differently. More specifically, when a note is played on an instrument, the fundamental frequency (f_o) of the note (440 Hz for “A”) and also the harmonics (nf_o, where n is an integer i.e. 880Hz, 1320Hz, ... for “A”) are excited. This is the reason why a piano key sounds more natural and rich than the basic sinusoidal tone. In this laboratory exercise, you will listen to sinusoidal tones as well as the sound produced by square waves to understand how the human perception of sound is related to the fundamental frequency and harmonic components of a periodic signal.

2. Objectives

- To hear and "see" periodic signals and learn how
    - volume of a tone is dependent on the amplitude of the sinusoidal waveform,
    - the pitch is related to the fundamental frequency of a periodic signal,
    - the timbre (richness) is dependent on the number of harmonics terms in a periodic signal.
- To use the amplitude spectrum to interpret the characteristics of a sound.
- Observe how the human hearing range acts as a filter that affects our perception of sound.

3. Prior Investigations

- Sketch the amplitude spectrum of $A\sin(2\pi f t + \phi)$ with proper labelling.
- Derive the Fourier complex exponential coefficient of the periodic square wave shown in Figure 1.

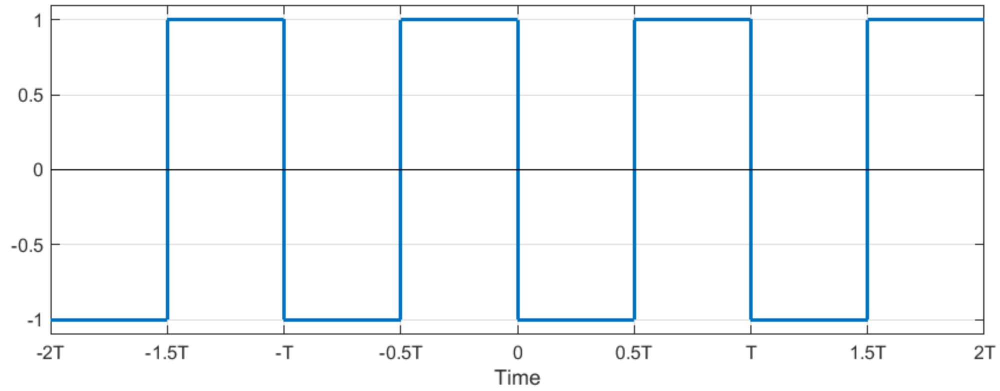

Figure 1 : Square wave with period of *T*

4. Listening to sine and square waveforms

The human hearing range, the range of frequencies that can be heard by humans, is commonly given as 20 Hz to 20,000 Hz (20 kHz). There is, however, a considerable variation between individuals, especially at high frequencies. It is also considered normal for hearing, especially high frequency tones, to gradually decline with age. The first exercise is to establish your hearing range.

- Create a temporary folder to store the files downloaded from CG2023 Workbin.
- Start Matlab2020a and change the working directory to the one you created.
- Drag and drop the file with name ***audible_signal.mdl*** from the Current Folder window to the Command Window.
- Set the volume of the computer to 50%, the amplitude of the sine waveform to unity and the frequency to 20 kHz. Can you hear the tone? Gradually reduce the frequency in steps of 1 kHz until you hear a tone. Record the frequency.
- A more comprehensive hearing test requires the measurement of the softest sound that each ear can hear at a number of specific frequencies. The test results are then presented as an audiogram, a plot of sound intensity versus frequency. Taking a sinusoidal with an amplitude of 0.1 as the reference for "perfect" hearing, sound intensity may be computed using the amplitude (A) of softest audible tone via the following equation:

$$\text{Intensity} = \log_{10}\frac{0.1}{A} \text{ decibel HL (dB HL)}$$

  - Set the amplitude of the sine wave to 0.1 and frequency to 500 Hz. Adjust the computer volume until the tone is barely audible.
  - Keeping the computer volume constant, gather data to complete Table 1. Plot the audiograms for your left and right ear.
  - Mark the area in the audiogram that represents sounds you can hear. Suppose sensitive hearing is defined as - 0.5 dB HL. Determine the range of frequencies that you are sensitive to.

Frequency (Hz)	Left Ear		Right Ear	
	Amplitude, A	Intensity (dB HL)	Amplitude, A	Intensity (dB HL)
500	0.1	0	0.1	0
5000				
10000				
12000				
14000				
15000				
16000				

Table 1 : Hearing Test Results

In this exercise, listen to the sound corresponding to the square wave shown in Figure 1 and compare with the tones generated by sine waveforms with the same fundamental frequencies.

- Compare the sound generated by a square wave of frequencies 400 Hz, 1000 Hz, 4000 Hz, 6000 Hz with sinusoidal waveforms of the corresponding fundamental frequencies.
- Do you hear any difference between the sine and the square waveform at 400 Hz, 800 Hz, 4000 Hz, 8000 Hz? Explain why by identifying the harmonic components of the square wave (Fourier Series coefficients for the square wave is derived in the Prior Investigation) and use your audiogram to determine if you can hear the higher harmonics.

5. Spectrum of square waveforms with different duty cycles

The duty cycle of a square waveform is the percentage of one period in which it has a non-zero value (See Figure 2). This section aims at exploring the effect of varying the duty cycle on the spectrum of the square waveform.

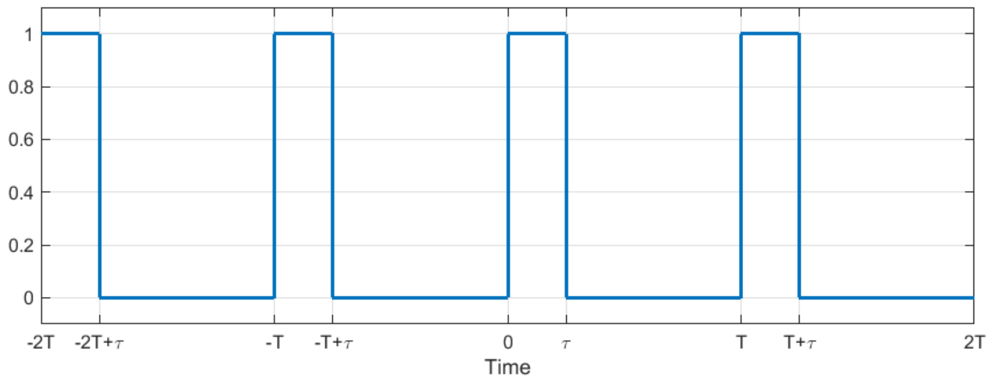

Figure 2: Square wave of period T and duty cycle $\frac{\tau}{T} \times 100\%$

- Open the ***task2.mdl*** Simulink model. It is programmed to generate and display the time domain waveforms and amplitude spectrum of a 2K Hz square waveform. The duty cycle can be varied by double-clicking on the "Pulse Wave" block.
- Generate the spectrum of square waveforms with duty cycles of 50%, 30% and 10%. Listen to the sounds generated by these 3 waveforms.
- By analysing the spectrum of the 3 waveforms, explain why the sound generated by the 50% duty cycle waveform is audibly different from the rest.

- Unlike the spectrum for the square wave in Part 4, there is a spectral component at 0 Hz. Which waveform has the largest spectral component at 0 Hz. Explain why.
- Can our ears "hear" the spectral component at 0 Hz?

6. Phenomenon of Beats

In acoustics, a beat is an interference between two pure sinusoidal tones of different frequencies, perceived as periodic variations in volume. Beats may be generated by mixing (multiplying) two sinusoidal waveforms of different frequencies. In this exercise, it is assumed that the frequency of one sinusoidal waveform is in the audible range, and the frequency of the second signal is much lower.

- Open the ***beats.mdl*** Simulink model.
- Write down the time-domain expression of the signal generated by the model.
- From the mathematical expression or by clicking on the "Scope" block to examine the time-domain waveform, describe what you expect to hear if the waveform is converted to sound.
- Derive and sketch the amplitude spectrum of the signal.
  Hint : $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
  What are the new frequencies created by the mixing process? These new frequencies are called heterodynes, and the mixing process is also known as heterodyning. Explain why it may be useful to create the new frequencies.

THE END

Please delete the folder and files that was created during the course of the experiment and submit your report before leaving the lab.