CG2023 TUTORIAL 5 (PROBLEMS)

Q.1 The signal $x(t)$ shown in Fig.Q.1 is sampled at 5 Hz to form a continuous-time signal $x_s(t)$. Let $X_s(f)$ be the spectrum of $x_s(t)$. Sketch $|X_s(f)|$ and $\angle X_s(f)$.

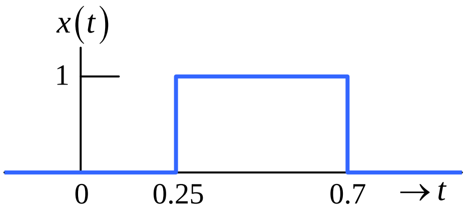


Fig.Q.1

Q.2 The signal $x(t)$ shown in Fig.Q.2 is sampled at 0.25 Hz to form a continuous-time signal $x_s(t)$. Let $X_s(f)$ be the spectrum of $x_s(t)$. Determine $X_s(f)$ and show that it has a repetition period of 0.25 Hz.

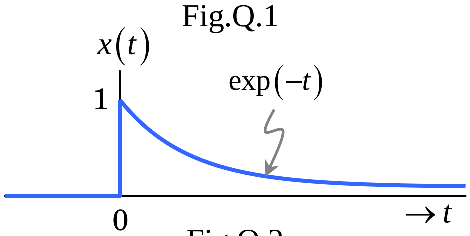


Fig.Q.2

Q.3 When a periodic signal $x(t)$ is modeled as

$$x(t) = \sum_{n=-\infty}^{\infty} g(t - nT),$$

we call $g(t)$ the generating function of $x(t)$. Let $X(f)$ and $G(f)$ be the Fourier transforms of $x(t)$ and $g(t)$, respectively, and c_k be the complex exponential Fourier series coefficients of $x(t)$.

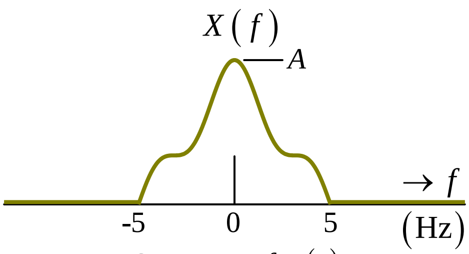
(a) Show how may $X(f)$ be obtained from $G(f)$ directly.

(b) Express c_k in terms of $G(f)$.

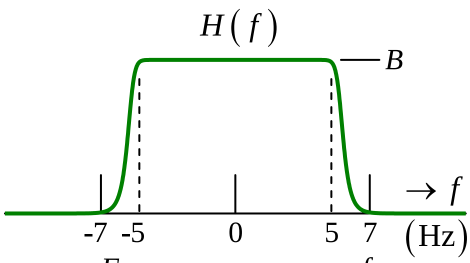
(c) Give an example to show that for a given $x(t)$, the generating function $g(t)$ is not unique.

Q.4 A signal $x(t)$ is sampled, stored and later reconstructed from the stored samples using a reconstruction filter. The spectrum of $x(t)$ and the frequency response of the reconstruction filter are shown in Fig.Q.4. What is the Nyquist sampling frequency for $x(t)$? What sampling frequency would you recommend so that $x(t)$ can be reconstructed from its samples without distortion, and why? Illustrate your answer.

Fig.Q.4



Spectrum of $x(t)$

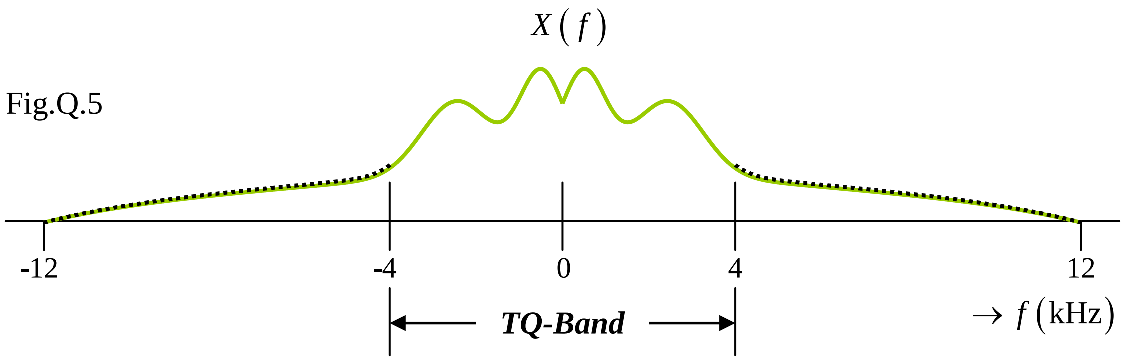


Frequency response of reconstruction filter

Q.5 Speech studies have shown that perceptual cues pertaining to speech intelligibility and speaker identity are mainly found within the first $4\,\text{kHz}$ of a speech spectrum. We shall refer to this frequency band as the *TQ-Band*, where *TQ* stands for '*Telephone Quality*'.

The spectrum of a speech signal $x(t)$ is shown in Fig.Q.5 where the *TQ-Band* is indicated.

Fig.Q.5



In order to send $x(t)$ over a digital telephone network, the first step is to sample $x(t)$ without corrupting its *TQ-Band*. Suggest a method for sampling $x(t)$ in each of the following situations.

Situation A: Anti-aliasing lowpass filter is not available and frequency aliasing is prohibited.

Situation B: Lowest sampling frequency must be used at all cost.

Situation C: Anti-aliasing lowpass filter is not available and frequency aliasing is permitted.

Comment on the advantage and disadvantage of each method.

Supplementary Problems

These problems will not be discussed in class.

S.1 A bandlimited lowpass signal $x(t)$ of bandwidth $20(kHz)$ is sampled at a rate of f_S samples/sec to form $x_S(t)$. $x_S(t)$ is then sent through a reconstruction filter having frequency response $H(f)$ so that $x(t)$ is exactly reproduced at the filter's output. Find the smallest applicable value of f_S and specify the corresponding $H(f)$.

Answer : $f_S = 40\ kHz, \quad H(f) = \frac{1}{40000} \cdot \text{rect}\left(\frac{f}{40000}\right)$

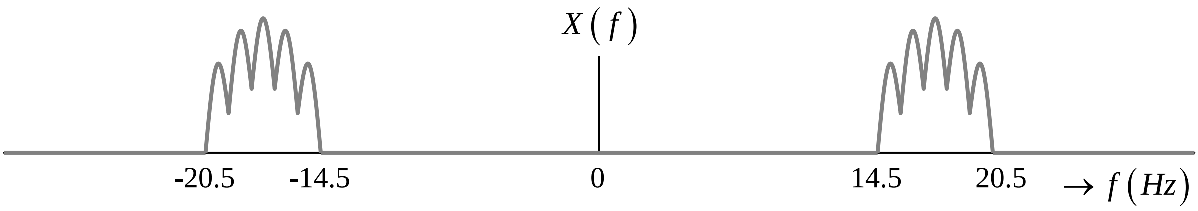
S.2 The sampled version of a signal $x(t)$ has the form $x_S(t) = \sum_{n=-5}^{5} x(5n)\delta(t-5n)$.

(a) Determine the sampling frequency used.

(b) If $x(t) = \text{tri}(t)$, can it be perfectly reconstructed from $x_S(t)$, and why?

Answer : (a) $0.2\ Hz$ (b) *No*

S.3 The spectrum of a bandpass signal $x(t)$ is shown in the figure below.



(a) What is the Nyquist sampling frequency for $x(t)$.

(b) Determine the lowest sampling frequency that can be used so that $x(t)$ may be reconstructed from its sampled version without distortion. Specify the reconstruction filter.

Answer : (a) $41\ Hz$ (b) $7\ Hz$

Below is a list of solved problems selected from ***Chapter 5*** *of* ***Hwei Hsu (PhD), 'The Schaum's series on Signals & Systems,' 2nd Edition***.

Selected solved-problems: ***5.58, 5.59, 5.60***

These solved problems should be treated as supplementary module material catered for students who find the need for more examples or practice-problems

---