

1. a) The PUN for both circuits are the same

$\bar{A}$  is connected in series with  $\bar{B} \Rightarrow \bar{A} \cdot \bar{B}$  (subnet 1)

Subnet 1 is connected in parallel with  $\bar{C}$  and  $\bar{D} \Rightarrow (\bar{A} \cdot \bar{B}) + \bar{C} + \bar{D}$

$$\boxed{OUT = (\bar{A} \cdot \bar{B}) + \bar{C} + \bar{D}}$$

PDN for Circuit A

A is connected in parallel with B  $\Rightarrow A + B$  (subnet 1)

subnet 1 is connected in series with C and D  $\Rightarrow (A + B) \cdot C \cdot D$

$$\overline{OUT} = (A + B) \cdot C \cdot D$$

$$OUT = \overline{(A + B) \cdot C \cdot D}$$

$$= \overline{A + B} + \bar{C} + \bar{D}$$

$$= (\bar{A} \cdot \bar{B}) + \bar{C} + \bar{D}$$

PDN for Circuit A is a dual network of PUN

PDN for Circuit B

D is connected in series with C  $\Rightarrow D \cdot C$  (subnet 1)

Subnet 1 is connected in series with A parallel with B  $\Rightarrow D \cdot C \cdot (A + B)$

$$\overline{OUT} = D \cdot C \cdot (A + B)$$

$$= (A + B) \cdot C \cdot D \quad (\text{commutativity of AND operator})$$

$$OUT = \overline{(A + B) \cdot C \cdot D}$$

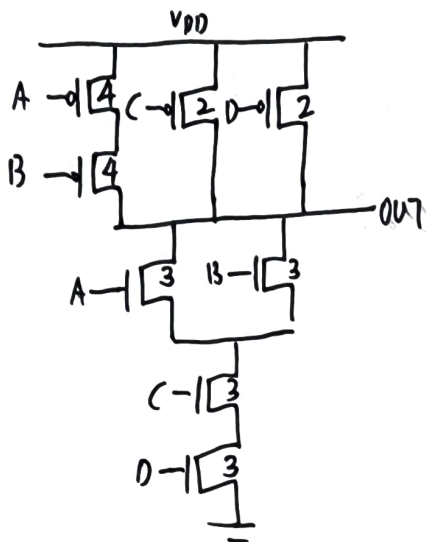
$$= (\bar{A} \cdot \bar{B}) + \bar{C} + \bar{D}$$

PDN for Circuit B is a dual network of PUN

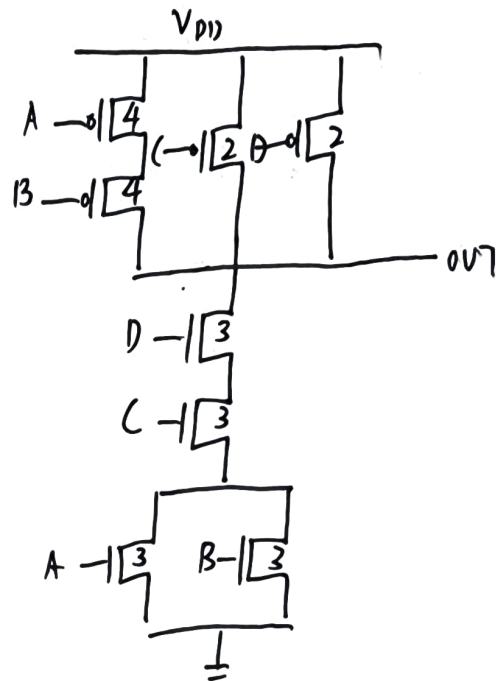
Both circuits implement the same logic function

b)

circuit A



circuit B



$$R \propto \frac{1}{W}$$

Let  $W_{PMOS}$  of INV1X be  $2\lambda$  and the resistance of this PMOS be  $R_p$   
 Let  $W_{NMOS}$  of INV1X be  $\lambda$  and the resistance of this NMOS be  $R_n$

PUN

short circuit paths through PUN

A-B  
C  
D

For A-B, each PMOS has resistance  $\frac{1}{2}R_p$  so  $\frac{1}{2}R_p + \frac{1}{2}R_p = R_p$

A and B should be assigned width  $4\lambda$

since width of PMOS A = 4  $\Rightarrow \lambda = 1$

A = 4, B = 4

For C and D, paths with a sink transistor should have same size as reference

C =  $2\lambda = 2$

D =  $2\lambda = 2$

A = 4

B = 4

C = 2

D = 2

## PDN

short circuit paths through PDN

A - C - D

B - C - D

For A - C - D, each NMOS has resistance  $\frac{1}{3}R_N$  so  $\frac{1}{3}R_N + \frac{1}{3}R_N + \frac{1}{3}R_N = R_N$

A, C, D should be assigned with 3

since  $\lambda = 1$ ,  $A = 3$ ,  $C = 3$ ,  $D = 3$

For B - C - D, since C and D have width 3 and resistance  $\frac{1}{3}R_N$  each

resistance of B needs to be  $R_N - \frac{1}{3}R_N - \frac{1}{3}R_N = \frac{1}{3}R_N$

B also has width 3

$$A = 3$$

$$B = 3$$

$$C = 3$$

$$D = 3$$

c)

when output is low, PDN is connected to GND

worst case output resistance of  $12k\Omega$  when each independent series path is turned ON only (A - C - D or B - C - D)

$\Rightarrow$  each NMOS has resistance  $12k\Omega \div 3 = 4k\Omega$

output resistance lowest when all paths are connected to GND

$\Rightarrow$  all inputs are high to turn ON each NMOS

$$A = 1$$

$$B = 1$$

$$C = 1$$

$$D = 1$$

$$R_{PDN} = R_A // R_B + R_C + R_D$$

$$= 2k\Omega + 4k\Omega + 4k\Omega$$

$$= \underline{10k\Omega}$$

d) when output is high, PUN is connected to VDD  
worst case output resistance of  $12k\Omega$  when each independent series connected path is turned ON only (A-B or C or D)

$$\text{Resistance of A and B} = 12k\Omega \div 2 = 6k\Omega$$

$$\text{Resistance of C} = 12k\Omega$$

$$\text{Resistance of D} = 12k\Omega$$

output resistance is lowest when all 3 paths are connected to GND

$\Rightarrow$  all inputs are low

$$A = 0$$

$$B = 0$$

$$C = 0$$

$$D = 0$$

$$\begin{aligned} R_{\text{min}} &= (R_A + R_B) // R_C // R_D \\ &= 12k\Omega // 12k\Omega // 12k\Omega \\ &= \underline{4k\Omega} \end{aligned}$$

e) best case  $t_{\text{PLH}}$  and  $t_{\text{PHL}}$  when output resistance is lowest

$t_{\text{PLH}}$  is the propagation delay when capacitor is charged through PUN

$$\begin{aligned} \text{best case } t_{\text{PLH}} &= 0.69 \times R_{\text{PUN}} \times C \\ &= 0.69 \times 4k\Omega \times 100\text{fF} \\ &= \underline{276\text{ps}} \end{aligned}$$

$t_{\text{PHL}}$  is the propagation delay when capacitor is discharged through PDN

$$\begin{aligned} \text{best case } t_{\text{PHL}} &= 0.69 \times R_{\text{PDN}} \times C \\ &= 0.69 \times 10k\Omega \times 100\text{fF} \\ &= \underline{690\text{ps}} \end{aligned}$$

2. a)

$$\begin{aligned}x &= A \cdot B + 0 \cdot \bar{B} \\ &= A \cdot B \quad (\text{AND})\end{aligned}$$

x is input to CMOS inverter

$$\text{OUT} = \overline{AB}$$

circuit performs NAND logic function

$$= \overline{A} + \overline{B}$$

b)

consider when  $A = V_{DD}$  and B is initially OFF

when B is switched ON, current flows from  $A = V_{DD}$  to charge x

The source of the PMOS transistor is at x and drain at  $V_{DD}$

so  $V_{GS} = V_{DS} = V_{DD}$  immediately after  $B = 1$

$x = V_S$  keeps increasing until  $I_D = 0$  when  $V_{GS} = V_{TH}$  or  $V_{DS} = 0V$

since  $V_{GS}$  will hit threshold first,  $V_S = x$  can only increase until  $V_{DD} - V_{TH}$

there is a  $V_{TH}$  voltage drop when trying to pull a 1

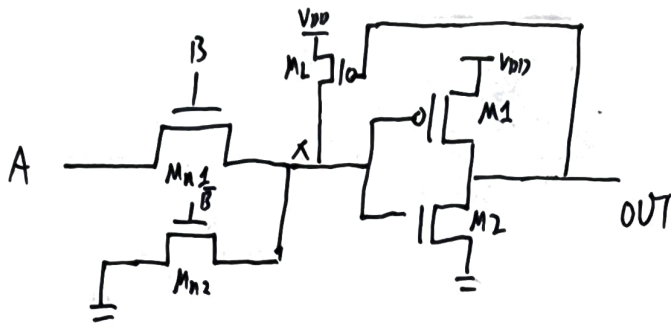
when x is not at  $V_{DD}$ , the PMOS transistor in CMOS inverter after the pull transistor network might not be completely off with  $V_{GS}$  is not 0

Hence M1 and M2 might both be on simultaneously which causes

a short circuit current to flow through which results in non-zero

static power dissipation

c)



we could add a PMOS transistor which acts as a level restoring transistor  $M_L$

To turn a weak 1 to a strong 1 at  $x$ , the inverter detects the weak 1 and passes a 0 to the level restoring PMOS which turns on and passes a strong 1  $V_{DD}$  to  $x$ .

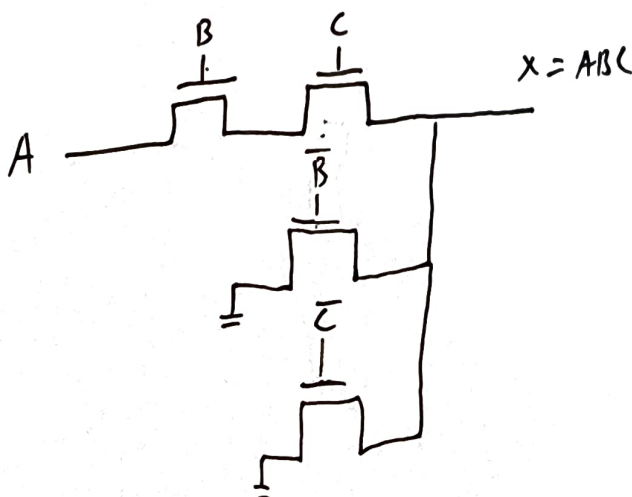
when  $A$  or  $B$  is switched to 0, the NMOS would want to pass a strong 0 while the level restoring PMOS wants to pass a strong 1

voltage at  $x$  is determined by relative values of resistance of NMOS and PMOS

To allow NMOS to pull  $x$  down to a voltage low enough such that the inverter sees a 0, resistance of PMOS has to be higher than NMOS

$M_L$  should have a small enough size so that  $M_{n1}$  or  $M_{n2}$  can pull node  $x$  to a low enough voltage so inverter output will switch and turn off PMOS

d)



$$x = ABC + 0\bar{B} + 0\bar{C} \\ = ABC$$