CG2023 Lab 1 : Spectrums and Sounds of Periodic Signals

3. a) $$x(t) = A\sin(2\pi f t + \phi)$$
$$= A\sin\left(2\pi f\left(t + \frac{\phi}{2\pi f}\right)\right)$$

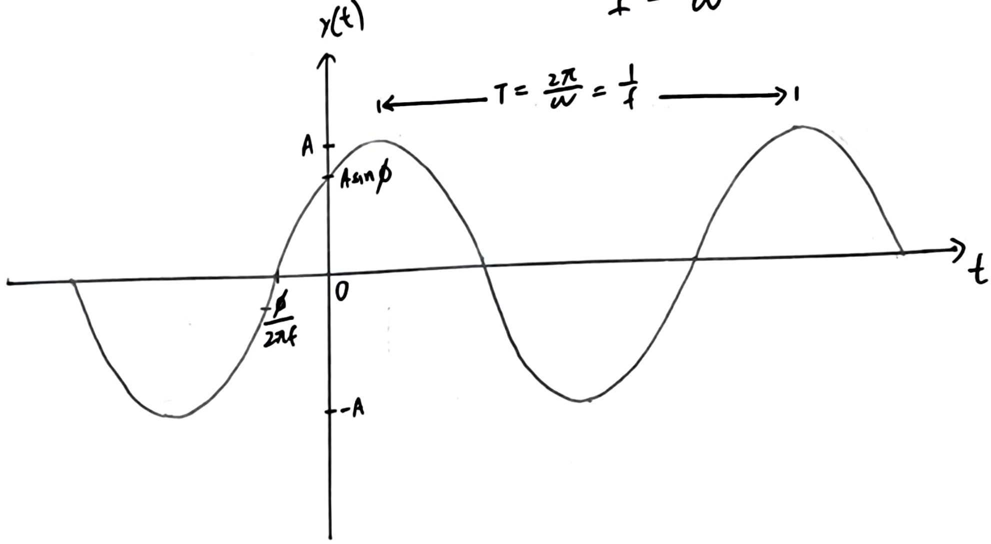
$A \Rightarrow$ Amplitude

$2\pi f = \omega \Rightarrow$ Angular Frequency

$f \Rightarrow$ Cyclic Frequency

$\phi \Rightarrow$ Phase

$\frac{1}{f} = \frac{2\pi}{\omega} = T \Rightarrow$ Fundamental Period



b) $$y(t) = \begin{cases} 1; & -\frac{T}{2} \le t < 0 \\ -1; & 0 \le t < \frac{T}{2} \end{cases}$$

square wave with period T

$$y(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi \frac{k}{T} t}$$

where $$c_k = \frac{1}{T}\int_{-T/2}^{T/2} y(t)\, e^{-j2\pi\frac{k}{T}t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{0} e^{-j2\pi\frac{k}{T}t}\, dt - \frac{1}{T}\int_{0}^{T/2} e^{-j2\pi\frac{k}{T}t}\, dt$$

$$= \frac{1}{T}\left[\frac{e^{-j2\pi\frac{k}{T}t}}{-j2\pi\frac{k}{T}}\right]_{-T/2}^{0} - \frac{1}{T}\left[\frac{e^{-j2\pi\frac{k}{T}t}}{-j2\pi\frac{k}{T}}\right]_{0}^{T/2}$$

$$= \frac{1}{T}\left[\frac{1 - e^{j\pi k} - e^{-j\pi k} + 1}{-j2\pi\frac{k}{T}}\right]$$

$$= \frac{2\cos(\pi k) - 2}{j2\pi k}$$

$$= \begin{cases} 0 & \text{if } k = \text{even} \\ -\frac{2}{j\pi k} & \text{if } k = \text{odd} \end{cases}$$

* y(t) is real & odd

$\Updownarrow$

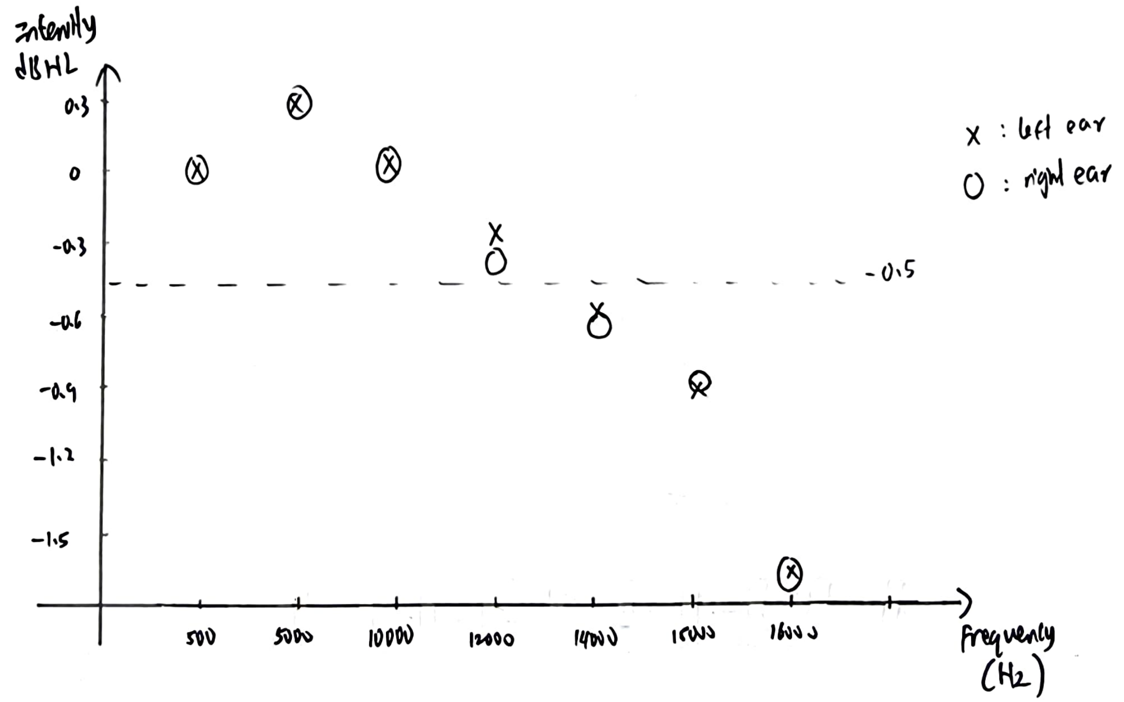
c_k is imaginary & odd

4. Listening to sine and square waveforms

Hearing range upper bound : ≈ 16 kHz

Frequency (Hz)	left ear		right ear	
	Amplitude, A	Intensity (dBHL)	Amplitude, A	Intensity (dBHL)
500	0.1	0	0.1	0
5000	0.05	0.30	0.05	0.30
10000	0.1	0	0.1	0
12000	0.2	-0.30	0.3	-0.48
14000	0.5	-0.70	0.6	-0.78
15000	1.1	-1.04	1.0	-1
16000	5	-1.70	5	-1.70

$$\text{Intensity} = \log_{10} \frac{0.1}{A} \text{ decibel HL (dBHL)}$$



The range I am sensitive to is up to about 12 kHz square wave vs sine wave

At lower frequencies like 400Hz and 1000Hz, square waves sound richer and harsher while sine waves sound smooth and clear. However, at higher frequencies like 4000 Hz and 6000Hz, both waveforms sound much more similar to each other.

A sine wave only has its fundamental frequency component while a square wave is made up of a sum of sine waves at whole odd numbered multiples of the fundamental frequency.

At lower frequencies, for example 400Hz, a 400Hz square wave will have frequency components of 400Hz, 1200Hz, 2000Hz, 2800Hz, ... which are all audible, hence the square wave sounds richer and harsher than the sine wave

However, at higher frequencies, for example 6000Hz, a square wave will have frequency components of 6000Hz, 18kHz, 30kHz, ..., but since the higher harmonics are out of audible range, only the 6000Hz component is audible, hence sounds similar to the 6000Hz sine wave.

5. Spectrum of square waveforms with different duty cycles

$$x(t) = \begin{cases} 1; & 0 \le t < \tau \\ 0; & \tau \le t < T \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi \frac{k}{T} t}$$

where
$$c_k = \frac{1}{T}\int_0^{\tau} e^{-j2\pi \frac{k}{T} t}\, dt$$
$$= \frac{1}{T}\left[\frac{e^{-j2\pi \frac{k}{T} t}}{-j2\pi \frac{k}{T}}\right]_0^{\tau}$$
$$= \frac{e^{-j2\pi k \frac{\tau}{T}} - 1}{-j2\pi k}$$

when $k=0$
$$c_0 = \frac{1}{T}\int_0^{\tau} 1\, dt$$
$$= \frac{\tau}{T}$$

50% duty cycle

$$\tau = \frac{T}{2}$$
$$c_0 = \frac{1}{2}$$
$$c_k = \frac{e^{-j\pi k} - 1}{-j2\pi k}$$
$$= \begin{cases} 0 & \text{if } k \text{ is even} \neq 0 \\ \frac{1}{j\pi k} & \text{if } k \text{ is odd} \end{cases}$$

30% duty cycle

$$\tau = 0.3T$$
$$c_0 = 0.3$$
$$c_k = \frac{e^{-j0.6\pi k} - 1}{-j2\pi k}$$
$$= \begin{cases} 0 & \text{if } k \text{ is multiple of } 10 \\ \frac{e^{-j0.6\pi k} - 1}{-j2\pi k} & \text{otherwise} \end{cases}$$

10% duty cycle

$$\tau = 0.1T$$
$$c_0 = 0.1$$
$$c_k = \frac{e^{-j0.2\pi k} - 1}{-j2\pi k}$$
$$= \begin{cases} 0, & \text{if } k \text{ is multiple of } 10 \\ \frac{e^{-j0.2\pi k} - 1}{-j2\pi k} & \text{otherwise} \end{cases}$$

Comparing the 3 duty cycles, 50% duty cycle has even harmonics with amplitude of 0 while 30% and 10% duty cycles have only frequencies that are multiples of 10 of the fundamental frequency with amplitude of 0, hence they sound different from 50% duty cycle. as they are made up of more frequency components.

50% duty cycle also has highest spectral component at 0Hz with $c_0 = \frac{1}{2}$ while 30% duty cycle has amplitude 0.3 at 0Hz and 10% duty cycle has amplitude 0.1 at 0Hz

compared to the square wave in part 4 which was centered around 0, this square wave has a DC offset which is stored in the 0 frequency value. This DC offset also equal to the average value of the signal.

our ears cannot hear the spectral component at 0Hz

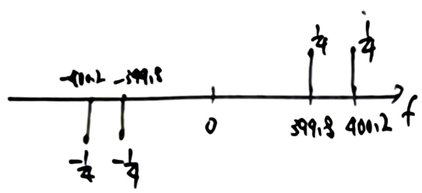
6. Phenomenon of Beats

$$x_1(t) = \sin(2\pi(400)t)$$
$$x_2(t) = \sin(2\pi(\tfrac{1}{5})t)$$

$$x_1(t)\cdot x_2(t) = \sin(800\pi t)\cdot \sin(\tfrac{2\pi}{5}t)$$
$$= \frac{1}{2}\left[\cos(799.6\pi t) - \cos(800.4\pi t)\right]$$
$$= \frac{1}{4}e^{j799.6\pi t} + \frac{1}{4}e^{-j799.6\pi t} - \frac{1}{4}e^{j800.4\pi t} - \frac{1}{4}e^{-j800.4\pi t}$$

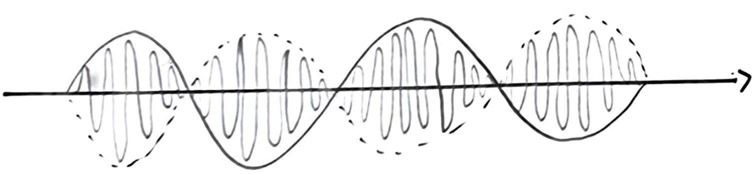
Magnitude spectrum

Spectral Plot {



⇒ new frequencies created

From the scope, the lower frequency sinusoid acts as an envelope to the peaks of the higher-frequency waveform



As expected, the amplitude of the tone oscillates at 2 times the frequency of the lower frequency sinusoid

A possible application of heterodyning is to generate new frequencies and move information from one frequency channel to another

It can also be used to tune instruments who have close but not identical pitch where the difference generates a beat acoustic