A0214561M

1. D 2. D 3. D 4. C 5. D 6. C 7. A

8. A 9. B 10. D 11. C 12. C

13. ABC 14. ABCD 15. BD

16. BD 17. ABC

18. a) 1. (Reflexivity)

1.1 Let $(a,b) \in A$

1.2 Then $ab = ab$

1.3 $(a,b) R (a,b)$

2. (Symmetry)

2.1 Let $(a,b), (c,d) \in A$ such that $(a,b) R (c,d)$

2.2 Then $ab = cd$ by definition of R

2.3 Then $cd = ab$

2.4 $(c,d) R (a,b)$

3. (Transitivity)

3.1 Let $(a,b), (c,d), (e,f) \in A$ such that $(a,b) R (c,d) \wedge (c,d) R (e,f)$

3.2 Then $ab = cd$ and $cd = ef$

3.3 Then $ab = cd = ef$; thus $ab = ef$

3.4 $(a,b) R (e,f)$

4. Since R is reflexive, symmetric and transitive, it is an equivalence relation

b)

$$[(1,1)] = \{(1,1)\}$$

$$[(4,3)] = \{(1,12), (2,6), (3,4), (4,3), (6,2), (12,1)\}$$

19. a)

$$\{\{3230, 2040, 1231\}, \{2103, 2030, 1101\}, \{2106, 2100\}\}$$

$$\{\{3230, 2040, 1231\}, \{2103, 2030\}, \{2106, 2100, 1101\}\}$$

b)

$$\{\{1231, 1101\}, \{2040, 2030, 2100\}, \{3230, 2103, 2106\}\}$$

$$\{\{1231, 1101\}, \{2100, 3230, 2103\}, \{2040, 2030, 2106\}\}$$

$$\{\{1231, 2030, 2100\}, \{1101, 2040\}, \{3230, 2103, 2106\}\}$$

$$\{\{1231, 2030, 2106\}, \{1101, 2040\}, \{3230, 2103, 2100\}\}$$

20. a)

$$Z \geq 3$$

$$r = 79 \quad n = 3$$

$$\binom{79+3-1}{79} = 3240$$

b) i) must contain both 1s

$${}^{6}C_{2} = 15$$

ii) total ways $= {}^{8}C_{4} = 70$

no duplicates $= 70 - 15 = 55$

c)

fifth term $\binom{10}{4}\left(\frac{1}{2\sqrt{x}}\right)^{6}\left(-\frac{1}{2}\right)^{4}$

$$\text{coefficient} = {}^{10}C_{4} \times \left(\frac{1}{2\sqrt{x}}\right)^{6} \times \left(-\frac{1}{2}\right)^{4}$$

$$= \frac{210}{1024} \times \frac{1}{x^3} = 105$$

$$x^3 = \frac{1}{512}$$

$$\boxed{x = \frac{1}{8}}$$

d) i)

0 edges $\Rightarrow$ 1

1 edge $\Rightarrow$ 6

2 edges $\Rightarrow$ 9

3 edges $\Rightarrow$ 18

6 edn $\Rightarrow$ 1

5 edn $\Rightarrow$ 6

4 ed $\Rightarrow$ 9

$\boxed{50}$

ii) $6+9+6+1 = \boxed{22}$

e) $$P(Y\,1994 \mid Y+G) = \frac{P(Y\,1994 \cap Y+G)}{P(Y+G)}$$

$P(Y+G) =$ yellow from 1994, green from 1996 + yellow from 1996, green from 1994

$$= \frac{2}{10} \times \frac{16}{100} + \frac{14}{100} \times \frac{1}{10} = \frac{23}{500}$$

$$P(Y\,1994 \cap Y+G) = \frac{2}{10} \times \frac{16}{100} = \frac{4}{125}$$

$$P(Y\,1994 \mid Y+G) = \frac{\frac{4}{125}}{\frac{23}{500}} \boxed{= \frac{16}{23}}$$

f)

Proof by contradiction

1. Assume every student has different non-negative score,

2. The lowest possible sum is if the students scored in the range $[0, 1, \dots 20]$

3. total is $\frac{20 \times 21}{2} = 210$

4. This is greater than the actual total score of 200, hence contradiction

5. There must be at least 2 students with the same score

21 a)

$\{g, e\}$ 9

$\{g, h\}$ 12

$\{b, d\}$ 29

$\{d, g\}$ 32

$\{f, g\}$ 38

$\{a, d\}$ 61

$\{c, e\}$ 70

7 edges ✓

total weight = 251

b)

complete graph ÷ 2

$$\boxed{\frac{n(n-1)}{4}}$$

c)

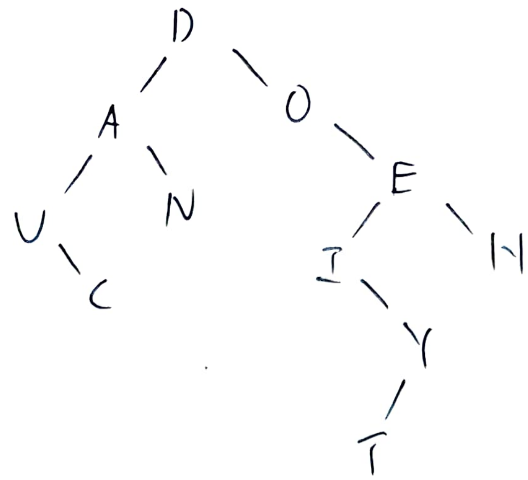
$$\# \text{ edges} = \frac{(4k+2)(4k+1)}{4} = \frac{16k^2+12k+2}{4}$$

$$= \frac{8k^2+6k+1}{2}$$

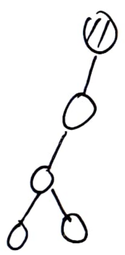
$$= \frac{2(4k^2+3k)+1}{2}$$

$4k^2+3k \in \mathbb{Z}$ by closure hence number of edges is not an integer

There is no self complementary graph with $4k+2$ vertices d)



e)



5 for 5 marks? HAHA

22.

$80x + 3y - 35z = 0$

$80 \cdot 2 + 3 \cdot 5 + -35 \cdot 5 = 0$

$x = 2$

$y = 5$

$z = 5$

$n = 212$

23

a) $n \mid a^k$ $\quad a^k = \ell n$ $\quad$ yes

1 equivalence class $\Rightarrow$ all of $1, \dots n-1$ divide n

n must be 2

n is prime

b)