★ Sketch the magnitude and Phase Spectra of

- $x(t) = 8\sin(12\pi t + \frac{\pi}{2})$

$= \frac{8}{2j}\left(\exp(j(12\pi t + \frac{\pi}{2})) - \exp(-j(12\pi t + \frac{\pi}{2}))\right)$

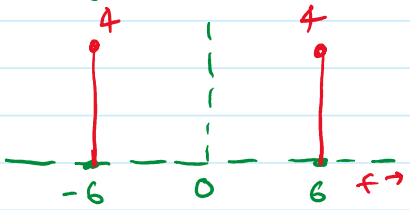
$= -4j\left(\exp(j12\pi t)\exp(j\frac{\pi}{2}) - \exp(-j(12\pi t))\exp(-j\frac{\pi}{2})\right)$

$= -4\exp(j\frac{\pi}{2})\exp(j12\pi t)\exp(j\frac{\pi}{2}) + 4\exp(j\frac{\pi}{2})\exp(-j12\pi t)\exp(-j\frac{\pi}{2})$

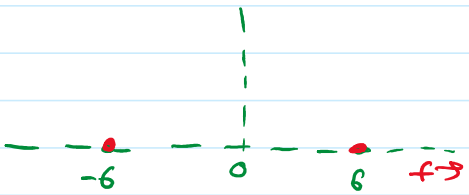
$= 4\exp(j\pi)\exp(j\frac{\pi}{2})\exp(j\frac{\pi}{2})\exp(j12\pi t) + 4\exp(j0)\exp(-j12\pi t)$

$= 4\exp(j2\pi)\exp(j2\pi(6)t) + 4\exp(j0)\exp(j2\pi(-6)t)$

Magnitude Spectrum



Phase Spectrum



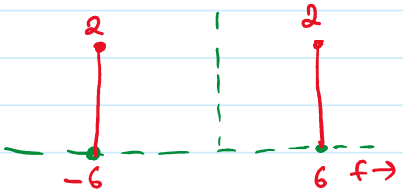
- $y(t) = 4\cos(12\pi t + \frac{\pi}{16})$

$= \frac{4}{2}\left(\exp(j(12\pi t + \frac{\pi}{16})) + \exp(-j(12\pi t + \frac{\pi}{16}))\right)$

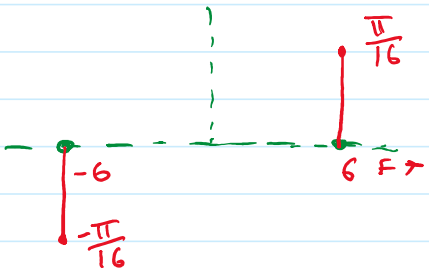
$= \frac{4}{2}\exp(j12\pi t).\exp(\frac{\pi}{16}) + \frac{4}{2}\exp(-j12\pi t).\exp(-j\frac{\pi}{16})$

$= \frac{4}{2}\exp(j2\pi(6)t).\exp(\frac{\pi}{16}) + \frac{4}{2}\exp(j2\pi(-6)t)\exp(-j\frac{\pi}{16})$

Magnitude Spectrum



Phase Spectrum

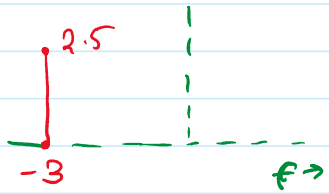


- $z(t) = 2.5 \exp\left(-j\left(6\pi t + \frac{\pi}{6}\right)\right)$

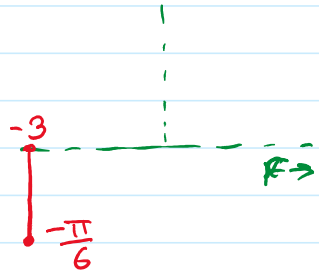
$= 2.5 \exp(-j6\pi t) \cdot \exp(-j\pi/6)$

$= 2.5 \exp(j2\pi(-3)t) \exp\left(-\frac{j\pi}{6}\right)$

Magnitude Spectrum



Phase Spectrum



★ Determine the magnitude and phase spectra of the following signals

- $x(t) = \sin^2 t$

$= \frac{1 - \cos 2t}{2}$

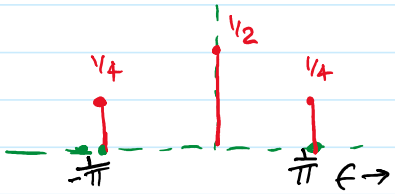
$= \frac{1}{2} - \frac{1}{2}\cos 2t$

$= \frac{1}{2} - \frac{1}{2}\left(\frac{\exp(j2t) + \exp(-j2t)}{2}\right)$

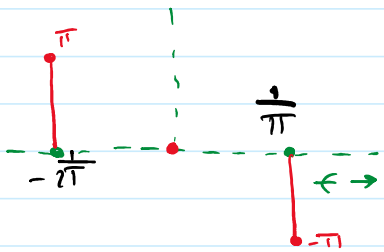
$= \frac{1}{2} - \frac{1}{4}\exp(j2t) - \frac{1}{4}\exp(-j2t)$

$= \frac{1}{2} + \frac{1}{4}\exp(j2t)\exp(j\pi) + \frac{1}{4}\exp(-j2t)\exp(-j\pi)$

Magnitude Spectrum



Phase Spectrum



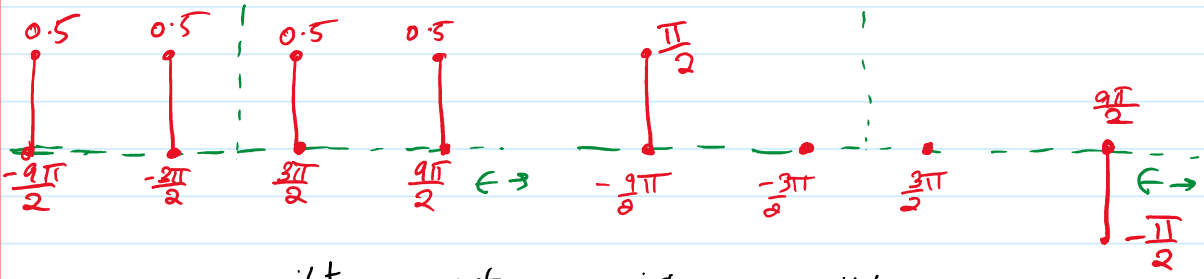
- $y(t) = \sin 9t + \cos 3t$

$= \frac{1}{2j}\left(e(j9t) - e(-j9t)\right) + \frac{1}{2}\left(e(j3t) + e(-j3t)\right)$

$= 0.5\exp(j9t)\exp(-j\frac{\pi}{2}) + 0.5\exp(-j9t)\exp(j\frac{\pi}{2}) + 0.5\exp(j3t) + 0.5\exp(j3t)$

$$= 0.5\exp(j9t)\exp(-j\tfrac{\pi}{2}) + 0.5\exp(-j9t)\exp(j\tfrac{\pi}{2}) + 0.5\exp(j3t) + 0.5\exp(j3t)$$

Magnitude Spectrum



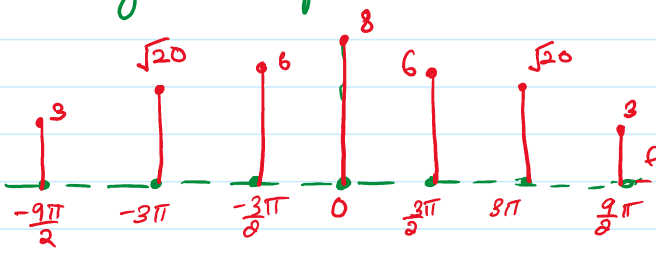
Phase Spectrum

$$x(t) = (4+2j)e^{-j6t} + 3je^{-j9t} + 8 - 3je^{j9t} + (4-2j)e^{j6t} + 6je^{j8t} - 6je^{-j3t}$$

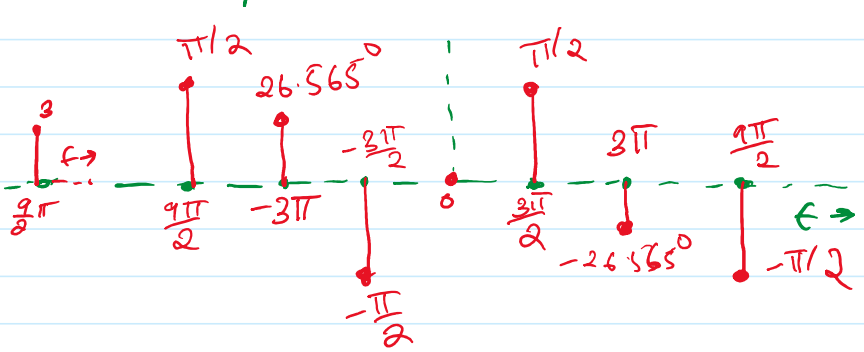
$$= 4e^{-j6t} + 2je^{-j6t} + 3je^{-j9t} - 6je^{-j3t} + 8 + 6je^{j3t} - 3je^{j9t} + 4e^{j6t} - 2je^{j6t}$$

$$= 4e^{-j6t} + 2e^{-j6t}\cdot e^{j\pi/2} + 3e^{-j9t}e^{j\pi/2} + 6e^{-j3t}e^{-j\pi/2} + 8e^{0} + 4e^{j6t} + 2e^{j6t}\cdot e^{-j\pi/2} + 3e^{j9t}e^{-j\pi/2} + 6e^{j3t}e^{j\pi/2}$$

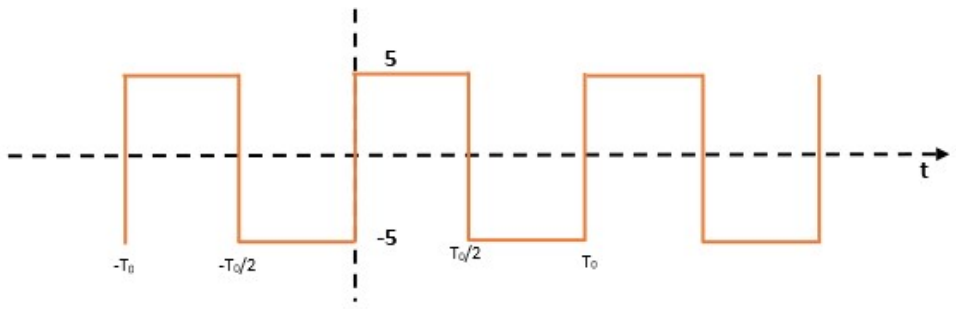
Magnitude Spectrum



Phase Spectrum



- X(t)➔



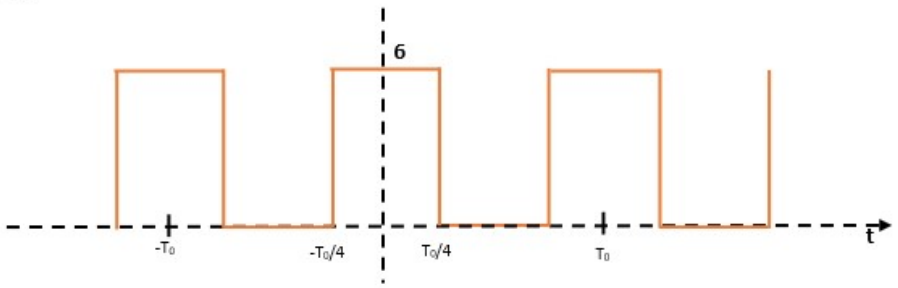
Solution – Signals & Systems – Schaum's Outline

Solved Example 5.6

Solved Example 5.6

signals-and-systems-2nd-edition-schaums-outline-series

- Y(t) →



Solution – Signals & Systems – Schaum's Outline

Solved Example 5.7

signals-and-systems-2nd-edition-schaums-outline-series