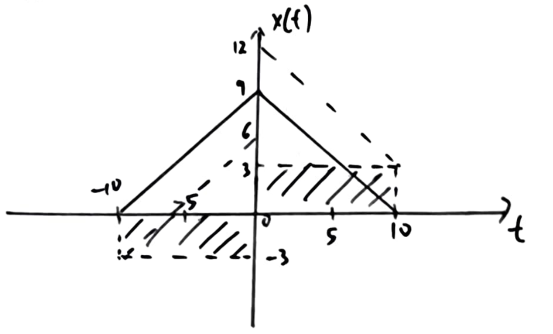
CG2023 Assignment 1

1. $x(t) = A\,\text{tri}\left(\frac{t}{\alpha}\right) + B\,\text{rect}\left(\frac{t-b}{\beta}\right) + C\,\text{rect}\left(\frac{t-c}{\chi}\right)$

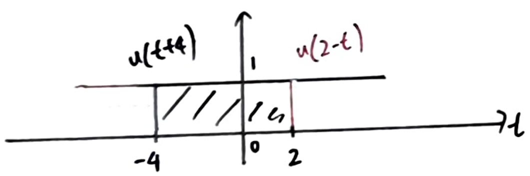


$x(t) = 9\,\text{tri}\left(\frac{t}{10}\right) + (-3)\,\text{rect}\left(\frac{t-(-5)}{10}\right) + 3\,\text{rect}\left(\frac{t-5}{10}\right)$

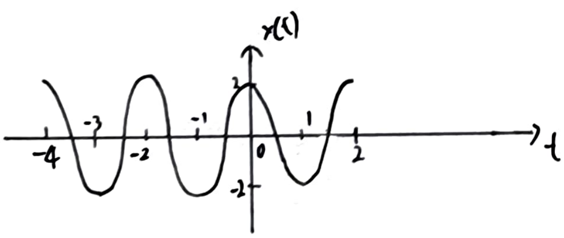
$\boxed{A = 9,\ \alpha = 10,\ B = -3,\ b = -5,\ \beta = 10,\ C = 3,\ c = 5,\ \chi = 10}$

2. a) $x(t) = 2\cos(\pi t)\,u(t+4)\,u(2-t)$

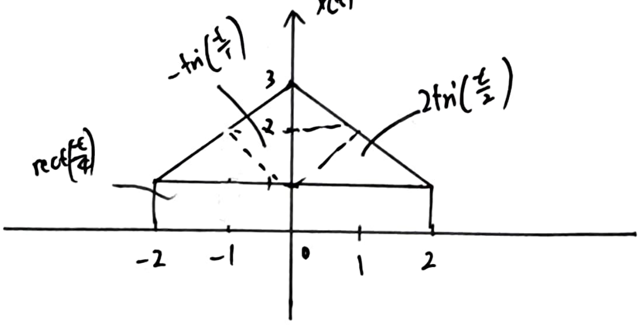
$T_p = \frac{2\pi}{\pi} = 2s$



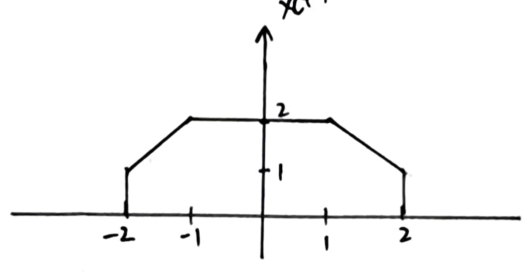
$x(t) = \begin{cases} 2\cos(\pi t); & -4 \le t \le 2 \\ 0; & \text{otherwise} \end{cases}$



b) $x(t) = \text{rect}\left(\frac{t}{4}\right) + 2\,\text{tri}\left(\frac{t}{2}\right) - \text{tri}\left(\frac{t}{1}\right)$



$\Rightarrow$



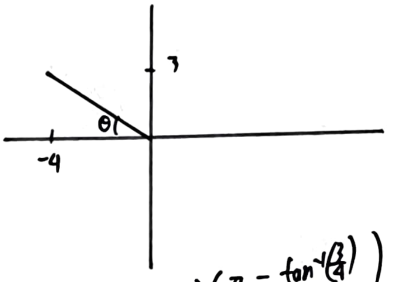
3. a)

$$x(t) = -3 + j4$$

$$y(t) = \sqrt{2}\, e^{j\frac{\pi}{4}}$$

$$= \sqrt{2}\cos\frac{\pi}{4} + j\sqrt{2}\sin\frac{\pi}{4}$$

$$= 1 + j$$



$$x(t) - y(t) = -3 + j4 - (1 + j)$$

$$= -4 + j3$$

$$5e^{j(\pi - \tan^{-1}(\frac{3}{4}))}$$

$$|x(t) - y(t)| = \sqrt{(-4)^2 + (3)^2}$$

$$= 5$$

$$\angle[x(t) - y(t)] = \pi - \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 0.795\pi$$

b)

$$x(t) \cdot y(t) = (-3 + j4) \cdot (1 + j)$$

$$= -3 - 3j + 4j + 4j^2$$

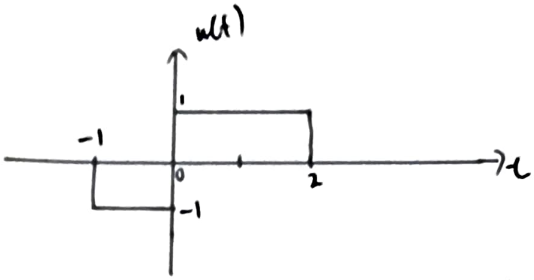
$$= -7 + j$$

$$\frac{x(t)}{y(t)} = \frac{-3 + j4}{1 + j} = \frac{(-3 + j4)(1 - j)}{(1 + j)(1 - j)}$$

$$= \frac{-3 + 3j + 4j + 4}{1 + 1}$$

$$= \frac{1 + j7}{2}$$

4.



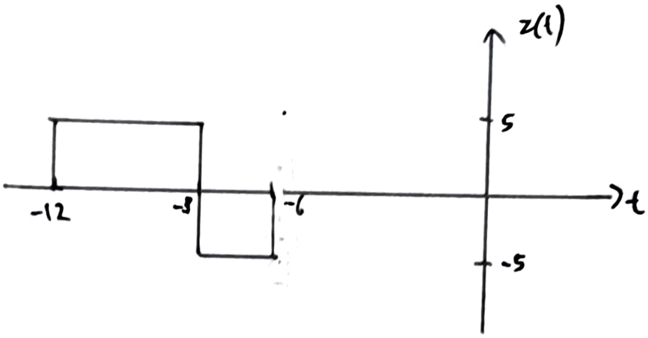
$z(t) = 5w\left(-\frac{t}{2} - 4\right)$

① shift right by 4 $[3, 4, 6]$

② expand along t axis by 2 $[6, 8, 12]$

③ reflect over vertical axis $[-12, -8, -6]$

④ multiply amplitude by 5



5.

① time scaling : contraction along t axis by $\alpha = \frac{3}{2}$ $[-2, 2, 4]$

$x_1(t) = x\left(\frac{3}{2}t\right)$

② time reversal : reflect over vertical axis $[-4, -2, 2]$

$x_2(t) = x\left(-\frac{3}{2}t\right)$

③ time shifting : shift right by 6 units $[2, 4, 8]$

$x_3(t) = x\left(-\frac{3}{2}(t-6)\right)$

$= x\left(-\frac{3}{2}t + 9\right)$

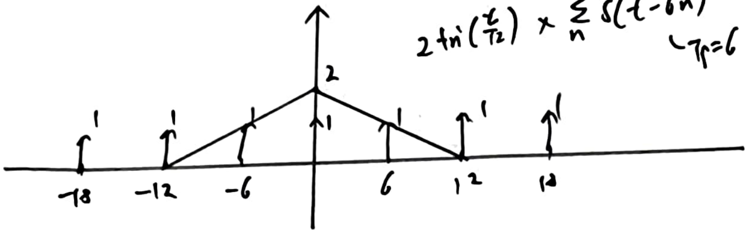
④ multiply amplitude by $\frac{3}{2}$

$y(t) = \frac{3}{2}x\left(-\frac{3}{2}t + 9\right)$ //

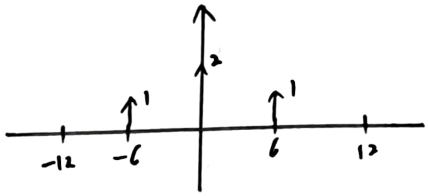
6. $y(t) = 3\,\mathrm{rect}\left(\frac{t}{8}\right) * \left[2\,\mathrm{tri}\left(\frac{t}{12}\right) \times \sum_n \delta(t-6n)\right]$

$2\,\mathrm{tri}\left(\frac{t}{12}\right) \times \sum_n \delta(t-6n)$

$T_p = 6$



$\Rightarrow$



sampling property of $\delta(t)$

$x(t)\,\delta(t-\lambda) = x(\lambda)\,\delta(t-\lambda)$

distributing

$$y(t) = 3\,\mathrm{rect}\left(\frac{t}{8}\right) * \left[\delta(t+6) + 2\delta(t) + \delta(t-6)\right]$$

$$= 3\,\mathrm{rect}\left(\frac{t}{8}\right) * \delta(t+6) + 3\,\mathrm{rect}\left(\frac{t}{8}\right) * 2\delta(t) + 3\,\mathrm{rect}\left(\frac{t}{8}\right)\delta(t-6)$$

$$= 3\,\mathrm{rect}\left(\frac{t+6}{8}\right) + 6\,\mathrm{rect}\left(\frac{t}{8}\right) + 3\,\mathrm{rect}\left(\frac{t-6}{8}\right)$$

replication property

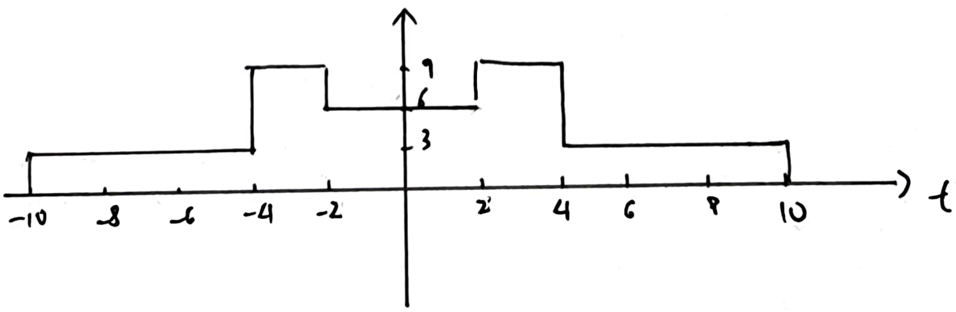
$x(t) * \delta(t-\lambda) = x(t-\lambda)$

$$\int_{-\infty}^{\infty} y(t)\,dt = \int_{-\infty}^{\infty} 3\,\mathrm{rect}\left(\frac{t}{8}\right) * \delta(t+6) + 6\,\mathrm{rect}\left(\frac{t}{8}\right)\delta(t) + 3\,\mathrm{rect}\left(\frac{t}{8}\right)\delta(t-6)\,dt$$

$$= \int_{-\infty}^{\infty} 3\,\mathrm{rect}\left(\frac{t+6}{8}\right) + 6\,\mathrm{rect}\left(\frac{t}{8}\right) + 3\,\mathrm{rect}\left(\frac{t-6}{8}\right)$$

$$= 3\times 8 + 6\times 8 + 3\times 8$$

$$= 96$$



CG2023 Assignment 2

$$\frac{1}{j} = -j = e^{-j\frac{\pi}{2}}$$

1. a) $x(t) = 8\sin\left(12\pi t + \frac{\pi}{2}\right)$

$$= \frac{8}{j2} e^{j(2\pi 6t + \frac{\pi}{2})} - \frac{8}{2j} e^{-j(2\pi 6t + \frac{\pi}{2})}$$

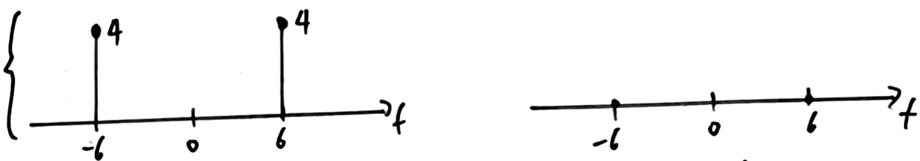
$$= 4e^{-j\frac{\pi}{2}} e^{j(2\pi 6t + \frac{\pi}{2})} + 4e^{j\frac{\pi}{2}} e^{-j(2\pi 6t + \frac{\pi}{2})}$$

$$= 4e^{j\cancel{(\frac{\pi}{2} - \frac{\pi}{2})}^{0}} e^{j2\pi(6)t} + 4e^{j\cancel{(\frac{\pi}{2} - \frac{\pi}{2})}^{0}} e^{j2\pi(-6)t}$$

OR $8\cos(12\pi t)$

$$= \frac{8e^{j12\pi t} + 8e^{-j12\pi t}}{2} \quad //$$

Spectral Plot:



Magnitude spectrum

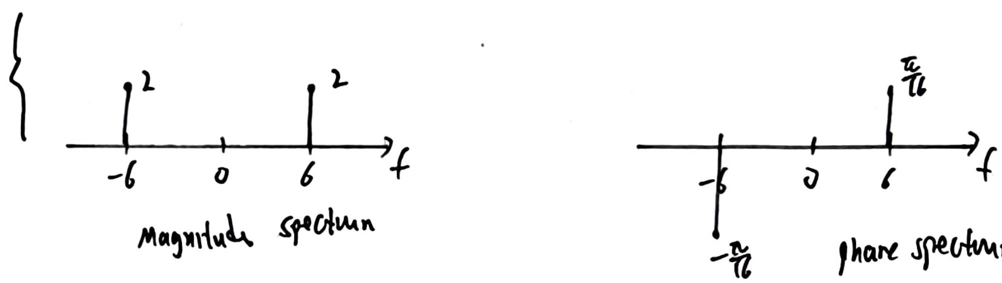
Phase spectrum

b) $y(t) = 4\cos\left(12\pi t + \frac{\pi}{16}\right)$

$$= \frac{4}{2} e^{j(2\pi 6t + \frac{\pi}{16})} + \frac{4}{2} e^{-j(2\pi 6t + \frac{\pi}{16})}$$

$$= 2e^{j(\frac{\pi}{16})} e^{j2\pi(6)t} + 2e^{j(-\frac{\pi}{16})} e^{j2\pi(-6)t}$$

Spectral Plot:



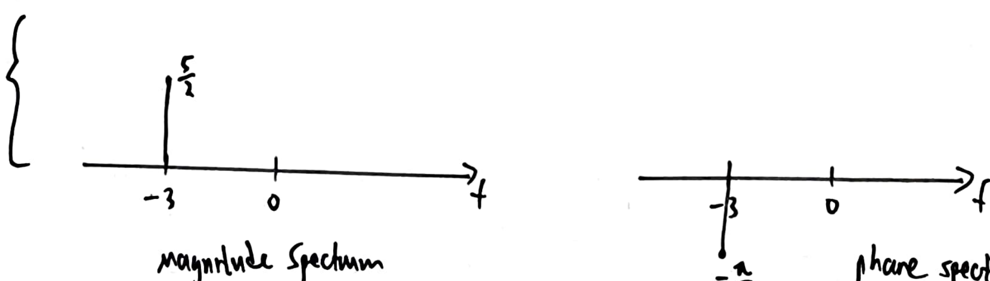
Magnitude spectrum

Phase spectrum

c) $z(t) = \frac{5}{2} e^{-j(6\pi t + \frac{\pi}{6})}$

$$= \frac{5}{2} e^{j(-\frac{\pi}{6})} e^{j2\pi(-3)t}$$

Spectral Plot:



Magnitude Spectrum

Phase spectrum

$$\cos 2t = 1 - 2\sin^2 t$$
$$\sin^2 t = \frac{1-\cos 2t}{2}$$

2. a) $x(t) = \sin^2 t$

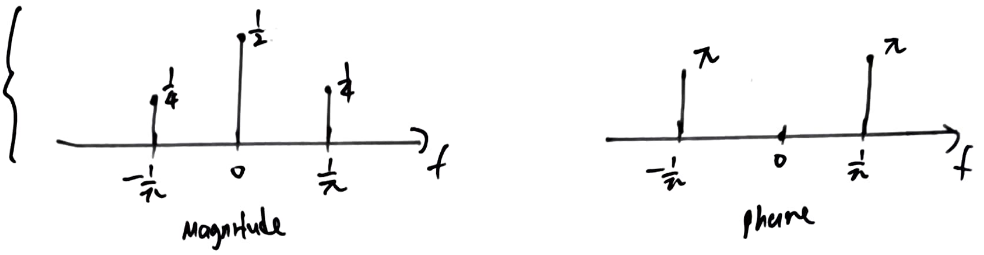
$$= \frac{1}{2} - \frac{\cos 2t}{2}$$

$$= \frac{1}{2} - \frac{1}{4}e^{j2\pi(\frac{1}{\pi})t} - \frac{1}{4}e^{-j2\pi(\frac{1}{\pi})t}$$

$$= \frac{1}{2} + \frac{1}{4}e^{j\pi}e^{j2\pi(\frac{1}{\pi})t} + \frac{1}{4}e^{j\pi}e^{j2\pi(-\frac{1}{\pi})t}$$

$$\boxed{\angle +\pi = -\pi}$$

Spectral Plot :

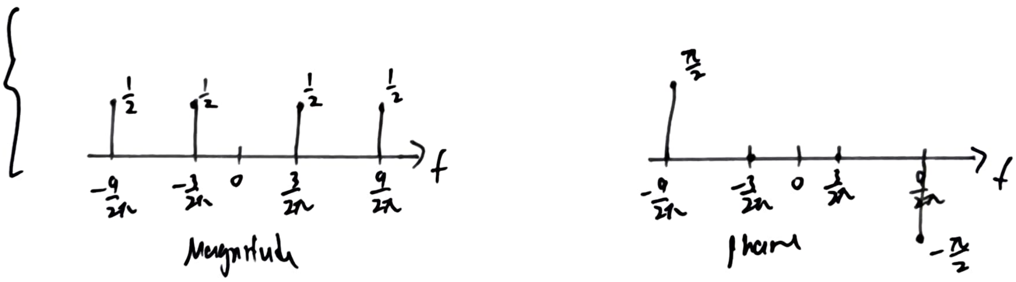


b) $y(t) = \sin 9t + \cos 3t$

$$= \frac{1}{j2}e^{j2\pi(\frac{9}{2\pi})t} - \frac{1}{j2}e^{-j2\pi(\frac{9}{2\pi})t} + \frac{1}{2}e^{j2\pi(\frac{3}{2\pi})t} + \frac{1}{2}e^{-j2\pi(\frac{3}{2\pi})t}$$

$$= \frac{1}{2}e^{j(\frac{\pi}{2})}e^{j2\pi(\frac{9}{2\pi})t} + \frac{1}{2}e^{j(\frac{\pi}{2})}e^{j2\pi(-\frac{9}{2\pi})t} + \frac{1}{2}e^{j2\pi(\frac{3}{2\pi})t} + \frac{1}{2}e^{j2\pi(-\frac{3}{2\pi})t}$$

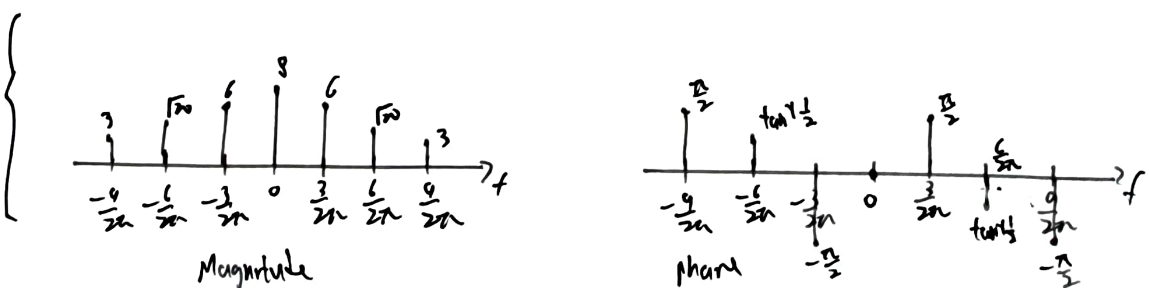
Spectral Plot :



c) $z(t) = (4+2j)e^{-j6t} + 3je^{-j9t} + 8 - 3je^{j9t} + (4-2j)e^{j6t} + 6je^{j3t} - 6je^{-j3t}$

$$= \sqrt{20}e^{j\tan^{-1}(\frac{1}{2})}e^{j2\pi(-\frac{3}{\pi})t} + 3e^{j\frac{\pi}{2}}e^{j2\pi(-\frac{9}{2\pi})t} + 8$$
$$+ 3e^{j(\frac{\pi}{2})}e^{j2\pi(\frac{9}{2\pi})t} + \sqrt{20}e^{j-\tan^{-1}(\frac{1}{2})}e^{j2\pi(\frac{3}{\pi})t}$$
$$+ 6e^{j\frac{\pi}{2}}e^{j2\pi(\frac{3}{2\pi})t} + 6e^{j-\frac{\pi}{2}}e^{j2\pi(-\frac{3}{2\pi})t}$$

Spectral Plot :



3. a) $x(t) = \begin{cases} 5; & 0 \le t < \frac{T_0}{2} \\ -5; & \frac{T_0}{2} \le t < T_0 \end{cases}$ period = T_0

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi \frac{k}{T_0} t}$$ where

$$c_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j2\pi\frac{k}{T_0}t}\, dt$$

$$= \frac{1}{T_0}\int_0^{\frac{T_0}{2}} 5e^{-j2\pi\frac{k}{T_0}t}\, dt + \frac{1}{T_0}\int_{\frac{T_0}{2}}^{T_0} -5e^{-j2\pi\frac{k}{T_0}t}\, dt$$

$$= \frac{5}{T_0}\left[\frac{e^{-j2\pi\frac{k}{T_0}t}}{-j2\pi\frac{k}{T_0}}\right]_0^{T_0/2} + \frac{5}{T_0}\left[\frac{e^{-j2\pi\frac{k}{T_0}t}}{j2\pi\frac{k}{T_0}}\right]_{T_0/2}^{T_0}$$

$$= \frac{5}{T_0}\left(\frac{e^{-j\pi k}}{-j2\pi\frac{k}{T_0}} - \frac{1}{-j2\pi\frac{k}{T_0}} + \frac{e^{-j2\pi k}}{j2\pi\frac{k}{T_0}} - \frac{e^{-j\pi k}}{j2\pi\frac{k}{T_0}}\right)$$

$$= \frac{5}{-j2\pi k}\left(2e^{-j\pi k} - \cancel{e^{-j2\pi k}}^{\,1} - 1\right)$$

$$= \frac{5}{j\pi k}\left(1 - e^{-j\pi k}\right)$$

when $k=0$: $c_0 = \frac{1}{T_0}\int_0^{\frac{T_0}{2}} 5\, dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} -5\, dt$

$$= \frac{5}{2} - \frac{5}{2} = 0$$

when k is odd: $c_k = \frac{5}{j\pi k}\left(1 - \cancel{e^{-j\pi k}}^{\,-1}\right)$

$$= \frac{10}{j\pi k}$$

when k is even: $c_k = \frac{5}{j\pi k}\left(1 - \cancel{e^{-j\pi k}}^{\,1}\right)$

$$= 0$$

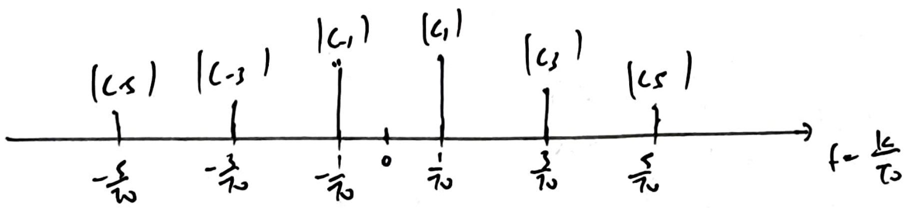
x(t) is real & odd

X_k is imaginary and odd

$c_k = -c_{-k} \Rightarrow$ no cosine terms

only odd functions

Spectrum :



b) $y(t) = \begin{cases} 6; & |t| \le \frac{T_0}{4} \\ 0; & \text{otherwise} \end{cases}$ Period $= T_0$

$$y(t) = \sum_{k=-\infty}^{\infty} c_k \, e^{j2\pi \frac{k}{T_0} t}$$ where

$$c_k = \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} 6 e^{-j2\pi \frac{k}{T_0} t} \, dt$$

$$= \frac{6}{T_0} \left[\frac{e^{-j2\pi \frac{k}{T_0} t}}{-j2\pi \frac{k}{T_0}} \right]_{-\frac{T_0}{4}}^{\frac{T_0}{4}}$$

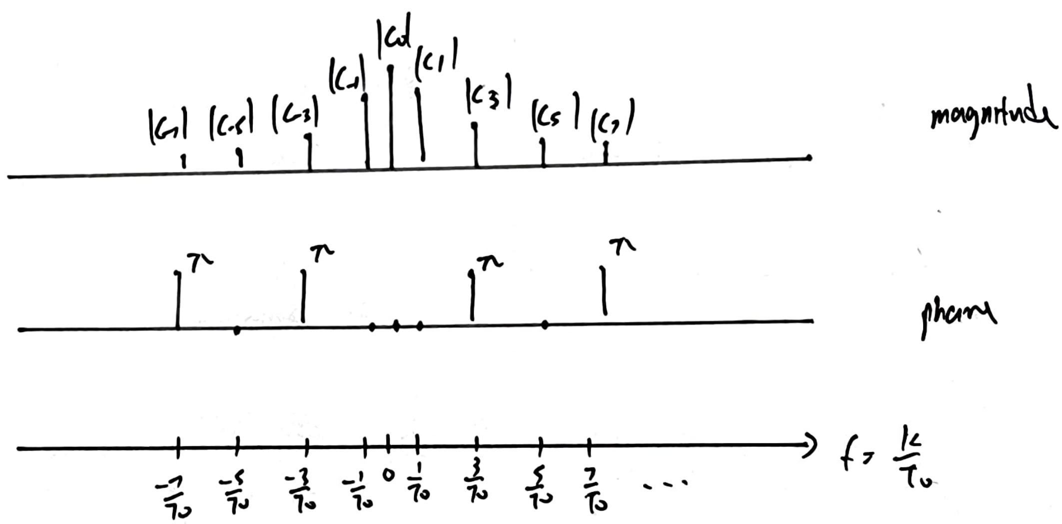
$$= \frac{6}{T_0} \left(\frac{e^{-j\frac{k\pi}{2}}}{-j2\pi\frac{k}{T_0}} - \frac{e^{j\frac{k\pi}{2}}}{-j2\pi\frac{k}{T_0}} \right)$$

$$= 3 \left(\frac{e^{j\frac{k\pi}{2}} - e^{-j\frac{k\pi}{2}}}{2j \frac{k\pi}{2}} \right) = 3 \operatorname{sinc}\left(\frac{k}{2}\right)$$

$c_k = c_{-k}$ and $c_k = 0$ when k is even $\left(\operatorname{sinc}(x) = \begin{cases} 0; & x \in \mathbb{Z} \\ 1; & x = 0 \end{cases} \right)$

$c_0 = 3$

Spectrum :



$$|c_k| = 3\left|\operatorname{sinc}\left(\frac{k}{2}\right)\right| = \frac{6}{\pi k}$$

$$\angle c_k = \begin{cases} 0; & c_k \ge 0 \\ \pm\pi; & c_k < 0 \end{cases}$$

CG2023 Assignment 3

1. $x(t) = e^{-\alpha t} u(t) \qquad X(f) = \frac{1}{\alpha + j2\pi f}$

a) $y(t) = e^{\alpha t} u(-t)$

$= x(-t)$

$Y(f) = \frac{1}{|-1|} X\left(\frac{f}{-1}\right)$ (time scaling property)

$= X(-f)$

$= \frac{1}{\alpha - j2\pi f}$

b) $z(t) = e^{-|t|}$

$Z(f) = \int_0^{\infty} e^{-t} e^{-j2\pi f t} dt + \int_{-\infty}^{0} e^{t} e^{-j2\pi f t} dt$

$= \int_0^{\infty} e^{-(1+j2\pi f)t} dt + \int_{-\infty}^{0} e^{(1-j2\pi f)t} dt$

$= \left[\frac{e^{-(1+j2\pi f)t}}{-(1+j2\pi f)}\right]_0^{\infty} + \left[\frac{e^{(1-j2\pi f)t}}{1-j2\pi f}\right]_{-\infty}^{0}$

$= \frac{1}{1+j2\pi f} + \frac{1}{1-j2\pi f}$

$= \frac{2}{1+4\pi^2 f^2}$

2. a) $w(t) = -\mathrm{rect}\left(t+\frac{1}{2}\right) + \mathrm{rect}\left(\frac{t-1}{2}\right)$

b) let $x(t) = \mathrm{rect}(t)$, then $X(f) = \mathrm{sinc}(f)$

$x\left(t+\frac{1}{2}\right) \leftrightarrow X(f)e^{-j2\pi f\left(-\frac{1}{2}\right)} = X(f)e^{j\pi f}$ (time shifting)

$-x\left(t+\frac{1}{2}\right) \leftrightarrow -X(f)e^{j\pi f}$ (linearity)

let $y(t) = \mathrm{rect}\left(\frac{t}{2}\right)$, then $Y(f) = 2\,\mathrm{sinc}(2f)$ (time shifting)

$y(t-1) \leftrightarrow Y(f)e^{-j2\pi f(1)}$

$$W(f) = -\mathrm{sinc}(f)e^{j\pi f} + 2\,\mathrm{sinc}(2f)\,e^{-j2\pi f}$$

c) $y(t) = -\mathrm{sinc}(t)e^{j\pi t} + 2\,\mathrm{sinc}(2t)e^{-j2\pi t}$

$\mathrm{sinc}(t) \leftrightarrow \mathrm{rect}(-f)$ (Duality)

$2\,\mathrm{sinc}(2t) \leftrightarrow \mathrm{rect}\left(-\frac{f}{2}\right)$

$\mathrm{sinc}(t)e^{j\pi t} \leftrightarrow \mathrm{rect}\left(-\left(f-\frac{1}{2}\right)\right)$ (frequency shifting)

$2\,\mathrm{sinc}(2t)e^{-j2\pi t} \leftrightarrow \mathrm{rect}\left(-\frac{(f+1)}{2}\right)$

$$Y(f) = -\mathrm{rect}\left(-f+\frac{1}{2}\right) + \mathrm{rect}\left(-\frac{(f+1)}{2}\right)$$ (linearity)

3. a)

$$X(f) = 2\,\text{rect}\left(\frac{f-\frac{1}{2}}{3}\right)$$

let $z(t) \;\square\; \text{rect}(f)$ $\qquad$ $(z(t) = \text{sinc}(t))$

$z(3t) \;\square\; \frac{1}{3}\,\text{rect}\left(\frac{f}{3}\right)$ $\qquad$ (time scaling)

$z(3t)e^{j2\pi(\frac{1}{2})t} \;\square\; \frac{1}{3}\,\text{rect}\left(\frac{f-\frac{1}{2}}{2}\right)$ $\qquad$ (freq shifting)

$6\,z(3t)\,e^{j2\pi(\frac{1}{2})t} \;\square\; 2\,\text{rect}\left(\frac{f-\frac{1}{2}}{3}\right)$ $\qquad$ (linearity)

$$x(t) = 6z(3t)\,e^{j2\pi(\frac{1}{2}t)}$$

$$= \underline{6\,\text{sinc}(3t)\,e^{j\pi t}}$$

b)

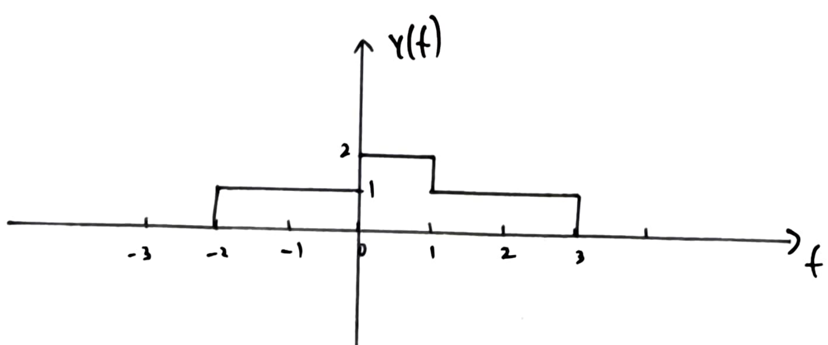
$\cos(2\pi t) \;\square\; \frac{1}{2}\left[\delta(f-1) + \delta(f+1)\right]$

multiplication in time domain $\Rightarrow$ convolution in freq domain

$$Y(f) = X(f) * \frac{1}{2}\left[\delta(f-1) + \delta(f+1)\right]$$

$= \frac{1}{2}X(f) * \delta(f-1) + \frac{1}{2}X(f) * \delta(f+1)$ $\qquad$ (distributive)

$= \frac{1}{2}X(f-1) + \frac{1}{2}X(f+1)$ $\qquad$ (replication)



c) $\cos(40\pi t) \;\square\; \frac{1}{2}[\delta(f-20)+\delta(f+20)]$

multiplication in time domain $\Rightarrow$ convolution in freq domain

$$z(f) = X(f) * \frac{1}{2}[\delta(f-20)+\delta(f+20)] \cdot (1+j)$$ (linearity)

$$= \left[\frac{1}{2}X(f)\,\delta(f-20) + \frac{1}{2}X(f)\,\delta(f+20)\right](1+j)$$ (Distributive)

$$= \left[\frac{1}{2}X(f-20) + \frac{1}{2}X(f+20)\right](1+j)$$

$$= \left[\mathrm{rect}\left(\frac{f-\frac{41}{2}}{3}\right) + \mathrm{rect}\left(\frac{f+\frac{39}{2}}{3}\right)\right](1+j) \rightarrow \sqrt{2}e^{j\frac{\pi}{4}}$$

4. a) $\delta(t) \;\square\; 1$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

$$u(t) \;\square\; \frac{1}{j2\pi f}\underbrace{X(f)}_{\rightarrow 1} + \frac{1}{2}\underbrace{X(0)}_{\rightarrow 1}\,\delta(f)$$ (Integration in time domain)

$$= \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

b) $\mathrm{sgn}(t) = 2u(t) - 1$

$$\mathrm{sgn}(t) \;\square\; 2\left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right] - \delta(f)$$ (linearity)

$$= \frac{1}{j\pi f} + \delta(f) - \delta(f)$$ (Duality)

$$= \frac{1}{j\pi f}$$

c) $h(t) = \frac{1}{\pi t}$

$\frac{1}{j\pi t} \leftrightarrow \operatorname{sgn}(-f)$ (Duality)

$\frac{1}{\pi t} \leftrightarrow j \operatorname{sgn}(-f)$ (linearity)

5. a)

$$x(t) = \underbrace{\sin\left(\tfrac{\pi}{4} t\right)}_{T = \frac{2\pi}{\pi/4} = 8} \cdot 4\,\mathrm{rect}\left(\tfrac{t-2}{4}\right) - \underbrace{\sin(\pi t)}_{T = \frac{2\pi}{\pi} = 2} \cdot \mathrm{rect}\left(t - \tfrac{9}{2}\right)$$

b)

$\sin\left(\frac{\pi}{4} t\right) \leftrightarrow \frac{-j}{2}\left[\delta\left(f - \frac{1}{8}\right) - \delta\left(f + \frac{1}{8}\right)\right]$

$\sin(\pi t) \leftrightarrow \frac{-j}{2}\left[\delta\left(f - \frac{1}{2}\right) - \delta\left(f + \frac{1}{2}\right)\right]$

$\mathrm{rect}\left(\frac{t-2}{4}\right) \leftrightarrow 4\,\mathrm{sinc}(4f)\, e^{-j4\pi f}$

$\mathrm{rect}\left(t - \frac{9}{2}\right) \leftrightarrow \mathrm{sinc}(f)\, e^{-j9\pi f}$

multiplication in time domain $\Rightarrow$ convolution in freq domain

$$X(f) = -8j\,\mathrm{sinc}(4f)\, e^{-j4\pi f} * \left[\delta\left(f - \tfrac{1}{8}\right) - \delta\left(f + \tfrac{1}{8}\right)\right]$$
$$+ \frac{j}{2}\,\mathrm{sinc}(f)\, e^{-j9\pi f} * \left[\delta\left(f - \tfrac{1}{2}\right) - \delta\left(f + \tfrac{1}{2}\right)\right]$$

$$= -8j\,\mathrm{sinc}\left(4\left(f - \tfrac{1}{8}\right)\right) e^{-j4\pi\left(f - \frac{1}{8}\right)} + 8j\,\mathrm{sinc}\left(4\left(f + \tfrac{1}{8}\right)\right) e^{-j4\pi\left(f + \frac{1}{8}\right)}$$
$$+ \frac{j}{2}\,\mathrm{sinc}\left(f - \tfrac{1}{2}\right) e^{-j9\pi\left(f - \frac{1}{2}\right)} - \frac{j}{2}\,\mathrm{sinc}\left(f + \tfrac{1}{2}\right) e^{-j9\pi\left(f + \frac{1}{2}\right)}$$

$$= 8\cos(4\pi f)\, e^{-j4\pi f}\left[\frac{1}{\frac{\pi}{4} - 16\pi f^2}\right] + \frac{1}{2}\cos(\pi f)\, e^{-j9\pi f}\left[\frac{1}{\pi f^2 - \frac{\pi}{4}}\right]$$

c) $z(t) = \sum_{n=-\infty}^{\infty} x(t-10n)$

$$= x(t) * \sum_{n=-\infty}^{\infty} \delta(t-10n)$$

convolution in time domain $\Rightarrow$ multiplication in freq domain

$$Z(f) = X(f) \cdot \frac{1}{10} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{10}\right)$$

$$= \frac{X(f)}{10} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{10}\right)$$ where $X(f)$ is result from b)

b

symmetric

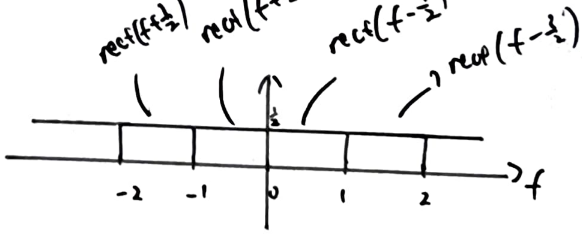
$\mathrm{sinc}(t) \supset \mathrm{rect}(-f) = \mathrm{rect}(f)$ (Duality)

$$y(t) = x(t) \times \sum_{n=-\infty}^{\infty} \delta(t-2n)$$

multiplication in time domain $\Rightarrow$ convolution in freq domain

$$Y(f) = \mathrm{rect}(f) * \frac{1}{2} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{2}\right)$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \mathrm{rect}\left(f - \frac{n}{2}\right)$$



$$= \frac{1}{2}$$

CG2023 Assignment 4

1. a) $x(t) = 2\,\mathrm{rect}\left(\frac{t-3}{6}\right)$

$X(f) = 2 \cdot 6 \,\mathrm{sinc}(6f) \cdot e^{-j6\pi f}$ (time shifting)

$= 12 \,\mathrm{sinc}(6f)\, e^{-j6\pi f}$

$|X(f)| = 12 \,\mathrm{sinc}(6f)$

$E_x(f) = |X(f)|^2 = \underline{144 \,\mathrm{sinc}^2(6f)}$

b) $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df$

easier to calculate E in time domain

$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$

$= \int_0^6 4 \, dt = \underline{24}$ Joules

c) $|X(f)| = 12 \,\mathrm{sinc}(6f)$ period $= \frac{1}{6}$

nulls of $|X(f)|$ occur at $f = \pm\frac{1}{6}, \pm\frac{2}{6}, \pm\frac{3}{6} \ldots$

1st null bandwidth of $x(t)$ occurs at $\underline{\frac{1}{6}\,Hz}$

2.

a) $x(f) = 3\,\mathrm{tri}\left(\frac{f}{3}\right) = \begin{cases} 3\left(1-\frac{|f|}{3}\right); & |f| \leq 3 \\ 0; & |f| > 3 \end{cases}$

$E_x(f) = |X(f)|^2 = \begin{cases} 9 - 6|f| + |f|^2; & |f| \leq 3 \\ 0; & |f| > 3 \end{cases}$

b) $E = \int_{-\infty}^{\infty} |X(f)|^2 \, df = 2\int_0^3 9 - 6f + f^2 \, df$

$= 2\left[9f - \frac{6f^2}{2} + \frac{f^3}{3}\right]_0^3$

$= \underline{18 \text{ Joules}}$

c) let f_B be 3-dB bandwidth of $x(t)$

$\frac{|x(f_B)|}{x(0)} = \frac{1}{\sqrt{2}} \Rightarrow \frac{3 - f_B}{3} = \frac{1}{\sqrt{2}}$

$3\sqrt{2} - \sqrt{2}\, f_B = 3$

$f_B = \frac{3 - 3\sqrt{2}}{-\sqrt{2}}$

$= \underline{0.87868 \text{ Hz}}$

3. a) $X(f) = \text{sinc}(f)$

$x(t) = \text{rect}(t)$

Easier to compute energy E in time domain

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot dt$$

$$= \underline{1 \text{ Joule}}$$

b) $y(t) = \text{sinc}(t)$

$Y(f) = \text{rect}(f)$

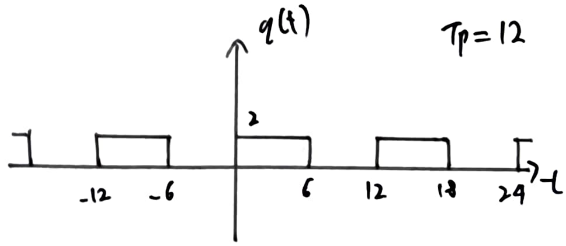
Easier to compute energy E in frequency domain

$$E = \int_{-\infty}^{\infty} |Y(f)|^2 df = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot df$$

$$= \underline{1 \text{ Joule}}$$

4. a) $x(t) = 2\,\text{rect}\left(\frac{t-3}{6}\right)$

$$q(t) = \sum_{n=-\infty}^{\infty} x(t-12n)$$



$$c_k = \frac{1}{12}\int_0^6 2e^{-j2\pi \frac{k}{12}t}\, dt = \frac{1}{6}\left[\frac{e^{-j\pi \frac{k}{6}t}}{-j\pi k/6}\right]_0^6$$

$$= \frac{1 - e^{-j\pi k}}{j\pi k}$$

$$= \begin{cases} \frac{2}{j\pi k} ; & k \text{ is odd} \\ 0 ; & k \text{ is even} \end{cases} \qquad c_0 = 1$$

$$P_q(f) = \sum_{k=-\infty}^{\infty} |c_k|^2 \delta\left(f - \frac{k}{12}\right) \quad \text{where } |c_k|^2 = \begin{cases} 1 ; & k=0 \\ \frac{4}{\pi^2 k^2} ; & k \text{ is odd} \\ 0 ; & k \text{ is even} \neq 0 \end{cases}$$

b) $$P = \sum_{k=-\infty}^{\infty} |c_k|^2 = 1 + \sum_{\text{odd } k} \frac{4}{\pi^2 k^2}$$

5. a) $$v(t) = 2 + (3+j)e^{j4\pi t} + 4e^{j8\pi t} + 5e^{j\frac{\pi}{4}} e^{j10\pi t}$$

$$= 2 + \sqrt{10}\, e^{j\tan^{-1}(\frac{1}{3})} e^{j2\pi(2)t} + 4e^{j2\pi(4)t} + 5e^{j\frac{\pi}{4}} e^{j2\pi(5)t}$$

Fundamental frequency = HCF $\{2, 4, 5\} = 1$ Hz

comparing with the Fourier series expansion $v(t) = \sum_k c_k e^{j2\pi kt}$

$$|c_k| = \begin{cases} \sqrt{10} ; & k = 2 \\ 4 ; & k = 4 \\ 5 ; & k = 5 \\ 0 ; & \text{otherwise} \end{cases}$$

$$P_v(f) = \sum_{k=-\infty}^{\infty} |c_k|^2 \delta(f-k) = 10\,\delta(f-2) + 16\,\delta(f-4) + 25\,\delta(f-5)$$

b) $$P = \sum_{k=-\infty}^{\infty} |c_k|^2 = 10 + 16 + 25$$

$$= \underline{51 \text{ watts}}$$