CS1231/CS1231S Assignment #2

AY2020/21 Semester 1

Deadline: Wednesday, 4 November 2020, 4:00pm

IMPORTANT: Please read the instructions below

This is a graded assignment worth 10% of your final grade. Please work on it by yourself, not in a group or in collaboration with anybody. Anyone found plagiarising (submitting other's work as your own), or sending your answers to others will be penalised with a straight zero for the assignment, and if found re-committing this offence, will be referred to the disciplinary board.

You are to submit your assignment to **LumiNUS Files**. A submission folder has been created for you at Files > Assignment #2 > Your tutorial group > Your personal folder.

Your answers may be typed or handwritten. Make sure that it is legible (for example, don't use very light pencil or ink, or very small font) or marks may be deducted.

You are to submit **a SINGLE pdf file**, where each page is A4 size. Do not submit multiple files or files in other format, or we will not accept your submission.

You may test out your submission before the deadline, but make sure you remove any test files you have submitted earlier.

Late submission will NOT be accepted, as the folder will automatically close on the dot. We will set the closing time to slightly later than 4pm to provide a grace period, but in your mind, you should treat **4pm** as the deadline. If you think you might be too busy on the day of the deadline, please submit earlier. Also, avoid submitting in the last minute; if everybody does that (and we have more than 1000 students in CS1231 and CS1231S) the system may get sluggish due to the overload, or worse, it may break down, and you will miss the deadline.

Note the following as well:

- Name your pdf file with your **Student Number** (eg: A0123456M)
- To keep the submitted document short, you may submit your answers without including the questions.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit **polished work**, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places.

To combine all pages into a single pdf document for submission, you may find the following scanning apps helpful if you intend to scan your handwritten answers:

* for Android: https://fossbytes.com/best-android-scanner-apps/

* for iphone: https://www.switchingtomac.com/tutorials/ios-tutorials/the-best-ios-scanner-apps-to-scan-documents-images/

If you need any clarification about this assignment, please post on the **LumiNUS > Assignments** forum.

Question 0. (2 marks)

This is an administrative question (and 2 free marks). Please follow these instructions carefully.

a. Remove any unnecessary files from your submission folder, INCLUDING your midterm submission AND Assignment 1 submission, if you haven't already done so. There should be only one file in your submission folder. (1 mark)

b. Name your file with your **Student Number** (eg: A0123456M) (1 mark)

Question 1. (9 marks)

Aiken and Betty are getting married, and as their best friend you have agreed to help them run various errands large and small.

a. Your first task is to plan the seating for the "Main Table", where the happy (but severely stressed) couple and their direct family members would be sitting. As COVID-19 will finally be over by the time they get married, you are assuming a traditional table of 10 persons around a round table. This is a difficult task whose consequences can be cataclysmic, so you must proceed carefully. You begin by working out how many possible seating arrangements you can make, so that Aiken and Betty can exhaustively consider all the options. **You must show your working in order to convince Aiken and Betty, or you don't get any marks from us.**

   i. Aiken and Betty themselves, and their respective pairs of parents must sit next to each other (i.e. Aiken's dad and mum have to sit together, and likewise with Betty's). However Aiken's mum and dad do not have to sit next to Betty's mum and dad. How many ways can you arrange the couple, their parents and four of their relatives (including Uncle Pete and Aunt Jemima whom we will meet in the next part) around the table? (2 marks)

   ii. (NOTE: This part depends on you solving the earlier question about seating Aiken and Betty's relatives. Please do that part first) Aiken now tells you that his Uncle Pete and Betty's Aunt Jemima are not on the best of terms, and if they sit next to each other, World War 3 will break out. How many ways can you now arrange their relatives around this table in order to forestall this apocalypse? (3 marks)

b. After an exhausting session of considering how to prevent an extinction-level event from occurring from poor seating choices, Aiken, Betty and their parents (6 in total) are now starving. Your next task is to buy food for them. Your options from the nearby food centre are (i) chicken rice, (ii) roast duck rice, (iii) vegetarian rice and (iv) vegetarian noodles. **Again if you do not show any working, you will not get any marks from us.**

   i. How many ways can you buy food for the Starving Six? (No food for you. Sorry.) We are buying 1 dish per person. (1 mark)

   ii. It turns out that Aiken's parents are vegetarians. Now how many ways can you buy food for the Starving Six? Again we are buying one dish per person. (3 marks)

Question 2. (6 marks)

Let $a \in \mathbb{Z}_{\geqslant 2}$. Suppose that for all $m, n \in \mathbb{Z}^+$, if $a \mid mn$, then $a \mid m$ or $a \mid n$. Show that a is prime.

Question 3. (3 marks)

Use the Euclidean Algorithm to find integers x and y that satisfy

$$42x + 15y = 6.$$

Show all computational steps.

Question 4. (12 marks)

Fix $n \in \mathbb{Z}_{\geqslant 2}$. Define a relation R on $\{0,1,\ldots,n-1\}$ by setting, for all $x, y \in \{0,1,,\ldots,n-1\}$,

$$x\ R\ y \quad \Leftrightarrow \quad \exists a \in \{1,2,\ldots,n-1\} \ \ ax \equiv ay \pmod{n}.$$

(a) Show that R is reflexive and symmetric. [2 marks]

(b) Suppose $n = 4$.

(i) Draw the following diagram.

$$0 \qquad 2$$

$$1 \qquad 3$$

For each pair $x, y \in \{0,1,2,3\}$, draw a straight line (without arrowheads) joining x and y in your copy of the diagram if $x \neq y$ and $x\ R\ y$. Draw no other line. [2 marks]

(ii) Is R an equivalence relation in this case? Briefly justify your answer. If R is an equivalence relation, then write out all the equivalence classes in roster notation without repetition. [3 marks]

(c) Suppose $n = 6$.

(i) Draw the following diagram.

$$\begin{array}{ccccc} & 0 & & 3 & \\ 4 & & & & 1 \\ & 2 & & 5 & \end{array}$$

For each pair $x, y \in \{0,1,\ldots,5\}$, draw a straight line (without arrowheads) joining x and y in your copy of the diagram if $x \neq y$ and $x\ R\ y$. Draw no other line. [2 marks]

(ii) Is R an equivalence relation in this case? Briefly justify your answer. If R is an equivalence relation, then write out all the equivalence classes in roster notation without repetition. [3 marks]

Explicit references to ~~Lemma 8.6.5~~ and ~~Proposition 8.6.13~~ may be omitted in your proofs and arguments.

congurence relation is an equivalance relatior (i.e. satisfying reflexive, symmetric and transitive property)

Theorem 7.10

Question 5. (5 marks)

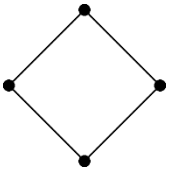
Let $\mathcal{C}$ be a partition of a set A. Show that there exist a set B and a surjection $f: A \to B$ such that

$$\mathcal{C} = \{\{x \in A : f(x) = y\} : y \in B\}.$$

Question 6. (3 marks)

For each $n \in \mathbb{Z}^+$, let $D_n = \{d \in \mathbb{Z}^+ : d \mid n\}$. Write down $a, b, c \in \mathbb{Z}^+$ such that the following are respectively Hasse diagrams for D_a, D_b, and D_c with respect to the partial order "divides":

(a) 

(b) 

(c)