CS1231(S) Tutorial 3: Sets

National University of Singapore

2020/21 Semester 1

When asked to 'find' a set in the following, the answer should involve a list of all of the elements in the set.

Questions for discussion on the LumiNUS Forum

Answers to these questions will not be provided.

1. Let $R = \{x : x \notin x\}$.
2. Case 1: suppose $R \in R$.
   2.1. Then $\sim(R \notin R)$ by the Double Negative Law.
   2.2. So $R \notin R$ by the definition of R on line 1.
   2.3. Line 2 and line 2.2 form a contradiction.
3. Case 2: suppose $R \notin R$.
   3.1. So $R \in R$ by the definition of R on line 1.
   3.2. Line 3 and line 3.1 form a contradiction.
4. Thus we have a contradiction in any case. ⁂

D1. Let $A = \{a, \varnothing\}$, where a is a letter. Which of the following are true? Which of them are false?

(a) $a \in A$.
(b) $\{a\} \in A$.
(c) $a \subseteq A$.
(d) $\{a\} \subseteq A$.
(e) $\varnothing \subseteq A$.
(f) $\varnothing \in A$.
(g) $\{\varnothing\} \subseteq A$.
(h) $\{\varnothing\} \in A$.

D2. Find two finite sets A, B such that $A \in B$ and $A \subseteq B$.

D3. Find the power set of each of the following sets:

(a) $\{x, y, z, w\}$;
(b) $\{a, \{a, b\}\}$;
(c) $\varnothing$;
(d) $\{\varnothing\}$;
(e) $\{\{a\}, \varnothing\}$.

Tutorial questions

1. Which of the following are true? Which of them are false?

(a) $\varnothing \in \varnothing$.
(b) $\varnothing \subseteq \varnothing$.
(c) $\varnothing \in \{\varnothing\}$.
(d) $\varnothing \subseteq \{\varnothing\}$.
(e) $\{\varnothing, 1\} = \{1\}$.
(f) $1 \in \{\{1, 2\}, \{2, 3\}, 4\}$.
(g) $\{1, 2\} \subseteq \{3, 2, 1\}$.
(h) $\{3, 3, 2\} \subsetneq \{3, 2, 1\}$.

2. Let $A = \{1, \{1,2\}, 2, \{1,2\}\}$. Find $|A|$.

3. Let $A = \{0,1,4,5,6,9\}$ and $B = \{0,2,4,6,8\}$. Find $|A|$, $|B|$, $|A \cap B|$, and $|A \cup B|$.

4. Let $A = \{2n+1 : n \in \mathbb{Z}\}$ and $B = \{2n-1 : n \in \mathbb{Z}\}$. Is $A = B$? Prove that your answer is correct.

5. Let $A = \{x \in \mathbb{Z} : 2 \leqslant x \leqslant 5\}$ and $B = \{x \in \mathbb{Q} : 2 \leqslant x \leqslant 5\}$. Is $A = B$? Prove that your answer is correct.

6. Let $U = \{5, 6, 7, \dots, 12\}$ and $M_k = \{n \in \mathbb{Z} : n = km \text{ for some } m \in \mathbb{Z}\}$ for each $k \in \mathbb{Z}$. Find:

   (a) $\{n \in U : n \text{ is even}\}$;
   (b) $\{n \in U : n = m^2 \text{ for some } m \in \mathbb{Z}\}$;
   (c) $\{-5, -4, -3, \dots, 5\} \setminus \{1, 2, 3, \dots, 10\}$;
   (d) $\overline{\{5,7,9\} \cup \{9,11\}}$, where U is considered the universal set;
   (e) $\big\{(x,y) \in \{1,3,5\} \times \{2,4\} : x + y \geqslant 6\big\}$;
   (f) $\mathcal{P}(\{2,4\})$.

7. Show that for all sets A, B, C,
$$A \cap (B \setminus C) = (A \cap B) \setminus C.$$

8. (2009/10 Semester 2 exam question B) Prove that for all sets A and B,
$$(A \cup \overline{B}) \cap (\overline{A} \cup B) = (A \cap B) \cup (\overline{A} \cap \overline{B}).$$

9. Let A, B be sets. Show that $A \subseteq B$ if and only if $A \cup B = B$.

10. For sets A and B, define $A \oplus B = (A \setminus B) \cup (B \setminus A)$.

    (a) Let $A = \{1,4,9,16\}$ and $B = \{2,4,6,8,10,12,14,16\}$. Find $A \oplus B$.
    (b) Show that for all sets A, B,
$$A \oplus B = (A \cup B) \setminus (A \cap B).$$

11. (2015/16 Semester 1 exam question 16(a)) Denote by $|x|$ the absolute value of the integer x, i.e.,
$$|x| = \begin{cases} x, & \text{if } x \geqslant 0; \\ -x, & \text{if } x < 0. \end{cases}$$
Given the set $S = \{-9, -6, -1, 3, 5, 8\}$, for each of the following statements, state whether it is true or false, with explanation.

    (a) $\exists z \in S \ \forall x, y \in S \ z > |x - y|$.
    (b) $\exists z \in S \ \forall x, y \in S \ z < |x - y|$.

12. For sets $A_m, A_{m+1}, \dots, A_n$, define
$$\bigcup_{i=m}^{n} A_i = A_m \cup A_{m+1} \cup \cdots \cup A_n \quad \text{and} \quad \bigcap_{i=m}^{n} A_i = A_m \cap A_{m+1} \cap \cdots \cap A_n.$$

    (a) Let $A_i = \{x \in \mathbb{Z} : x \geqslant i\}$ for each $i \in \mathbb{Z}$. Write down $\bigcup_{i=2}^{5} A_i$ and $\bigcap_{i=2}^{5} A_i$ in roster notation.
    (b) Let $B_1, B_2, \dots, B_k, C_1, C_2, \dots, C_\ell$ be sets such that
$$\bigcup_{i=1}^{k} B_i \subseteq \bigcap_{j=1}^{\ell} C_j.$$
Show that $B_i \subseteq C_j$ for all $i \in \{1, 2, \dots, k\}$ and all $j \in \{1, 2, \dots, \ell\}$.