

**EE2023 TUTORIAL 1 (SOLUTIONS)****Solution to Q.1**

Write  $z$  in polar form:

$$z = x + jy = |z| \exp(j\angle z).$$

Since adding integer multiples of  $2\pi$  to  $\angle z$  does not affect the value of  $z$ , we may also express  $z$  as

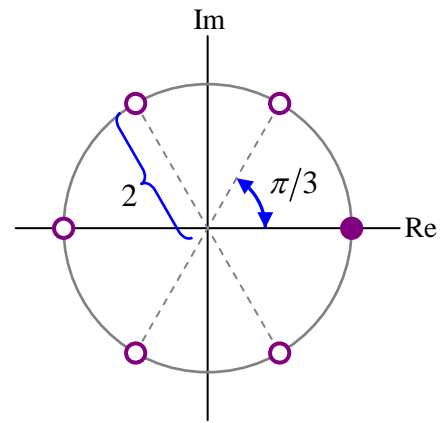
$$z = |z| \exp(j(\angle z + 2k\pi))$$

where  $k$  is an integer. This leads to

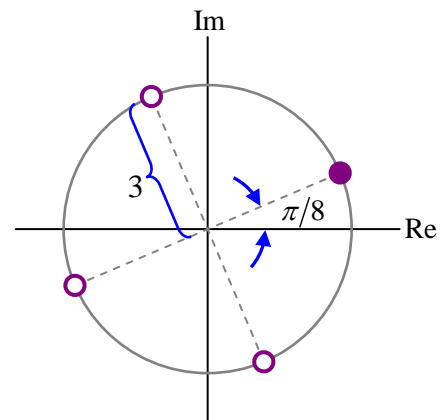
$$z^{1/N} = |z|^{1/N} \exp\left(j\left(\frac{\angle z}{N} + \frac{2k\pi}{N}\right)\right); \quad k = 0, 1, \dots, N-1,$$

which yields the  $N$  distinct values of  $z^{1/N}$ .

$$64^{1/6}: \quad \left\{ \begin{array}{l} z = 64 \rightarrow \begin{cases} |z| = 64 \\ \angle z = 0 \end{cases} \\ 64^{1/6} = |z|^{1/N} \exp\left(j\left(\frac{\angle z}{N} + \frac{2k\pi}{N}\right)\right) \Big|_{z=64, N=6} \\ = 2 \exp\left(j\left(\frac{k\pi}{3}\right)\right); \quad k = 0, 1, \dots, 5 \\ = \begin{cases} 2; 2 \exp\left(j\left(\frac{\pi}{3}\right)\right); 2 \exp\left(j\left(\frac{2\pi}{3}\right)\right); \\ -2; 2 \exp\left(j\left(\frac{4\pi}{3}\right)\right); 2 \exp\left(j\left(\frac{5\pi}{3}\right)\right) \end{cases} \end{array} \right.$$



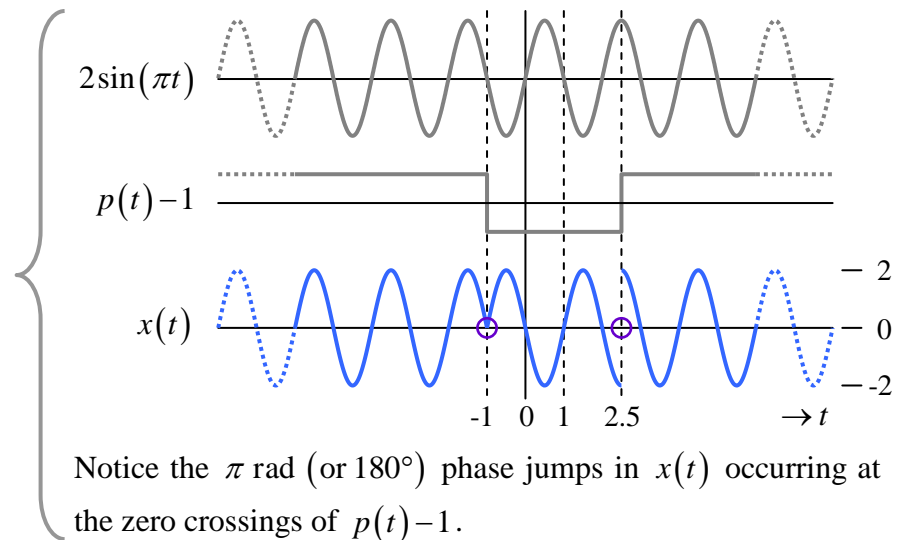
$$(j81)^{1/4}: \quad \left\{ \begin{array}{l} z = j81 \rightarrow \begin{cases} |z| = 81 \\ \angle z = \frac{\pi}{2} \end{cases} \\ (j81)^{1/4} = |z|^{1/N} \exp\left(j\left(\frac{\angle z}{N} + \frac{2k\pi}{N}\right)\right) \Big|_{z=81, N=4} \\ = 3 \exp\left(j\left(\frac{\pi}{8} + \frac{k\pi}{2}\right)\right); \quad k = 0, 1, \dots, 3 \\ = \begin{cases} 3 \exp\left(j\left(\frac{\pi}{8}\right)\right), 3 \exp\left(j\left(\frac{5\pi}{8}\right)\right), \\ 3 \exp\left(j\left(\frac{9\pi}{8}\right)\right), 3 \exp\left(j\left(\frac{13\pi}{8}\right)\right) \end{cases} \end{array} \right.$$



**Solution to Q.2**

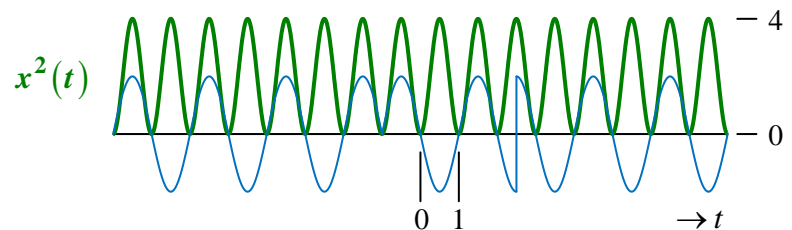
(a)  $p(t) = 2 - 2\text{rect}\left(\frac{t-0.75}{3.5}\right)$

(b) By inspection,  $x(t)$  is not periodic.



(c)

$$\begin{aligned} x^2(t) &= 4\sin^2(\pi t) \underbrace{(p(t)-1)^2}_1 \\ &= 4\sin^2(\pi t) \\ &= 2(1 - \cos(2\pi t)) \end{aligned}$$



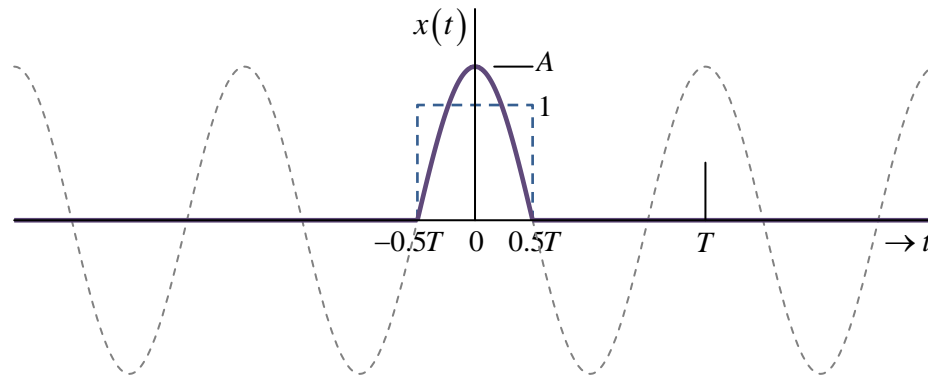
Note that  $x^2(t)$  is periodic with a period of  $T = 1$ .

Average Power: 
$$\left\{ \begin{aligned} P &= \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt = \int_{-0.5}^{0.5} 2(1 - \cos(2\pi t)) dt = 2 \\ &\underbrace{\hspace{1cm}}_{x^2(t) \text{ is periodic. } \therefore P \text{ can be obtained by averaging over one period.}} \end{aligned} \right.$$

(d) Since the average power of  $x(t)$  is finite, its total energy must be infinite.  $x(t)$  is an aperiodic power signal.

### Solution to Q.3

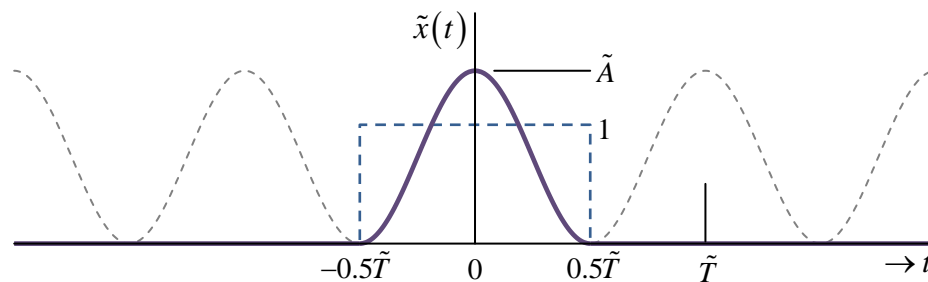
**Half-cosine pulse:**  $x(t) = A \cos\left(\frac{\pi t}{T}\right) \text{rect}\left(\frac{t}{T}\right)$



$$x^2(t) = \frac{A^2}{2} \left[ 1 + \cos\left(\frac{2\pi t}{T}\right) \right] \text{rect}\left(\frac{t}{T}\right)$$

**Energy :**  $E = \frac{A^2}{2} \underbrace{\int_{-0.5T}^{0.5T} 1 + \cos\left(\frac{2\pi t}{T}\right) dt}_{\substack{\text{over one} \\ \text{period} = 0}} = \frac{1}{2} A^2 T$

**Raised-cosine pulse:**  $\tilde{x}(t) = \frac{\tilde{A}}{2} \left( 1 + \cos\left(\frac{2\pi t}{\tilde{T}}\right) \right) \text{rect}\left(\frac{t}{\tilde{T}}\right)$



$$\tilde{x}^2(t) = \frac{\tilde{A}^2}{4} \left[ \frac{3}{2} + 2\cos\left(\frac{2\pi t}{\tilde{T}}\right) + \frac{1}{2}\cos\left(\frac{4\pi t}{\tilde{T}}\right) \right] \text{rect}\left(\frac{t}{\tilde{T}}\right)$$

**Energy :**  $\tilde{E} = \frac{\tilde{A}^2}{4} \underbrace{\int_{-0.5\tilde{T}}^{0.5\tilde{T}} \frac{3}{2} + 2\cos\left(\frac{2\pi t}{\tilde{T}}\right) dt}_{\substack{\text{over one} \\ \text{period} = 0}} + \frac{1}{2} \underbrace{\int_{-0.5\tilde{T}}^{0.5\tilde{T}} \cos\left(\frac{4\pi t}{\tilde{T}}\right) dt}_{\substack{\text{over two} \\ \text{periods} = 0}} = \frac{3}{8} \tilde{A}^2 \tilde{T}$

**Both  $x(t)$  and  $\tilde{x}(t)$  will have the same energy if  $A^2 T = \frac{3}{4} \tilde{A}^2 \tilde{T}$ .**

**Solution to Q.4**

$$(a) \quad x(t) = \cos(3.2t) + \sin(1.6t) + \exp(j2.8t) \quad \cdots \quad \begin{cases} \cos(3.2t) & \text{has a frequency of } 3.2 \text{ rad/s} \\ \sin(1.6t) & \text{has a frequency of } 1.6 \text{ rad/s} \\ \exp(j2.8t) & \text{has a frequency of } 2.8 \text{ rad/s} \end{cases}$$

Highest common factor (HCF) of  $\{3.2, 1.6, 2.8\}$  exists and is equal to 0.4. Thus,  $x(t)$  is periodic. Since there is no harmonic cancellation among the sinusoidal components of  $x(t)$ , the fundamental frequency and period of  $x(t)$  are 0.4 rad/s (or  $0.2/\pi$  Hz) and  $5\pi$  s, respectively.

**REMARKS:** Although  $x(t)$  is periodic with a fundamental frequency of 0.4 rad/s, it does not contain the fundamental frequency component itself.

$$(b) \quad x(t) = \cos(4t) + \sin(\pi t) \quad \cdots \quad \begin{cases} \cos(4t) & \text{has a frequency of } 4 \text{ rad/s} \\ \sin(\pi t) & \text{has a frequency of } \pi \text{ rad/s} \end{cases}$$

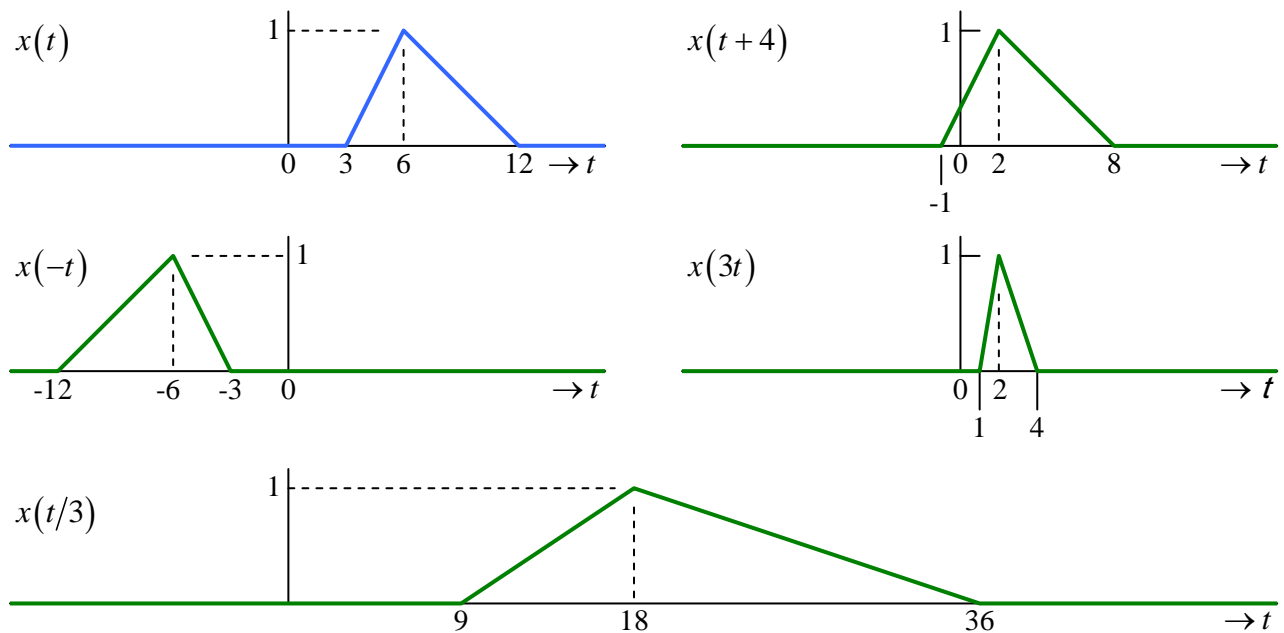
Highest common factor (HCF) of  $\{4, \pi\}$  does not exist. Thus,  $x(t)$  is not periodic.

**REMARKS:** Summing sinusoids does not necessarily lead to a periodic signal unless the frequencies of the sinusoids are harmonics of a common fundamental frequency.

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**Solution to Q.5**

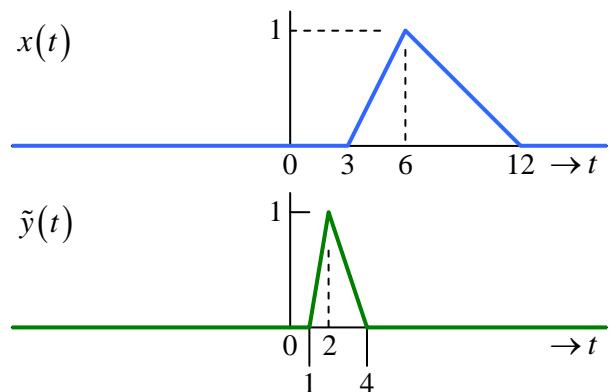
(a)



(b) We observe that  $y(t)$  is a time-scaled, -reversed and -shifted version of  $x(t)$ .

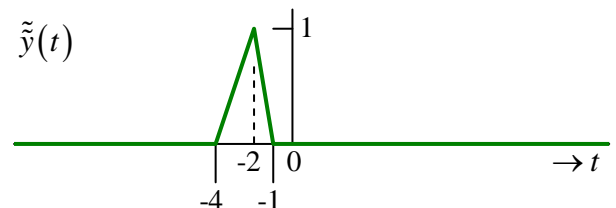
For problems of this nature, we should start with time-scaling first since it involves linear warping of the time axis. If we were to start with time-shifting and/or time-reversal, we may have to redo them after time-scaling. However, this sequence of operation need not be followed if we are sketching the signal from the mathematical expression.

Comparing  $x(t)$  and  $y(t)$ , we note that  $y(t)$  involves time-scaling (or contraction) of  $x(t)$  by a factor of 3.



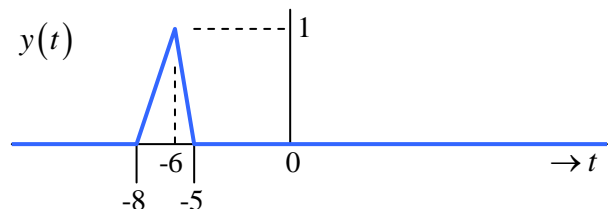
Time-scaling of  $x(t)$ :  $\tilde{y}(t) = x(3t)$

Time-reversal of  $\tilde{y}(t)$ :  $\tilde{\tilde{y}}(t) = \tilde{y}(-t) = x(-3t)$



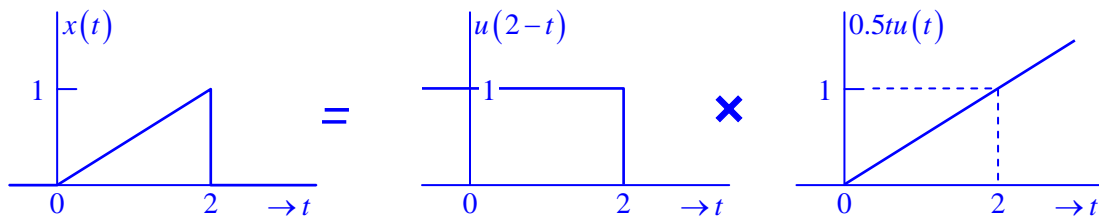
Time shifting of  $\tilde{\tilde{y}}(t)$ : 
$$\begin{cases} y(t) = \tilde{\tilde{y}}(t+4) \\ \quad = x(-3(t+4)) \end{cases}$$

$\therefore y(t) = x(-3t-12)$

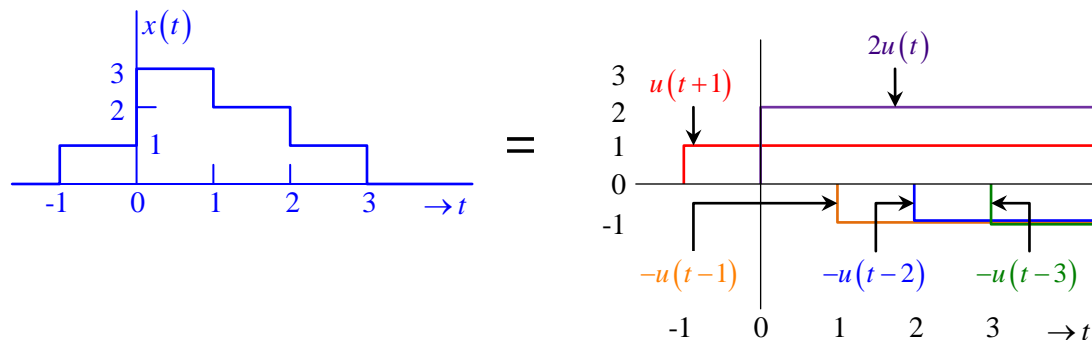


**Solution to S.1**

$$(a) \quad x(t) = u(2-t) \cdot 0.5tu(t) = u(2-t) \cdot \int_{-\infty}^t 0.5u(\tau) d\tau$$



$$(b) \quad x(t) = u(t+1) + 2u(t) - u(t-1) - u(t-2) - u(t-3)$$

**Solution to S.2**

$$(a) \quad x(t) = \cos(2t + 0.25\pi) = \cos\left(2\pi \frac{1}{\pi} t + 0.25\pi\right) \text{ is a sinusoid of amplitude 1 and frequency } \frac{1}{\pi}.$$

$\rightarrow$  periodic, period  $= \pi$ , power  $= 1/2$

$$(b) \quad x(t) = \cos^2(t) = 0.5[1 + \cos(2t)] = 0.5 + 0.5\cos\left(2\pi \frac{1}{\pi} t\right) \text{ is a sinusoid of amplitude 0.5 and frequency } \frac{1}{\pi}, \text{ plus a 0.5 dc value.}$$

$$\rightarrow \text{periodic, period} = \pi, \text{ power} = \frac{0.5^2}{2} + 0.5^2 = \frac{3}{8}$$

$$(c) \quad x(t) = \cos(2\pi t)u(t) \text{ does not satisfy } x(t) = x(t+T) \quad \forall t \text{ where } T \text{ is a finite constant.}$$

$\rightarrow$  non-periodic

$$(d) \quad x(t) = \exp(j\pi t) = \exp\left(j2\pi \frac{1}{2} t\right) \text{ is a complex sinusoid of amplitude 1 and frequency } \frac{1}{2}.$$

$\rightarrow$  periodic, period  $= 2$ , power  $= 1$

### Solution to S.3

(a) When  $t < 0$ :  $\int_{-\infty}^t \cos(\tau)u(\tau)d\tau = 0$

When  $t \geq 0$ :  $\int_{-\infty}^t \cos(\tau)u(\tau)d\tau = \int_0^t \cos(\tau)d\tau = \sin(\tau)\Big|_0^t = \sin(t)$

Combining the 2 cases:  $\int_{-\infty}^t \cos(\tau)u(\tau)d\tau = \sin(t)u(t)$

(b) When  $t < 0$ :  $\int_{-\infty}^t \cos(\tau)\delta(\tau)d\tau = 0$

When  $t \geq 0$ :  $\int_{-\infty}^t \cos(\tau)\delta(\tau)d\tau = 1$

Combining the 2 cases:  $\int_{-\infty}^t \cos(\tau)\delta(\tau)d\tau = u(t)$

(c)  $\int_{-\infty}^{\infty} \cos(t)u(t-1)\delta(t)dt = 0$  because  $u(t-1)\delta(t) = 0 \forall t$

(d)  $\underbrace{\int_0^{2\pi} t \sin\left(\frac{t}{2}\right)\delta(\pi-t)dt}_{\text{sifting property of } \delta\text{-function}} = \pi \sin\left(\frac{\pi}{2}\right) = \pi$

### Solution to S.4

(a)  $x(t) = u(t) = \begin{cases} 1; & t \geq 0 \\ 0; & t < 0 \end{cases}$

$$x_e(t) = 0.5[u(t) + u(-t)] = \begin{cases} 1; & t = 0 \\ 0.5; & t \neq 0 \end{cases}$$

$$x_o(t) = 0.5[u(t) - u(-t)] = \begin{cases} 0; & t = 0 \\ 0.5; & t > 0 \\ -0.5; & t < 0 \end{cases}$$

(b)  $x(t) = \sin\left(\omega_c t + \frac{\pi}{4}\right)$

$$\begin{aligned} x_e(t) &= 0.5\left[\sin\left(\omega_c t + \frac{\pi}{4}\right) + \sin\left(-\omega_c t + \frac{\pi}{4}\right)\right] \\ &= \sin\left(\frac{\pi}{4}\right)\cos(\omega_c t) = \frac{1}{\sqrt{2}}\cos(\omega_c t) \end{aligned}$$

$$\begin{aligned} x_o(t) &= 0.5\left[\sin\left(\omega_c t + \frac{\pi}{4}\right) - \sin\left(-\omega_c t + \frac{\pi}{4}\right)\right] \\ &= 0.5\left[\sin\left(\omega_c t + \frac{\pi}{4}\right) + \sin\left(\omega_c t - \frac{\pi}{4}\right)\right] \\ &= \sin(\omega_c t)\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\sin(\omega_c t) \end{aligned}$$

where we make use of the trigonometric relationship  $\sin(A) + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)$ .