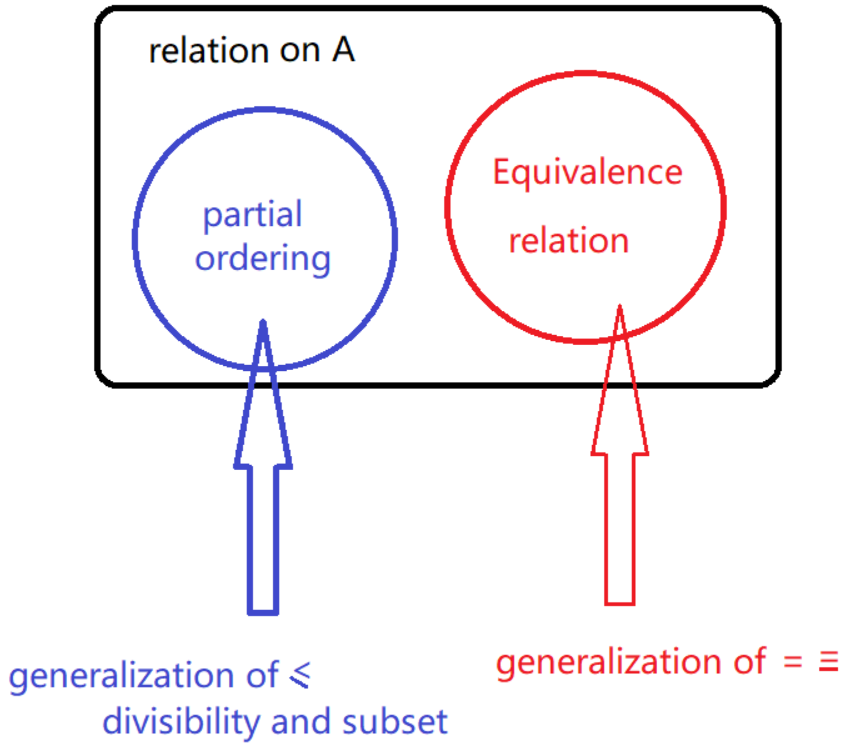
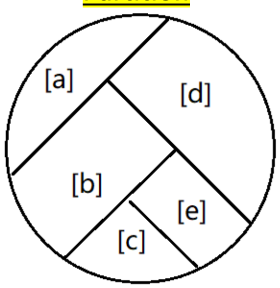
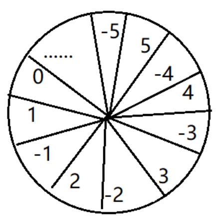
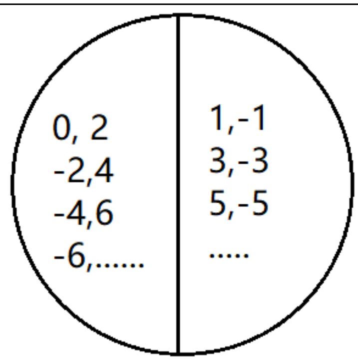
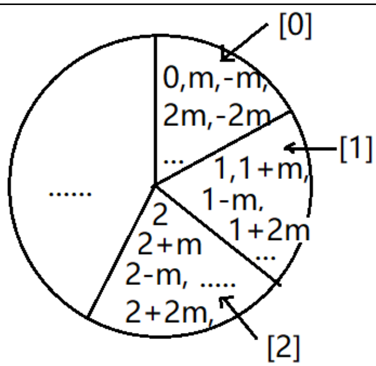
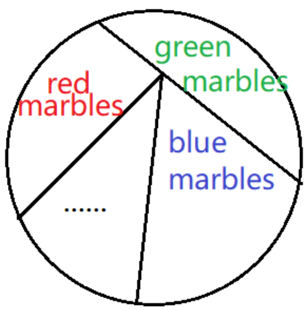
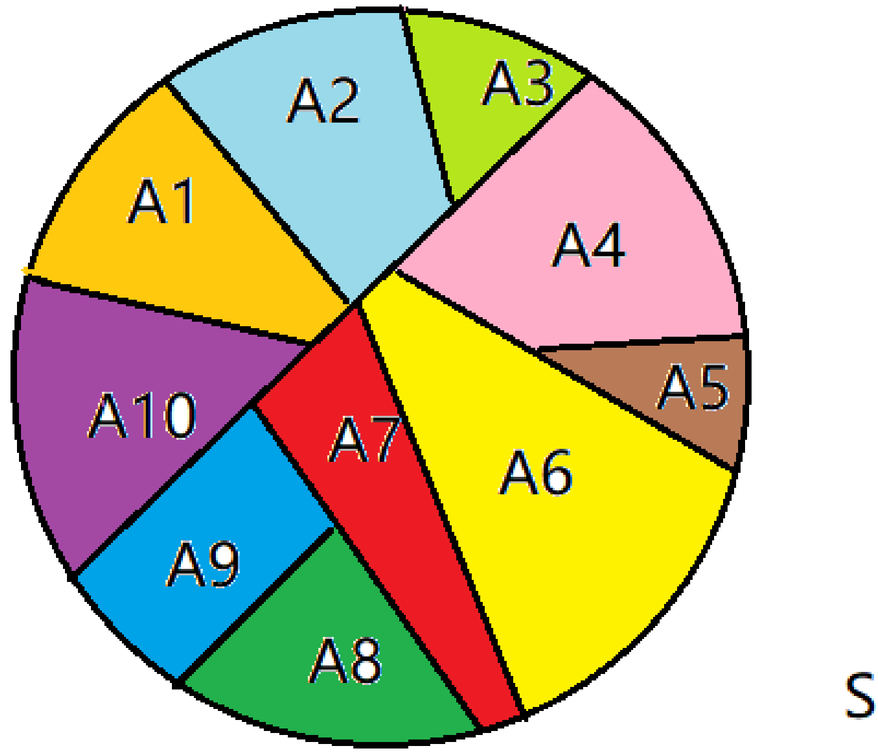
Relation R on A	**Example** On **Z** (set of integers) a=b	On **Z** (set of integers) $a \le b$	On **Z** (set of integers) $a < b$	On **Z** (set of integers) $a \equiv b\ (mod\ 3)$ Congruent modulo 3	On Z^+(**set of positive integers**) $a \vert b$	On sets, $A \subseteq B$
Reflexive $\forall x \in A, (x,x) \in R$	$\forall x \in Z, x = x$	$\forall x \in Z, x \le x$	$\forall x \in Z, x < x$ **False**	$\forall x \in Z, x \equiv x\ (mod\ 3)$ (By the definition of congruence relation and because $3 \vert (x-x)$ i.e. $3 \vert 0$)	$\forall x \in Z^+, x \vert x$	$\forall A, A \subseteq A$
Symmetric $\forall x, y \in A, (x,y) \in R \to (y,x) \in R$	$\forall x, y \in Z, x = y \to y = x$	$\forall x \in Z, x \le y \to y \le x$ (**False**, because $1 \le 2$, but $2 \nleq 1$)	$\forall x \in Z, x < y \to y < x$ (**False**, because $1 < 2$, but $2 \nless 1$)	$\forall x, y \in Z, x \equiv y\ (mod\ 3) \to y \equiv x\ (mod\ 3)$ (By the definition of congruence relation and because $3 \vert (x-y) \to 3 \vert (y-x)$)	$\forall x, y \in Z^+, x \vert y \to y \vert x$ (**False**, because $1 \vert 2$ but $2 \nmid 1$)	$\forall A, B, A \subseteq B \to B \subseteq A$ (**False**, because $\{1\} \subseteq \{1,2\}$ but $\{1,2\} \nsubseteq \{1\}$)
Antisymmetric $\forall x, y \in A, (x,y) \in R \wedge (y,x) \in R \to x = y$	$\forall x, y \in Z, x = y \wedge y = x \to x = y$	$\forall x, y \in Z, x \le y \wedge y \le x \to x = y$	$\forall x, y \in Z, x < y \wedge y < x \to x = y$ True, because "$x < y \wedge y < x$" is always false	$\forall x, y \in Z, x \equiv y\ (mod\ 3) \wedge y \equiv x\ (mod\ 3) \to x = y$ (**False**, because $1 \equiv 4\ (mod\ 3)$, $4 \equiv 1\ (mod\ 3)$ But $1 \ne 4$.)	$\forall x, y \in Z^+, x \vert y \wedge y \vert x \to x = y$	$\forall A, B, A \subseteq B \wedge B \subseteq A \to A = B$
Transitive $\forall x, y, z \in A, (x,y) \in R \wedge (y,z) \in R \to (x,z) \in R$	$\forall x, y, z \in Z, x = y \wedge y = z \to x = z$	$\forall x, y, z \in Z, x \le y \wedge y \le z \to x \le z$	$\forall x, y, z \in Z, x < y \wedge y < z \to x < z$	$\forall x, y, z \in Z, x \equiv y\ (mod\ 3) \wedge y \equiv z\ (mod\ 3) \to x \equiv z\ (mod\ 3)$	$\forall x, y, z \in Z^+, x \vert y \wedge y \vert z \to x \vert z$	$\forall A, B, C, A \subseteq B \wedge B \subseteq C \to A \subseteq C$

				(By the definition of congruence relation and because $3\|(x-y) \wedge 3\|(y-z) \to 3\|[(x-y)+(y-z)]$ i.e. $3\|(x-z)$)		



Equivalence relation R on A	**Example** On **Z (set of integers)** a=b	On **Z (set of integers)** $a \equiv b \ (mod\ 2)$ Congruent modulo 2	On **Z (set of integers)** $a \equiv b \ (mod\ m)$	On **marbles**  A and B have the same color
Reflexive $\forall x \in A, (x, x) \in R$	$\forall x \in Z, x = x$	$\forall x \in Z, x \equiv x \ (mod\ 2)$ (By the definition of congruence relation and because $2\|(x - x)$ i.e. $2\|0$)	$\forall x \in Z, x \equiv x \ (mod\ m)$ (By the definition of congruence relation and because $m\|(x - x)$ i.e. $m\|0$)	For all marble x, x and x have the same color.
Symmetric $\forall x, y \in A, (x, y) \in R \rightarrow (y, x) \in R$	$\forall x, y \in Z, x = y \rightarrow y = x$	$\forall x \in Z, x \equiv y \ (mod\ 2) \rightarrow \ y \equiv x \ (mod\ 2)$ (By the definition of congruence relation and because $2\|(x - y) \rightarrow 2\|(y - x)$)	$\forall x \in Z, x \equiv y \ (mod\ m) \rightarrow y \equiv x \ (mod\ m)$ (By the definition of congruence relation and because $m\|(x - y) \rightarrow m\|(y - x)$)	For all marbles x and y, if x and y have the same color, then y and x have the same color
Transitive $\forall x, y, z \in A,$ $(x, y) \in R \land (y, z) \in R \rightarrow (x, z) \in R$	$\forall x, y, z \in Z,$ $x = y \ \land y = z \rightarrow x = z$	$\forall \ x, y, z \in Z,$ $x \equiv y \ (mod\ 2) \land y \equiv z \ (mod\ 2) \rightarrow x \equiv z \ (mod\ 2)$ (By the definition of congruence relation and because $2\|(x - y) \land 2\|(y - z) \rightarrow 2\|[(x - y) + (y - z)]$ i.e. $2\|(x - z)$)	$\forall \ x, y, z \in Z,$ $x \equiv y \ (mod\ m) \land y \equiv z \ (mod\ m) \rightarrow x \equiv z \ (mod\ m)$ (By the definition of congruence relation and because $m\|(x - y) \land m\|(y - z) \rightarrow m\|[(x - y) + (y - z)]$ i.e. $m\|(x - z)$)	For all marbles x,y and z, If x and y have the same color and y and x have the same color, Then x and z have the same color.

Equivalent classes $[a] = [a]_R$ $= \{x \in A \mid (a,x) \in R\}$ $A/R = \{[a]_R \mid a \in A\}$	$[a] = [a]_=$ $= \{x \in Z \mid a = x\}$ $= \{a\}$ $Z/=$ $= \{\{a\} \mid a \in Z\}$	$[a] = \{x \in Z \mid a \equiv x \ (mod\ 2)\}$ $[0] = \{x \in Z \mid 0 \equiv x \ (mod\ 2)\}$ $= set\ of\ even\ numbers$ $[1] = \{x \in Z \mid 1 \equiv x \ (mod\ 2)\}$ $= set\ of\ odd\ numbers$ $Z/\equiv = \{[0],[1]\}$ $= \{set\ of\ even\ numbers,$ $set\ of\ odd\ numbers\}$	$[a] = \{x \in Z \mid a \equiv x \ (mod\ m)\}$ $[0] = \{x \in Z \mid 0 \equiv x \ (mod\ m)\}$ $= set\ of\ multiples\ of\ m$ $[1] = \{x \in Z \mid 1 \equiv x \ (mod\ m)\}$ $= set\ of\ integers\ with$ $remainder\ 1,$ $when\ divided\ by\ m$ .... $[m-1]$ $= \{x \in Z \mid m-1 \equiv x \ (mod\ m)\}$ $= set\ of\ integers\ with$ $remainder\ m-1,$ $when\ divided\ by\ m$ $Z/\equiv = \{[0],[1],\ldots,[m-1]\}$	$[a]_R$ = {x is a marble | a and x have the same color} A/R $= set\ of\ different$ $color\ classes\ of\ marble$
Partition 				



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