CG2023 Signals & Systems

AY2020/21-2

Online Final Exam

Date: 30 April 2021 Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES:

1. This paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages.
2. Answer **ALL** questions in both Sections A and B.
3. The maximum mark for this paper is 80.
4. Write the answers for each question on a new page and paginate your answer script.
5. This is an **OPEN BOOK** exam. You are not allowed to consult, discuss and/or collaborate with any other person regarding the exam.
6. The "**Exam Declaration Form**" is located in the APPENDIX.
7. For the purpose of this examination, please take note of the digits ***a***, ***b***, ***c*** and ***d*** in your student number A0xx***abcd***X. These digits will be used in the questions in this quiz.

At the end of the exam, please do the following:

a. Enter your matriculation number and the total number of submitted pages in the boxes below.

Matriculation №:	

Number of pages in this PDF file (including this cover page and the Exam Declaration Form): i.e. 2 + number of answer pages	

b. Use a scanning app (such as Microsoft Office Lens or CamScanner) to capture this cover page, your answer pages (in page number sequence) and the filled/signed Exam Declaration Form in the **Appendix**.

c. Generate a consolidated PDF file consisting of this cover page, followed by the **signed** Exam Declaration Form, followed by your answer pages using the filename "**A0123456J-CG2023-Exam.pdf**" where you replace **A0123456J** with your **matriculation number**. Upload the consolidated PDF file to the

Your Group Number ↓

LumiNUS → CG2023 → Files → EXAM → Exam Submission Group (#)

folder within **20 minutes** from the time the examination ended. The Exam Submission Group (#) is the newly created exam groups and **not your lecture group**.

For examiner use only →

Question	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	TOTAL
Marks							

QUESTIONS

ATTENTION!

- **Justify each of your answers by showing the working or providing the reason that leads to it.**
- **Ensure that the parameter values are accurately derived from your student number and substituted into your answers.**

A penalty of up to 100% of the marks may be imposed for answer without justification or question solved without using the correct parameter values.

SECTION A

Answer ALL questions in this section (Each question carries 10 marks)

Q.1 A signal $x(t)$ is shown in the Figure Q1.

Another signal $y(t)$ is obtained by repeating the signal $x(t)$ after every $2W$ seconds, in both positive as well as negative time.

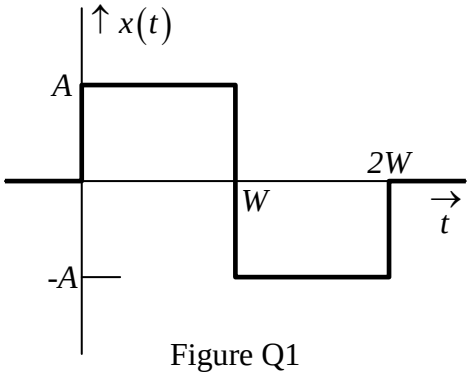

Figure Q1

The value of $A = W = a + b + c + d + 10$, where a, b, c and d are the values from your student number.

(a) Express $x(t)$ in terms of rectangle function, $\text{rect}(t)$.

(2 marks)

(b) Express $y(t)$ in terms of $x(t)$ and Dirac Delta function, $\delta(t)$.

(2 marks)

(c) Find the Fourier transform $X(f)$ for signal, $x(t)$.

(4 marks)

(d) Find the average power for signal, $y(t)$.

(2 marks)

Q.2 A signal $x(t)$ has a frequency spectrum as shown in Figure Q2(a).

The value of $W = a + b + c + d + 10$, where a, b, c and d are the values from your student number.

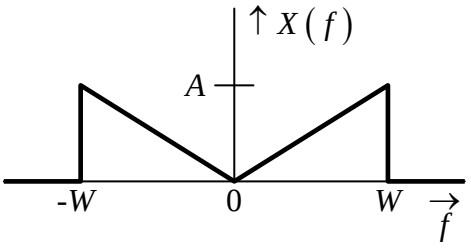

Figure Q2(a)

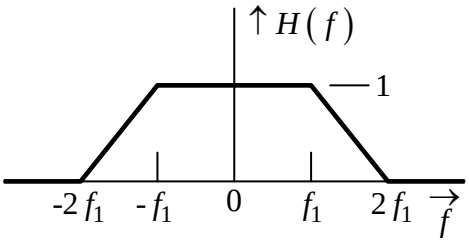

Figure Q2(b)

(a) Sketch the spectrum for signal $y(t) = x(t) \times 2\cos(2\pi W t)$.

(4 marks)

(b) What is the Nyquist sampling frequency for the signal $y(t)$?

(2 marks)

(c) The signal $y(t)$ is sampled, stored and passed through a reconstruction filter. The reconstruction filter's frequency response is given in Figure Q2(b). For perfect reconstruction of $y(t)$,

i. What must be value of f_1 for the reconstruction filter?

ii. What must be the sampling frequency, f_s?

(4 marks)

Q.3 The transfer function for an LTI system is as given by

$$\tilde{H}(s) = \frac{C \cdot s \cdot (s + D)}{\left(s^2 + 42s + E^2\right)}$$

The values of the system parameters are obtained as

$$C = a + 25 + b, \quad D = c + 2 \quad \text{and} \quad E = a + 2c + d + 6,$$

where a, b, c and d are the values from your student number.

(a) Describe the low-frequency and high frequency asymptotic behaviour of the magnitude slope and phase of $\tilde{H}(s)$.

(3 marks)

(b) Is the system BIBO stable and why?

(2 marks)

(c) Let $x(t)$ and $y(t)$ be the input and output signals of the system, respectively. Find $y(t)$ if $x(t) = 10 + 50 \cdot \cos(5t + 90°)$

(5 marks)

Q.4 The straight-line Bode magnitude plot for a LTI system $\tilde{H}(s)$, which consists of real poles and zeros, is shown in Figure Q4.

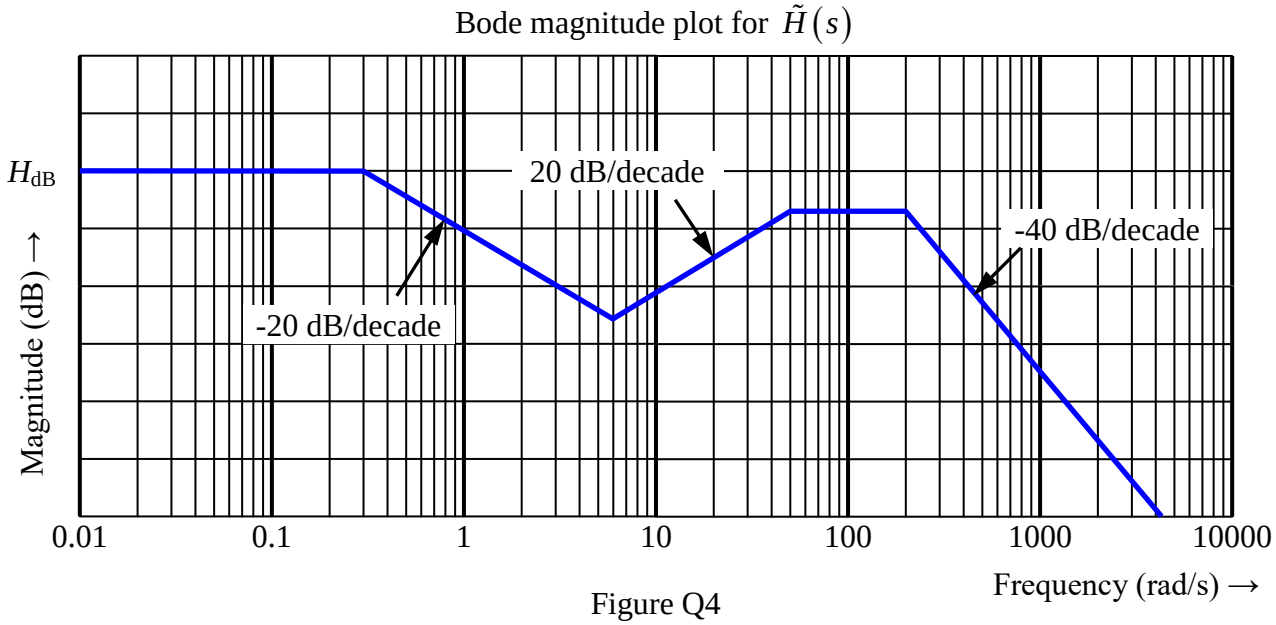

Figure Q4

The value of the syetem parameter is obtained as

$$H = c + 10 + d$$

where c and d are the values from your student number.

(a) How many poles and zeros are present in $\tilde{H}(s)$?

(2 marks)

(b) What is the value of the DC gain constant K_{dc}?

(3 marks)

(c) Identify $\tilde{H}(s)$ using Figure Q4.

(5 marks)

SECTION B

Answer ALL questions in this section (Each question carries 20 marks)

Q.5 A badly designed built-in equalizer of a hifi system, which is meant to boost the bass and treble of its audio input, has a transfer function given by

$$\tilde{H}(s) = K\frac{(s+2\beta)(s+\gamma)^2}{(s+\beta)\left(s^2+39600s+4\gamma^2\right)}.$$

The values of the system parameters K, β, and γ are obtained as

$$\beta = 20\times(10a+b+99), \quad \gamma = 100\times(10b+c+99) \quad \text{and} \quad K = 0.2\times(10c+d+1),$$

where a, b, c and d are the values from your student number.

(a) Derive the DC gain of $\tilde{H}(s)$.

(3 marks)

(b) Is the 2nd-order factor in $\tilde{H}(s)$ overdamped, critically damped, underdamped or undamped? Justify your answer.

(3 marks)

(c) Sketch the Bode magnitude plot for $\tilde{H}(s)$. You need not sketch on a semilog-x grid nor sketch to scale. However, you are required to label your sketch to indicate the following:

- the slope in dB/decade of each straight-line segments;
- the frequency in rad/s at each occurrence of slope change;
- the dB value where the plot intersects the $\omega = 0.5\beta$ line.

(9 marks)

(d) To enhance the bass response of the system, an integrator is cascaded with it to form a new system $\tilde{H}_1(s) = \frac{\beta}{s}\tilde{H}(s)$. Suppose the input to $\tilde{H}_1(s)$ is $x(t) = \cos(\omega_o t)$ where $\omega_o << \beta$. Find an approximate expression for the system steady state output $y(t)$ in terms of ω_o.

(5 marks)

Q.6 A periodic signal $s(t)$ is shown in the Figure Q6 below, where T_1, T_2 and T_3 are positive constants in seconds, $T_1 << T_2$ and $T_3 = T_1/2$. The second signal is

$$x(t) = 1000 \times B \times \text{sinc}^2\left(10^3 t - 0.5\right).$$

The third signal $y(t) = x(t) \times s(t)$.

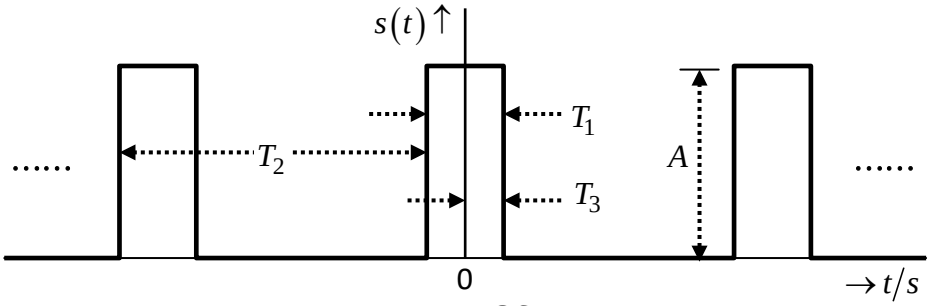

Figure Q6

The values of the parameters A and B are obtained as

$$A = 5 \times (a + 1) \quad \text{and} \quad B = 2 \times (b + c + 1),$$

where a, b and c are the values from your student number.

(a) Derive the Fourier transform $S(f)$ of $s(t)$, and the Fourier transform $X(f)$ of $x(t)$.

(6 marks)

(b) Derive the Fourier transform $Y(f)$ of $y(t)$.

(4 marks)

(c) One wants to reconstruct $x(t)$ from $y(t)$. Find out the maximal possible value of T_2 that allows $x(t)$ to be perfectly reconstructed from $y(t)$ with ideal filtering.

(5 marks)

(d) Find the bandwidth of the ideal filter used in (c) and its passband gain if $T_1 = 10^{-5}$ seconds.

(5 marks)

END OF PAPER

APPENDIX

Exam Declaration Form

Please read sections A, B and C below. Sign and submit this declaration form together with your answers.

A. Academic, Professional and Personal Integrity

1. *The University is committed to nurturing an environment conducive for the exchange of ideas, advancement of knowledge and intellectual development. Academic honesty and integrity are essential conditions for the pursuit and acquisition of knowledge, and the University expects each student to maintain and uphold the highest standards of integrity and academic honesty at all times.*

2. *The University takes a strict view of cheating in any form, deceptive fabrication, plagiarism and violation of intellectual property and copyright laws. Any student who is found to have engaged in such misconduct will be subject to disciplinary action by the University.*

3. *It is important to note that all students share the responsibility of protecting the academic standards and reputation of the University. This responsibility can extend beyond each student's own conduct, and can include reporting incidents of suspected academic dishonesty through the appropriate channels. Students who have reasonable grounds to suspect academic dishonesty should raise their concerns directly to the relevant Head of Department, Dean of Faculty, Registrar, Vice Provost or Provost.*

B. I have read and understood the rules of the assessments stated below:

a. *Students should attempt the assessments on their own. There should be no discussion or communication, via face to face or communication devices, with any other person during the assessment.*

b. *Students should not reproduce any assessment materials, e.g. by photography, videography, screenshots, copying down of questions, etc. Posting on public forums, e.g. social media and websites, is prohibited.*

C. I understand that by breaching any of the rules above, I would have committed offences under clause 3(I) of the NUS Statute 6, Discipline with Respect to Students, which is punishable with disciplinary action under clause 10 or clause 11 of the said statute.

3) Any student who is alleged to have committed or attempted to commit, or caused or attempted to cause any other person to commit any of the following offences, may be subject to disciplinary proceedings:

(I) plagiarism, giving or receiving unauthorised assistance in academic work, or other forms of academic dishonesty.

I have read and will abide by the NUS Code of Student Conduct (in particular, (A) Academic, Professional and Personal Integrity), B and C when attempting this assessment.

Signature: _____ Date: _____

Matric. No.: _____