CG2023 TUTORIAL 4 (PROBLEMS)

Q.1 A very simplified version of a car suspension system is shown in Fig.Q.1(a). The transfer function of the simplified car suspension system is

$$\frac{\tilde{X}_o(s)}{\tilde{X}_i(s)} = \frac{bs+k}{ms^2+bs+k}.$$

where $\tilde{X}_i(s)$ and $\tilde{X}_o(s)$ are the Laplace transforms of $x_i(t)$ and $x_o(t)$, respectively.

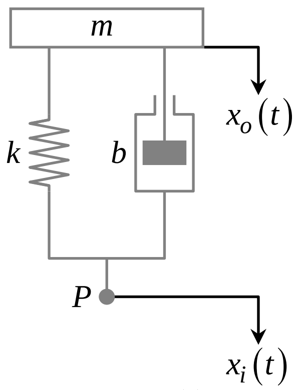


Fig.Q.1(a)

Suppose a car $\left(m = 1kg,\ k = 1\frac{N}{m} \text{ and } b = \sqrt{2}\frac{N}{m/s}\right)$ is travelling on a road that has speed reducing stripes and the input to the simplified car suspension system, $x_i(t)$, may be modelled by the periodic square wave shown in Fig.Q.1(b).

Determine the steady-state displacement of the car body, $x_o(t)$.

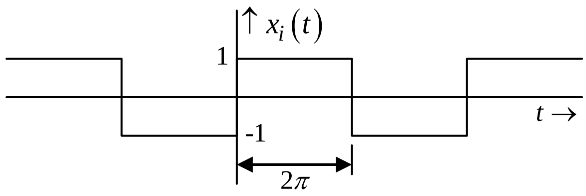


Fig.Q.1(b)

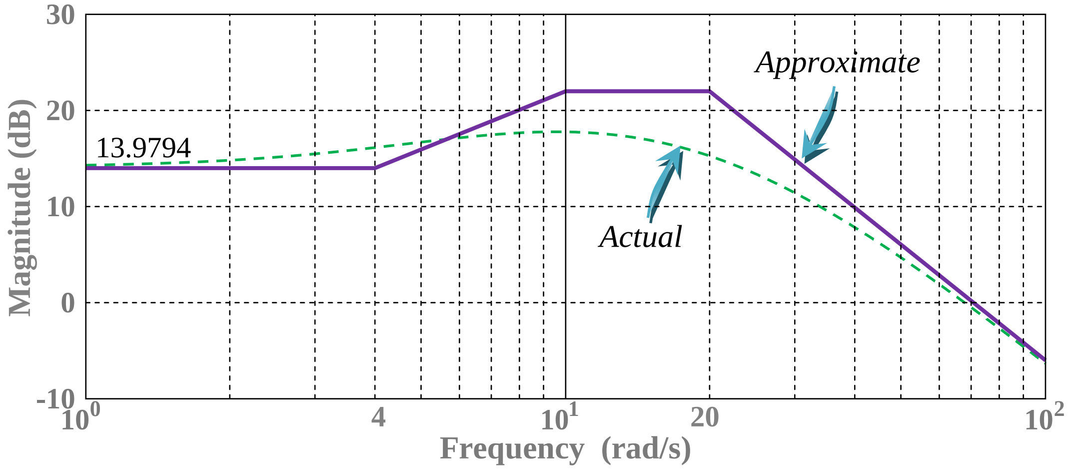
Hint: The Fourier Series representation of the periodic square wave shown in Q.1(b) is

$$x_i(t) = \frac{4}{\pi}\left[\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \cdots\right].$$

Q.2 A high speed recorder monitors the temperature of an air stream as sensed by a thermocouple. The recorded steady-state temperature may be expressed as $50 + 2\sin(4\pi t)$. If the system (thermocouple and high speed recorder) transfer function is $\tilde{H}(s) = \frac{1}{1+s}$, estimate the actual maximum and minimum air temperatures.

Q.3 The magnitude plot of a LTI system $\tilde{H}(s) = \frac{K(s+\alpha)}{(s+\beta)(s+\gamma)(s+\lambda)}$ is shown in Fig.Q.3. Using the approximate (straight line asymptotes) magnitude response, determine $K,\ \alpha,\ \beta,\ \gamma$ and λ.

Fig.Q.3



Q.4

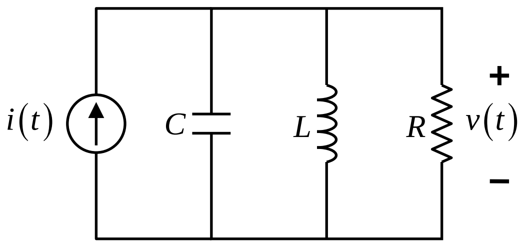


Fig.Q.4: RLC Circuit $\left(R = 40\Omega \quad C = 10^{-4}F \quad L = 1H\right)$

Fig.Q.4 shows a parallel RLC circuit which when driven by a current source, $i(t)$, produces a voltage drop, $v(t)$, across its elements. Let $\tilde{I}(s)$ and $\tilde{V}(s)$ be the Laplace transform of $i(t)$ and $v(t)$, respectively.

(a) Find the system transfer function $\tilde{H}(s) = \dfrac{\tilde{V}(s)}{\tilde{I}(s)}$.

(b) Find $v(t)$ if $i(t) = 5\cos(100t)$.

(c) Draw the straight-line Bode plots for $\tilde{H}(s)$.

Supplementary Problems

These problems are for self practice.

S.1 Find the response of the first order system, $\tilde{H}(s)=\dfrac{2}{0.2s+1}$, to an input sinusoid of $\sin(3t)$.

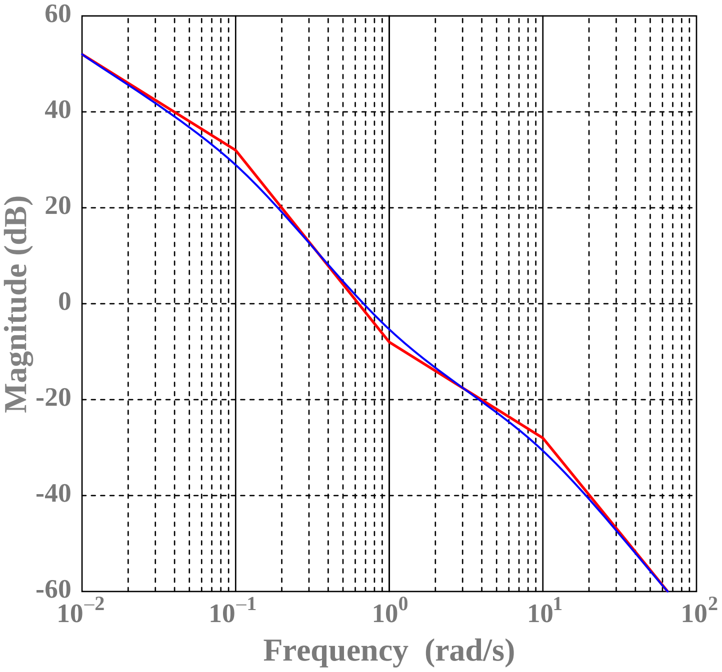
Answer: $1.71\sin(3t-0.54)$

S.2 The steady-state output of a first order system, $\tilde{H}(s)$, is $4.5\sin(5t-38°)$. Assuming that $\left|\tilde{H}(j5)\right|=0.75$ and $\angle\tilde{H}(j5)=-68°$, find the input signal.

Answer: $6\sin(5t+30°)$

S.3 The magnitude response for the system $\tilde{H}(s)$ is shown in Fig.S.3.

Fig.S.3



(a) What is the slope of the high frequency asymptote of the magnitude response?

Answer: -40 db/decade

(b) $\tilde{H}(s)$ has how many poles, zeros and integrators?

Answer: 3 Poles, 1 Zero and 1 Integrator

(c) The low frequency asymptote of the magnitude response is $\dfrac{K}{s}$. Find the value of K.

Answer: $K=4$

S.4 The frequency response of a system is given by $\tilde{H}(j\omega)=\dfrac{50}{5-2\omega^2+j11\omega}$. Draw the straight-line Bode magnitude plot for this system.

Answer: Horizontal line at *10dB* from low frequency up to $0.5\ rad/s$. Straight line of slope $-20\ dB/decade$ between $0.5\ rad/s$ and $5\ rad/s$. Straight line of slope $-40\ dB/decade$ from $5\ rad/s$ onwards.

S.5 Fig.S.5 shows the Bode diagram of a system whose transfer function is

$$\tilde{H}(s)=\frac{K(s+a)}{(s+b)\left(s^2+2\zeta\omega_n s+\omega_n^2\right)}.$$

What are the values of K, a, b, ζ and ω_n?

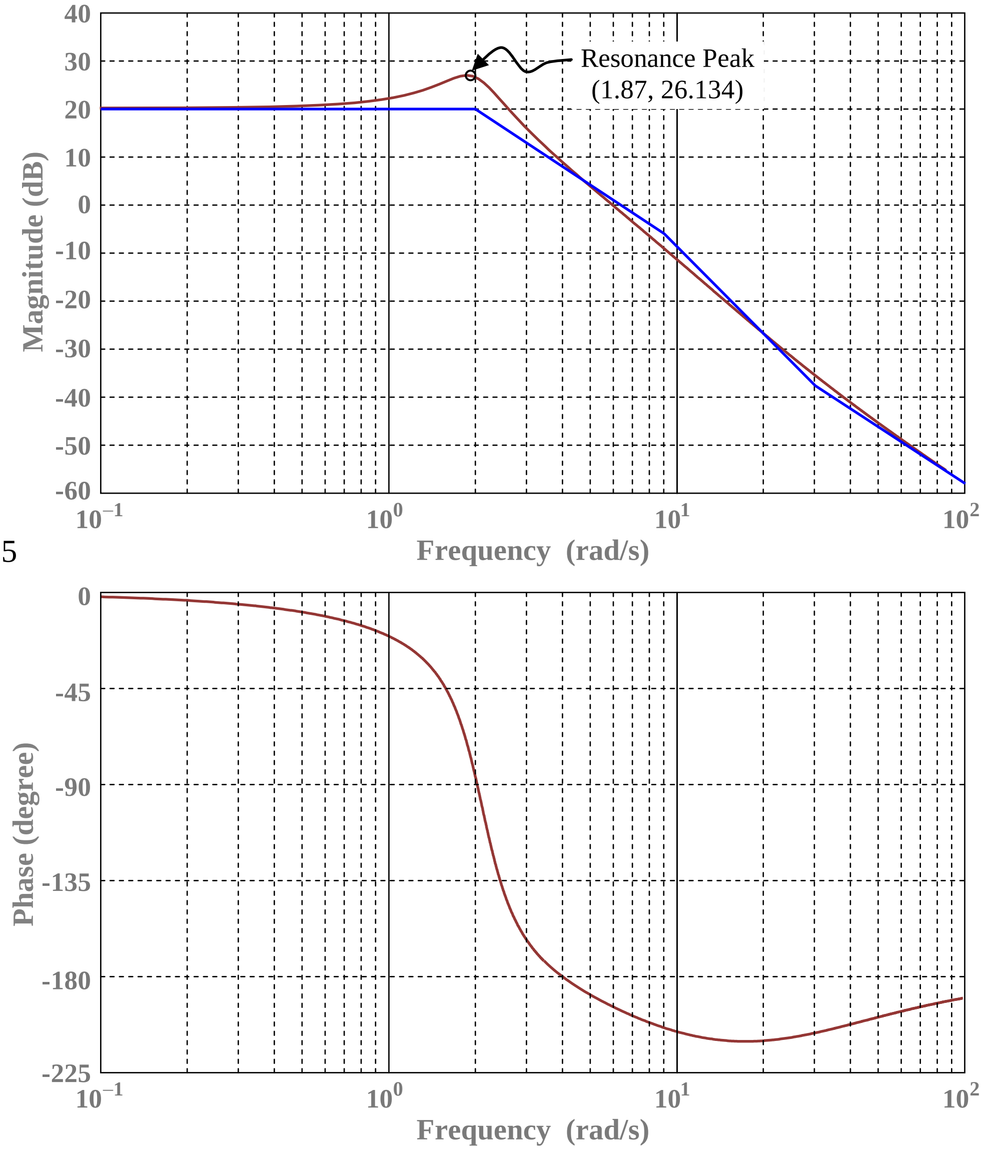


Fig.S.5

Answer: $K=12,\ a=30,\ b=9,\ \zeta=0.25,\ \omega_n=2$