CS1231

Discrete Structures

Revision Session 2

2018/19 Sem I PYP (1810)

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Question 16

Q16. Counting and Probability (14 marks)

Answer the following parts. Working is not required.

(a) (2 marks) What is the probability that on three rolls of a fair six-sided die, at least one 6 shows up? Leave your answer as a fraction or marks will be deducted.

- **Solution 1 Using Addition Rule:**

P(At least 1 6) = P(one 6) + P(two 6) + P(three 6)

$$= \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{25}{216} + \frac{5}{216} + \frac{1}{216} = \frac{31}{216}$$

Question 16

- **Solution 2 Using Subtraction Rule:**
  - Find combinations that have no 6's. I.e. all 5 faces except 6. Probability of each throw is 5/6. Three throws:

$$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

  - So probability of at least one 6 = $1 - \frac{125}{216} = \frac{91}{216}$

TWO ANSWERS ARE DIFFERENT! ONE OF THE SOLUTIONS IS WRONG!

Question 16

- **Solution 1 using Addition Rule (Corrected):**

case I: Probability of getting 1 '6':

Possiblities: Throw a 6, then two non-6. Or throw a non-6, then a '6', then a non-6. Throw two non-6, then a 6. So three possible ways of getting a 6.

Probability = $3 \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{75}{216}$

Combinatorially, this is equivalent to choosing ONE position for the '6'. So

P(one 6) = $\binom{3}{1} \times \frac{1}{6} \times \frac{5}{6}$

Case II: Probability of getting 2 '6's:

P(two 6) = $\binom{3}{2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{15}{216}$

Case III: Probability of getting 3 '6's:

P(three 6) = $\binom{3}{3} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

Total = P(one 6) + P(two 6) + P(three 6) = $\frac{75}{216} \div \frac{15}{216} \div \frac{1}{216} = \frac{91}{216}$

Question 16

(b) (2 marks) Figure 1 below shows a combination lock with 40 positions.

Figure 1: Combination Lock.

To open the lock, you rotate to a number in a clockwise direction, then to a second number in the counterclockwise direction, and finally to a third number in the clockwise direction. If consecutive numbers in the combination cannot be the same, how many combinations of three-number codes are there?

Question 16

- **Straightforward, covered in tutorial:**

  First number = 40 choices.

  Second number = 40 – choice for first number = 39

  Third number = 40 – choice for second number = 39

  Total choices = 40 x 39 x 39 = 60,840 possibilities.

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Question 16

(c) (2 marks) There are 12 slips of paper in a bag. Some of the slips have a 2 written on them, and the rest have a 7 written on them. If the expected value of the number shown on a slip randomly drawn from the bag is 3.25, how many slips have a 2 written on them?

- Let n be the number of slips with 2 written on it.
- Probability of drawing a slip with '2' = $\frac{n}{12}$
- Probability of drawing a slip with '7' = $\frac{12-n}{12}$
- Expected value = $2\times\frac{n}{12} + 7\times\frac{12-n}{12} = 3.25$
- So $\frac{2n+84-7n}{12} = 3.25$
- Solve for n gives us n = 9.

Question 16

(d) A bowl contains three coins. Two of them are normal coins and one of them is a two-headed coin.

i. (2 marks) You pick one coin at random and toss it. What is the probability that you get a head? Write your answer as a fraction.

ii. (2 marks) You pick one coin at random, toss it and get a head. What is the probability that the coin is the two-headed coin? Write your answer as a fraction.

- Let C1, C2 and C3 be the event that you threw coin 1, coin 2 (both normal) and coin 3 (two-headed), H be the event that you threw a head, and T be the event that you threw a tails. We assume that it is equally likely to pick each of the coins. I.e. P(C1) = P(C2) = P(C3) = $\frac{1}{3}$

$$P(H) = P(H \cap C1) + P(H \cap C2) + P(H \cap C3)$$
$$= P(H|C1)P(C1) + P(H|C2)P(C2) + P(H|C3)P(C3)$$
$$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} + 1 \times \frac{1}{3}$$
$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3}$$

Question 16

- **We want to find P(C3 | H):**

By conditional probabilities: $P(C3|H) = \frac{P(C3 \cap H)}{P(H)}$

$$= \frac{P(H|C3)P(C3)}{P(H)}$$

$$= \frac{1 \times \frac{1}{3}}{\frac{2}{3}}$$

$$= \frac{1}{3} \times \frac{3}{2}$$

$$= \frac{1}{2}$$

Question 16

(e) (4 marks) A row of houses are randomly assigned distinct numbers between 1 and 50 inclusive. What is the minimum number of houses to ensure that there are 5 houses numbered consecutively?

To receive full credit, you must define the pigeon and pigeonholes.

- **Straightforward, already covered in tutorial:**
  - We want to have a group of 5 houses. Hence break the number range 1-50 into ranges (or blocks) of 5. This gives us 10 blocks – the pigeonholes.
  - Now we need our pigeons:

  Question statement: At least one block of five consecutive houses.

  Rephrasing it to suit the generalized PHP: "Given k < n/m, there is some range where there will be k+1 = 5 houses". Hence k + 1 = 5, and k = 4.

  Let n be the number of houses:

  $4 < \frac{n}{10}, or\ n > 40$. Since n is an integer (cannot have part a house), $n = 41$.

Question 17

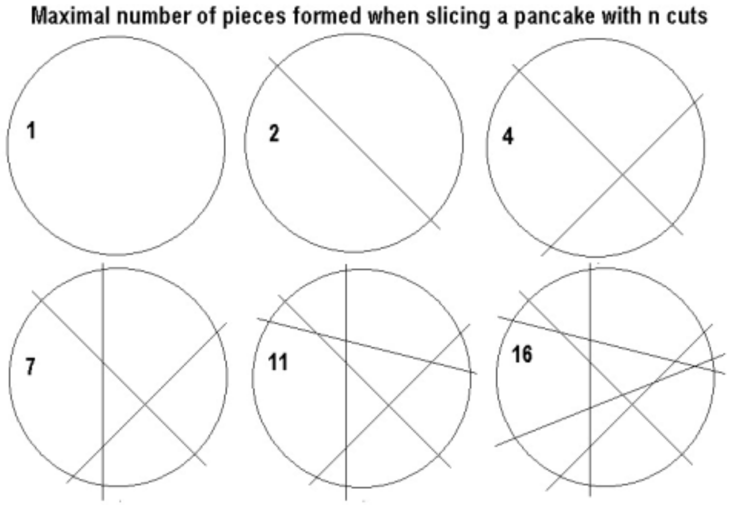
Q17. Graphs (14 marks)

The *lazy caterer's sequence* describes the number of maximum pieces of a pancake (or pizza) that can be made with a given number of straight cuts.

For example, with three straight cuts, you get seven pieces as shown in Figure 2 below.

Figure 2: Pancake (Photo credit: Wikipedia).

Figure 3 below shows the first few values in the lazy caterer's sequence starting with $n = 0$ where n is the number of straight cuts. The sequence is 1, 2, 4, 7, 11, 16, ...



Question 17

We may model this problem by using a graph. Figure 4 shows the graph corresponding to $n = 3$, where the vertices are the intersections among the cuts and the boundary of the pancake.

We may define the following functions:

$P(n)$: number of pieces of pancakes with n cuts

$V(n)$: number of vertices of a graph corresponding to a pancake with n cuts

$E(n)$: number of edges of a graph corresponding to a pancake with n cuts

In Figure 4, $P(3) = 7, V(3) = 9$ and $E(3) = 15$.

Question 17

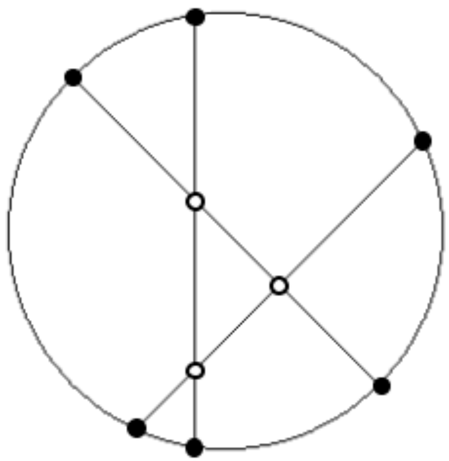


Figure 4: Graph representation.

The vertices of the graph are of two types: those with degree of three (which lie on the boundary of the circle, shown as black dots) and those with degree of four (which lie inside the circle, shown as white dots). Let's define two more functions:

$V_3(n)$: number of vertices with degree three

$V_4(n)$: number of vertices with degree four

In Figure 4, $V_3(3) = 6$ and $V_4(3) = 3$.

Answer the following parts. Working is not required.

Question 17

(a) (2 marks) Express $E(n)$ in terms of $V_3(n)$ and $V_4(n)$.

- **Note: Based on the examples given, we assume that new cuts will intersect existing cuts, and that cuts do not intersect existing vertices. I.e. a new cut will always add vertices where they meet existing cuts. Thus:**
  - A vertex at the perimeter will always have degree 3: The cut itself and the perimeter now split into 2 by the cut.
  - A vertex at the intersection of two cuts will be of the form X and always have degree 4, with each edge being the two halves of the intersecting lines.
- **Total degrees = 2 x # of edges**

  Total degrees = $3\times V_3(n) + 4\times V_4(n)$, since we know that:

  i. The graph consists solely of degree 3 and 4 vertices.

  ii. $V_3(n)$ and $V_4(n)$ are the number of vertices with degree 3 and 4 respectively

  Hence # of edges $E(n) = \frac{3\times V_3(n)+4\times V_4(n)}{2}$

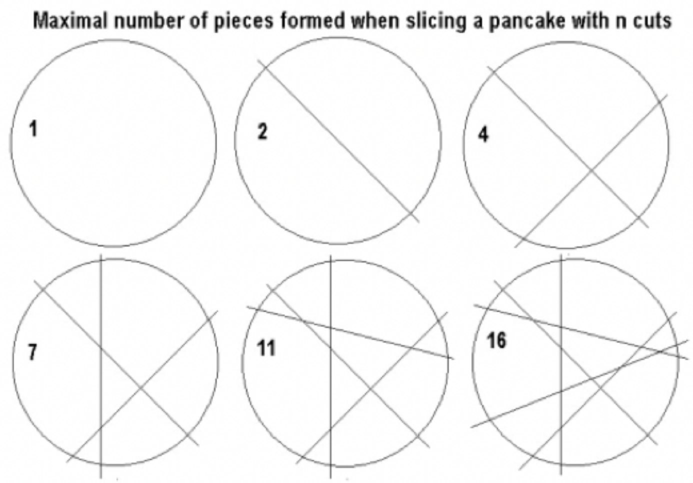
Question 17

(b) (2 marks) Write the recurrence relation for $V(n)$. The base case is $V(0) = 0$.

- Each new cut will add the following NEW vertices to the existing ones:
  - Two new vertices on the perimeter.
  - One new vertex where it intersects each of the preceding *n-1* cuts.

Hence:

$$V(n) = V(n-1) + 2 + n - 1$$
$$= V(n-1) + n + 1$$



Question 17

(c) (2 marks) Write the closed form formula for $V(n)$.

- **We begin by writing the values for various V(n) as sums of previous V(n) using V(n) = V(n-1) + n + 1:**

n	V(n)	n	V(n)
0	0	3	0 + 2 + 3 + 4 = 9
1	0 + 2 = 2	4	0 + 2 + 3 + 4 + 5 = 14
2	0 + 2 + 3 = 5	5	0 + 2 + 3 + 4 + 5 + 6 = 20

- **We observe that the sequence is**

$$V(n) = \left(\sum_{i=1}^{n+1} i\right) - 1$$

$$= \frac{(n+1)(n+2)}{2} - 1$$

$$= \frac{n^2 + 3n + 2 - 2}{2}$$

$$= \frac{n^2 + 3n}{2}$$

Question 17

(d) (2 marks) Write the recurrence relation for $E(n)$. The base case is $E(0) = 0$.

Every new cut added:

- Adds 2 new edges to perimeter since it slices it at two places.
- Slices each of the existing n-1 edges into additional n – 1 new edges.
- Is itself sliced into n – 1 + 1 = n edges
- So total added = 2 + n – 1 + n – 1 + 1 = 2n + 1 edges.

So:

$$E(n) = E(n-1) + 2 + (n-1) + (n-1+1)$$
$$= E(n-1) + 2n + 1$$

Question 17

(e) (2 marks) Write the closed form formula for $E(n)$.

As before we tabulate the edges for each n:

n	E(n)	n	E(n)
0	0	3	0 + 3 + 5 + 7 = 15
1	0 + 3 = 3	4	0 + 3 + 5 + 7 + 9 = 24
2	0 + 3 + 5 = 8	5	0 + 3 + 5 + 7 + 9 + 11 = 35

We observe that this is equivalent to $E(n) = \sum_{i=1}^{n}(2i+1)$

$$= \sum_{i=1}^{n} 2i + \sum_{i=1}^{n} 1$$

$$= 2\times \sum_{i=1}^{n} i + n$$

$$= 2\times \frac{n(n+1)}{2} + n$$

$$= n^2 + 2n$$

Question 17

(f) (2 marks) Euler's formula is given as $v - e + f = 2$. Relate v, e and f with the functions defined in this question.

$$v = V(n), e = E(n), f = P(n) + 1$$

Since f includes the surface outside of the pizza.

So:

$$V(n) - E(n) + P(n) + 1 = 2$$
$$V(n) - E(n) + P(n) = 1$$

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Question 17

(g) (2 marks) From part (f), or otherwise, derive the closed form formula for $P(n)$.

$$\begin{aligned}
P(n) &= 1 + E(n) - V(n) \\
&= 1 + n^2 + 2n - \frac{n^2 + 3n}{2} \\
&= \frac{2n^2 + 4n - (n^2 + 3n)}{2} + 1 \\
&= \frac{2n^2 - n^2 + 4n - 3n}{2} + 1 \\
&= \frac{n^2 + n}{2} + 1 \\
&= \frac{n^2 + n + 2}{2}
\end{aligned}$$

Question 18

Q18. Functions (12 marks)

Private cars in Singapore have license plates (see Figure 5) in the format: $S\alpha_1\alpha_2\ x_1x_2x_3x_4\ c$, where each α_1 and α_2 is a single letter taken from the usual English alphabet (excluding I and O), and each $x_1, \ldots, x_4$ is a single digit taken from $\{0, 1, \ldots, 9\}$. The last letter c is a checksum letter, ie. a function of the preceding letters and numbers. Its purpose is to serve as a quick check on the validity of the license plate.

Figure 5: A typical Singapore car license plate. (Photo Credit: Wikipedia)

Let $\mathcal{L}$ denote the set of all possible strings of the form: $\alpha_1\alpha_2x_1x_2x_3x_4$. The possible values of α_i and x_j are as described above. Also, let $\mathcal{K} = \{A, Z, Y, X, U, T, S, R, P, M, L, K, J, H, G, E, D, C, B\}$. Then the checksum function may be defined as $f : \mathcal{L} \to \mathcal{K}$, where $f(\alpha_1\alpha_2x_1x_2x_3x_4)$ is calculated in three steps:

Question 18

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F1. Let n_1 be the positional value of α_1 in the English alphabet, ie. $A = 1,\ B = 2,\ C = 3, \ldots,\ Z = 26$. And let n_2 be the positional value of α_2. (Note that since I and O are not allowed, $n_1, n_2 \notin \{9, 15\}$.)

F2. Compute $t = 9n_1 + 4n_2 + 5x_1 + 4x_2 + 3x_3 + 2x_4$, and $r = t\%19$. That is, r is the remainder of t modulo 19, which means $0 \le r < 19$.

F3. The checksum letter $c =$ the letter in $\mathcal{K}$ indexed by r, where $0 = A, 1 = Z, 2 = Y, \ldots, 18 = B$. (Here, we are treating $\mathcal{K}$ as an ordered set, in which its elements are indexed by position, starting from 0.)

Using the example in Figure 5:

F1. $\alpha_1 = D,\ \alpha_2 = N,\ x_1 = 7,\ x_2 = 4,\ x_3 = 8,\ x_4 = 4$; and so $n_1 = 4,\ n_2 = 14$.

F2. Then $t = 9 \cdot 4 + 4 \cdot 14 + 5 \cdot 7 + 4 \cdot 4 + 3 \cdot 8 + 2 \cdot 4 = 175$, and so $r = 175\%19 = 4$.

F3. Hence $c = U$.

Question 18

(a) (2 marks) Determine the checksum letter for $CS1231$. (No working needed.)

$$C = 3, S = 19$$
$$t = (9\times3 + 4\times19 + 5\times1 + 4\times2 + 3\times3 + 1\times2)$$
$$= 27 + 76 + 5 + 8 + 9 + 2$$
$$= 127$$
$$r = 127\%19$$
$$= 13$$
$$The\ letter\ in\ \mathcal{K}\ at\ position\ 13\ is\ H.$$
$$Note\ that\ A\ is\ in\ position\ 0, not\ 1.$$
$$\{A, Z, Y, X, U, T, S, R, P, M, L, K, J, H, G, E, D, C, B\}$$

Question 18

(b) (2 marks) Show that f is not one-to-one by finding another $y \in \mathcal{L}$ such that $f(y)$ is the same checksum letter as that in Figure 5. (No working needed. Just state a suitable y.)

$$t = 9n_1 + 4n_2 + 5x_1 + 4x_2 + 3x_3 + 2x_4,$$

We note that n_2 and x_2 have the same weight. This means we can increase n_2 by 1 and decrease x_2 by 1 and get back the same value 127.

So we use *CT*1131:

$$t = 9\times 3 + 4\times 20 + 5\times 1 + 4\times 1 + 3\times 3 + 2\times 1$$
$$= 27 + 80 + 5 + 4 + 9 + 2$$
$$= 127$$
$$r = 127\%19 = 13$$

So both CS1231 and CT1131 map to H. The function is not injective.

(NOTE: I should have used DN7484 instead of CS1231. In this case we cannot increase N to O because O is not permitted, so we decrease N to M and increase 4 to 5, giving us DM7584)

Question 18

(c) (8 marks) (Difficult) Is f onto? Prove or disprove.

1. We begin by constructing this table for $g(x_3, x_4) = (3x_3 + 2x_4)\ modulo\ 19$ and show that it is surjective.

x_3/x_4	0	1	2	3	4	5	6	7	8	9
0	0	2	4	6	8	10	12	14	16	18
1	3	5	7	9	11	13	15	17	0	2
2	6	8	10	12	14	16	18	1	3	5
3	9	11	13	15	17	0	2	4	6	8
4	12	14	16	18	1	3	5	7	9	11
5	15	17	0	2	4	6	8	10	12	14
6	18	1	3	5	7	9	11	13	15	17
7	2	4	6	8	10	12	14	15	18	1
8	5	7	9	11	13	15	17	18	2	4
9	8	10	12	14	16	18	1	2	5	7

Question 18

To prove surjectivity of the overall function, we only need to show that for each element in $\mathcal{K}$, there is some string $n_1n_2x_1x_2x_3x_4$ that maps to it. So we can construct such a string:

2. We start by constructing a string *S* that results in a partial value that is divisible by 19:

   2.1 We take S = BB03, and find $t_1 = 9\times n_1 + 4\times n_2 + 5\times x_1 + 4\times x_2$

   2.1.1 BB03: $t_1 = 9\times 2 + 4\times 2 + 5\times 0 + 4\times 3$
   $= 18 + 8 + 0 + 12$
   $= 38$
   $= 2\times 19$

   2.2 We can now create any modulo in the table by concatenating the respective *x3* and x*4* to *S*. That is, $r = g(x_3, x_4)$. Proof:

   2.2.1 We set $t_2 = 3x_3 + 2x_4$. So:
   $t = t_1 + t_2$
   $= 2\times 19 + t_2$

Question 18

2.2.2 Using the division theorem, we can rewrite t_2 as $19a + b$. Since $g(x_3, x_4) = t_2$ *modulo* 19, $g(x_3, x_4) = b$

2.2.3 Then $t = 2\times19 + 19\times a + b$

$$= (2 + a)\times19 + b$$

$$r = t \ mod\ 19 = b$$

$$= g(x_3, x_4)$$

3. Thus by fixing the first four characters as BB03, $r = g(x_3, x_4)$. Since $g(x_3, x_4)$ is surjective as shown in the table earlier, f is surjective.

E.g. BB0312: $t = 9\times2 + 4\times2 + 5\times0 + 4\times3 + 3\times1 + 2\times2$

$= 18 + 8 + 0 + 12 + 3 + 4$

$= 45$

$r = 7$

Question 18

x_3/x_4	0	1	2	3	4	5	6	7	8	9
0	0	2	4	6	8	10	12	14	16	18
1	3	5	7	9	11	13	15	17	0	2
2	6	8	10	12	14	16	18	1	3	5
3	9	11	13	15	17	0	2	4	6	8
4	12	14	16	18	1	3	5	7	9	11
5	15	17	0	2	4	6	8	10	12	14
6	18	1	3	5	7	9	11	13	15	17
7	2	4	6	8	10	12	14	15	18	1
8	5	7	9	11	13	15	17	18	2	4
9	8	10	12	14	16	18	1	2	5	7

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MCQ

Q1. How many permutations are there for this 8-letter word "ICanDoIt"?

A. 7!

B. (8/2)!

C. 8!

D. 8!/2

E. 8×7

Q2. Which of the following arguments are valid?

(I) No mammals lay eggs.
Duck-billed platypus lays eggs.
Therefore, duck-billed platypus is not a mammal.

(II) You get A grade if and only if you get more than 90 marks.
If you fail your midterm test, you don't get more than 90 marks.
Therefore, if you don't fail your midterm test, you get A grade.

(III) Water is a necessary condition for air.
Water is a sufficient condition for ice.
Therefore air only if ice.

A. (I) only.

B. (I) and (II) only.

C. (I) and (III) only.

D. (II) and (III) only.

E. All of (I), (II) and (III).

MCQ

Q3. How many integer solutions are there for the following equation, where each $x_i \geq 2$?

$$x_1 + x_2 + x_3 = 20.$$

A. 14

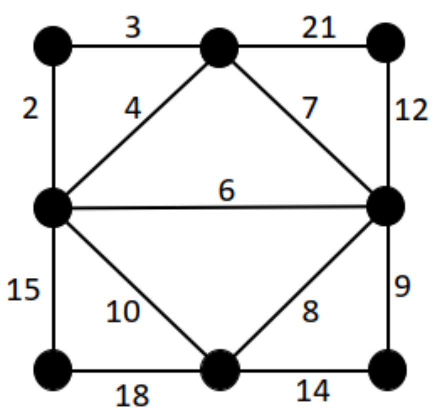
B. 120

C. 230

D. 680

E. None of the above.

Q4. What is the total weight of the minimum spanning tree of the graph shown below?



A. 39

B. 47

C. 50

D. 55

E. 60

MCQ

Q5. Which of the following statements are TRUE about the graph in Q 4?

A. The graph contains an Euler circuit.

B. The graph contains an Euler trail but not an Euler circuit.

C. The graph does not contain an Euler trail nor an Euler circuit.

D. The graph does not contain a Hamiltonian circuit.

E. None of the above.

Q6. The preorder traversal and inorder traversal of a binary tree with vertices A, B, C, D, E, F, G, H, I, J, K and L are given below:

Preorder:	A	B	D	G	I	J	C	E	H	K	L	F
Inorder:	D	I	G	J	B	A	E	K	H	L	C	F

What is the postorder traveral of this binary tree?

A. A B C D E F G H I J K L

B. L K J I H G F E D C B A

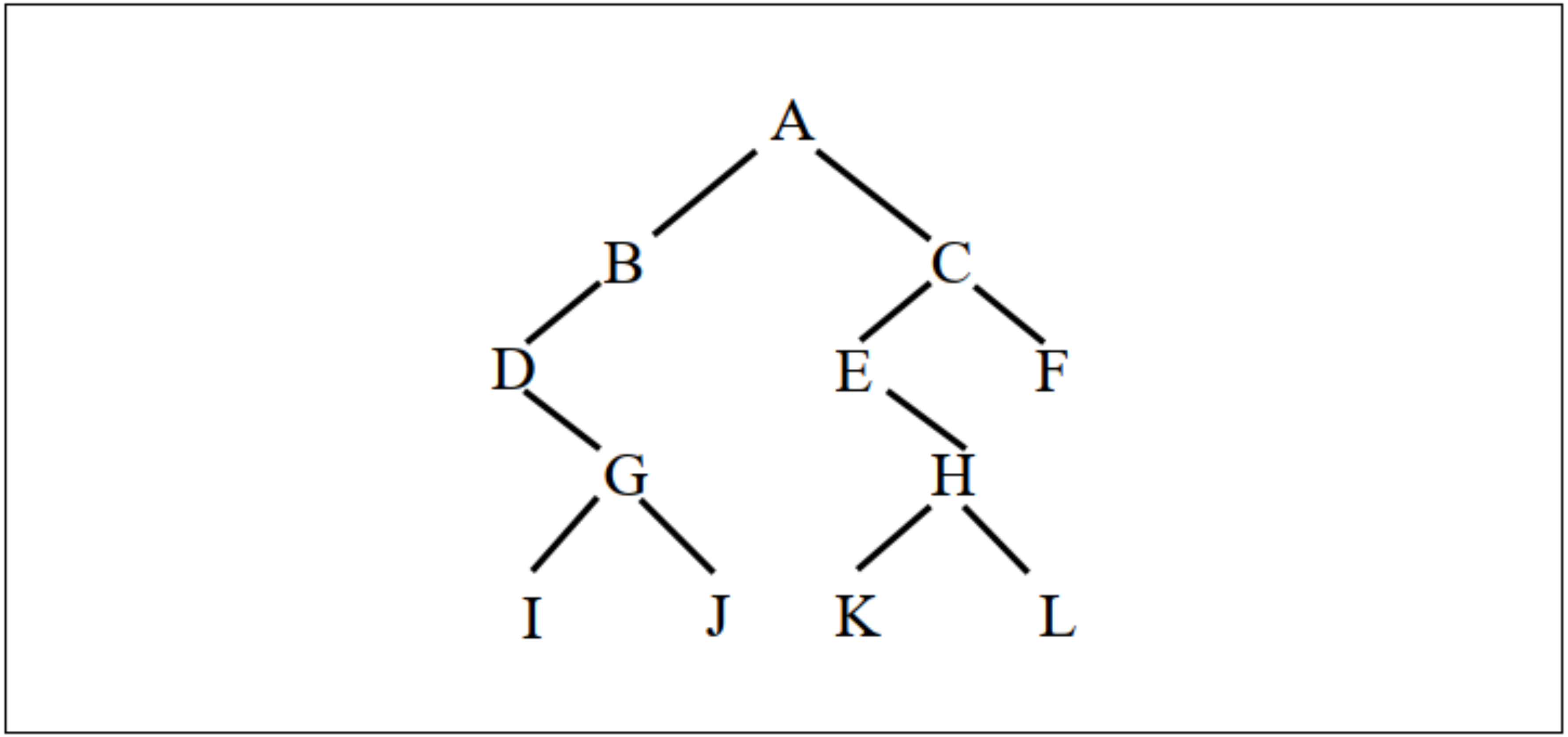
C. I J G D B K L H E C F A

D. I J G D B K L H E F C A

E. None of the above.

Solution: The tree is as shown.

MCQ



MCQ

Q7. A sequence of integers u_n starts with $u_0 = 0$, and for all positive integers n, $u_n = u_{n-1} + n$. Determine u_{251}.

Solution: These are the Triangle numbers: $0, 1, 3, 6, 10, \ldots$
The closed form formula is $u_n = \binom{n+1}{2}$. Hence $u_{251} = \binom{252}{2} = 31626$.

A. 502.

B. 31626.

C. 251!.

D. 2^{251}.

E. None of the above.

Alternative:

$u_1 = 0 + 1 = 1$

$u_2 = u_1 + 2 = 0 + 1 + 2 = 3$

$u_3 = u_2 + 3 = 0 + 1 + 2 + 3 = 6$

…

$u_n = \sum_{i=0}^{n} i = \frac{n(n+1)}{2}$.

$u_{251} = \frac{251 \times 252}{2} = 31626$.

The next six questions (Q8 – Q13) refer to the following definitions:

MCQ

$$\text{Define} \quad U = \mathbb{Z} - \{0\}, \quad A = \{-1, 0, 1\}, \quad B = \{0, 1\}.$$

and also define the bitstring S = the set of all non-empty strings over B. That is, S contains strings of the form: $0, 10, 1011, 0011010011, \ldots$, etc. By definition, ε, the empty string, is *not* in S. Further define:

$\mathcal{R}_1 \subseteq U \times U$ such that $\forall x, y \in U\ (x\ \mathcal{R}_1\ y \ \leftrightarrow\ \gcd(x, y) = 1)$.

$\mathcal{R}_2 \subseteq \mathbb{R} \times \mathbb{R}$ such that $\forall x, y \in \mathbb{R}\ (x\ \mathcal{R}_2\ y \ \leftrightarrow\ x^2 = y^2)$.

$\mathcal{R}_3 \subseteq A \times A$ such that $\forall x, y \in A\ (x\ \mathcal{R}_3\ y \ \leftrightarrow\ y \mid x)$.

$\mathcal{R}_4 \subseteq S \times S$ such that $\forall x, y \in S\ (x\ \mathcal{R}_4\ y \ \leftrightarrow\ x, y$ satisfy conditions $T1$ and $T2$).

$T1$: both x, y have equal length n,

$T2$: if $x = x_1 x_2 \ldots x_n$, and $y = y_1 y_2 \ldots y_n$, then each x_i is the "opposite letter" of y_i, where 1 is the opposite of 0, and vice versa.

Examples: $100\ \mathcal{R}_4\ 011$, but $0101\ \not\mathcal{R}_4\ 1111$.

Final definition: given any binary relation $\mathcal{R} \subseteq X \times Y$, define its *relation complement*, denoted $\mathcal{R}^c$, by:

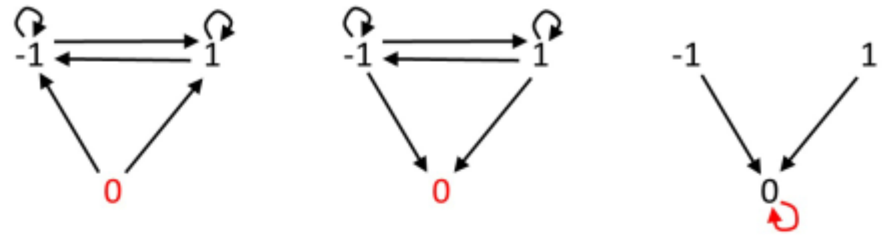
$$\mathcal{R}^c = (X \times Y) - \mathcal{R}.$$

Note that $\mathcal{R}^c \subseteq X \times Y$ is also a binary relation.

MCQ

Solution: The table below summarizes the properties. Note that $\mathcal{R}_4$ is a bijection: it "flips the bits" of its input string, eg. 1011 becomes 0100. Note also that in $\mathcal{R}_3$, the definition of divisibility does not permit a divisor to be 0, ie. $0 \nmid 0$.

Relation	Function?	Reflexive?	Symmetric?	Anti-symmetric?	Transitive?
$\mathcal{R}_1$, $\mathcal{R}_1^{-1}$ and $\mathcal{R}_1^c$	×	×	✓	×	×
$\mathcal{R}_2$ and $\mathcal{R}_2^{-1}$	×	✓	✓	×	✓
$\mathcal{R}_3$ and $\mathcal{R}_3^{-1}$	×	×	×	×	✓
$\mathcal{R}_4$ and $\mathcal{R}_4^{-1}$	✓	×	✓	×	×
$\mathcal{R}_2^c$	×	×	✓	×	×
$\mathcal{R}_3^c$	✓	×	×	✓	✓
$\mathcal{R}_4^c$	×	✓	✓	×	×



$\mathcal{R}_3$ $\mathcal{R}_3^{-1}$ $\mathcal{R}_3^c$

MCQ

Q8. If 10 $\mathcal{R}_4$ y, then y could be:

A. 01.

B. 10.

C. 11.

D. 20.

E. 101.

Q9. Recall that the elements of a binary relation are ordered pairs. Which of the following binary relations has $(1,0)$ as its element?

A. $\boldsymbol{\mathcal{R}_2^c}$**.**

B. $\mathcal{R}_1^{-1}$.

C. $\mathcal{R}_3 \circ \mathcal{R}_3$.

D. $\mathcal{R}_4^c$.

E. All of the above.

Note: Composition of relations is not covered and not tested.

MCQ

Q10. Which of the following are functions?

A. $\mathcal{R}_3^c$.

B. $\mathcal{R}_4^c$.

C. $\mathcal{R}_4$ and $\mathcal{R}_4^{-1}$.

D. $\mathcal{R}_2^{-1}$ and $\mathcal{R}_3^{-1}$.

E. None of the above.

Q11. Which of the following are transitive?

(I) $\mathcal{R}_1^c$ (II) $\mathcal{R}_2$ (III) $\mathcal{R}_4^c$ (IV) $\mathcal{R}_3^{-1}$.

A. (II) only.

B. (II) and (IV) only.

C. (III) and (IV) only.

D. (I), (II) and (III) only.

E. None of the above.

MCQ

Q12. Which of the following is a partial order?

A. $\mathcal{R}_3$.

B. $\mathcal{R}_2^c$.

C. $\mathcal{R}_1^{-1}$.

D. $\mathcal{R}_4^c$.

E. None of the above.

Q13. Which relation has the property that its composition with itself equals itself, ie. $\mathcal{R} \circ \mathcal{R} = \mathcal{R}$?

A. $\mathcal{R}_1$.

B. $\mathcal{R}_1^{-1}$.

C. $\mathcal{R}_3^c$**.**

D. $\mathcal{R}_3^{-1}$**.**

E. All of the above.

MCQ

Q14. Let $A = \{1, 2, 3\}$, $D = \{1, \{1\}, 2, \{3, 2\}, \{\varnothing\}\}$, and $E = \{1, 2\}$. Which of the following statements are TRUE?

(I) $A \subseteq D$.
(II) $\mathcal{P}(E) \subseteq D$.
(III) $3 \in A - E$.
(IV) $\mathcal{P}(E) \cap D = \{\{1\}\}$.

A. (II) only.
B. (IV) only.
C. (III) and (IV) only.
D. (I), (II) and (III) only.
E. All of (I), (II), (III) and (IV).

Q15. Let $f : \mathbb{Z} \to \mathbb{Q}$ be defined by $f(z) = z/2$, and let $g : \mathbb{Q} \to \mathbb{R}$ be defined by $g(q) = \pi \cdot q$. Which of the following statements are FALSE?

(I) f is onto.
(II) $g \circ f$ is one-to-one.
(III) $(g \circ f)(6) = 3\pi$.
(IV) g^{-1} is not a function.

A. (I) only.
B. (II) only.
C. (IV) only.
D. (I) and (IV) only.
E. None of (I), (II), (III) and (IV).