A0214561M

Part A

1. B 2. D 3. C 4. C 5. C 6. D 7. E

Part B

8. A 9. D 10. ABCD 11. AD 12. C 13. B

14. C 15. ABD 16. BC 17. D 18. BC 19. BCD

Part C

20. Proof (by contradiction)

1. Suppose $\exists p, q, r$ such that the statement is not a tautology
2. Then LHS $= (p \to q) \land (q \to r)$ must be false and RHS $= (p \to r) \to (r \to p)$ has to be false

2.1 For RHS to be false, $(p \to r)$ must be True and $(r \to p)$ must be false (by implication) (both sides false for $\vee$)

2.2 Thus, for $r \to p$ to be false, r is True, & p is false.

2.3 $(p \to r)$ is True with the above value of r & p

2.4 Hence RHS is false.

2.5 For LHS to be false, either $(p \to q)$ is False or $(q \to r)$ is false

2.6 As p is false from 2.2, $p \to q$ is vacuously true always

2.7 Hence $q \to r$ must be false (by elimination)

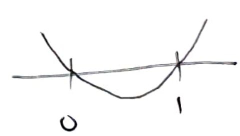
2.8 But r is true so $q \to r$ is always true

2.9 Hence LHS is true

3. There's a contradiction, hence the statement is a tautology

21.

1. Prove $\forall x \in \mathbb{R}\ ((x^2 > x) \to (x < 0) \vee (x > 1))$
2. Let $k \in \mathbb{R}$ such that $k^2 > k$



2.1 Then $k^2 - k > 0$

2.2 Then $k(k-1) > 0$

2.3 Thus both k and $k-1$ are positive or both are negative (by T25)

2.4 Case 1: both k and $k-1$ are positive

2.4.1 Then $k > 0 \land k > 1$

2.4.2 Thus $k > 1$ (definition of $\land$)

A0214561M

2.5 case 2: both k and k-1 are negative

2.6 Then $k < 0 \wedge k < 1$

2.7 Thus $k < 0$ (by definition of $\wedge$)

2.6 From both cases $k < 0 \vee k > 1$

3 Thus if $x^2 > x$, then $x < 0$ or $x > 1$

22. Proof (by Mathematical Induction)

1. let $P(n) = (a_n = 2^{n+1} + 2^n - 2)$ $\forall n \in \mathbb{Z}_{\geq 0}$

2. Base case: $n = 0$

2.1 $P(0)$: $a_0 = 1 = 2^{0+1} + 2^0 - 2 = 2+1-2 = 1$

2.2 Thus $P(0)$ is true

3. Inductive step: For any $k \in \mathbb{Z}_{\geq 0}$

3.1 Assume $P(k)$ is true, ie $a_k = 2^{k+1} + 2^k - 2$

3.1.1 consider the k+1 case:

3.1.2 $a_{k+1} = 2a_k + 2$ (from recursive definition of $a_{n+1} = 2a_n + 2$)

$= 2 \cdot 2^{k+1} + 2 \cdot 2^k - 4 + 2$

$= 2^{(k+1)+1} + 2^{k+1} - 2$ (by basic algebra)

3.1.3 Thus $P(k+1)$ is true

4. Therefore $P(n)$ is true for any $n \in \mathbb{Z}_{\geq 0}$, by MI